

# Nucleation kinetics at various conditions

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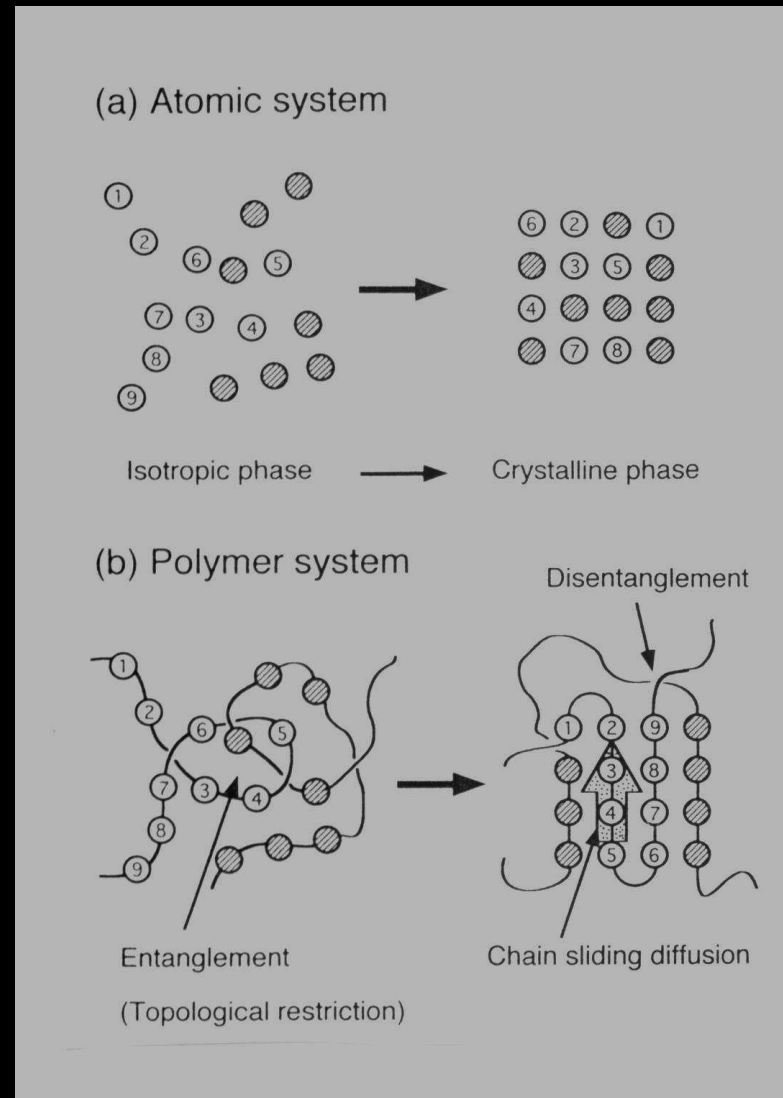
# Introduction

**Nucleation** → process leading to the formation of a new phase (solid, liquid) within metastable original phase (undercooled melt, supersaturated vapor or solution)

at

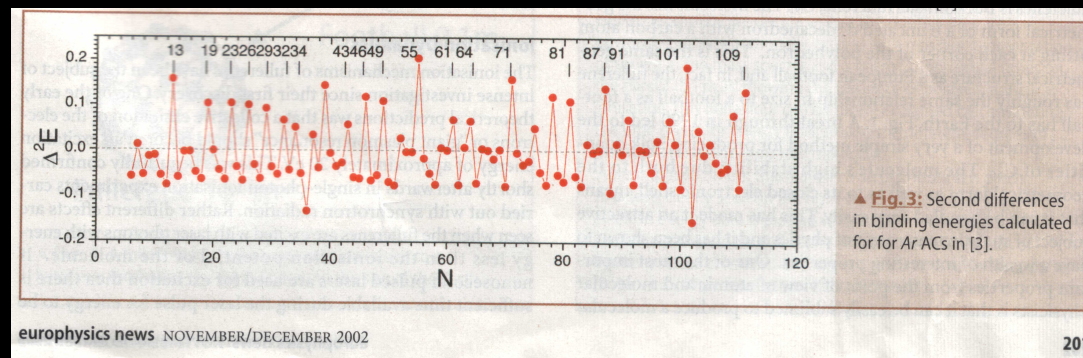
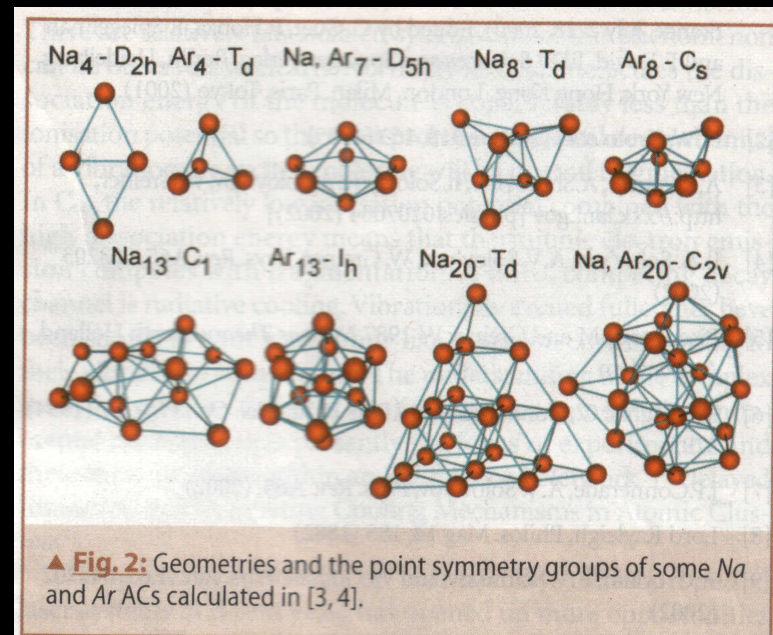
- random sites in the bulk of a mother phase  
(**homogeneous nucleation**)  
special case: **nucleation in closed systems**
- on substrate, surface of ampule, impurities, etc.  
(**heterogeneous nucleation**)  
special case: **nucleation on active centers**

# Introduction



M. Nishi et al.: Polymer Journal 31 (1999) 749.

# Introduction



Jean-Patrick Commerade: *The science of clusters: An emerging field,*  
*Euromphysics news 33/6 (2002) 200.*

# Motivation

Cooperation with experimental groups (Hiroshima Univ., etc.)

- **Nucleation in microemulsions**

**Microemulsion** → thermodynamically stable dispersion of one liquid phase into another (oil-in-water, water-in-oil)

Droplet diameter about 100 nanometers

**Theoretical approaches — problem to explain measurements**  
(Only thermodynamical aspects of nucleation in close systems, no connection with kinetics of formation of a new phase.)

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- Nucleation in closed systems

- Nucleation on active centers



# Thermodynamical aspects

Energy of formation of nuclei:

$$\Delta G(n) = G_{NP}(n) - G_{MP}(n) = \Delta G_V(n) + \Delta G_S(n)$$

Critical size  $n^*$ :

$$\frac{\partial \Delta G(n)}{\partial n} = 0 \Rightarrow n^*$$

Capillarity approximation:

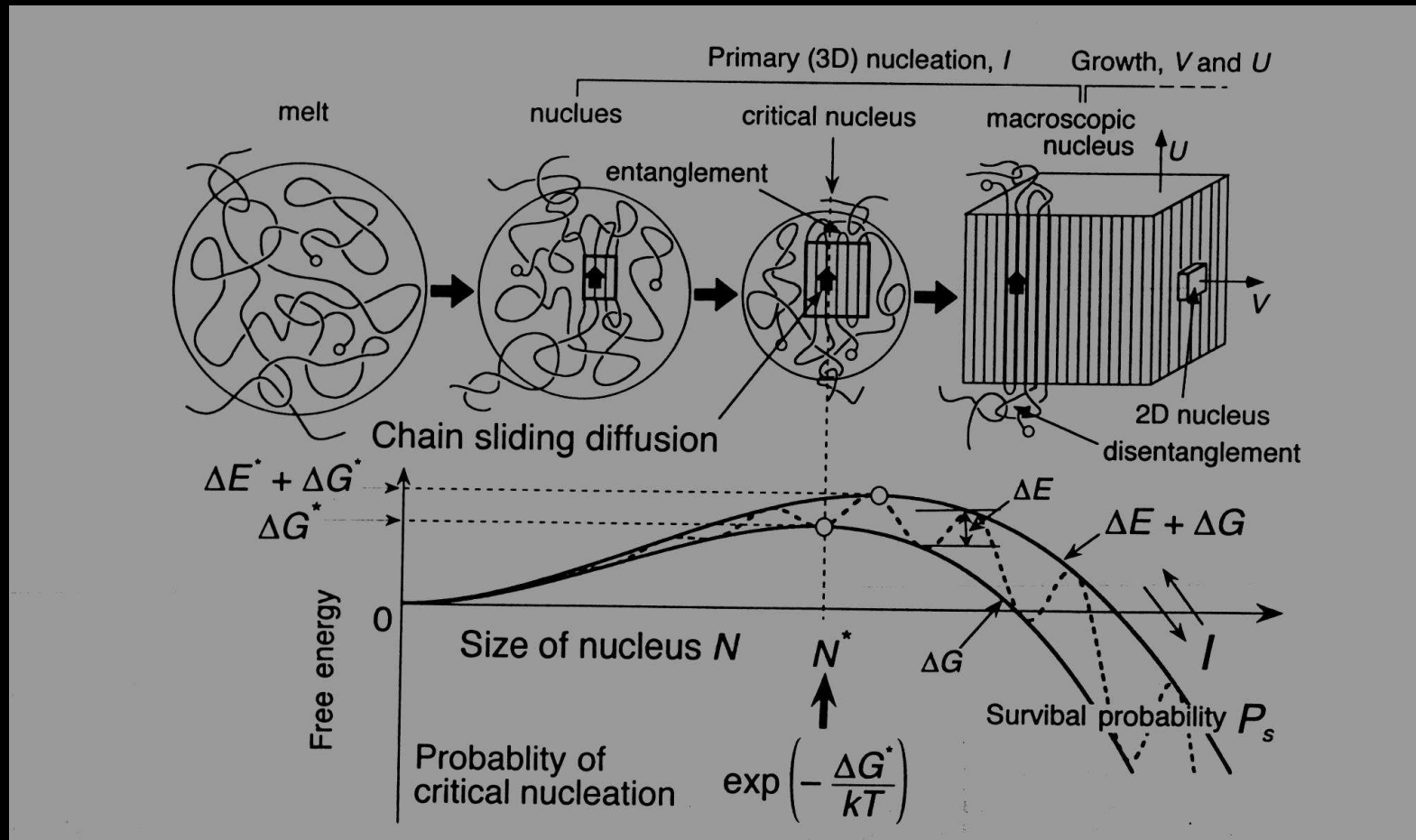
$$\Delta G(n) = -n\Delta\mu + \sigma S_n$$

For  $S_n = \gamma n^{2/3} \Rightarrow$

$$n^* = \left( \frac{2\gamma\sigma}{3\Delta\mu} \right)^3$$

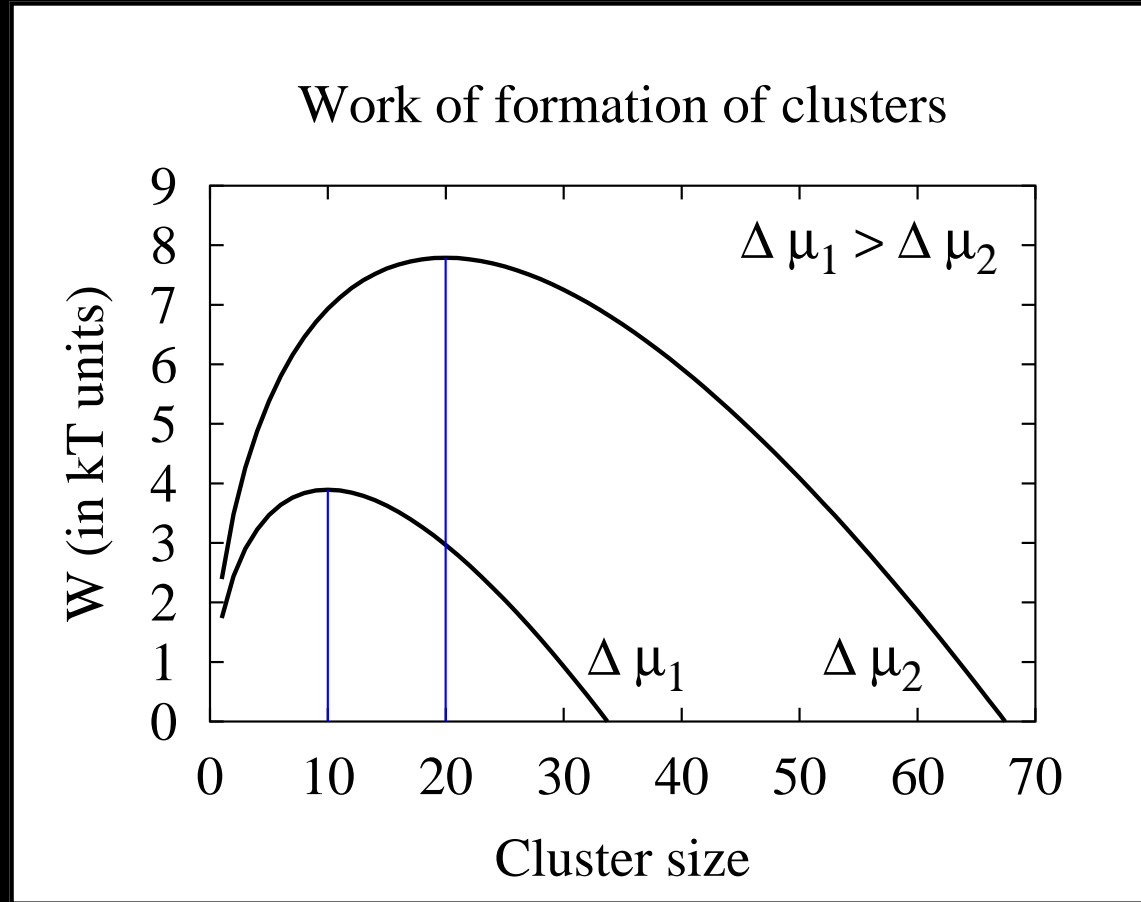
# Thermodynamical aspects

## Polymer systems



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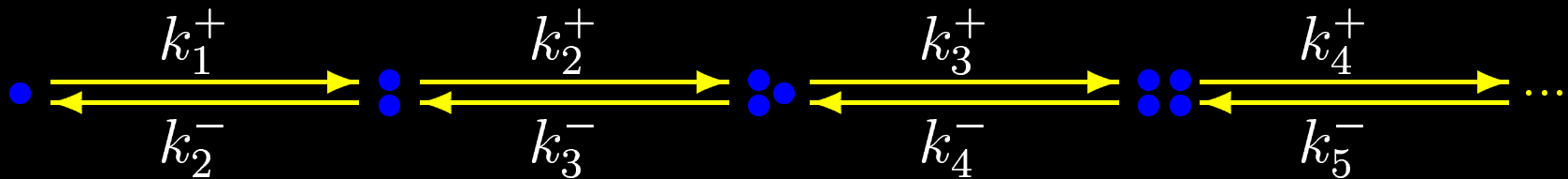
# Thermodynamical aspects



$$J^S = \text{const.} \exp\left(-\frac{\Delta G^*}{kT}\right)$$

nucleation rate

# Nucleation kinetics



$k_n^+$  ( $k_n^-$ ) – attachment (detachment) frequencies of molecules

- **Coalescence is neglected**  
attachment (resp. detachment) of *growth units* plays dominant role in nucleation and growth process
- **Nucleation starts at any nucleation center** (monomer, active center) in the bulk of supersaturated mother phase

# Basic equations

$$\frac{dF_n}{dt} = J_{n-1}(t) - J_n(t)$$

where cluster flux density (nucleation rate for  $n^*$ )

$$J_n(t) = k_n^+ F_n(t) - k_{n+1}^- F_{n+1}(t)$$

Total number of nuclei greater than  $m$

$$Z_m(t) = \sum_{n>m} F_n(t) = \int_0^t J_m(t') dt'$$

$F_n$  – number density of nuclei of size  $n$

# Basic equations

## Initial and boundary conditions

$$F_{n>1}(t = 0) = 0$$

$$F_{n \rightarrow \infty}(t) = 0$$

$N_1 (\equiv F_1)$  – number of nucleation centers

$$N_1 \gg \sum_{n>1} n F_n(t) \implies N_1 = \text{const.}$$

(open system)

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$$N_1(t) = N_0 - \sum_{n>1} F_n(t) \quad (\text{active centers})$$



# Transient frequencies

Vapor → Liquid

$$k_n^+ = \frac{P}{\sqrt{2\pi mkT}} S_n$$

Vapor → Solid

$$k_n^+ = \frac{P}{\sqrt{2\pi mkT}} S_n \exp\left(-\frac{E}{kT}\right)$$

Liquid → Solid

$$k_n^+ = \varrho_S \left(\frac{kT}{h}\right) S_n \exp\left(-\frac{E}{kT}\right) \exp\left(-\frac{q\Delta g_n}{kT}\right)$$

$$\Delta g_n = \Delta G_{n+1} - \Delta G_n; \quad q = \frac{1}{2}[1 + \text{sign}(\Delta g_n)]$$

$\varrho_S$  - surface density of monomers

# Avrami model

$$\frac{dN(t)}{dt} = J_A(t)[N_0 - N(t)]$$

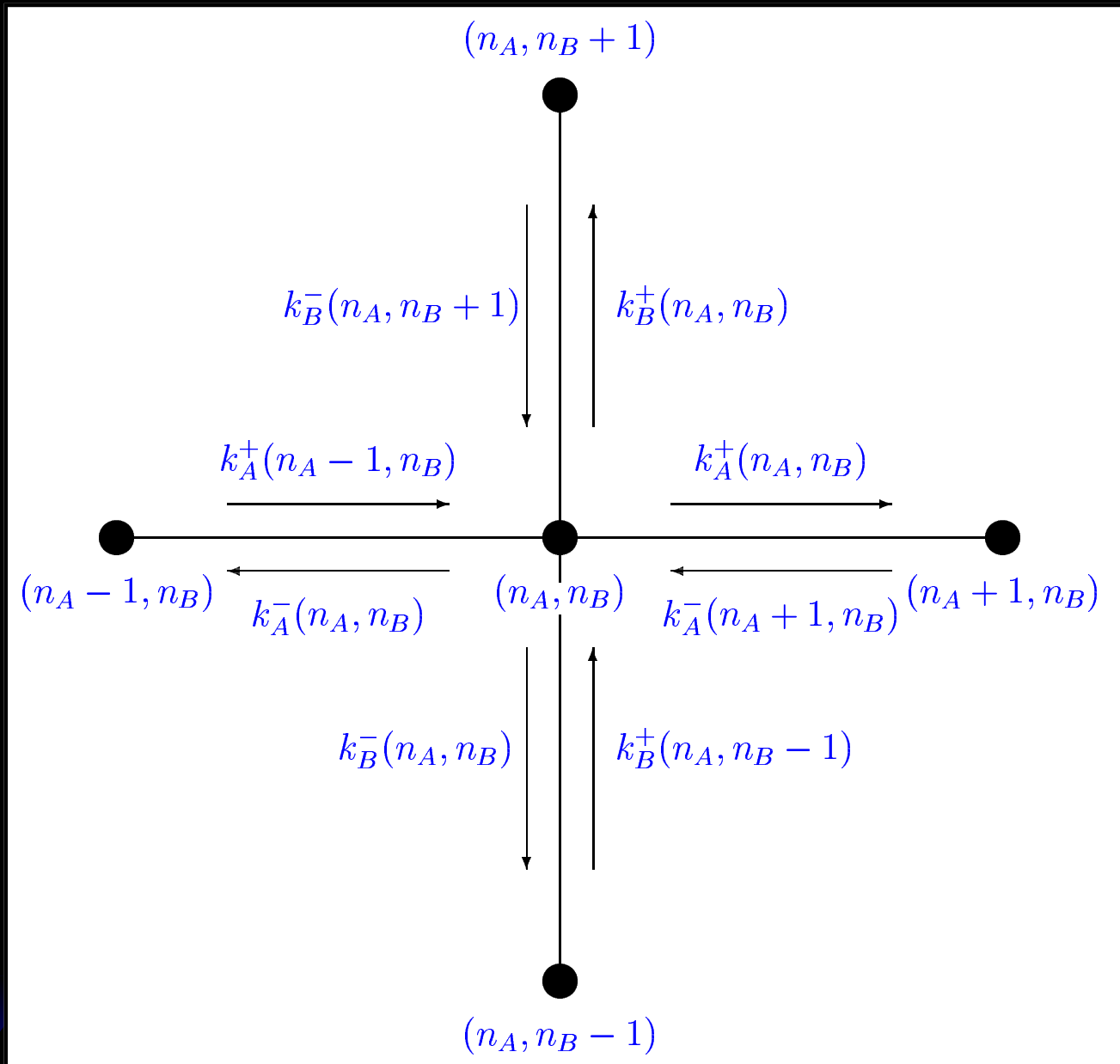
$N(t)$  – total number of nuclei

$J_A(t)$  – time dependent nucleation rate per active center

$$N(t) = N_0 \left[ 1 - \exp \left( - \int_0^t J_A(t') dt' \right) \right]$$

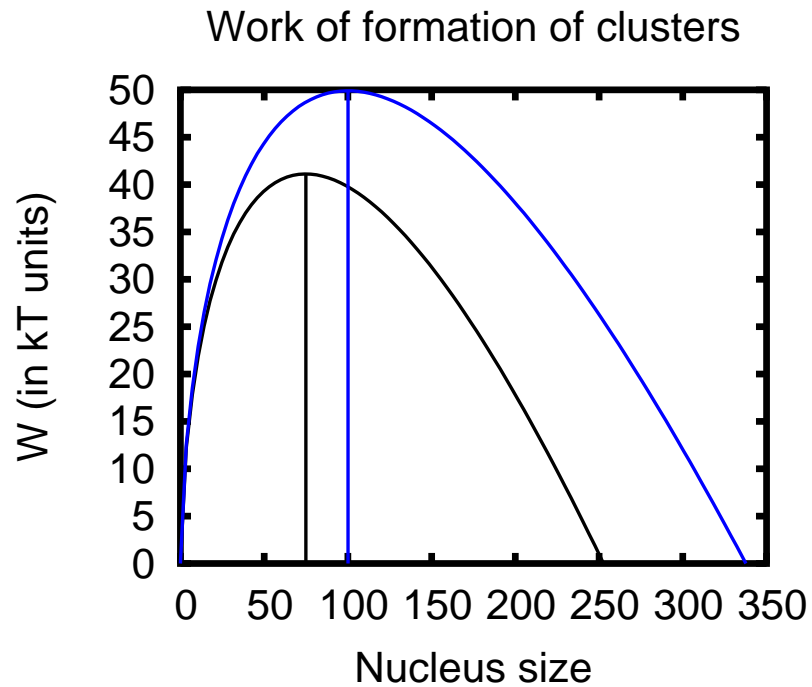
$$F_1^0 = N_0$$

# Binary nucleation

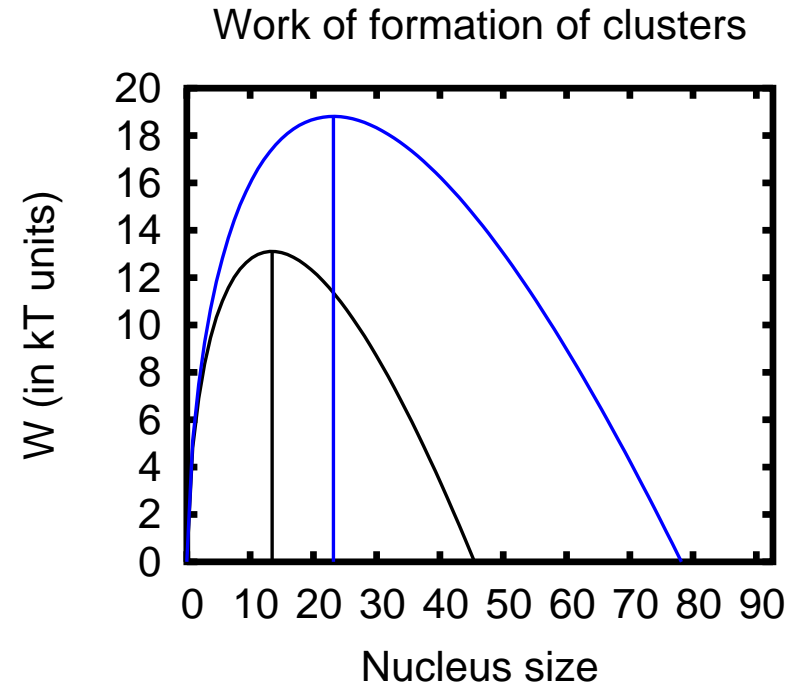


# Closed systems

$S=3$

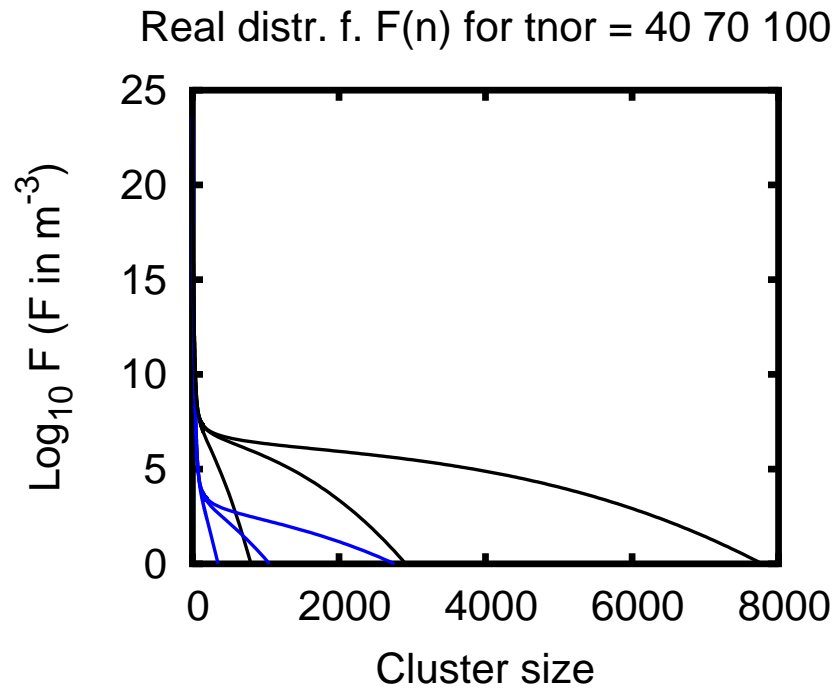


$S=7$

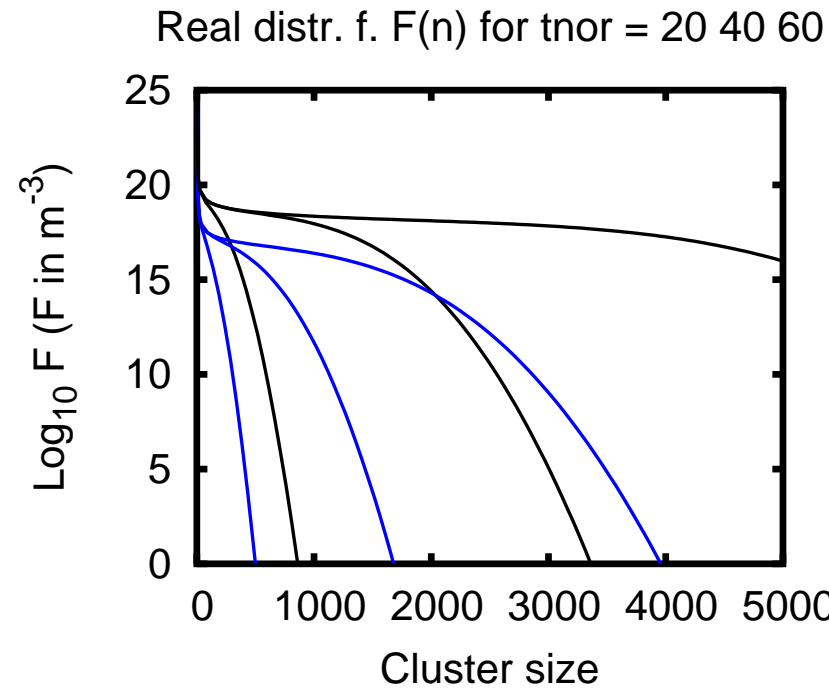


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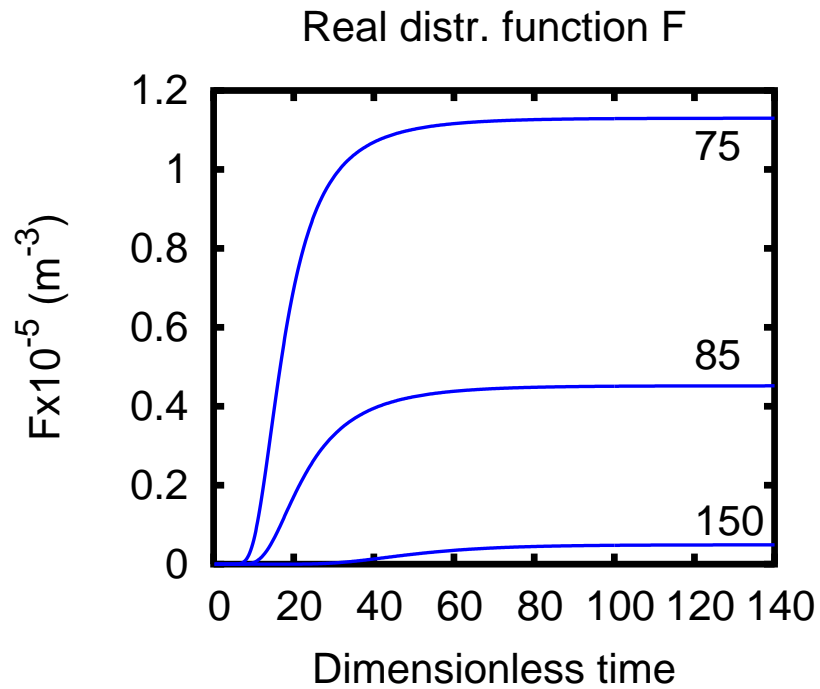


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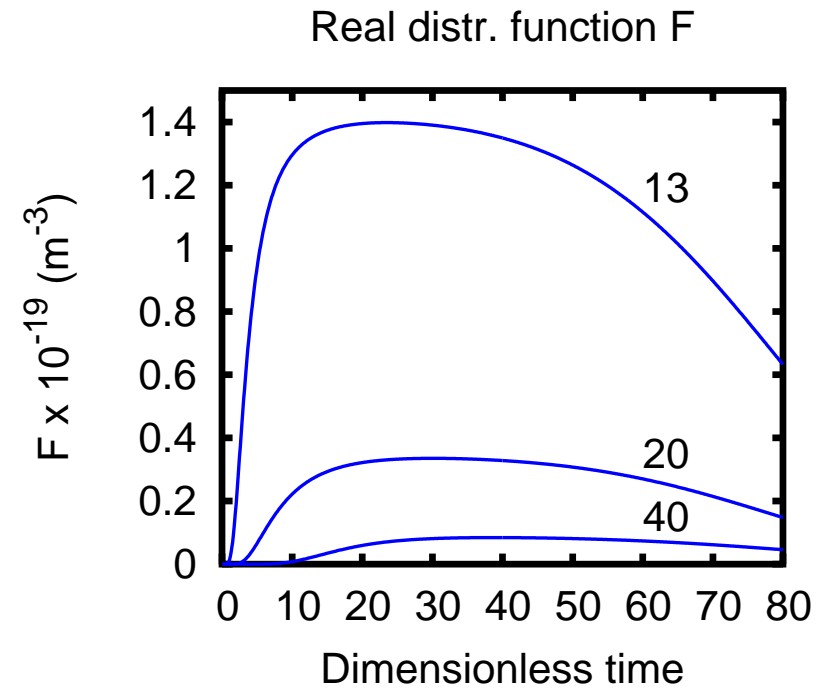


# Closed systems

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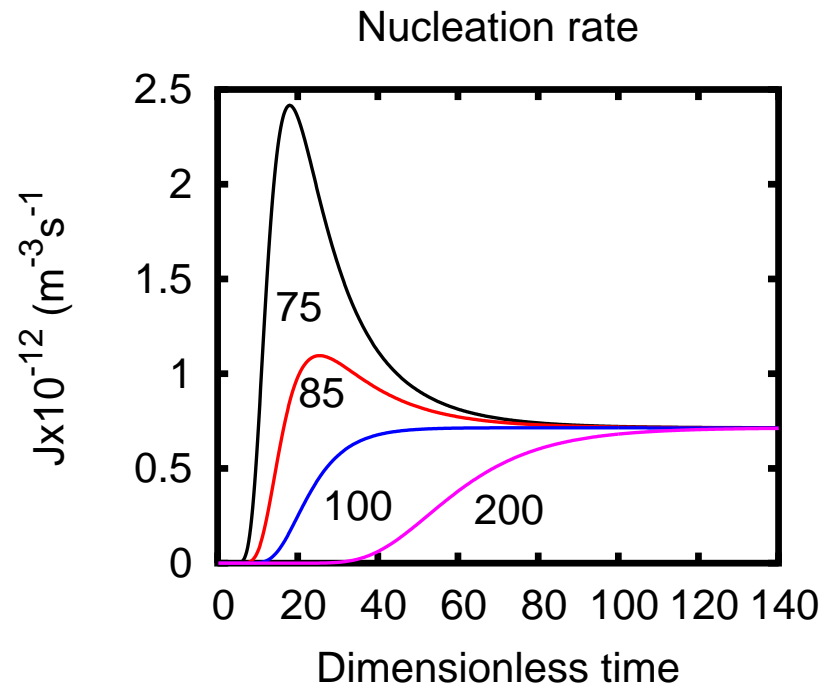


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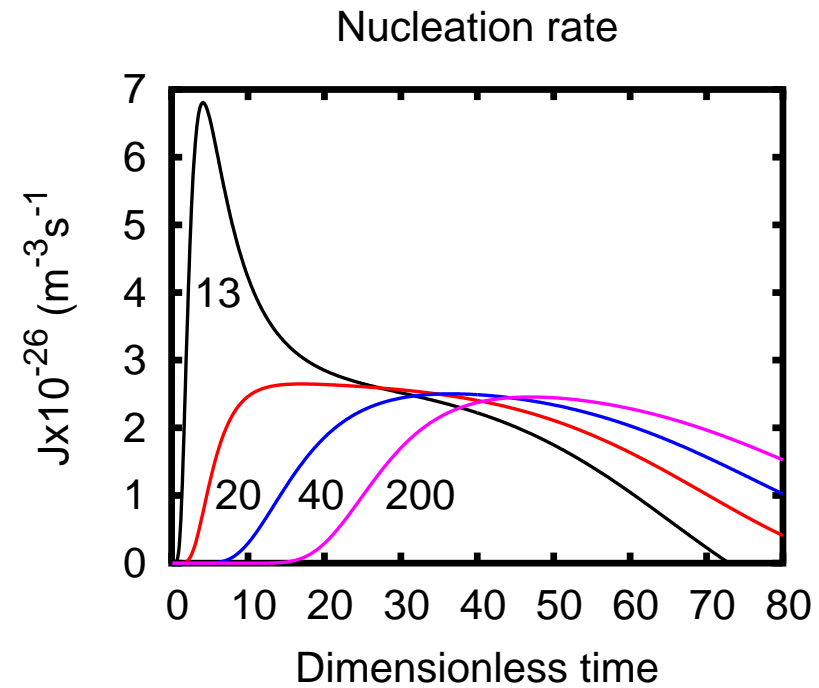


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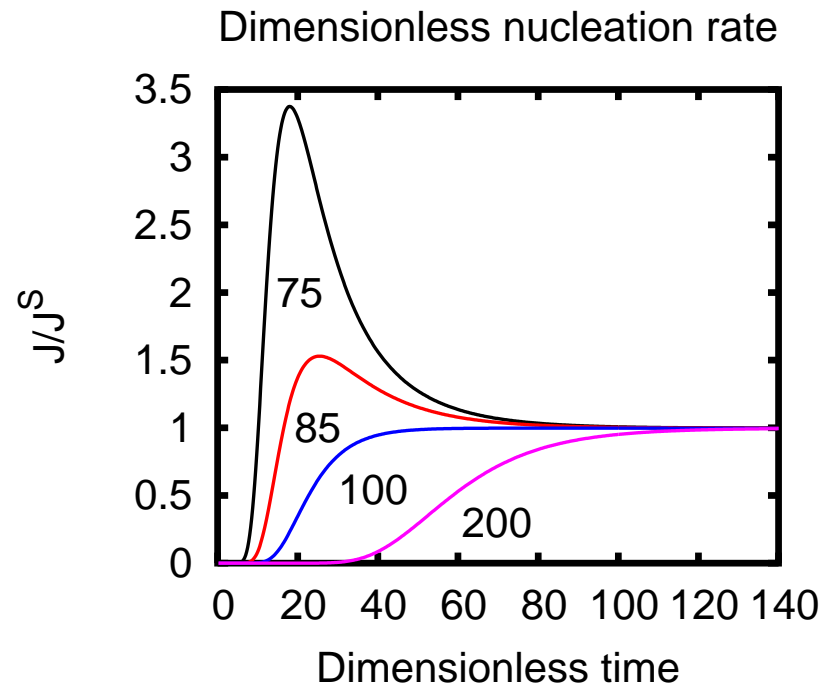


S=7

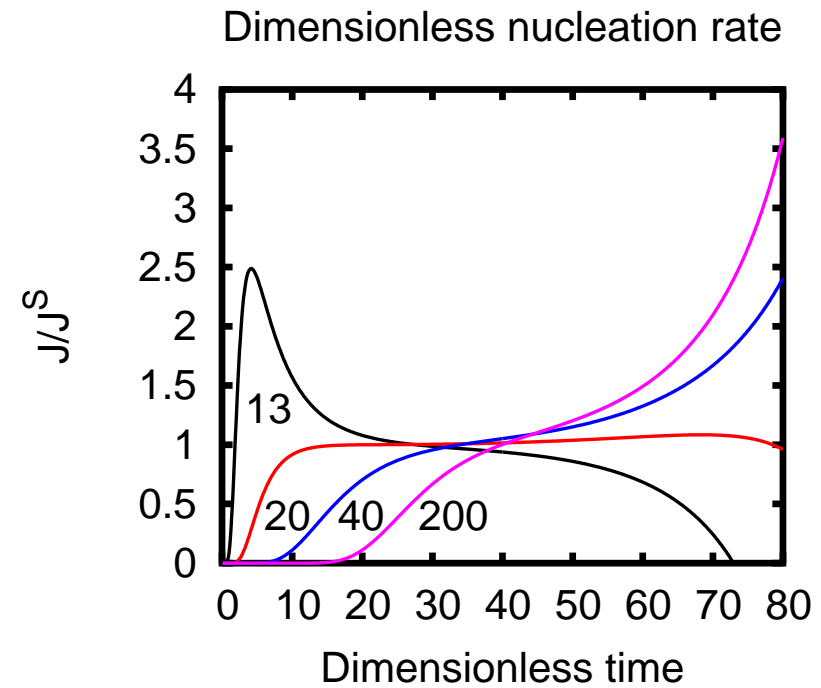


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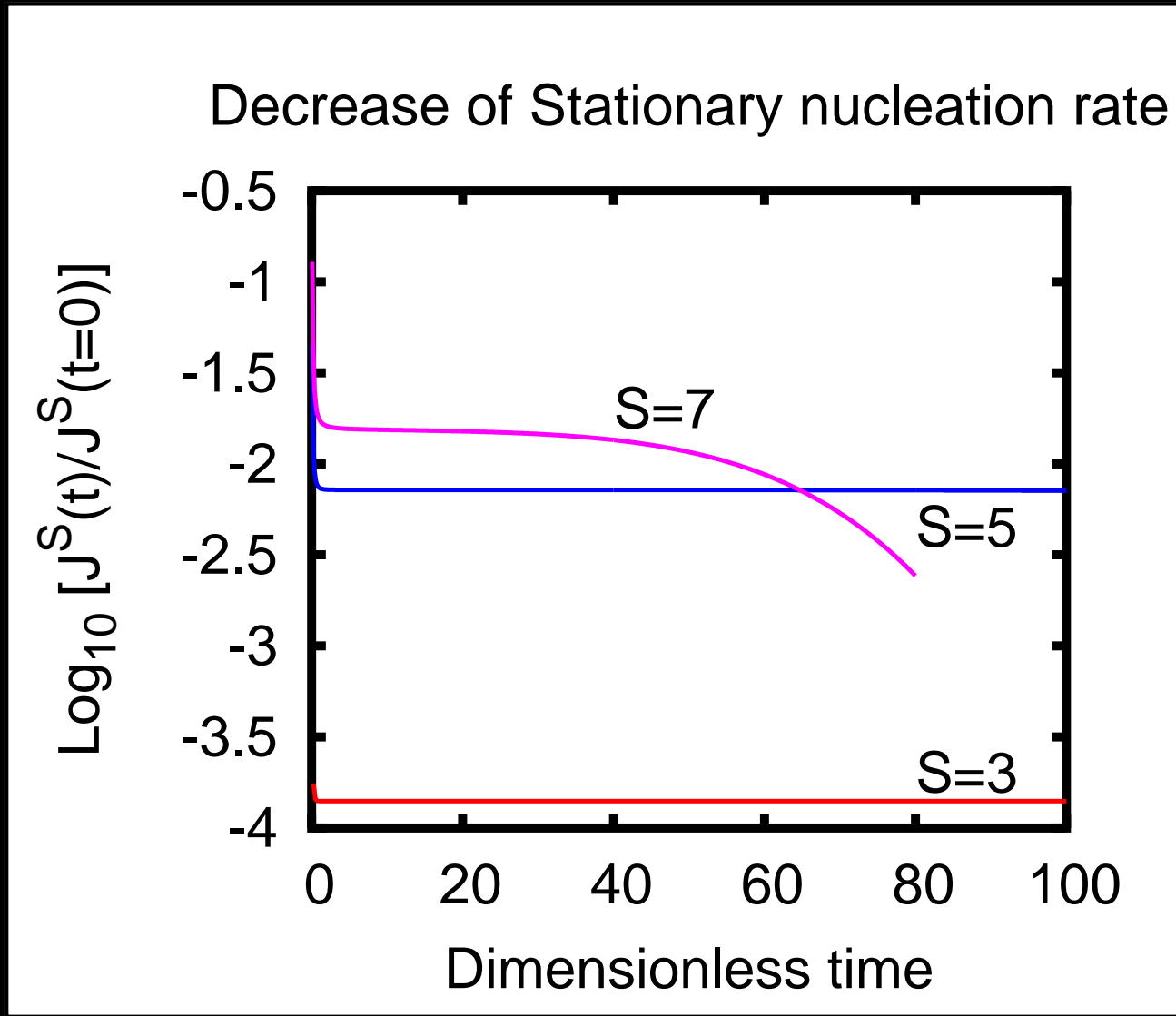


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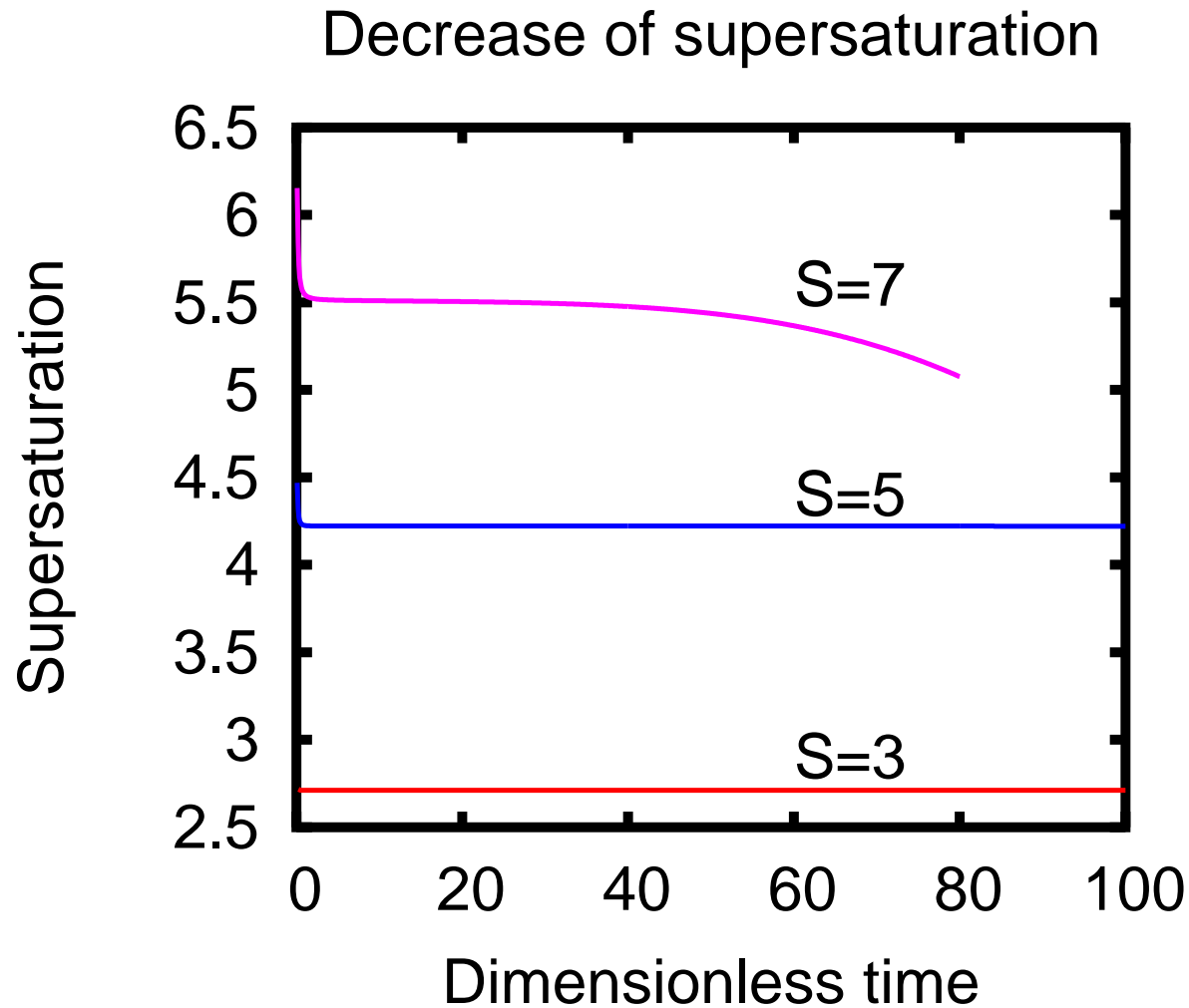




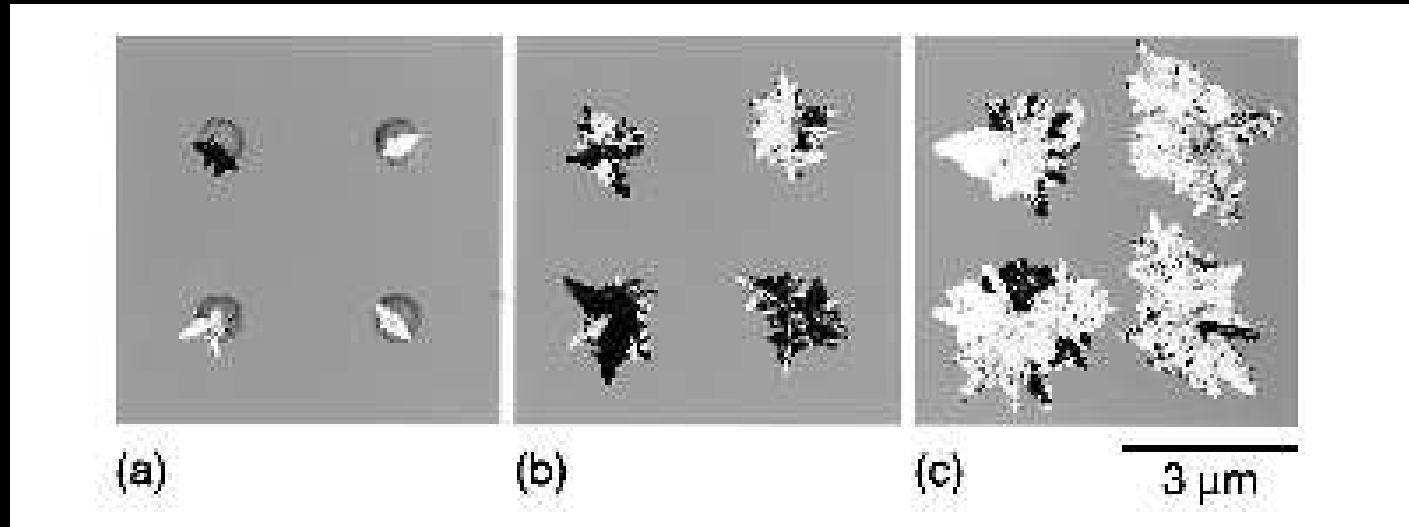
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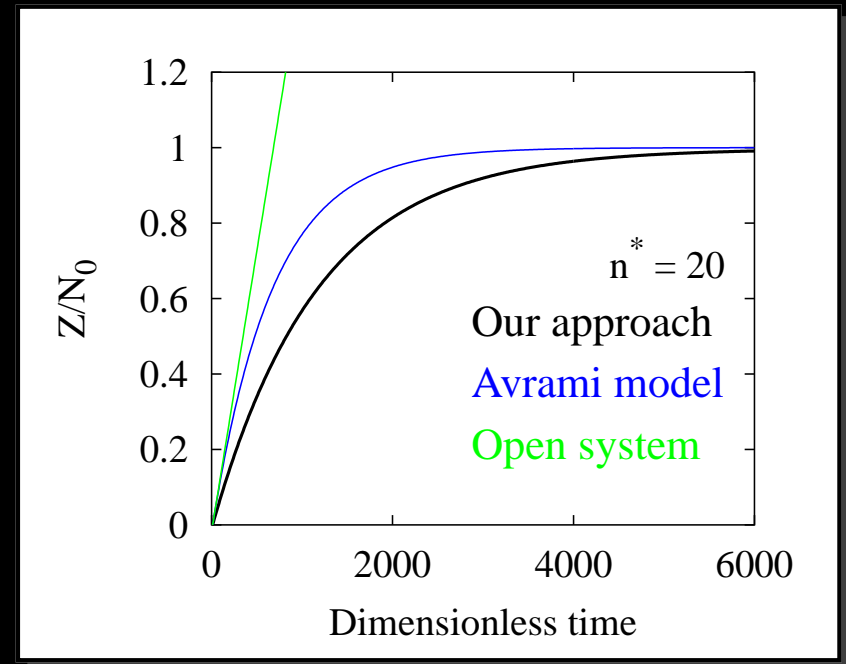
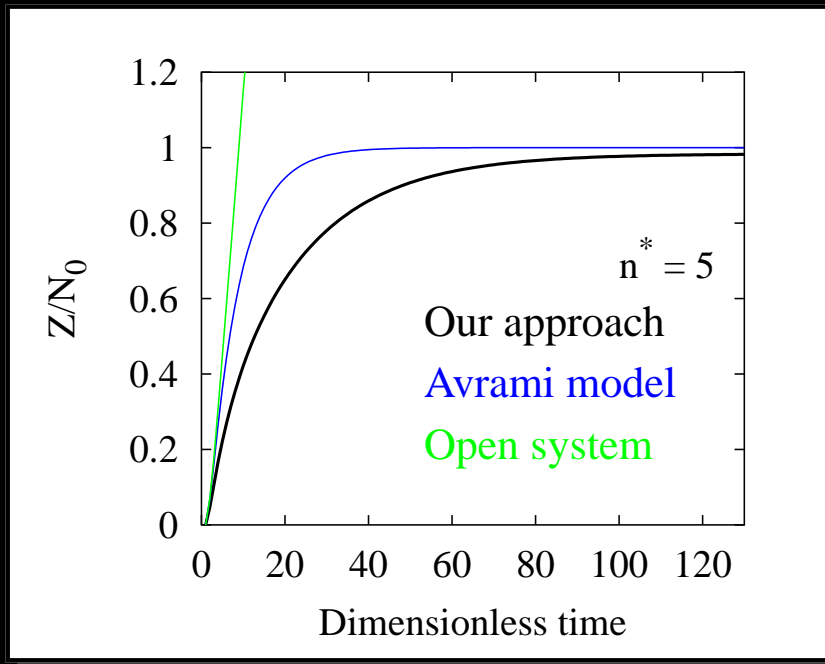
# Nucleation on active centers



H. Kumomi and F. G. Shi: Phys. Rev. Lett. 82 (1999) 2717.

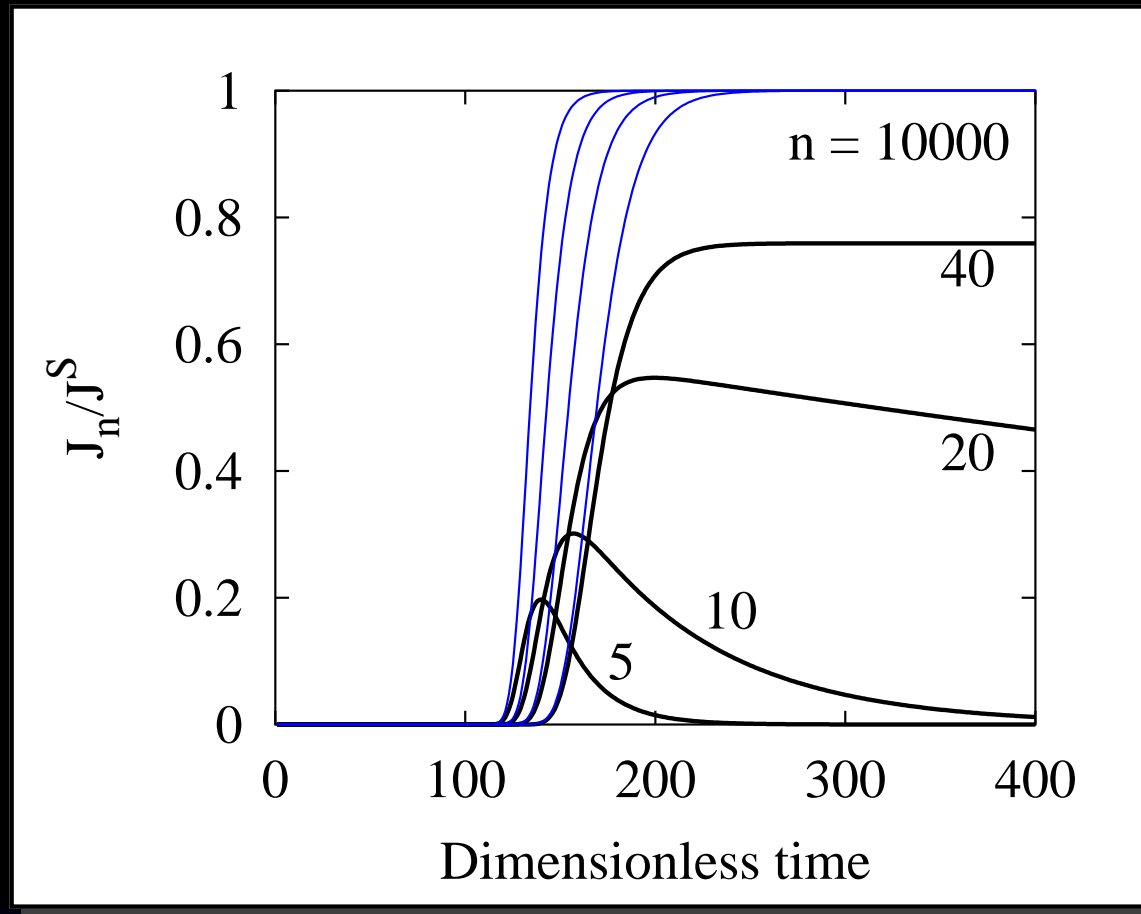
# Nucleation on active centers

## Total number of supercritical nuclei



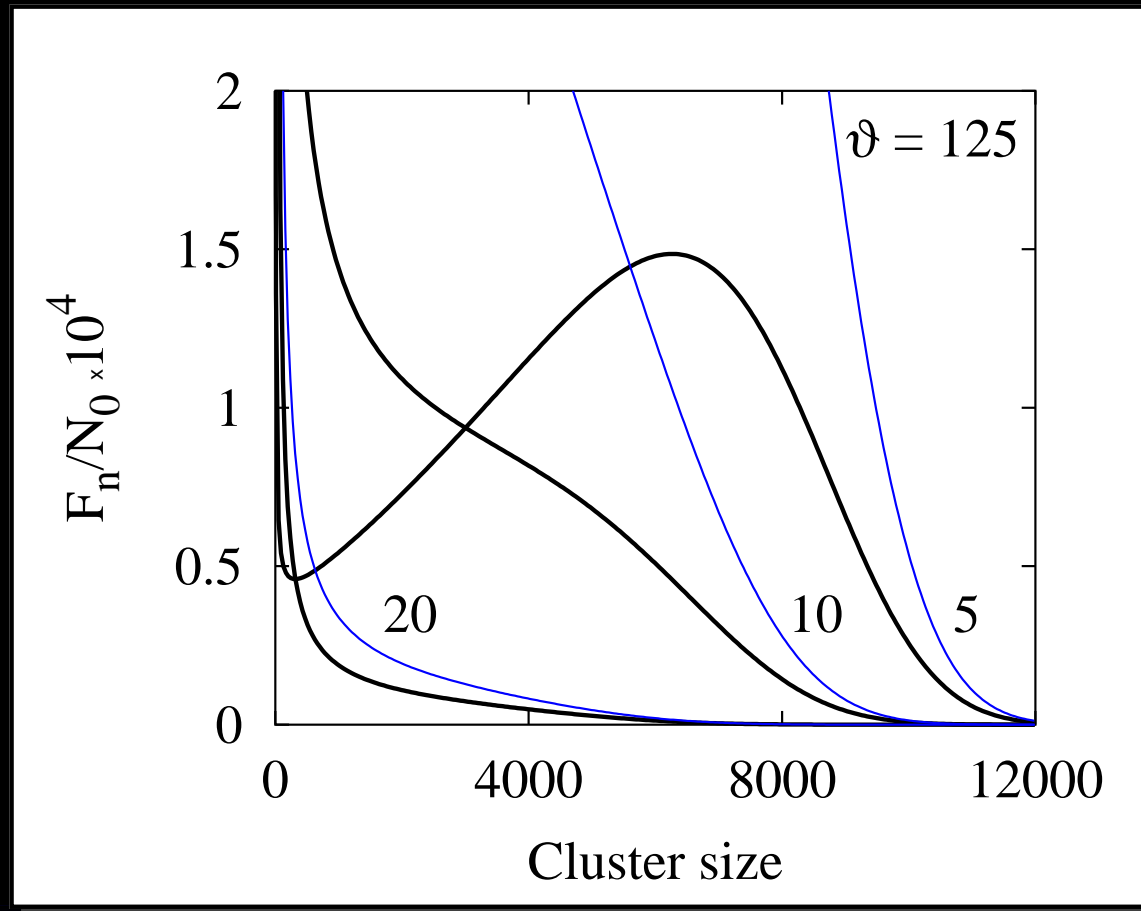
# Nucleation on active centers

Nucleation rate

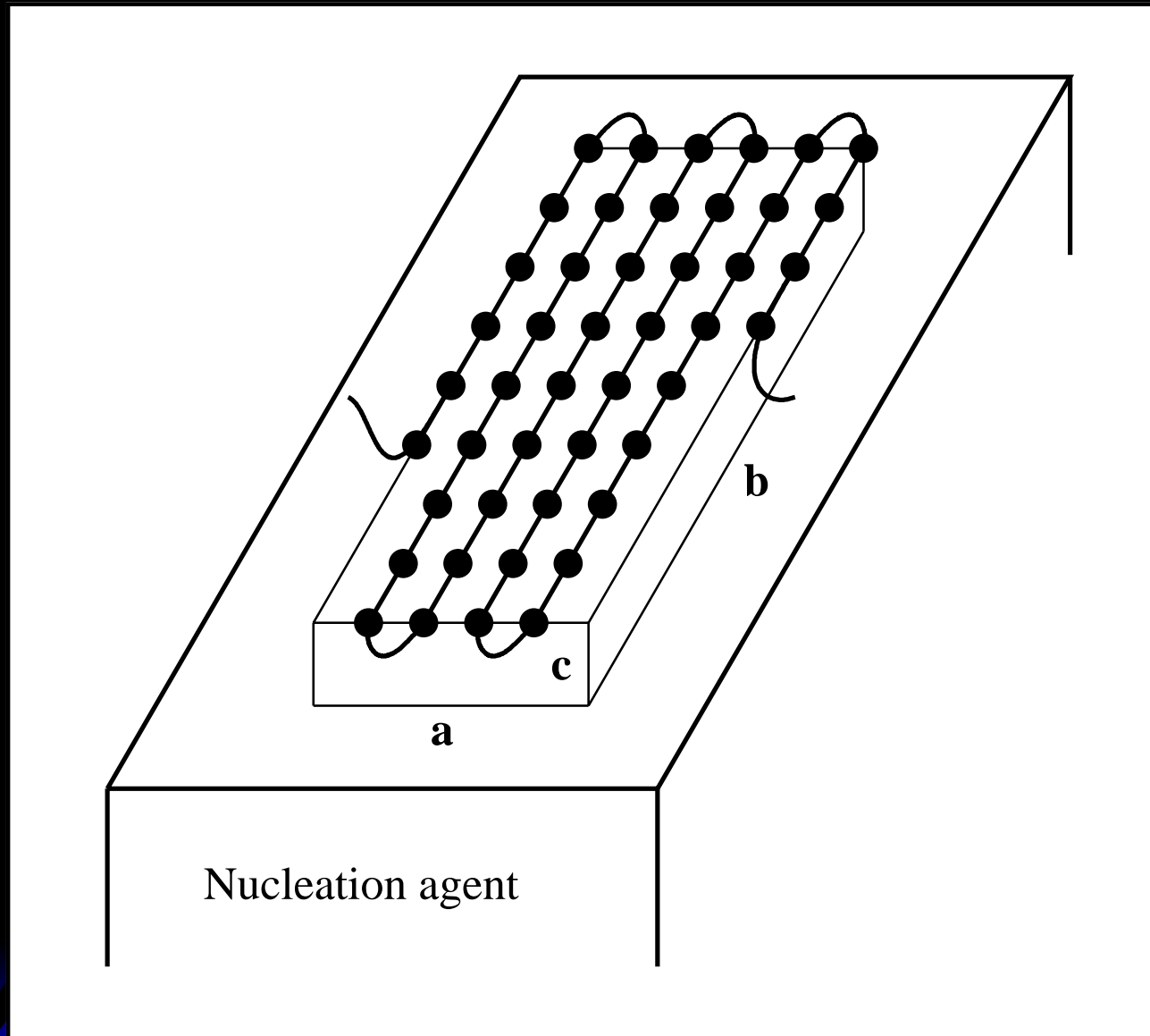


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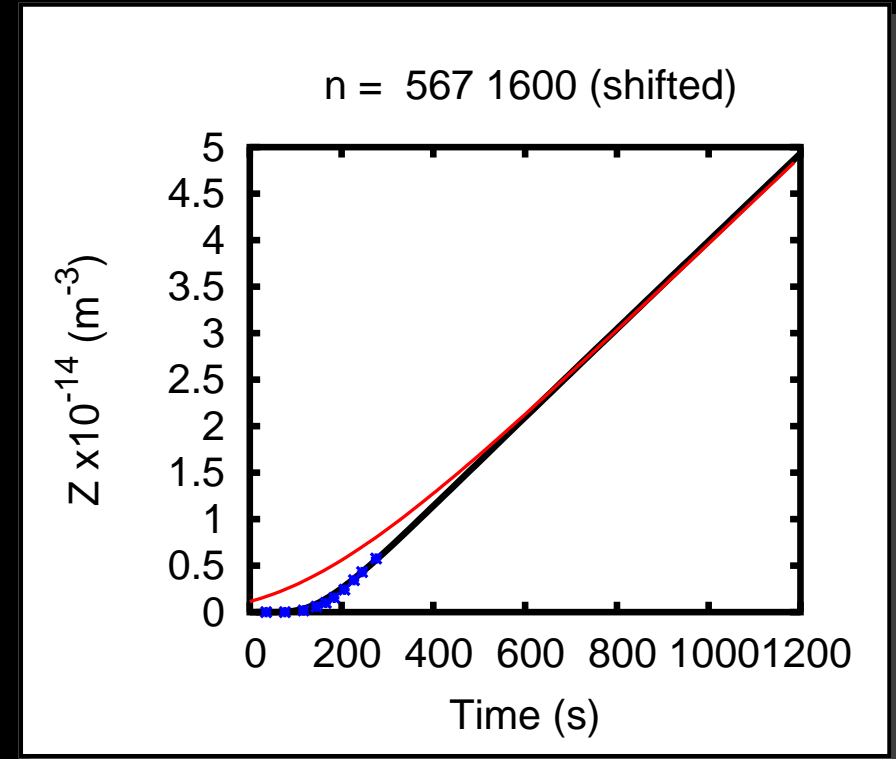
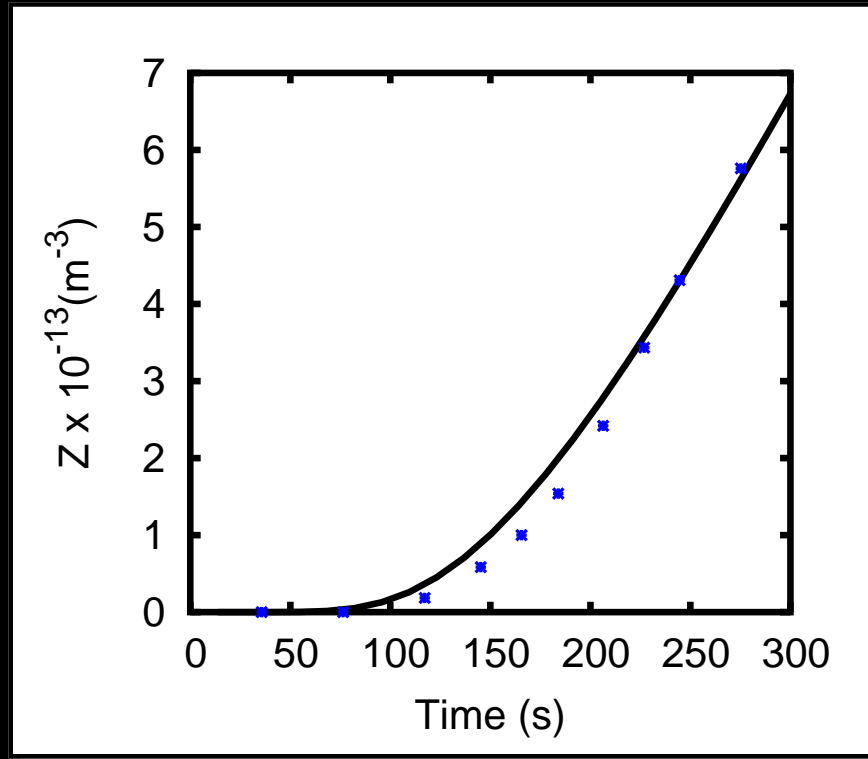
## Size distribution



# Nucleation on active centers

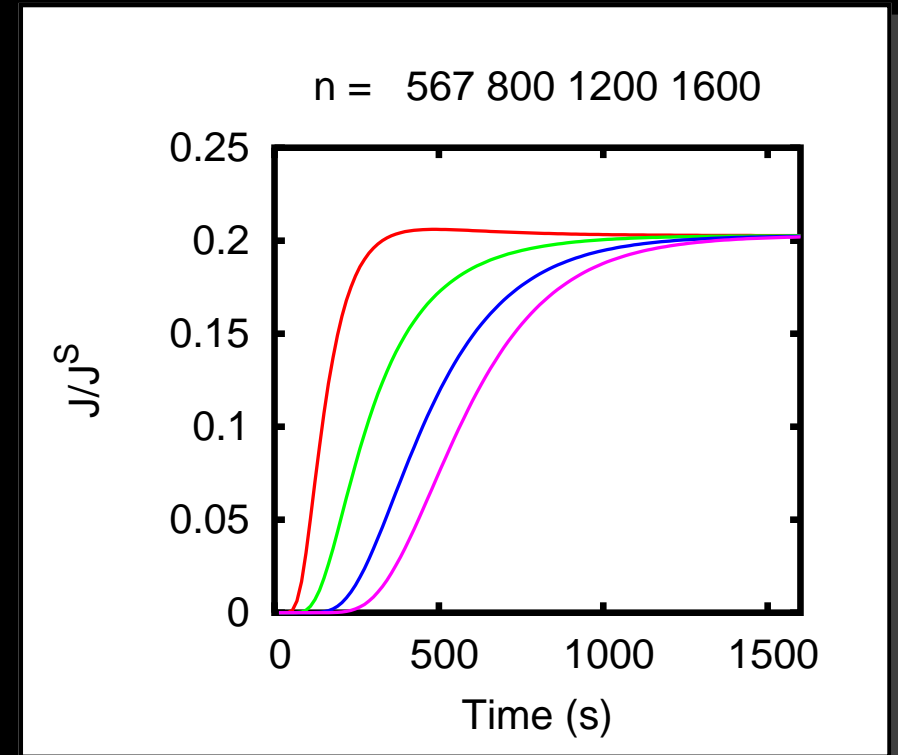
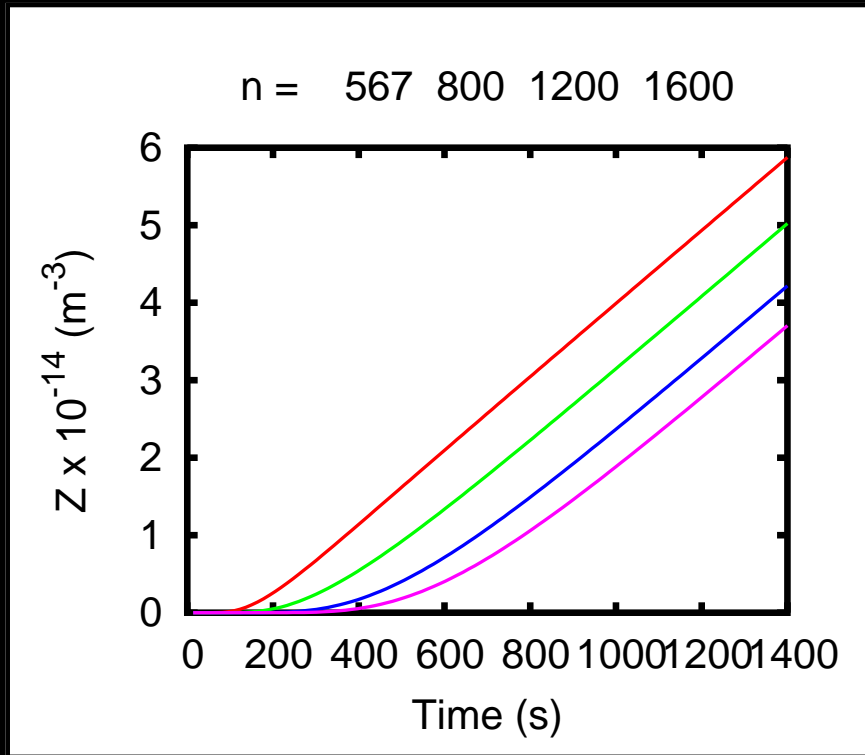


# Nucleation on active centers

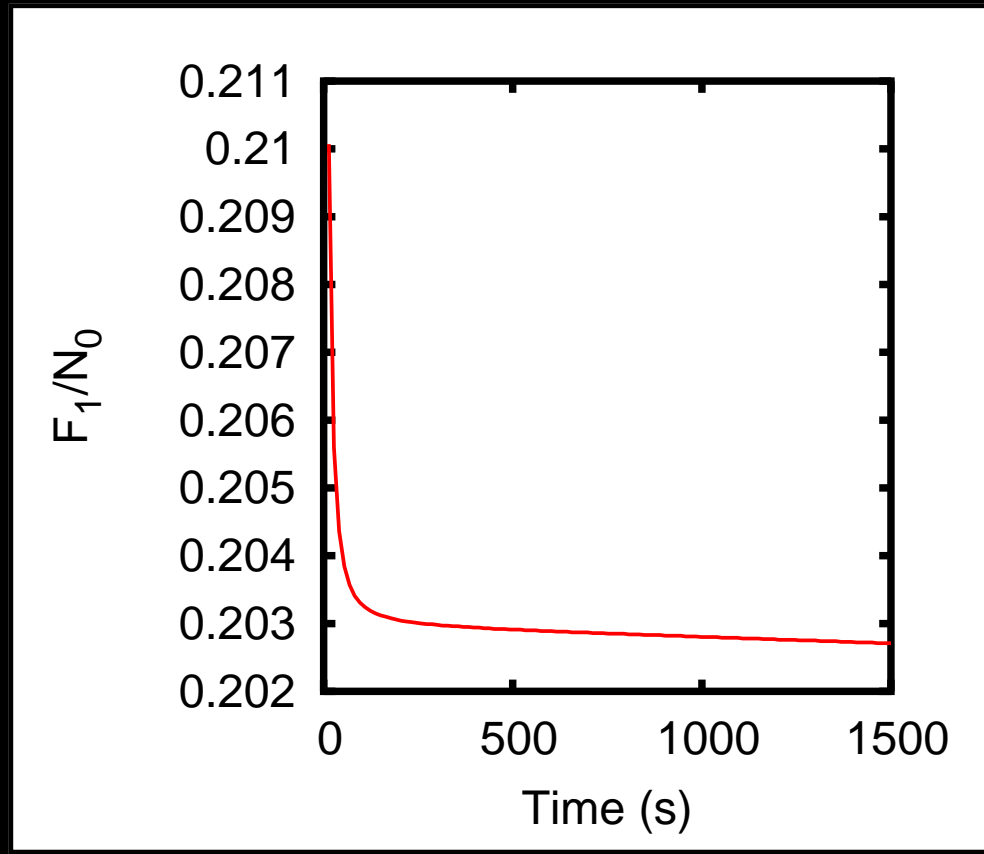




# Nucleation on active centers



# Nucleation on active centers



# Conclusions

## Closed systems

- At low supersaturations quasistationary regime is reached.
- At higher supersaturations formation of nuclei is fully nonstationary process.
- Standard approaches based only on thermodynamics can not describe nucleation in close systems.

## Nucleation on active centers

- Modified standard kinetic model including the depletion of the active centers fits well nucleation of FCC polyethylene at low supersaturation.