Efficiency of interacting Brownian motors

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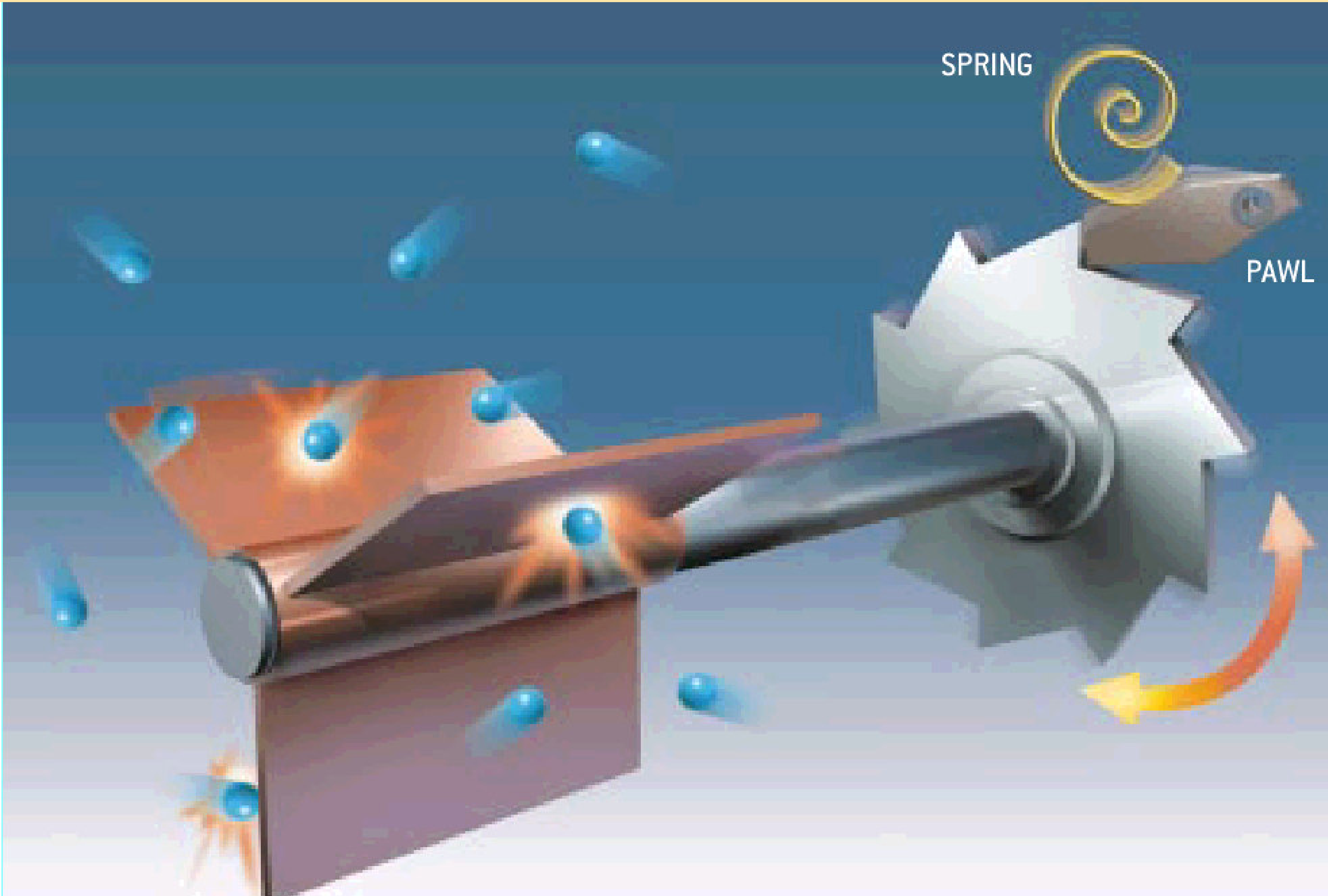
- Ratchet effect
- Realisations
- Rocking ratchet with interaction
- Current and energetics
- Thanks to GAČR 202/03/0551

Ratchet effect (Smoluchowski, Feynman,...)



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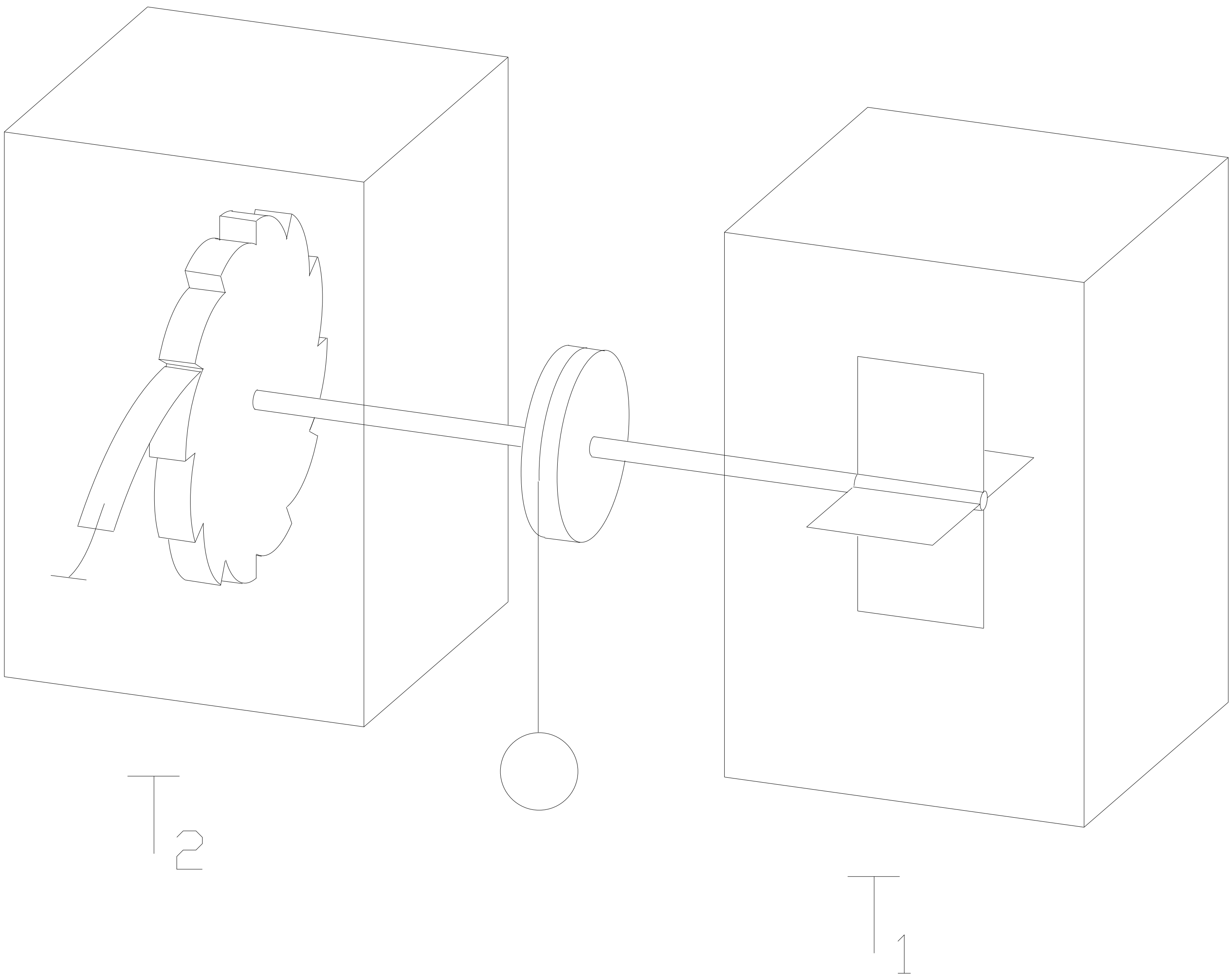

Brownian motor [P. Reiman, Phys. Rep. 361, 57 (2002); P. Hänggi et al., cond-mat/0410033]

ingredients:

1) Non-equilibrium (open) system $\left\{ \begin{array}{l} \text{On-off} \\ \text{Rocking} \end{array} \right.$

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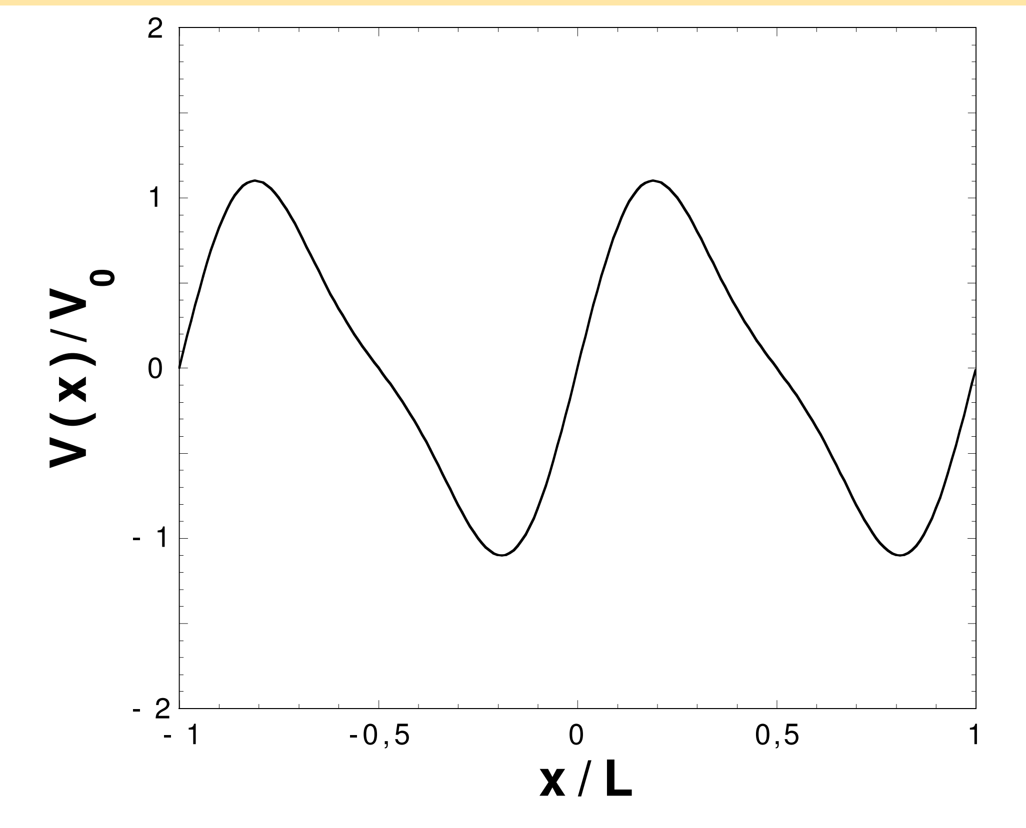
2) spatial asymmetry



Free movement

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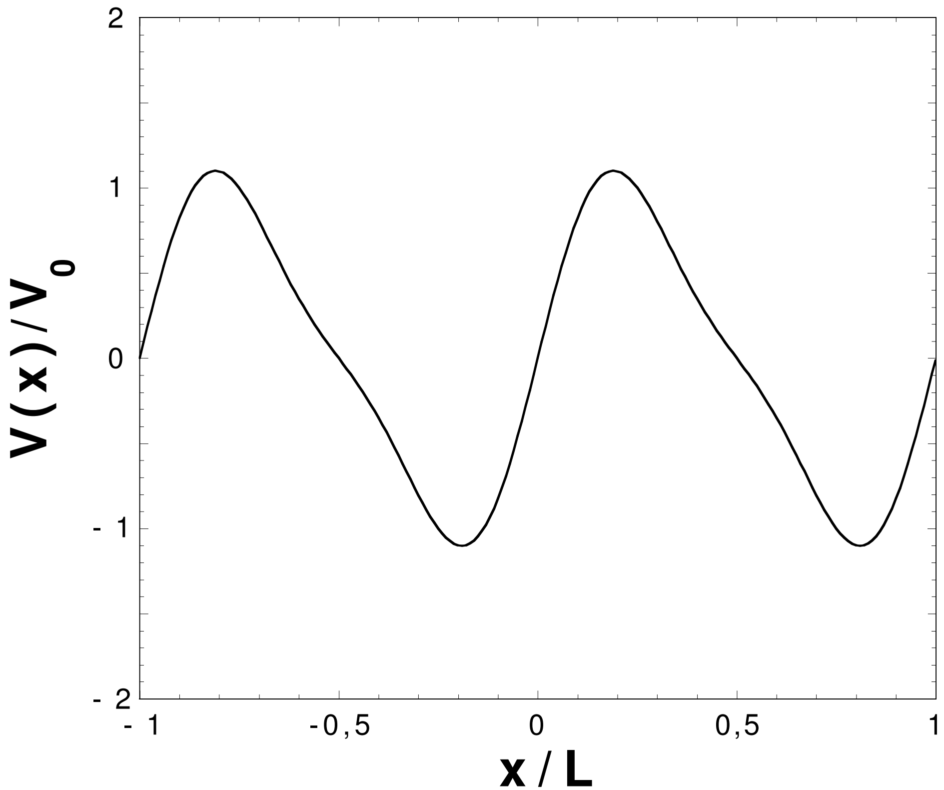

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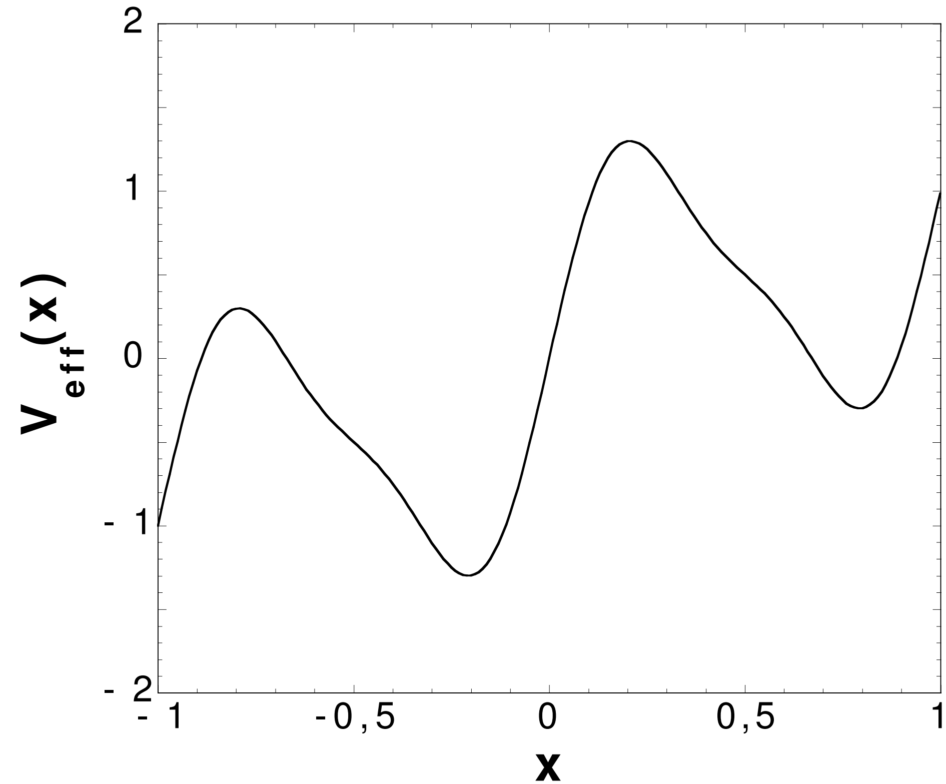
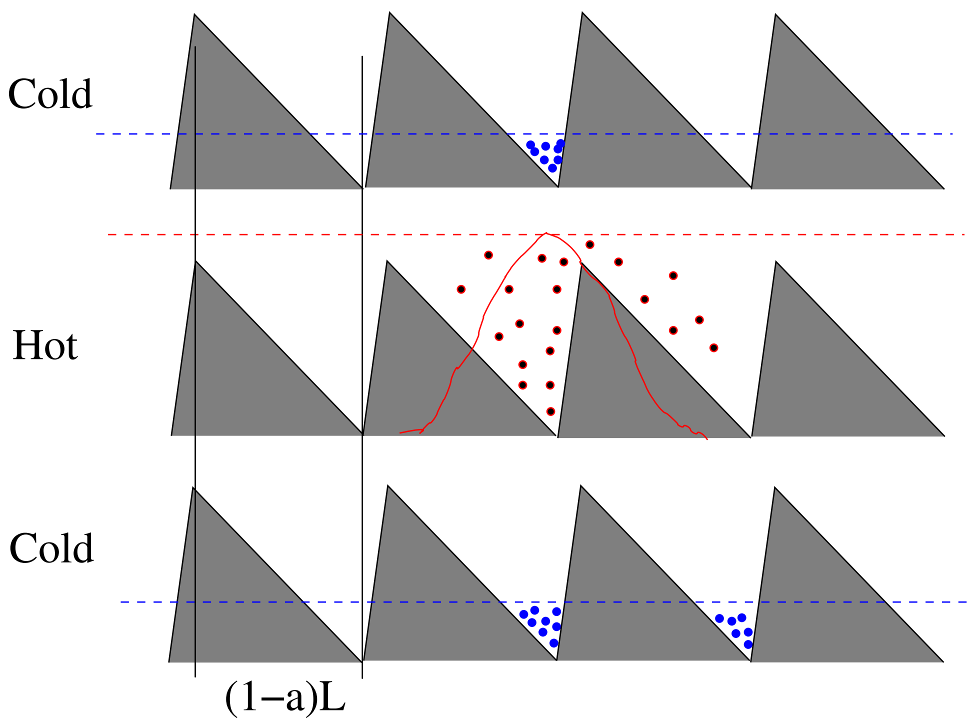
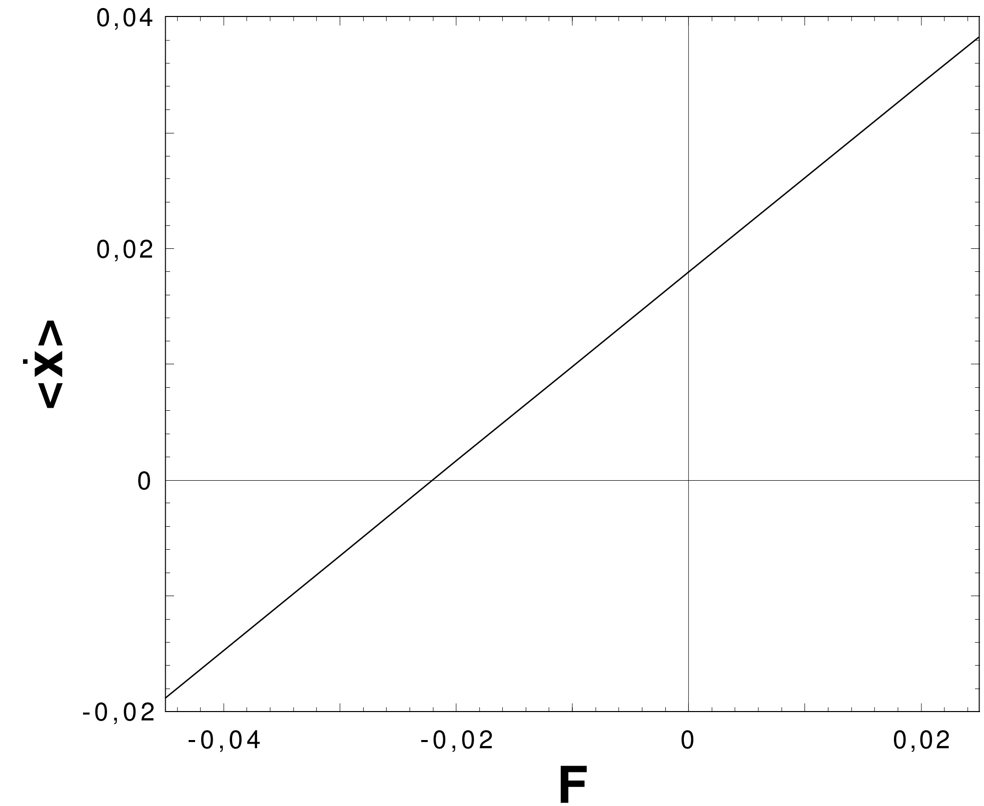


Load attached

Thermal ratchet

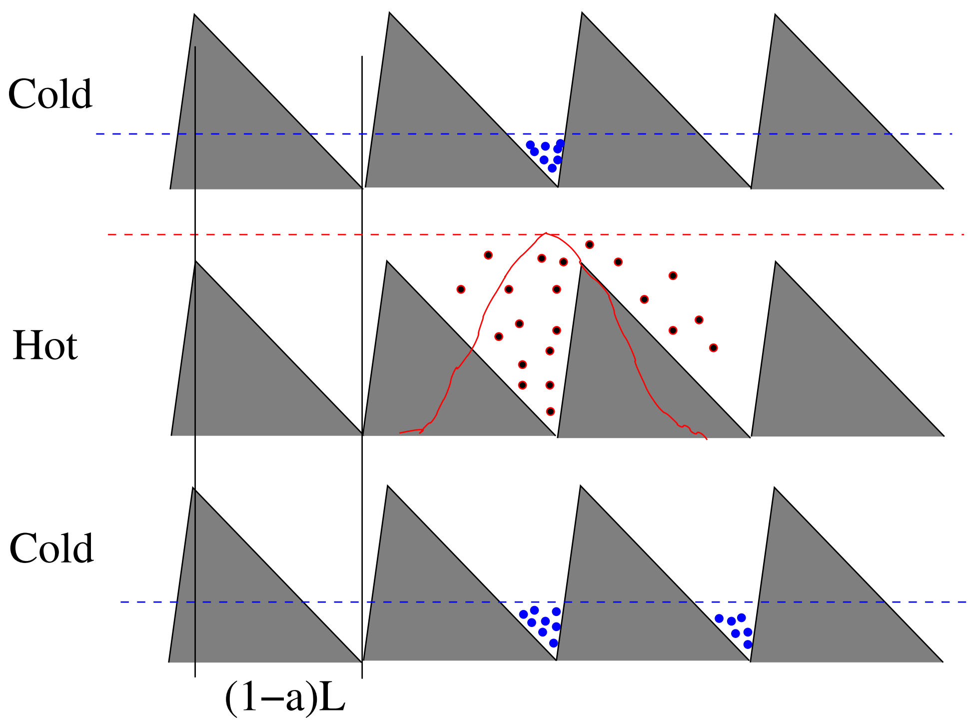


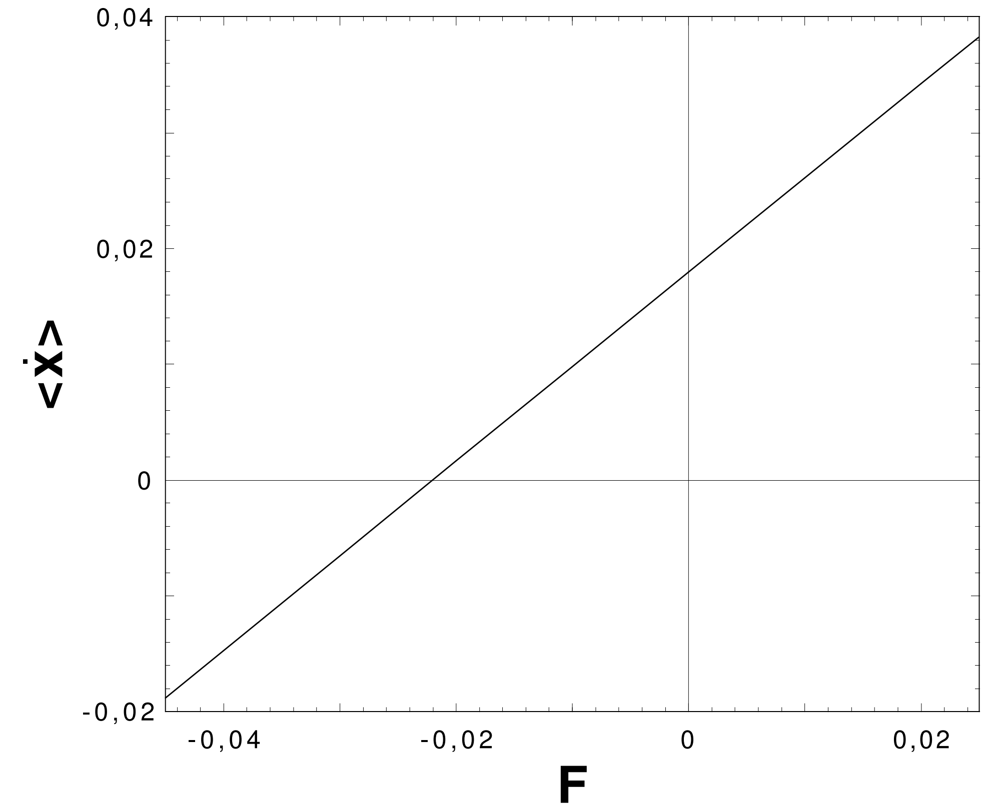


Diffusion

Current

Thermal ratchet





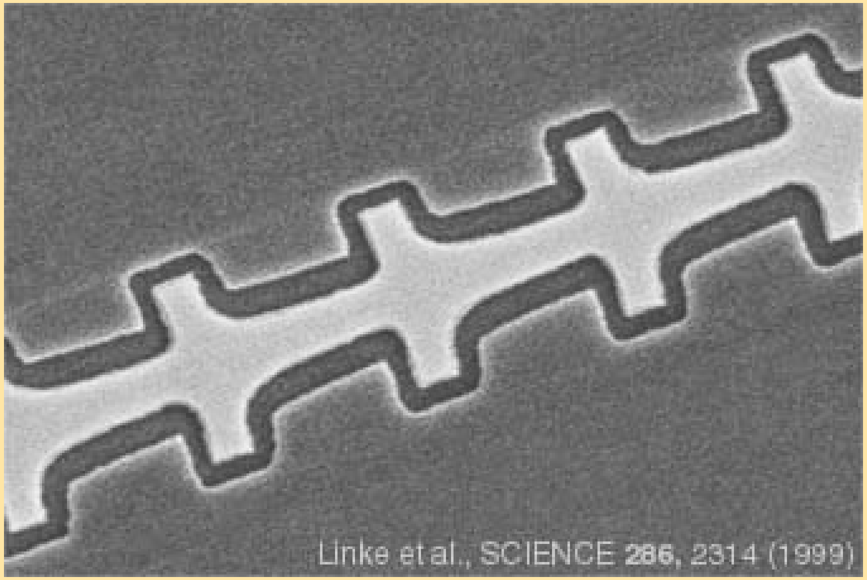
Diffusion

Current

Rocking ratchet

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = -\frac{\partial V(x)}{\partial x} + F_{\mathrm{load}} + F_0 \sin \omega t + \xi(t)$$

Realisations: technological



GaAs/AlGaAs heterostructure. Period $L \simeq 1.2\mu\mathrm{m}$.

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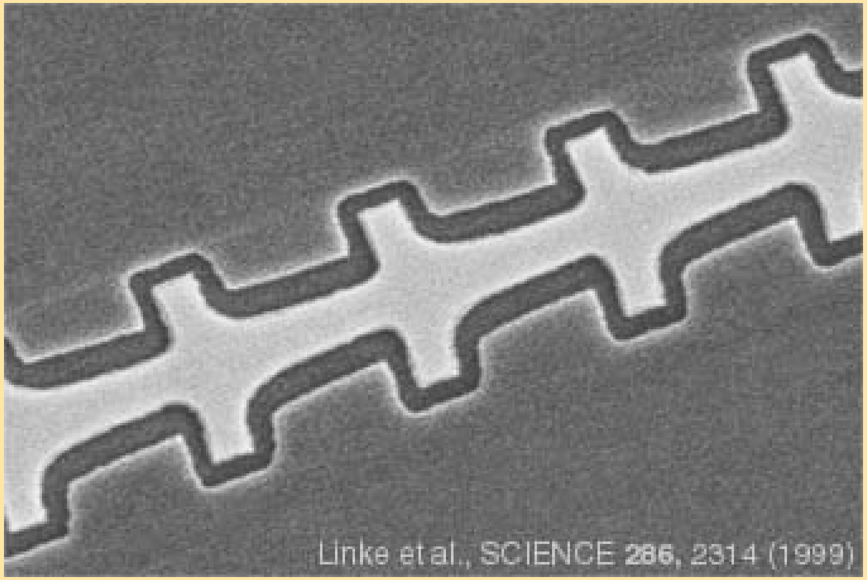

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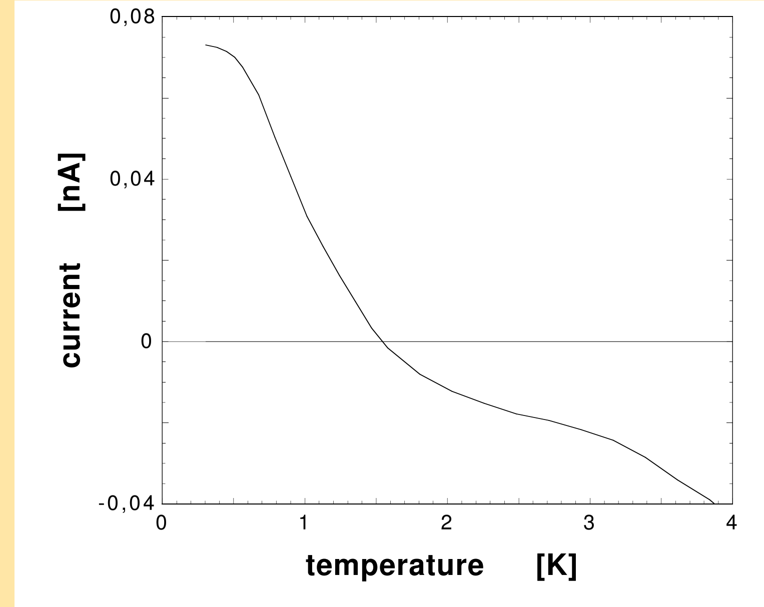

Rocking voltage, jumps $\pm$1mV, f=191Hz

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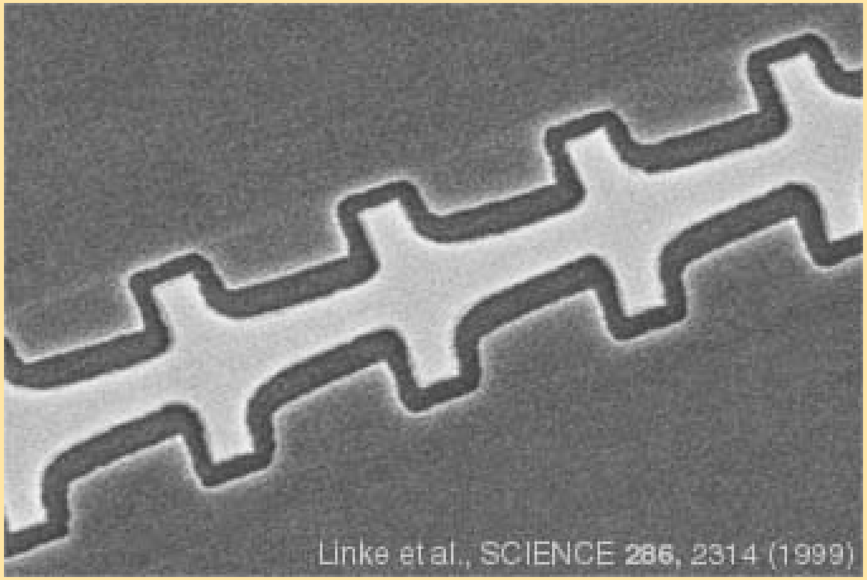

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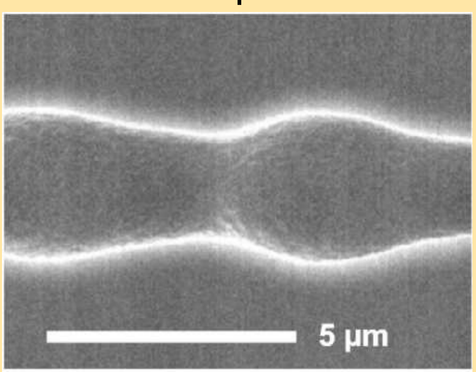
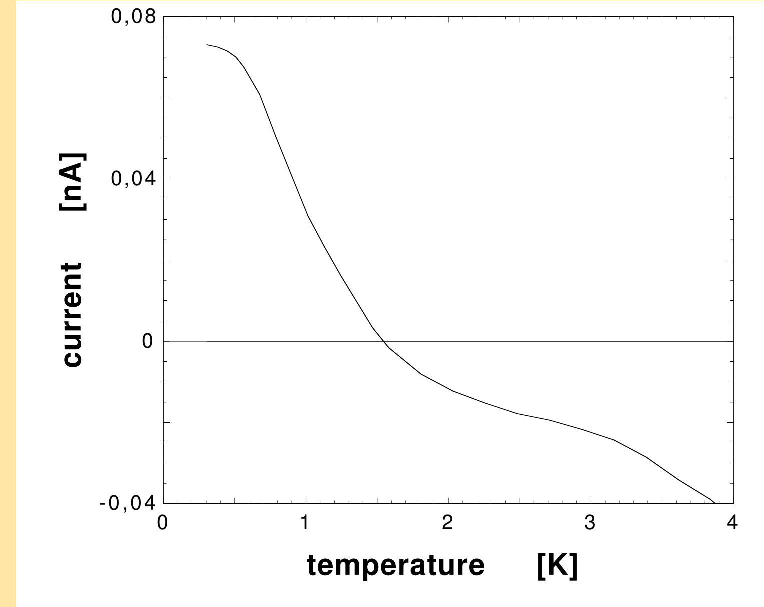
silicon nanopore





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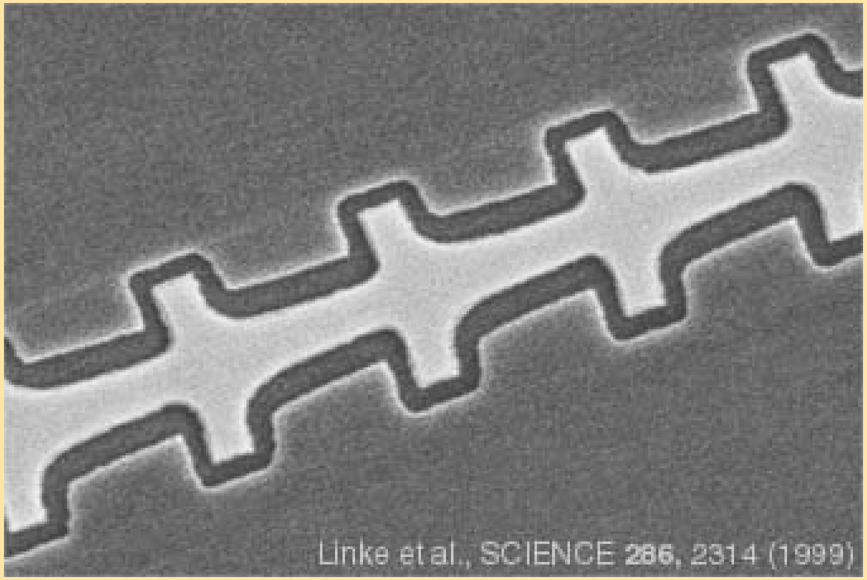

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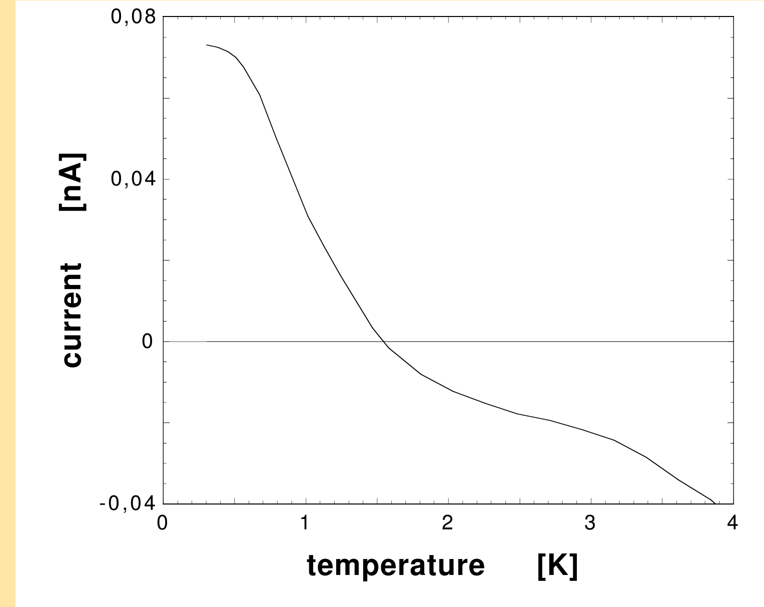

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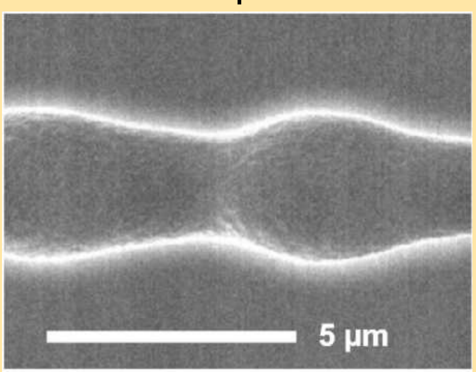
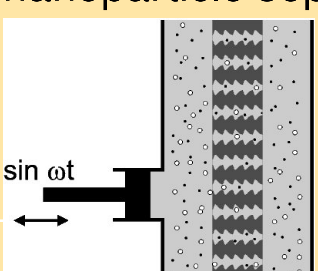


nanoparticle separation pump



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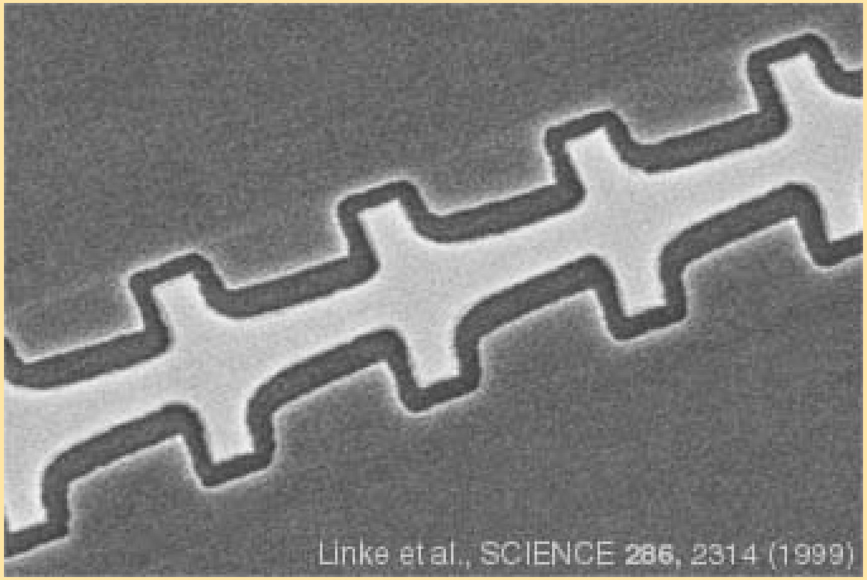

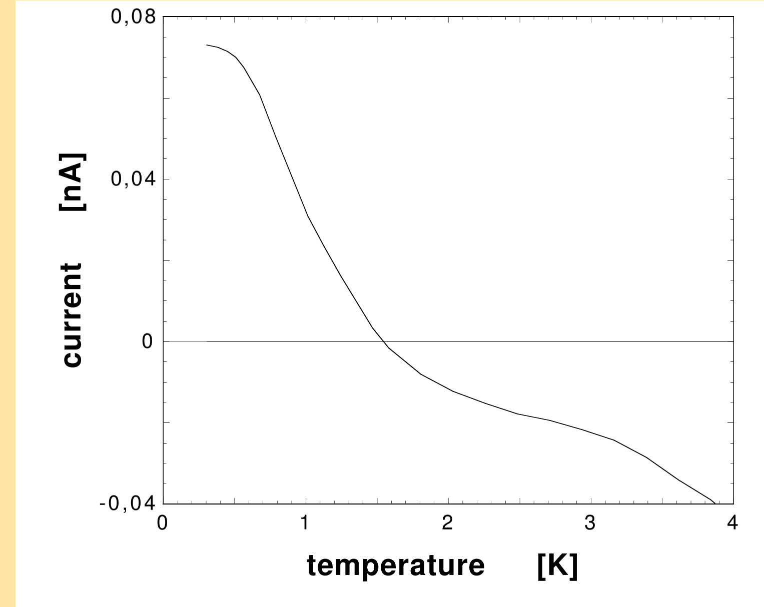

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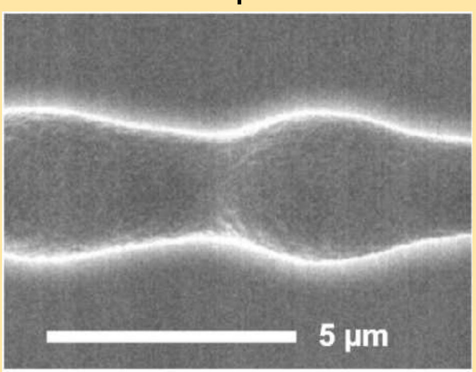

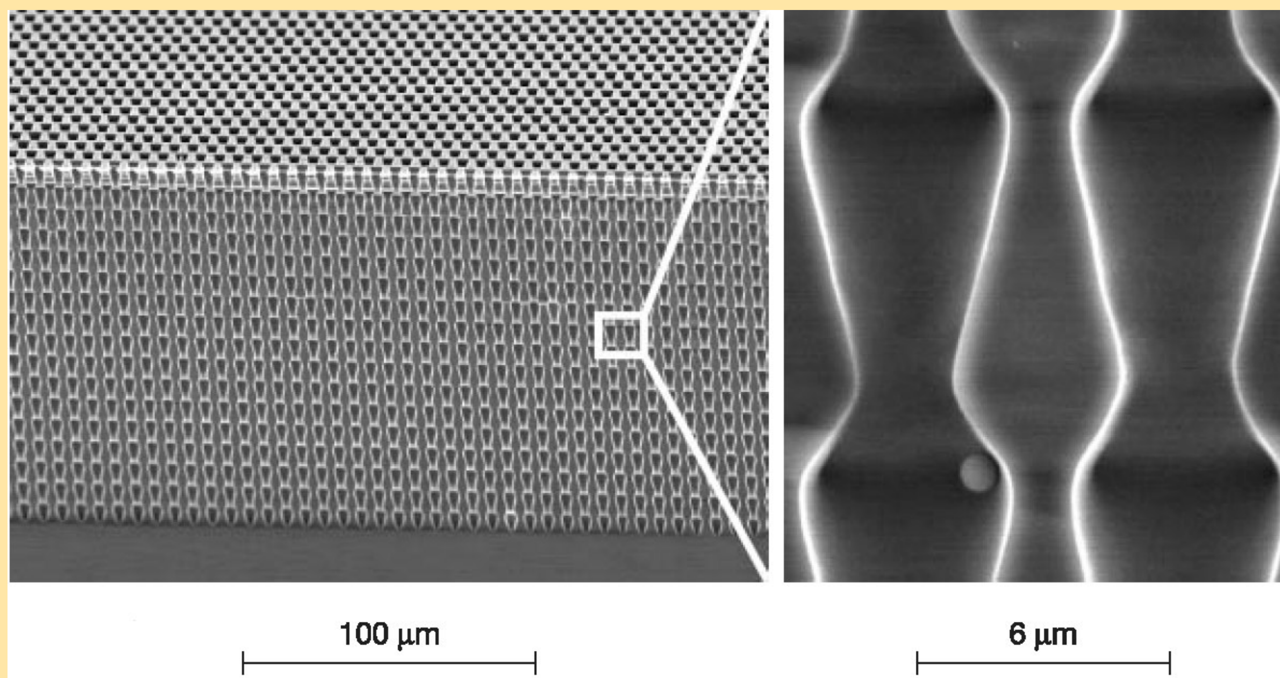

[Sven Matthias and Frank Müller, Nature 424, 53 (2003)]

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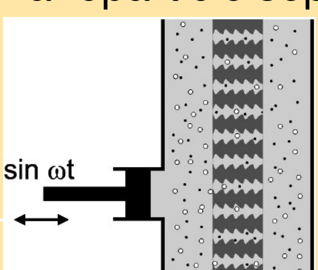
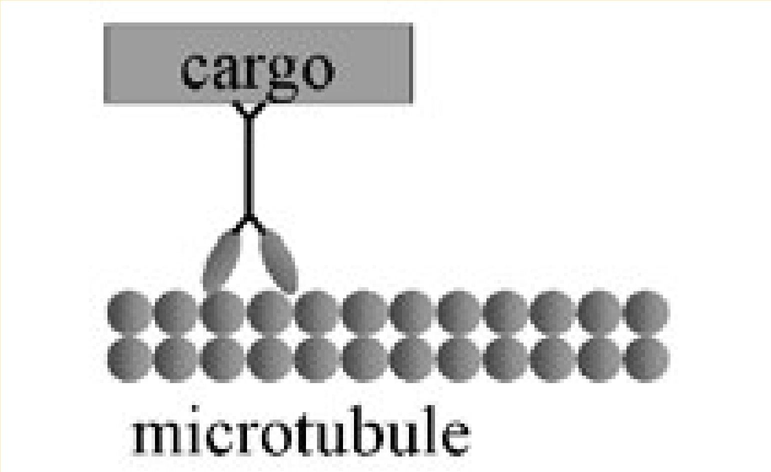
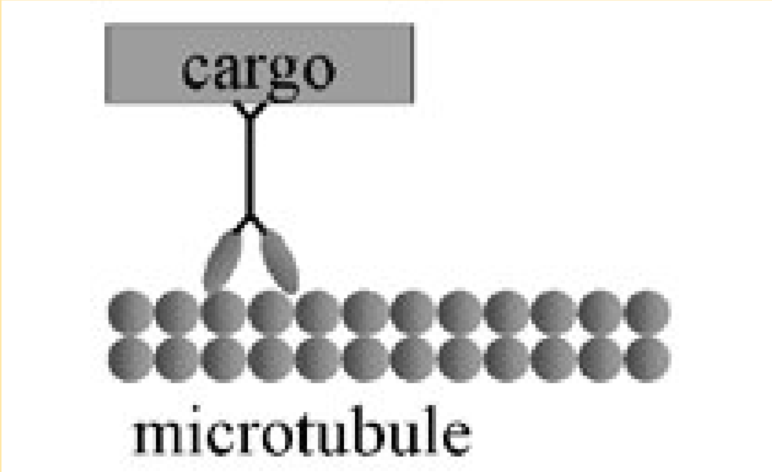


Realisations: biological



Kinesin carrying vesicles

Realisations: biological



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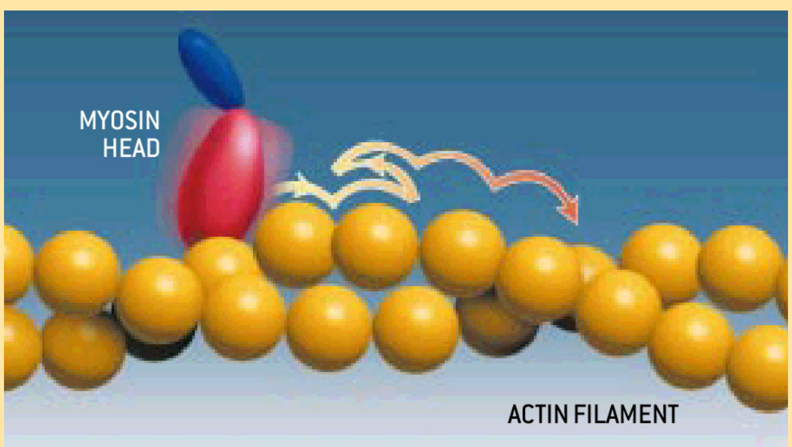
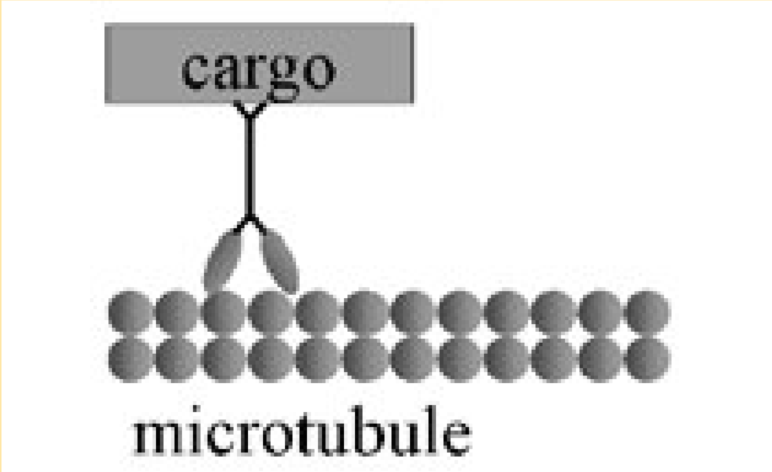


Myosin moving the muscle

Realisations: biological



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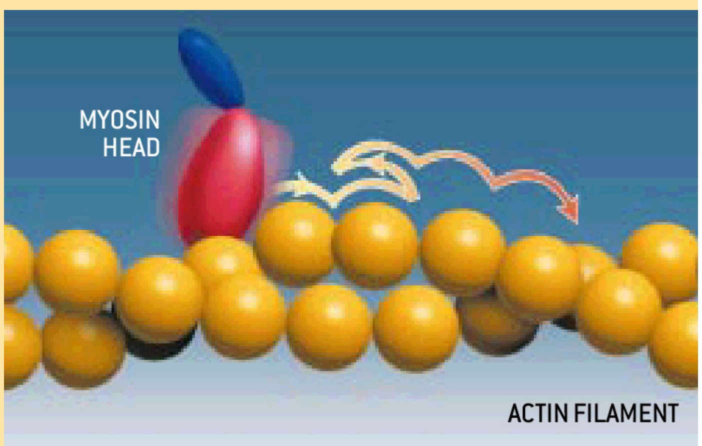

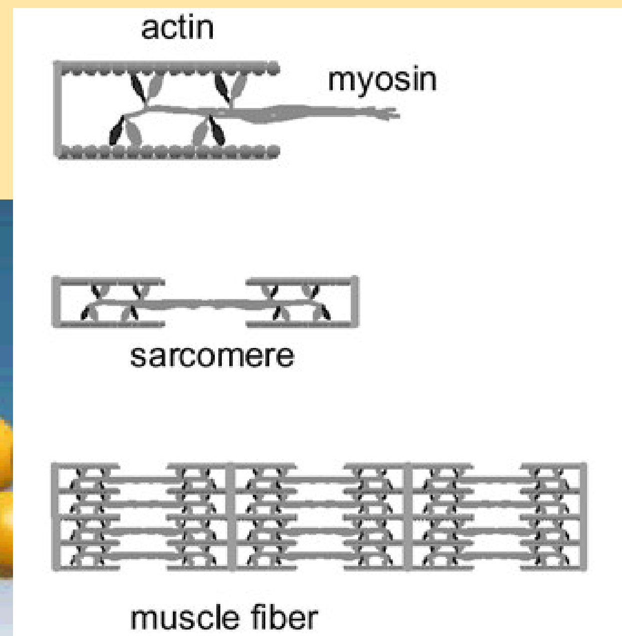

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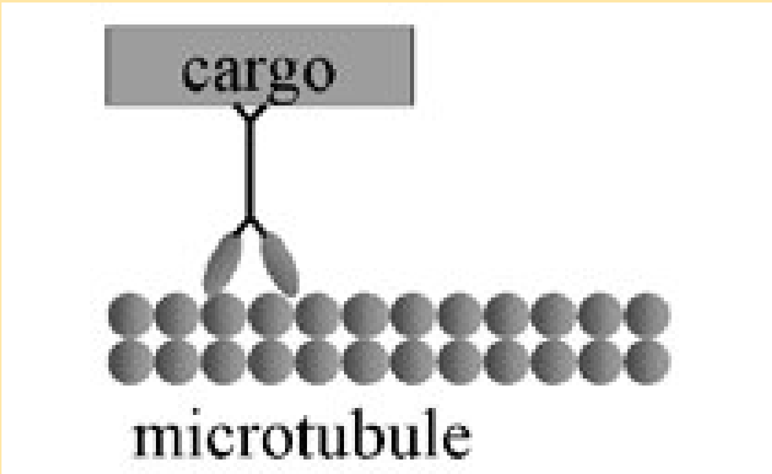

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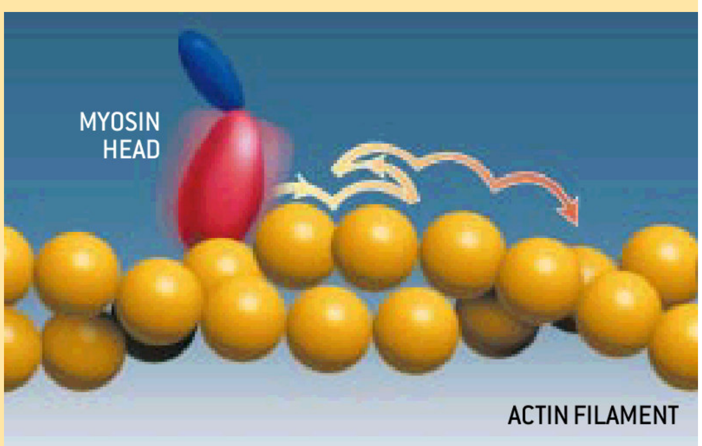

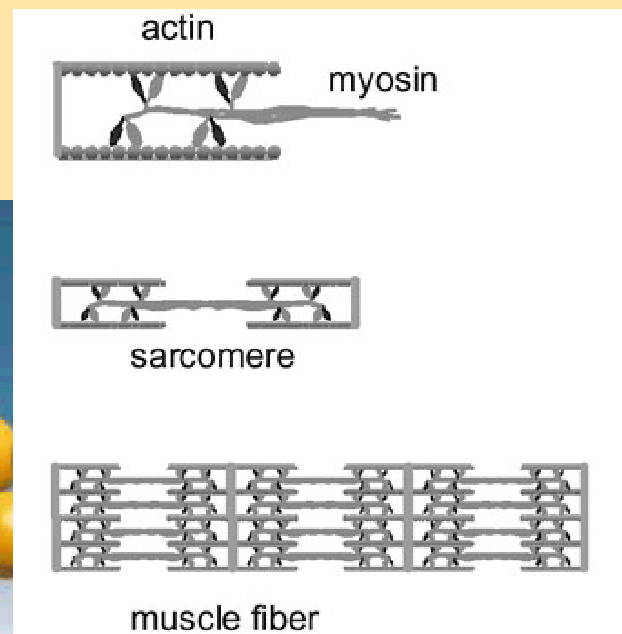

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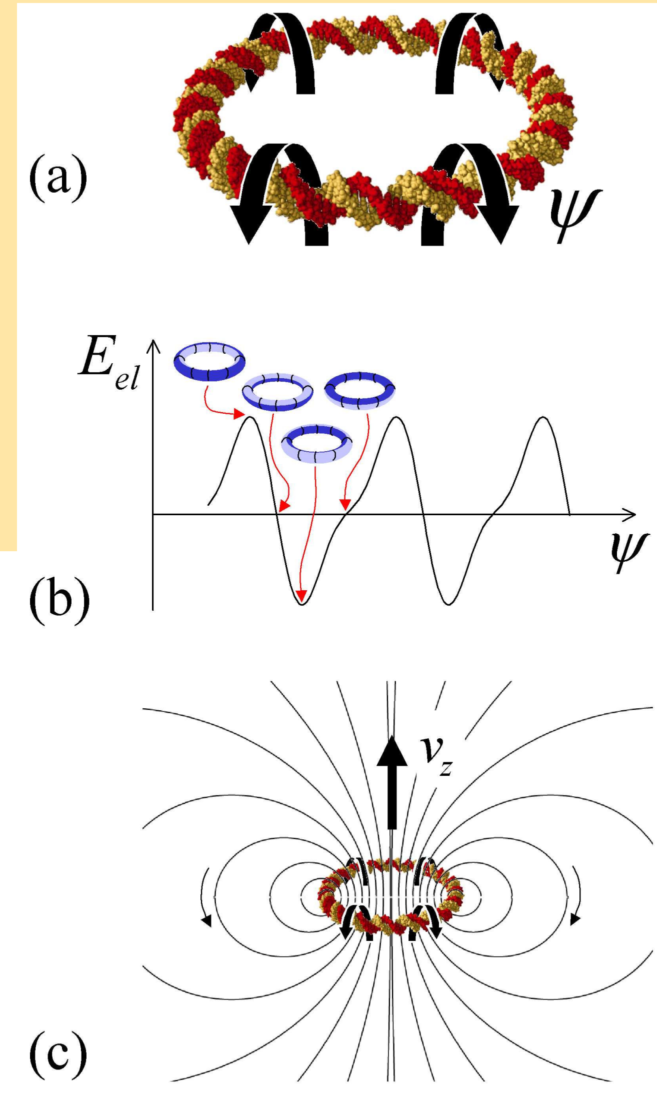
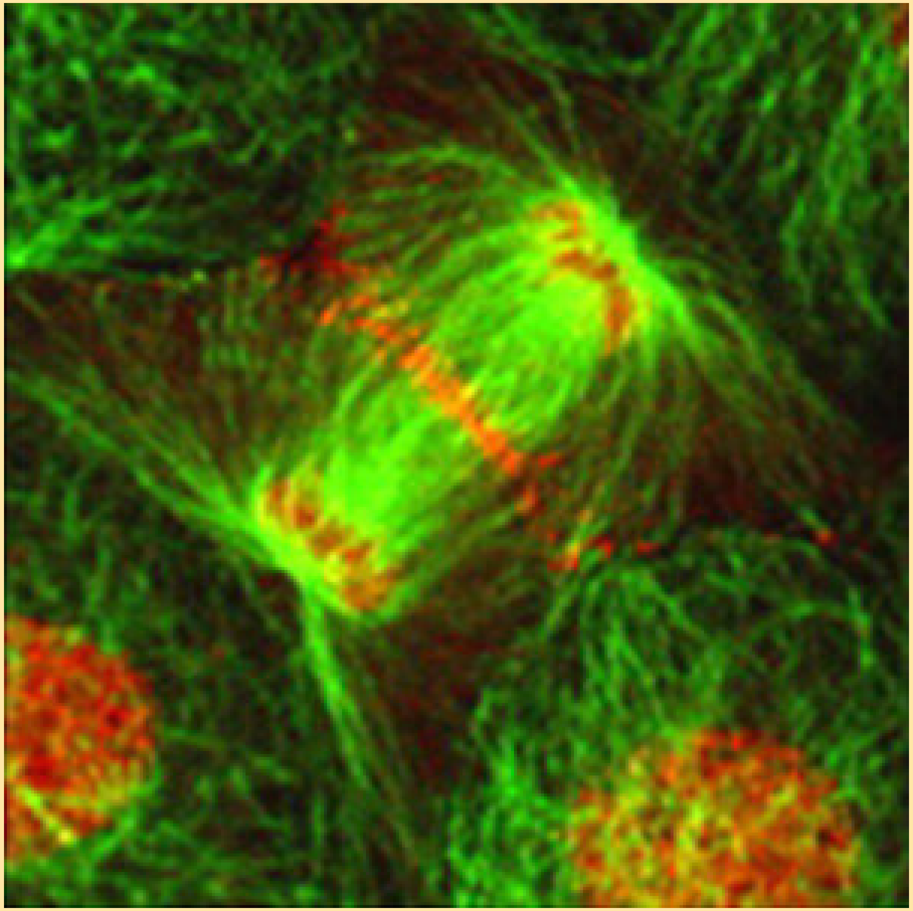
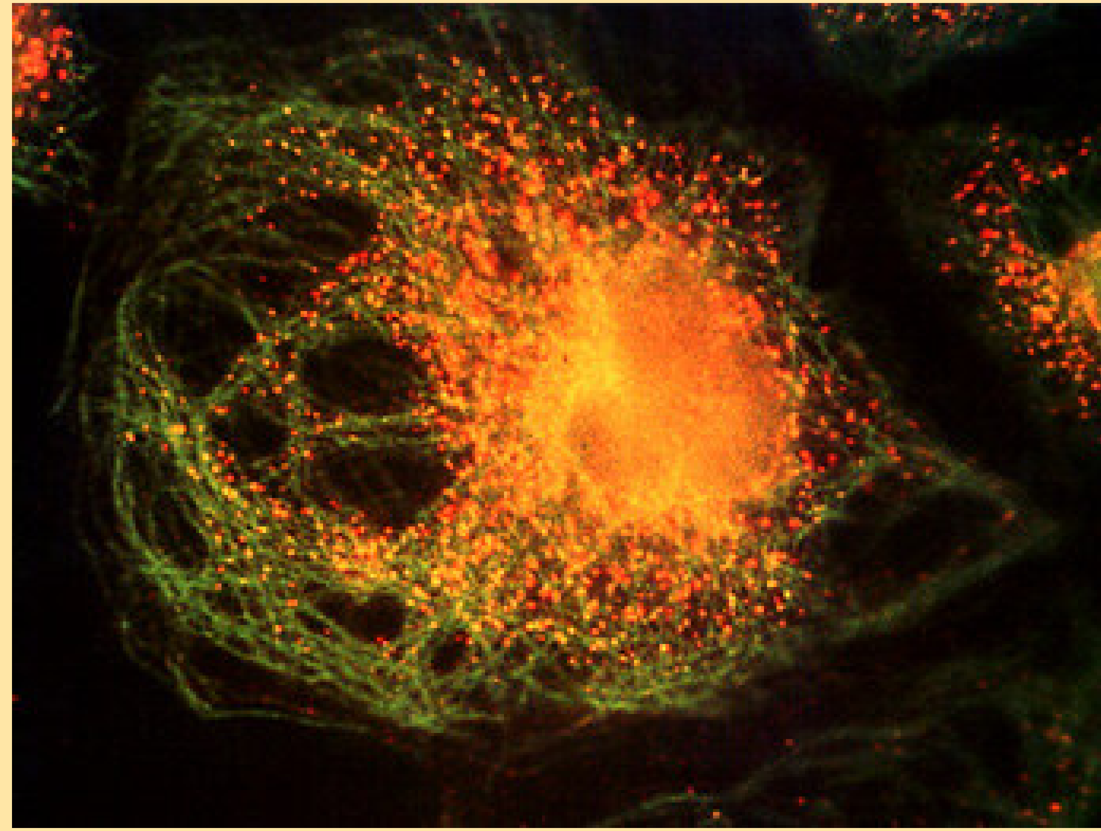


[Igor M. Kulic, Rochish Thaokar, Helmut Schiessel, cond-mat/0410197]

Within the cell [The Kinesin Home Page http://www.proweb.org/kinesin//index.html]

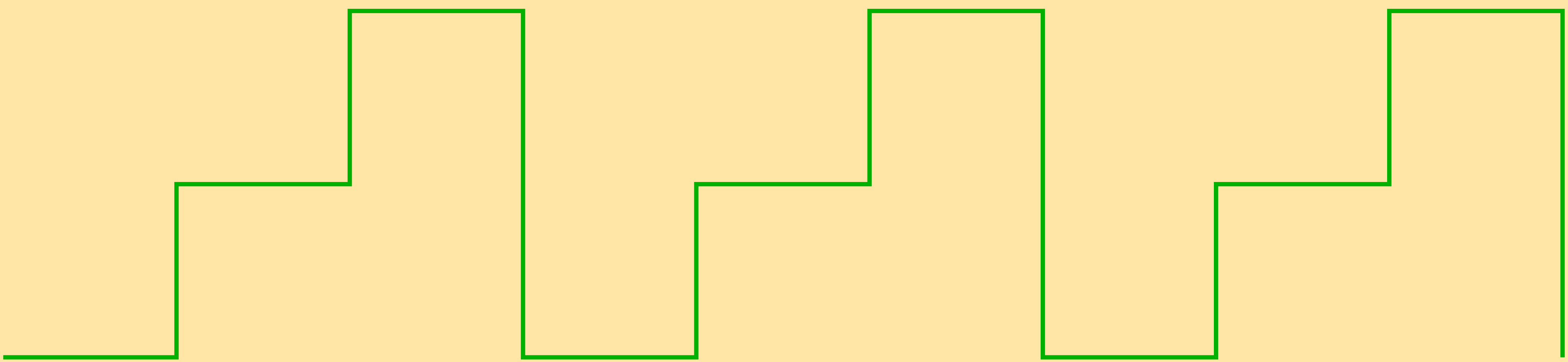


Kinesin proteins that associate with chromosomes. Kinesin (red) and microtubules (green)



Kinesin moving membranes. Microtubules (green) and Xklp1 (red).

Rocking ratchet with interaction



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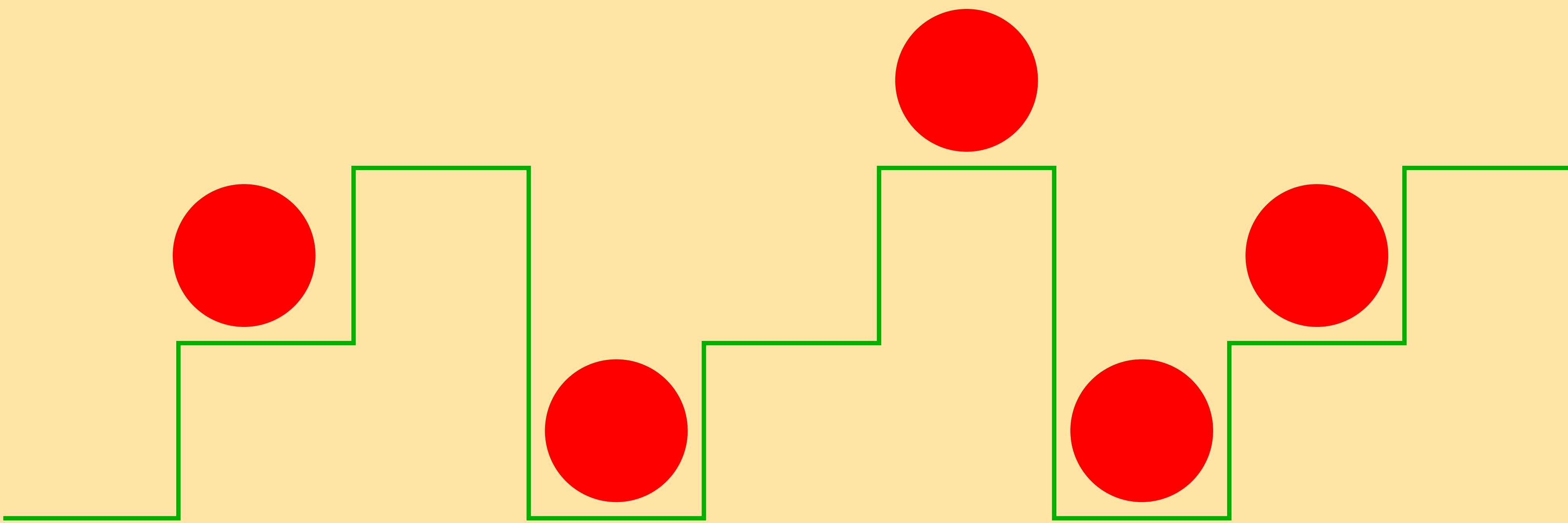

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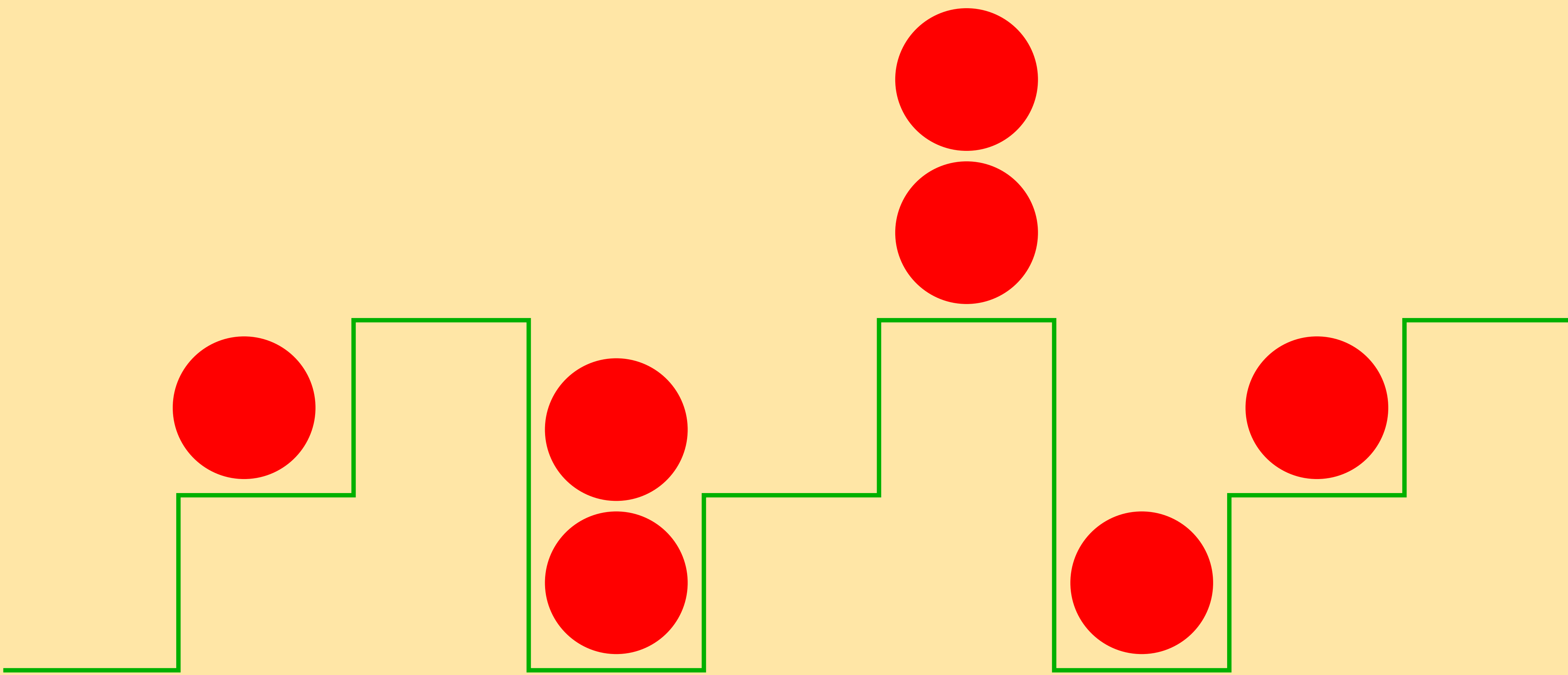

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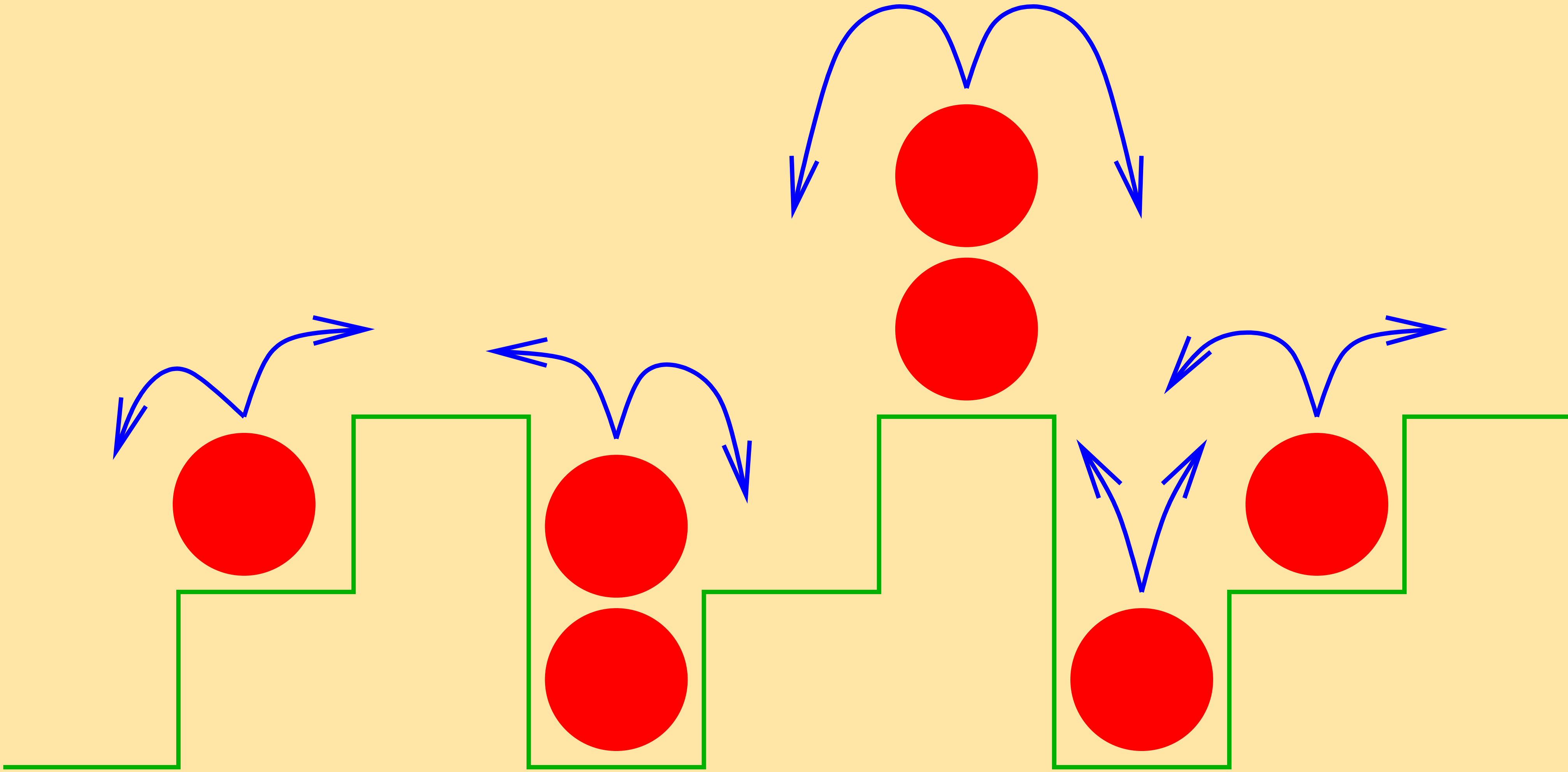

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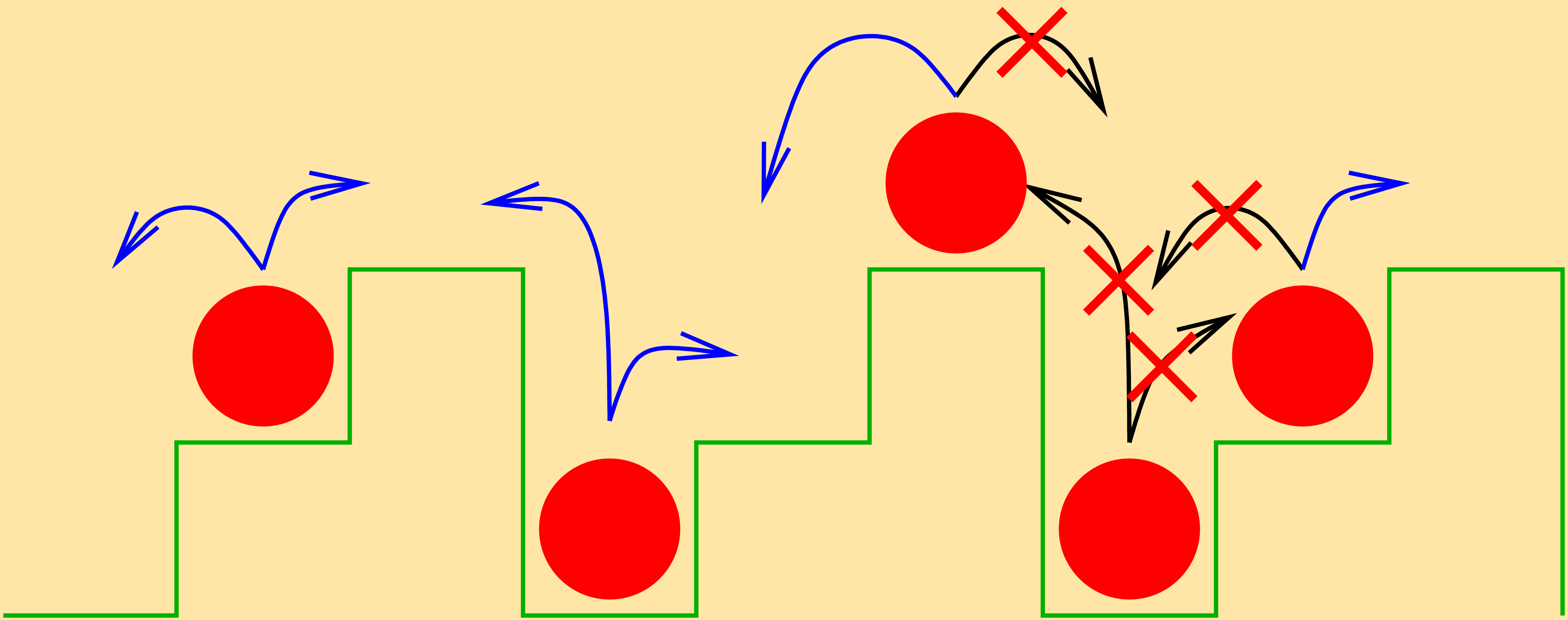
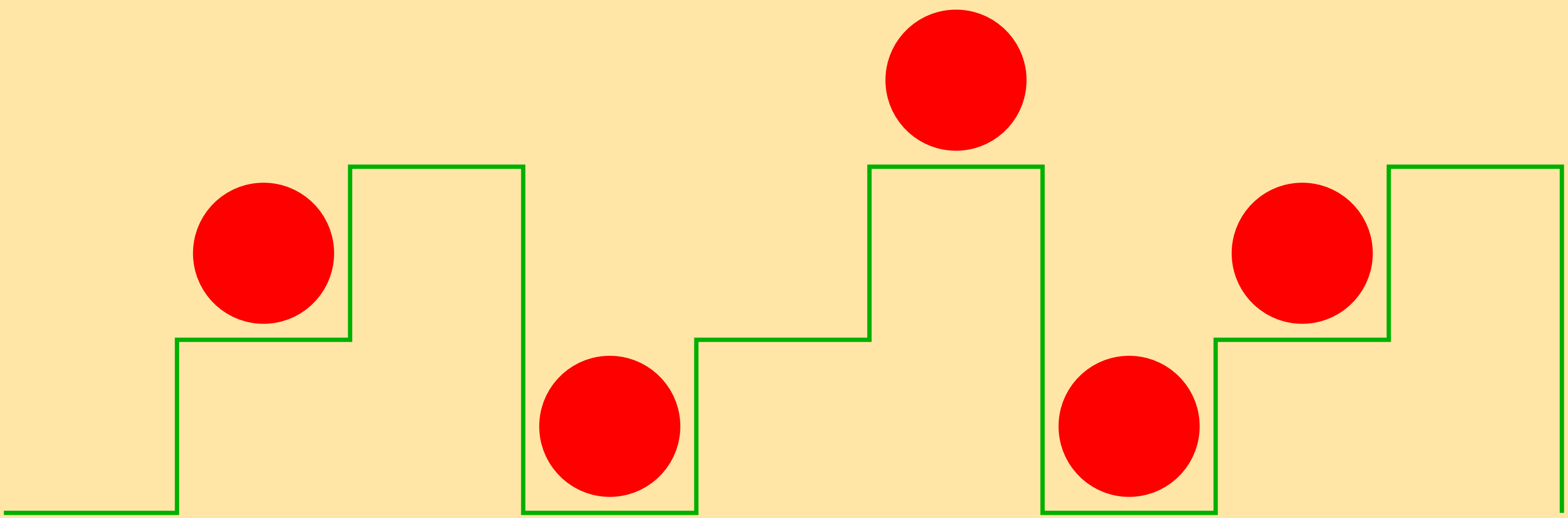


Periodic homogeneous force



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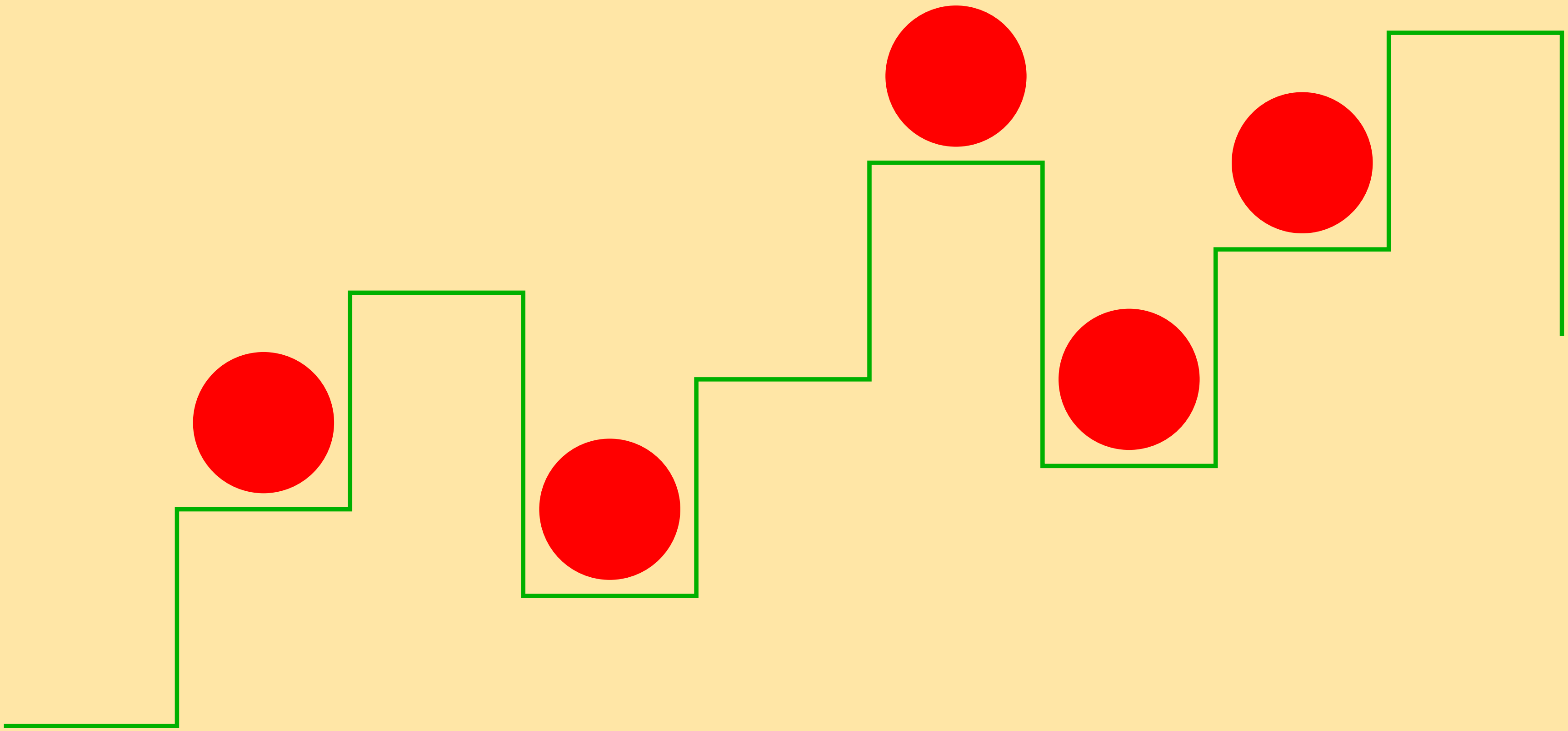

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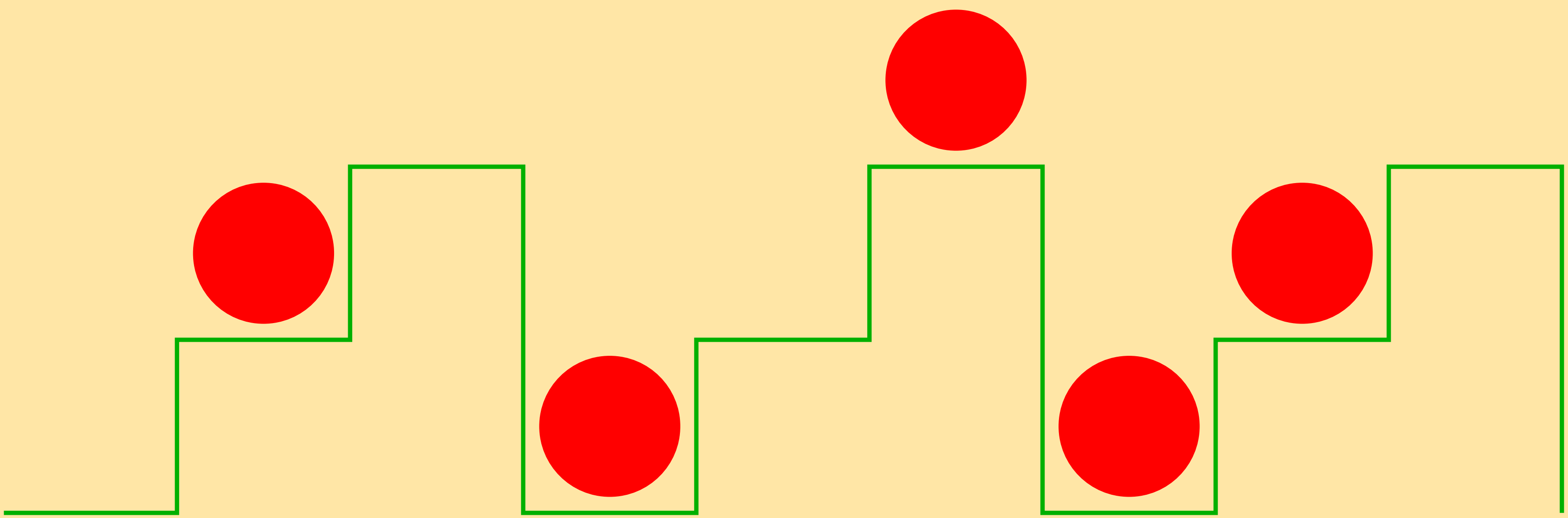

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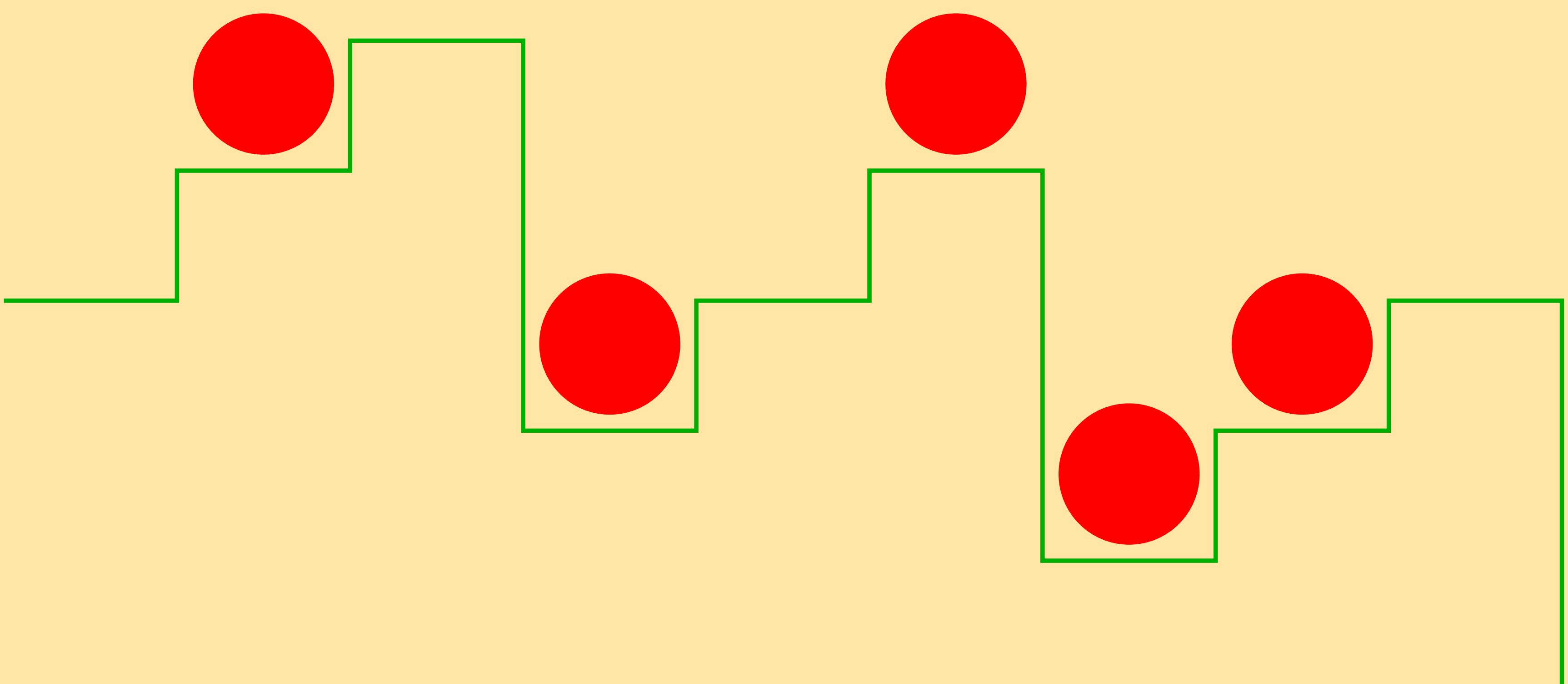

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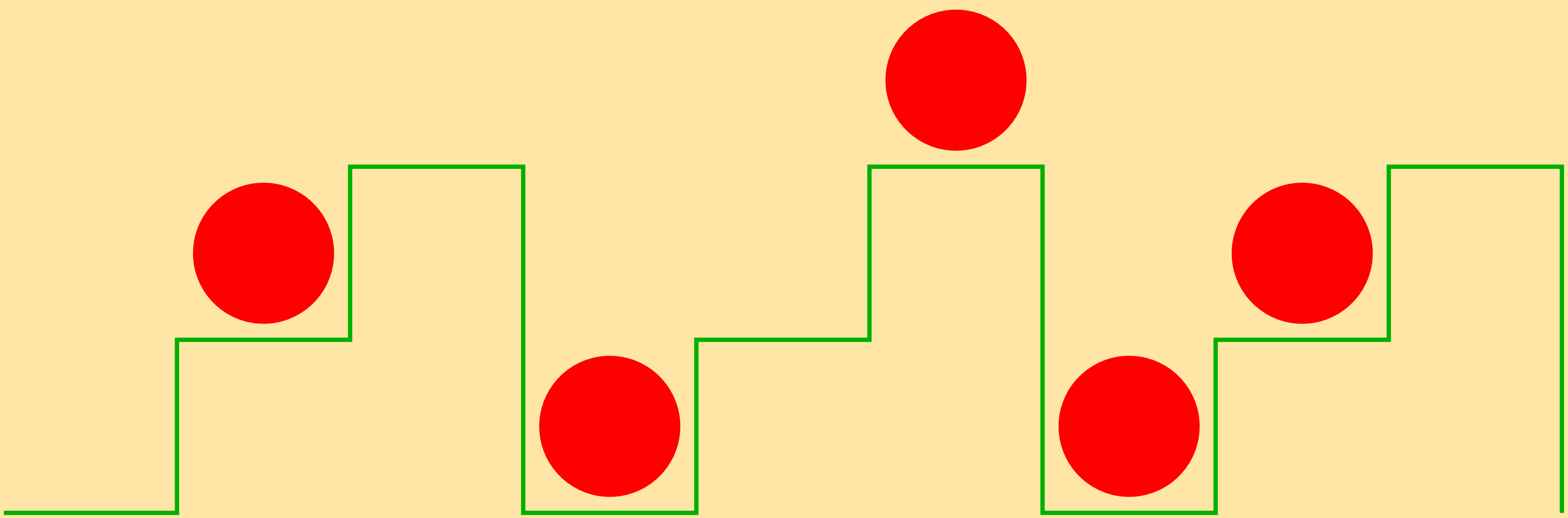

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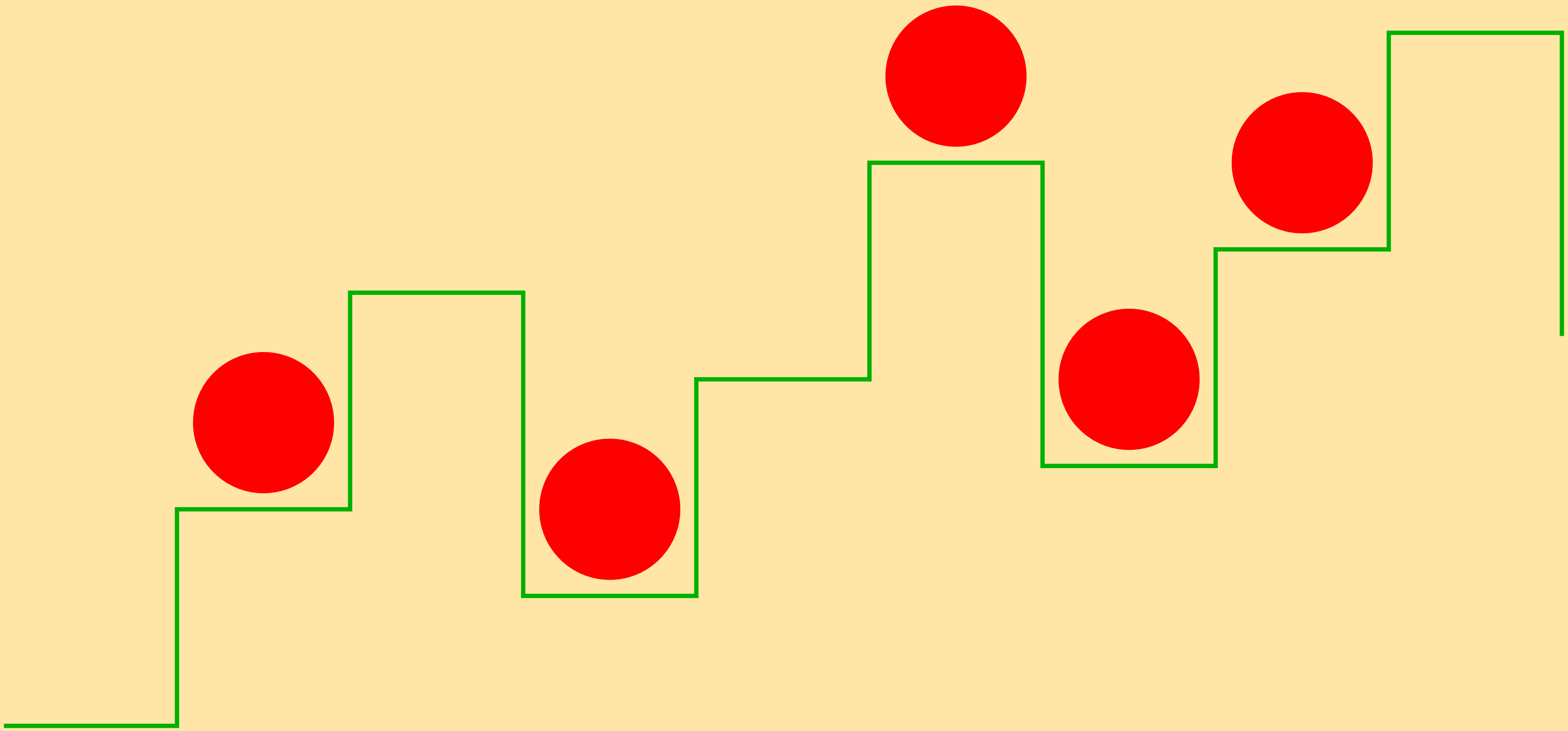
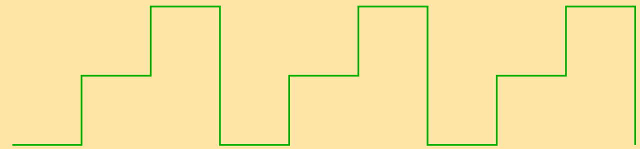


Algorithm

N particles on stripe of lenth L. Periodic b.c.

Potential $V(x) = x \pmod 3$, 

Average density $\alpha = N/L$

Temperature T, interaction strength $g \in [0, 1]$.

Number of particles on site x: $n(x) = \sum_{i=1}^{N} \delta(x_i - x)$

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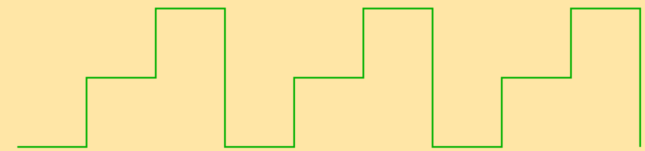
Potential $V(x) = x \pmod 3$, 

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- Attempted move from x to $x_{\text{new}} = x \pm 1$

$$\Delta E = V(x_{\text{new}}) - V(x) + (x_{\text{new}} - x)\,[F_{\text{load}} + F_0 \cos \omega t]$$

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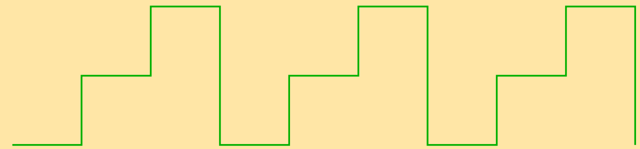
Potential $V(x) = x \pmod 3$, 
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- Probability to accept move

$$\max(1, \exp(-\Delta E/T))$$

Metropolis

Algorithm

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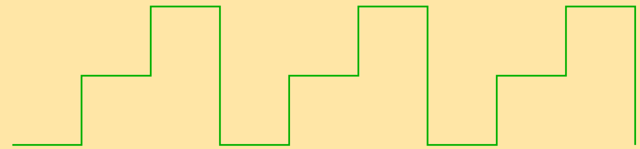
Potential $V(x) = x \pmod 3$, 

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Metropolis — Interaction - exclusion

Algorithm

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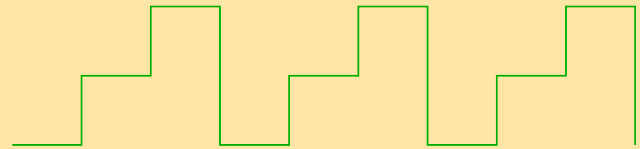
Potential $V(x) = x \pmod 3$, 
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Metropolis — Interaction - exclusion

Launch simulation

Measured quantities

Current

$$J = \langle \sum_{i=1}^{N} x_i(t+1) - x_i(t) \rangle$$

Measured quantities

Current

$$J = \langle \sum_{i=1}^{N} x_i(t+1) - x_i(t) \rangle$$

Efficiency

$$\eta = \frac{\Delta W}{\Delta U}$$

Measured quantities

Current

$$J = \langle \sum_{i=1}^{N} x_i(t+1) - x_i(t) \rangle$$

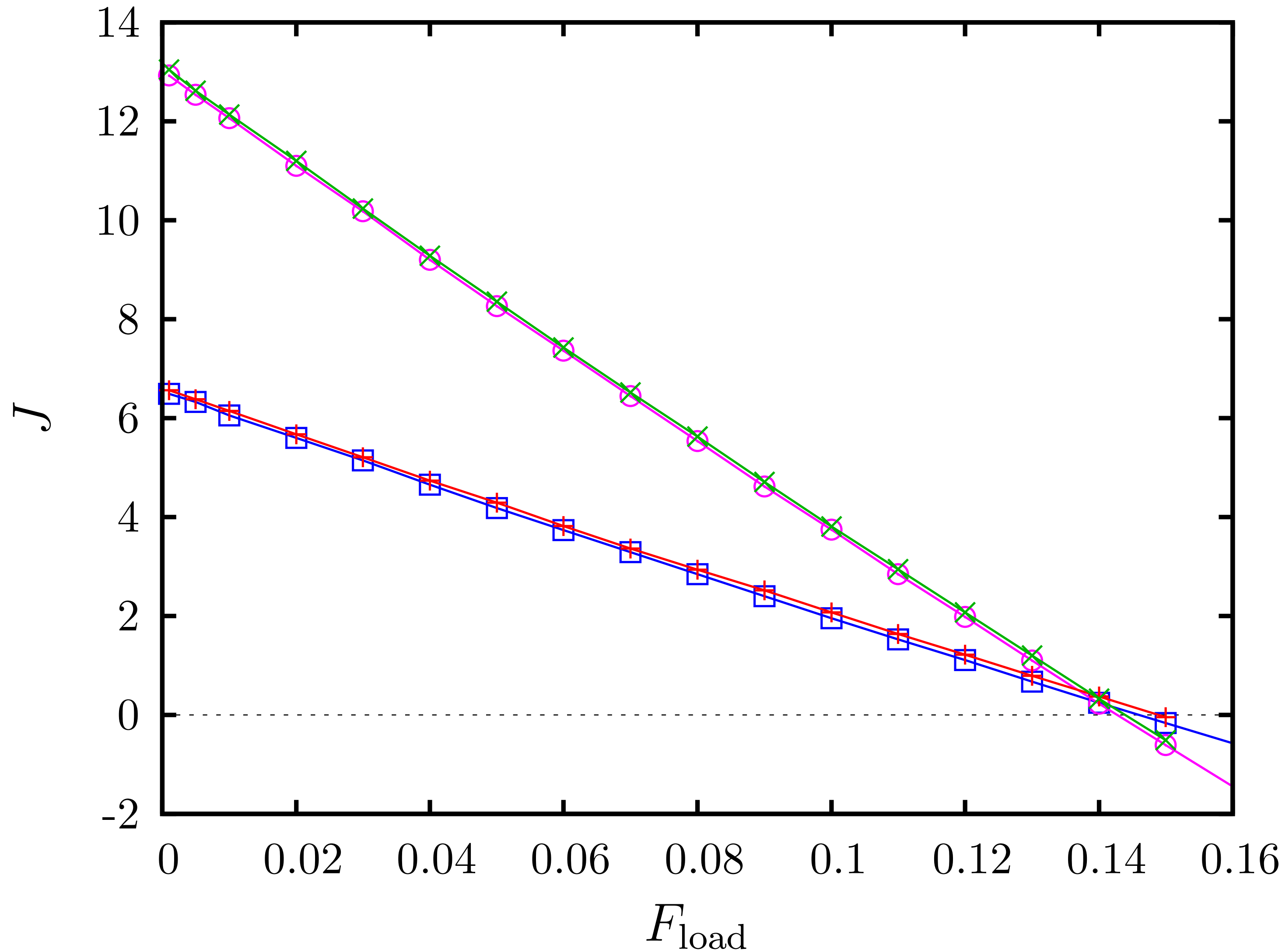
Efficiency

$$\eta = \frac{\Delta W}{\Delta U}$$

$$\Delta W = -\langle \sum_{i=1}^{N} (x_i(t+1) - x_i(t)) F_{\mathrm{load}} \rangle$$ useful work

$$\Delta U = \langle \sum_{i=1}^{N} (x_i(t+1) - x_i(t)) F_0 \cos \omega t \rangle$$ input from external source

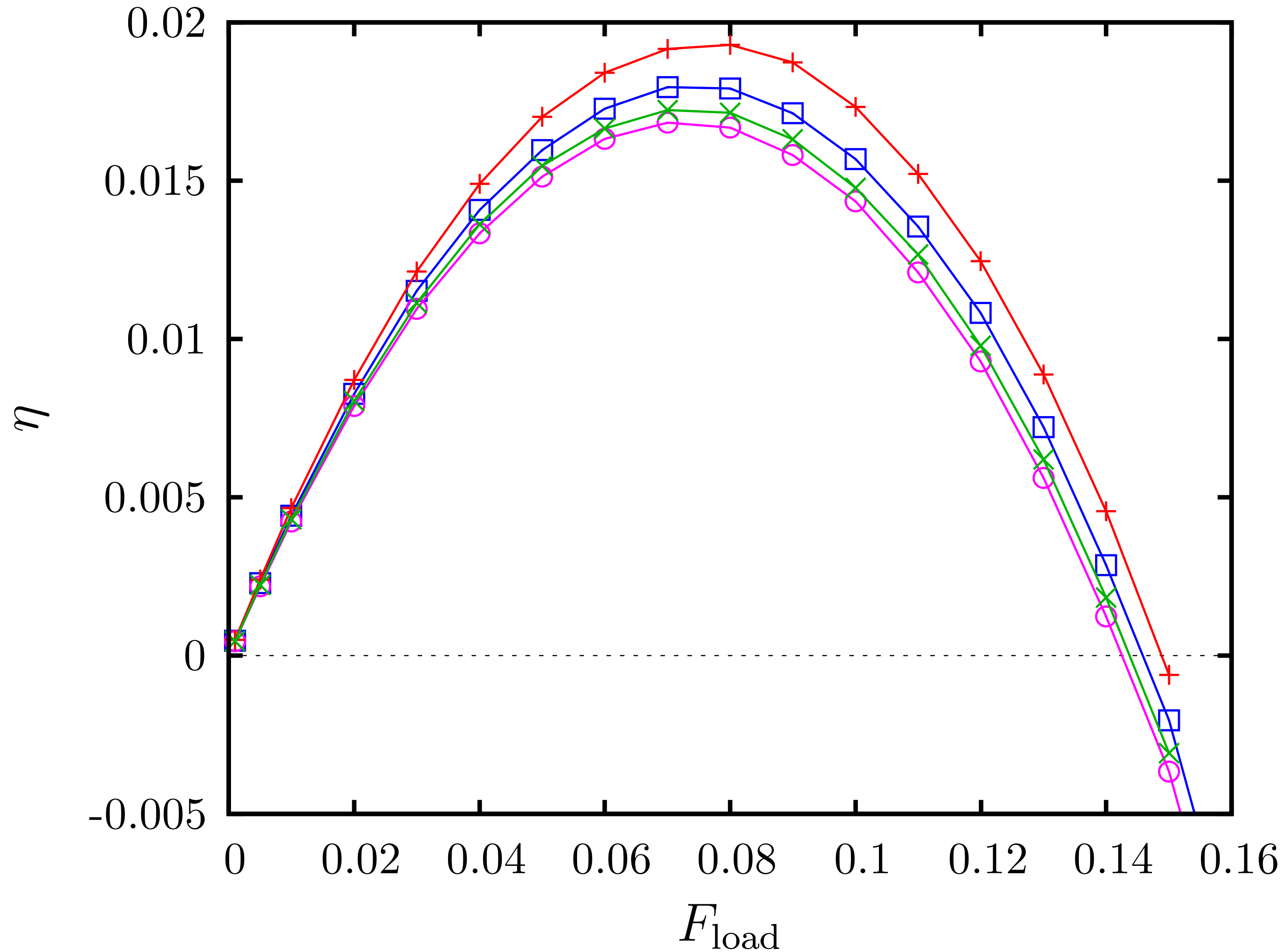
Results: Current depending on the external load.



Parameters: $L = 1000$, $\alpha = 0.5$, $T = 150$, $F_0 = 0.9$.

Further: + $\omega = 0.01$, $g = 1$; × $\omega = 0.01$, $g = 0$; ⊡ $\omega = 0.1$, $g = 1$; ⊙ $\omega = 0.1$, $g = 0$.

Efficiency depending on the external load.



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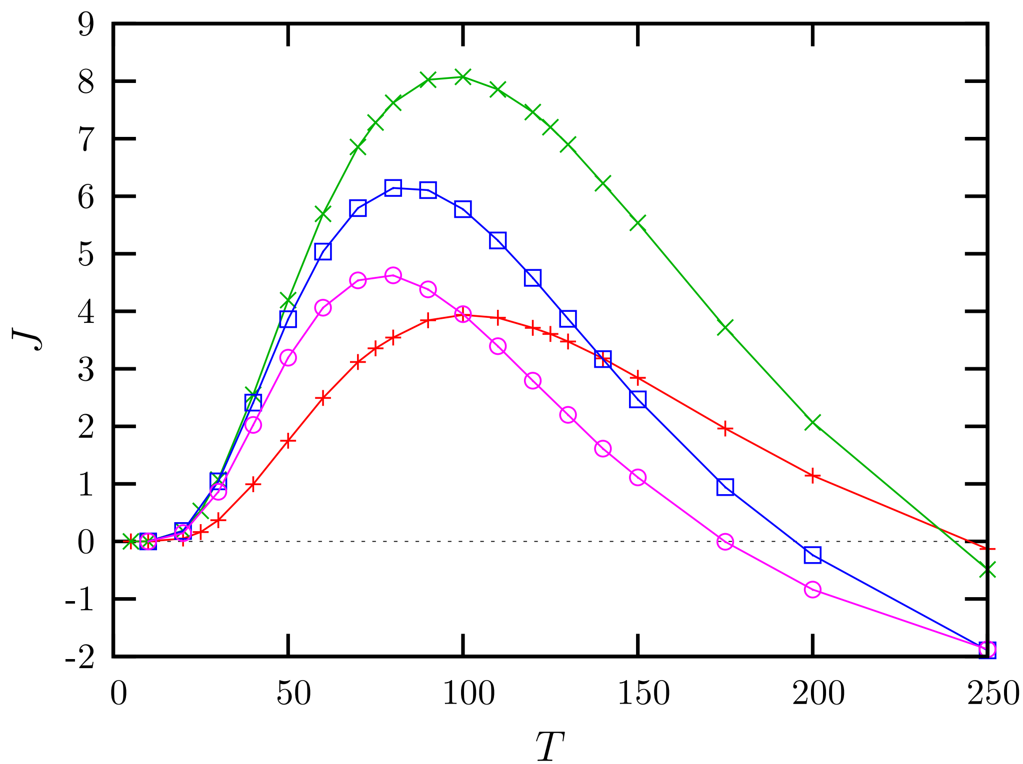
Temperature dependence

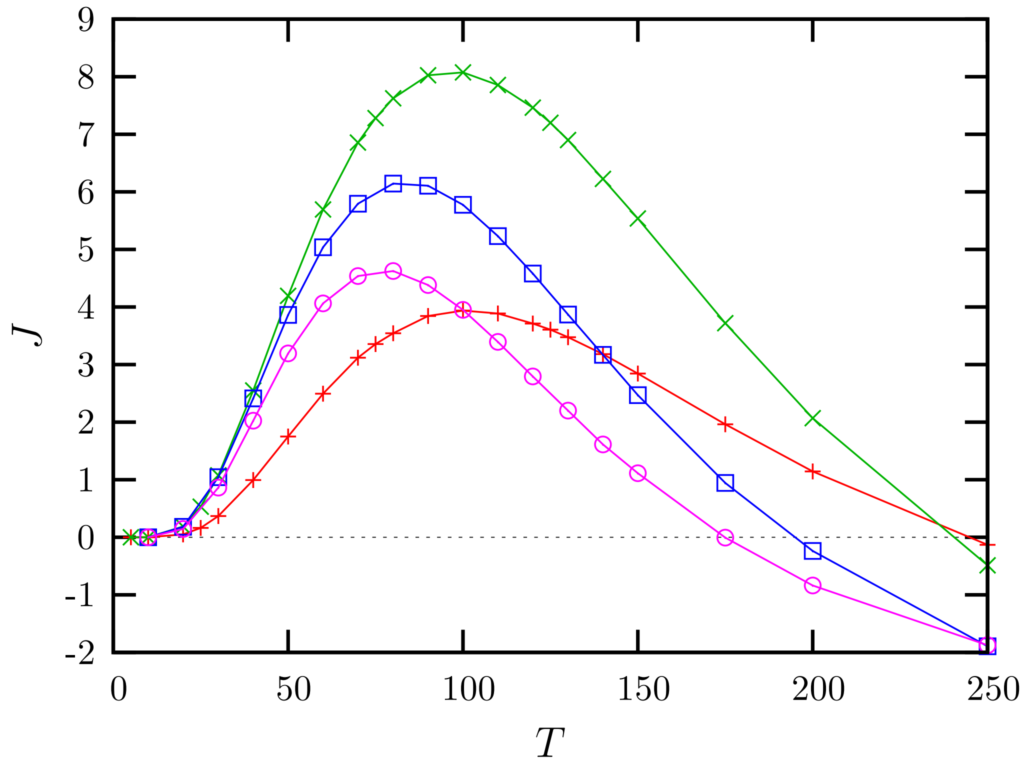


Current

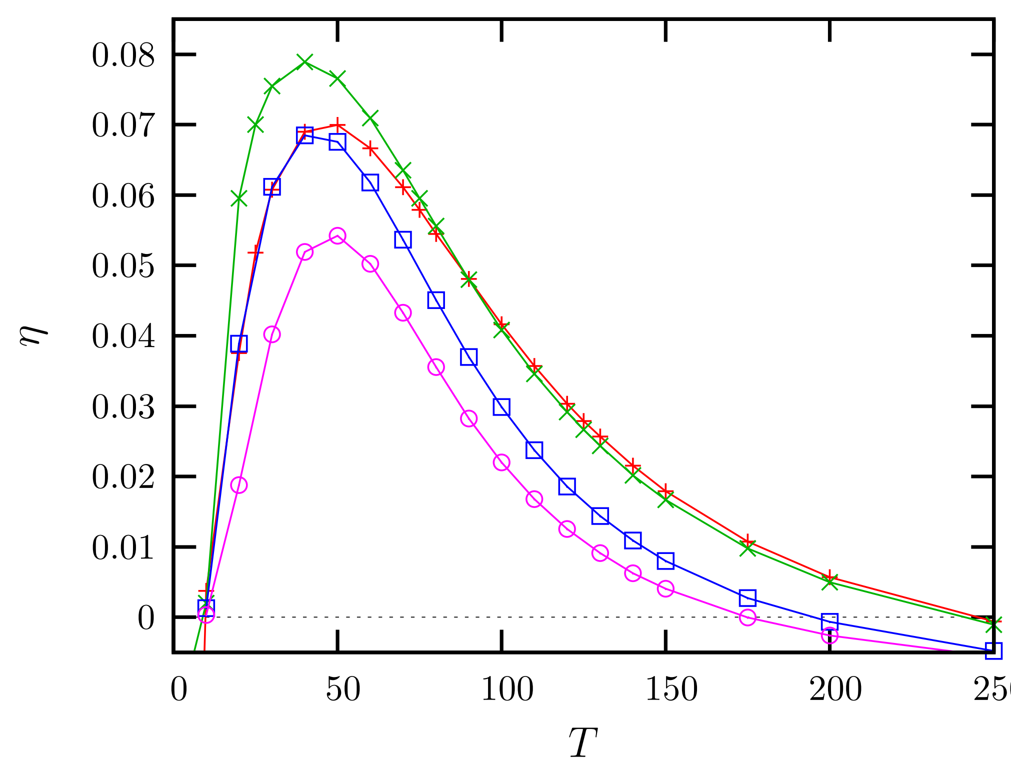
Parameters: $L = 1000$, $\alpha = 0.5$, $F_0 = 0.9$ $\omega = 0.1$, $F_{\text{load}} = 0.08$.

Further: + $g = 1$; × $g = 0$; ⊡ $g = 0.5$; ⊙ $g = 0.75$.

Temperature dependence



Current

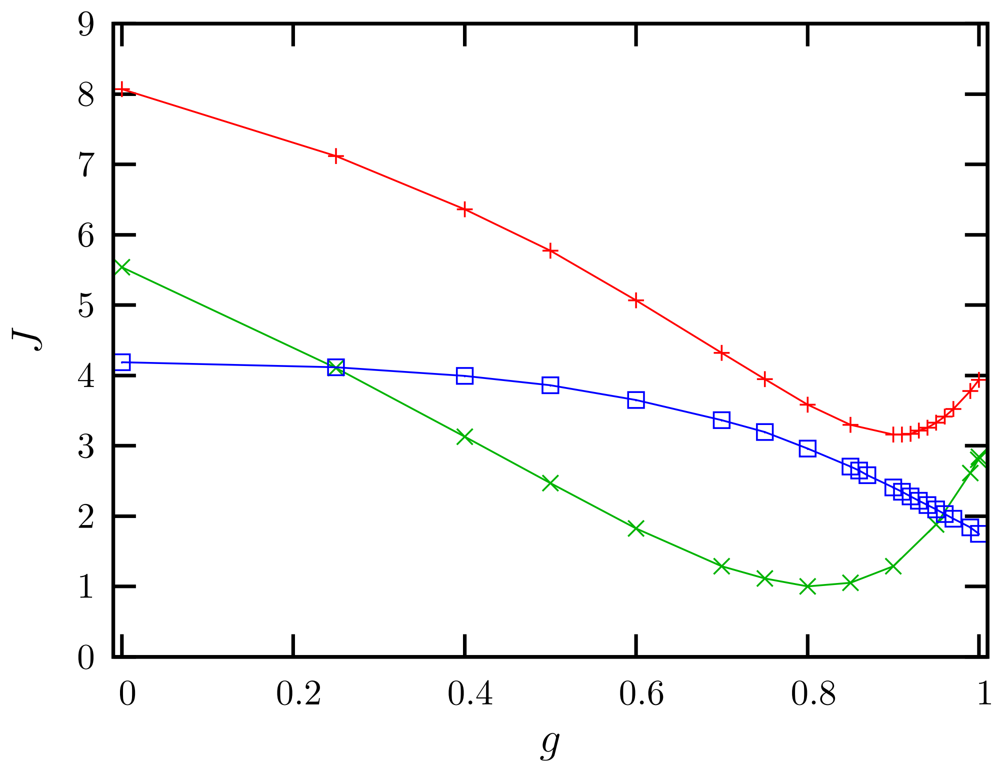


Efficiency

Parameters: $L = 1000$, $\alpha = 0.5$, $F_0 = 0.9$ $\omega = 0.1$, $F_{\mathrm{load}} = 0.08$.

Further: + $g = 1$; × $g = 0$; ⊡ $g = 0.5$; ⊙ $g = 0.75$.

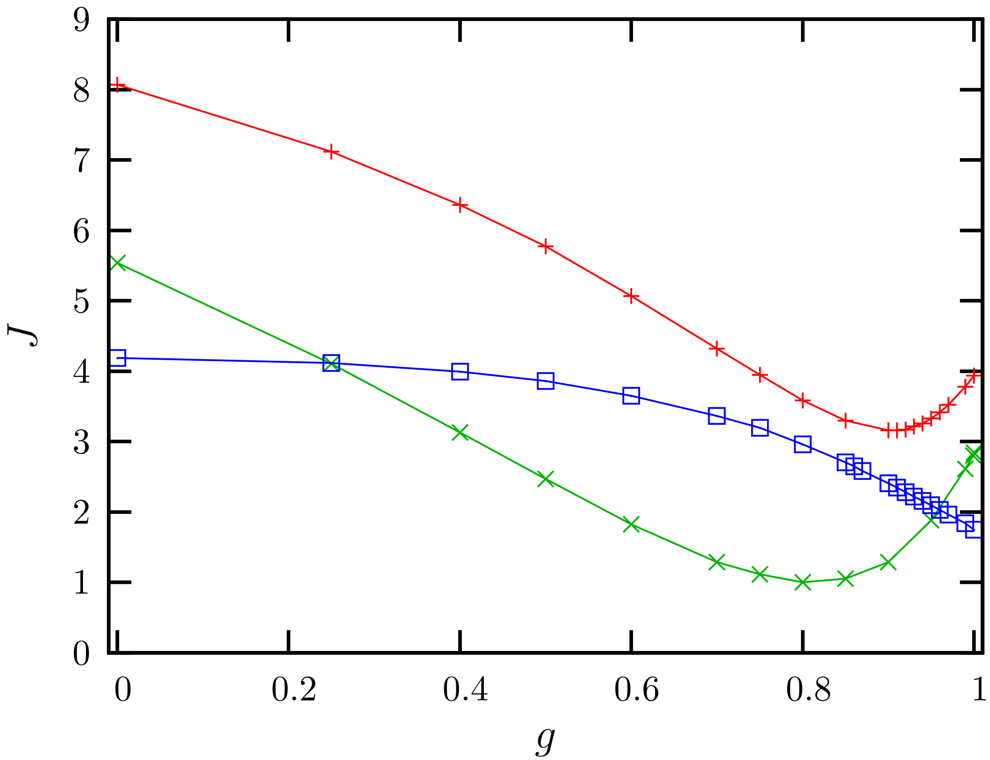
Dependence on interaction



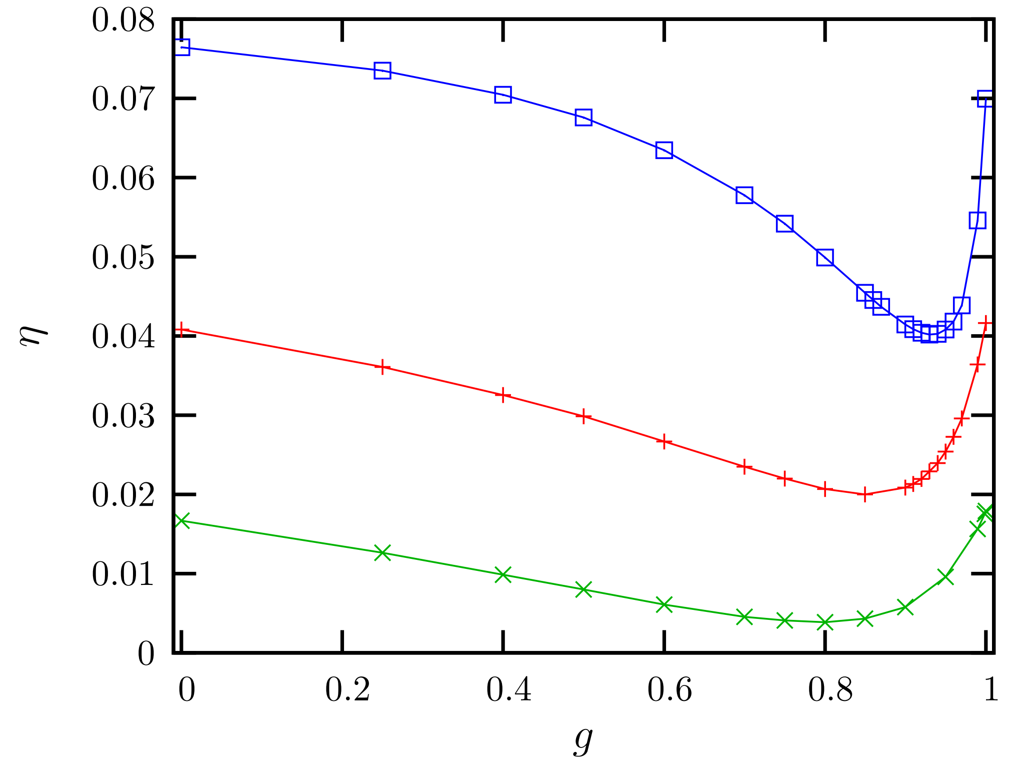
Current

Parameters: $L = 1000$, $\alpha = 0.5$, $F_0 = 0.9$ $\omega = 0.1$, $F_{\mathrm{load}} = 0.08$.
Further: $\times$ $T = 150$; $+$ $T = 100$; $\boxdot$ $T = 50$;

Dependence on interaction



Current

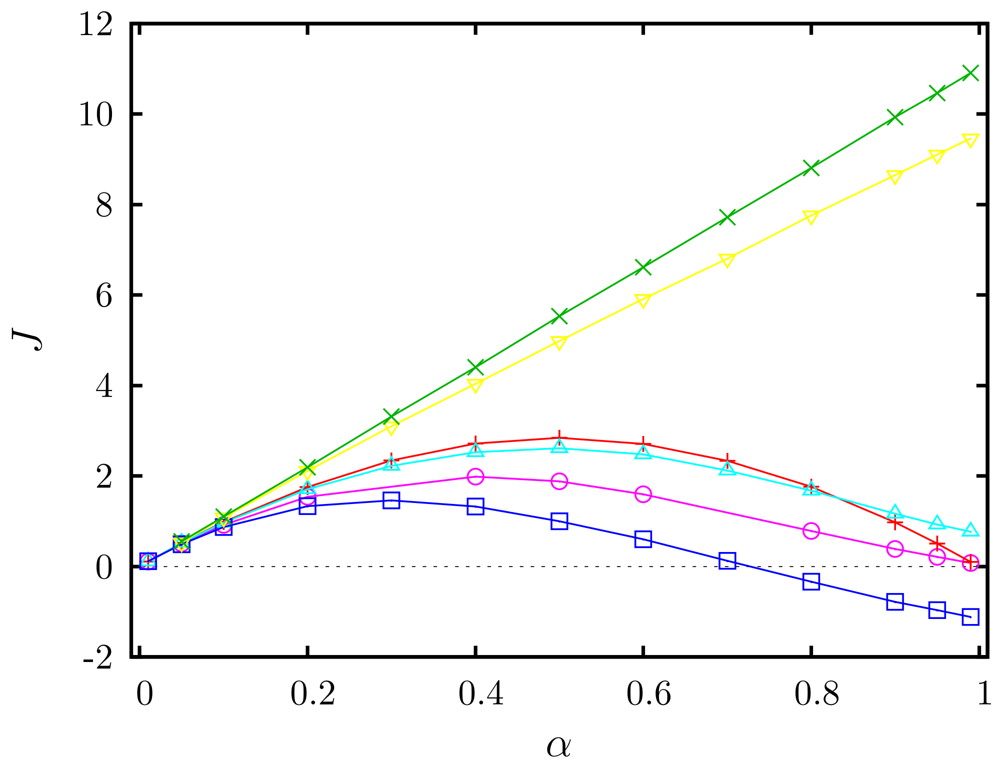


Efficiency

Parameters: $L = 1000$, $\alpha = 0.5$, $F_0 = 0.9$ $\omega = 0.1$, $F_{\mathrm{load}} = 0.08$.

Further: × $T = 150$; + $T = 100$; ⊡ $T = 50$;

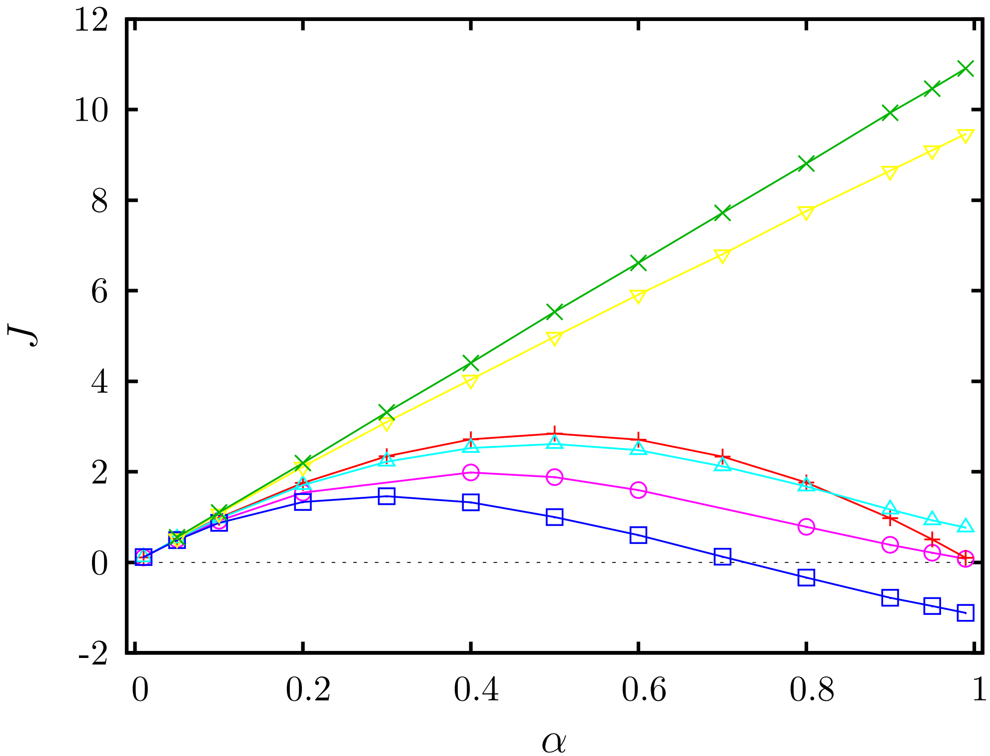
Dependence on density



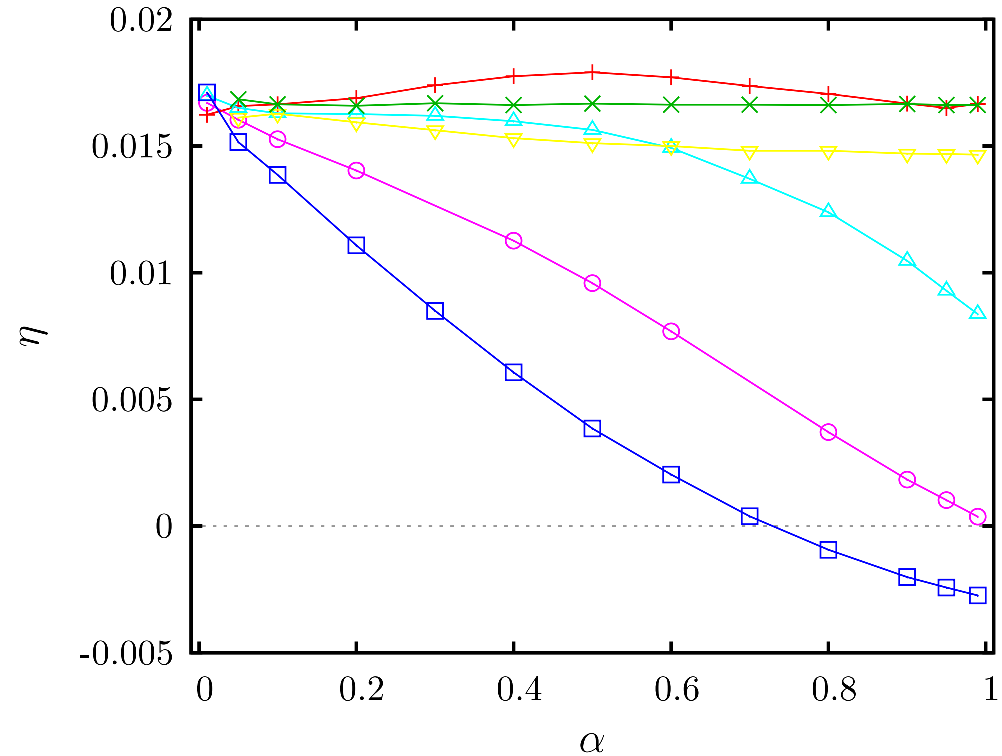
Current

Parameters: $L = 1000$, $F_0 = 0.9$ $\omega = 0.1$, $T = 150$, $F_{\text{load}} = 0.08$.
Further: × $g = 0$; ▽ $g = 0.1$; ⊡ $g = 0.8$; ⊙ $g = 0.95$; △ $g = 0.99$;
+ $g = 1$;

Dependence on density



Current



Efficiency

Parameters: $L = 1000$, $F_0 = 0.9$ $\omega = 0.1$, $T = 150$, $F_{\mathrm{load}} = 0.08$.

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Outlook

- True 2-dimensional simulation of a realistic structure.