Limits of systems of N particles

Claude Bardos
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This is a report on a joint program with many coworkers including Ducomet, Erdos, Golse, Gottlieb, Mauser and Yau.

The Boltzmann equation and then Euler and Navier Stokes equations can be derived from the classical Hamiltonian mechanic with the introduction of the finite BBJKY Hierarchy and then the infinite Boltzmann Hierarchy.

I have the feeling that these tools may have a much broader range of applications. Therefore I would like to consider the derivation from quantum systems in the weak so called weak coupling limit (this avoids the appearance of irreversibility).

The mathematical proofs rely on several tools : The space of trace class operators, or of compressed trace class operators to handle the physical case of the Coulomb potential with a version of the Hardy inequality due to Leray.

Several type of regimes are considered corresponding to factorized initial data (Hartree approximation) or fermionic approximation top be consistent with the Pauli exclusion principle.

Multi-dimensional stability of planar dynamical phase boundaries

Sylvie Benzoni-Gavage, Lyon

This talk is concerned with the linearized stability of sharp interfaces propagating in compressible fluids like van der Waals gases. The equations governing a flow involving a single interface are the Euler equations on either side, supplemented with jump conditions across the interface. It is not difficult to find planar interfaces, propagating at a constant speed and involving uniform states of the fluid on either side, which solve those equations. The issue here is to solve the Cauchy problem for initial data close to a given planar interface, by means of a perturbed interface and modified (nonuniform) states on either side. This is a free boundary value problem.

The analogous problem for classical shock waves in gases received considerable attention in the last decades. By means of so-called normal modes analysis, stability criteria were derived by D'yakov, Erpenbeck and others in the 1950's. In the 1980's, Majda pointed out some analogy with hyperbolic Initial Boundary Value problems, and interpreted stability criteria in terms of Kreiss-Lopatinksii determinants. His major result showed that there exist a local-in-time solution close to any reference shock front that satisfies the *uniform Kreiss-Lopatinksii condition.* His work was later revisited by Métivier and collaborators, who significantly improved both the result and the proof by means of para-differential calculus.

There are two main differences between classical shock waves and propagating phase boundaries. The first one is that the most interesting propagating phase boundaries are subsonic, and thus *undercompressive.* This feature requires an additional jump condition. One of the basic issues in the modeling of phase boundaries is to determine that additional jump condition. For isothermal phase boundaries, it is usually written as a so-called *kinetic relation*, which can be derived from the viscosity-capillarity approach of Slemrod/Truskinovsky for instance. The other special feature of phase boundaries is superficial tension. In several space dimensions, superficial tension together with curvature of the interface are known to play a role in the momentum jump condition.

The talk will review results from normal modes analysis of dynamical phase boundaries, starting from the ideal case of reversible (i.e. nondissipative) phase transitions and neglecting superficial tension at first. The influence of, on the one hand, dissipation and, on the other hand, superficial tension/curvature will then be discussed in some details. Special attention will be paid to cases when the uniform Kreiss-Lopatinskii condition fails. Indeed, in those cases, the derivation of energy estimates is more delicate and leads to a loss of derivatives. This question will be addressed carefully. Part of the results reported here are a joint work with Jean-François Coulombel (ENS Lyon) and Heinrich Freistühler (Max Planck Institute, Leipzig).

A posteriori analysis of a penalty finite element method for the Navier–Stokes problem

C. Bernardi, Paris

with V. Girault and F. Hecht

Laboratoire Jacques-Louis Lions, C.N.R.S. & Université Pierre et Marie Curie

We consider a discretization of the Stokes and Navier–Stokes equations obtained by using the penalty technique and applying the finite element method to the penalized problem. The aim of this work is to exhibit two types of error indicators, the first ones linked to the penalty parameters and the second ones linked to the finite elements, and to prove optimal a posteriori estimates and upper bounds for the indicators: this means that, up to some terms only involving the data, the error is equivalent to the Hilbertian sum of the indicators. This allows to optimize the choice of the penalty parameters when working with adaptive meshes, and the efficiency of the algorithm is illustrated by some numerical simulations.

Numerical models for atmospheric boundary layer flows problems

Tomáš Bodnár, Prague
with K. Kozel
Department of Technical Mathematics, Faculty of Mechanical Engineering, Czech Technical University of Prague

The work presents mathematical and numerical models for investigation of the atmospheric boundary layer (ABL) flows over 3D complex terrain. Different models based on Navier-Stokes equations and its simplifications will be presented. Together with these models numerical methods will be proposed for the solution. The results of numerical tests of the explicit FV and semiimplicit FD schemes will be presented. Finally some examples of practical problems solution will be given.

For all methods also additional transport equation for concentration has been considered in order to study the dispersion of passive pollutants over the complex 3D topography. The simple algebraic turbulence closure is used with the Reynolds-averaged models.

Existence of viscous compressible barotropic flow in a moving domain with free upper surface, via Galerkin method

Bum Ja Jin, Ferrara

Adaptive refinement of the finite element mesh versus refinement adjusted to the singularity in 2D fluid flow problem

Pavel Burda[1/]

with J. Novotný[2/], M. Jelínek[1/], B. Sousedík[1/]
[1/] Czech Technical University in Prague, Department of Mathematics
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In the paper we first discuss the problem of geometrical singularities caused by the non-convex internal angles of the fluid domain. We consider the incompressible fluid described by the Navier-Stokes equations. We deal with the stationary problem solved by means of the finite element method. In the paper we discuss two ways of refinement of the finite element mesh. First one is the adaptive refinement based on a posteriori error estimates. Second one is the refinement that utilizes the asymptotic behavior of the singularity near a corner. In this case we have suggested and imlemented an algorithm of refinement which is adjusted to the singularity. Both approaches are applied to the steady state incompressible fluid problem in 2D domain with forward and backward steps. Numerical results are given.

Loss of smoothness for large solutions to the 3D-Navier-Stokes equations and applications to a gravitational model

Marco Cannone, Université de Marne la Vallée

with P. Biler and G. Karch

This lecture will be based on the following result. There exists a divergence free function v_0 such that for any initial data $u_0 = \epsilon v_0$ with ϵ sufficiently small, then the Navier-Stokes equations admit a unique smooth solution. On the other hand if $\epsilon = 1$, then the corresponding solution is singular at zero for all time.

Numerical solution of compressible Navier–Stokes equations with the aid of the discontinuous Galerkin method

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We deal with the numerical simulation of viscous compressible flows. Our former computational methods were based on a general class of flux vector splitting schemes. They combine the finite volume (FV) method with finite element (FE) method. The disadvantage was the low order of accuracy and the necessity to construct two grids which is rather complicated particularly in 3D.

Another approach representing a simultaneous generalization of the FV as well as FE methods is the discontinuous Galerkin finite element (DG FE) method. It is based on the idea to approximate the solution by piecewise polynomial functions over a FE mesh without any requirement on interelement continuity.

In presented numerical method, the stabilization is achieved by adding some stabilization terms vanishing for regular solution. In order to avoid spurious oscillations near discontinuities or steep gradients, a suitable limitation based on jump indicator is applied. Furthermore we use superparametric finite elements for triangles adjacent to a nonpolygonal boundary of the computational domain in order to avoid unphysical behaviour of the numerical solution. We present numerical examples of a scalar convection diffusion equation and the application the method for the numerical solution of the system of the compressible Navier–Stokes equations.

Asymptotic behaviour for viscous compressible heat-conducting media with nonmonotone equation of state

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We consider the system of quasilinear equations for 1d-motion of viscous compressible heat-conducting media. The state function has the form $p(\eta,\theta) = p_0(\eta) + p_1(\eta)\theta$, with general nonmonotone p_0 and p_1, which allows us to treat both nuclear fluids and thermoviscoelastic solids (for fluids, p, η, and θ are the pressure, specific volume, and temperature). For an initial-boundary value problem with large data, we establish stabilization as $t \to \infty$: pointwise and in L^q for η, in L^q for v (the velocity), for any $q \in [2,\infty)$, and in L^2 for θ.

On the existence of solutions to the Navier-Stokes equations of a two-dimensional compressible flow

Radek Erban, Prague

We consider the Navier-Stokes equations for compressible, barotropic flow in two space dimensions, i.e.

$$\frac{\partial \varrho}{\partial t} + \mathrm{div}\ (\varrho \vec{u}) = 0,$$

$$\frac{\partial \varrho \vec{u}}{\partial t} + \mathrm{div}\ (\varrho \vec{u} \otimes \vec{u}) + \bigtriangledown p(\varrho) = \mu \Delta \vec{u} + (\lambda + \mu) \bigtriangledown (\mathrm{div}\ \vec{u}) + \varrho \vec{f},$$

where the density $\varrho = \varrho(t,x)$ and the velocity $\vec{u} = [u^1(t,x), u^2(t,x)]$ are functions of the time $t \in (0,T)$ and the spatial coordinate $x \in \Omega$ where $\Omega \subset R^2$ is a bounded regular domain, $\vec{f} = [f^1(t,x), f^2(t,x)]$ is a given external force, μ and λ are viscosity coefficients and $p(\varrho)$ is the pressure. We consider the pressure satisfying $p(\varrho) = a\varrho \log^d(\varrho)$ for large ϱ. Here a and d are positive constants and $d > 1$. We introduce useful facts from the theory of Orlicz spaces. Then we prove the existence of globally defined finite energy weak solutions.

The compressible Navier-Stokes equations in moving domains

Eduard Feireisl, Prague

Analysis of the Discontinuous Galerkin Finite Element Method for Nonlinear Convection-Diffusion Problems

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The subject matter of the paper is the derivation and analysis of the DGFEM for nonlinear convection-diffusion problems. The goal is to develop a sufficiently accurate and robust method for the solution of compressible flow. The method is theoretically analyzed on a model nonlinear scalar conservation law equation with a diffusion term. Namely, error estimates are investigated for two versions of the DGFEM combined with a finite volume approach to the discretization of convection terms. The attention is also paid to the choice of a new limiting of the order of the method in the vicinity of discontinuities in order to avoid spurious oscillations.

Unified analysis of adaptive finite volume methods for hyperbolic conservation laws[1]

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The cell-based three-dimensional upwinding schemes have in recent years been very successful in generating good numerical solutions to problems governed by systems of conservation laws. On the adaptivity side, much effort has been put into mesh adaptation procedures based on various types of mesh refinement/coarsening indicators including a grid alignment. We present the unified analysis of the use of shock indicator, residual error indicator, superconvergence error indicator and interpolation error indicator which were developed for transonic compressible flow.

Numerical Solution of Inviscid and Viscous Transonic Flows Through Turbine Cascades

Jaroslav Fořt, Prague
with J. Dobeš, J. Fürst, J. Halama, K. Kozel,
Department of Technical Mathematics, Fac. of Mechanical Eng., TU Prague, Czech Republic

This work deals with numerical solution of transonic flow through radial and axial turbine cascades described by the system of Euler or Navier-Stokes equations. Radial geometry of cascade involves a specific problem with proper formulation and numerical approximation of boundary conditions and some phenomena, like influence of body forces, can be studied only mumerically. Numerical results for design as well as off-design conditions achieved by several central and upwind type schemes are presented, some properties of proposed methods are tested and validated. The influence of grid quality is dicussed for inviscid and viscous flow models.

A Stokes-Like System for Mixtures II

Jens Frehse, Bonn

We consider the system presented in the first part of the contribution given by S. Goj (joint work with her and J. Málek) and discuss the problems which arise from more general boundary conditions, interaction terms and pressure laws.

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Applications of WENO schemes on unstructured meshes for internal aerodynamics

Jiří Fürst, Prague

Two-grid schemes for Navier-Stokes equations

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France

Two-grid schemes have been developed over the past ten years for solving efficiently nonlinear partial differential equations (mostly time-independent) and sometimes also some linear problems. For steady nonlinear problems, the idea of the scheme is:

- Step One - Solve the fully nonlinear equations on a coarse grid.

- Step Two - Linearize the problem on a fine grid by inserting the value computed at Step One into the nonlinear term.

Depending on the nature of the differential operator and the nonlinearity, and on the way in which it is linearized, it is possible that the result of Step Two has the same accuracy as if the fully nonlinear problem had been solved directly on the fine grid. When this is the case, this algorithm has a substantial reduction in complexity.

Such two-grid schemes have been applied to a variety of steady semi-linear equations by J.Xu (cf.[7], [8]), to steady nonlinear saddle point problems with nonlinear constaints by A.Niemisto in his thesis [6], and to steady Navier-Stokes equations by W.Layton (cf.[3]), by W.Layton and W.Lenferink (cf.[4], [5]), and later on by V.Girault and J.-L.Lions with particular emphasis on the three-dimensional problem on domains with corners (cf.[1]).

There has been much less studies on time-dependent problems. V.Girault and J.-L.Lions have investigated semi-discrete two-grid schemes for the time-dependent Navier-Stokes equations in three dimensions (cf.[2]). Both for the time-dependent and steady problem, they prove that, if the fine mesh-size is approximately the square of the coarse mesh-size, for currently used finite-elements of low degree on regular fmilies of triangulations, the accuracy of the two-grid scheme is the same as if the nonlinear problem is solved by the same finite elements on the fine grid. In contrast to the steady-state Navier-Stokes problem, extending this analysis of the time-dependent problem to polyhedral domains is still an open problem.

Here is a list of some relevant references on the subject. The thesis of A.Niemisto has an extensive list of references up to 1995.

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A Stokes-Like System for Mixtures I

Sonja Goj, Bonn

A result of a joint work with J. Frehse and J. Málek is presented.
We consider a stationary system of compressible Navier-Stokes like equations which model mixtures of fluids in the whole space R^3. The system consists of N balance equations for the momentum and N mass conservation equations, which are coupled via the viscosity terms, the pressure law and interaction terms.
The convective terms are neglected. The densities ρ_i of the species and their velocity fields $u^{(i)}$ are prescribed at infinity: $\rho_i|_\infty = \rho_{i\infty} > 0$, $u^{(i)}|_\infty = 0$. The authors obtain the existence of weak solutions $u \in H_0^1, \rho \in \rho_\infty + L^2$ with the additional regularity property $\rho \in L^q_{loc}, \nabla u \in L^q_{loc}$ for all $q < \infty$.

Nonlinear stability of Benard problem

Giovanna Guidoboni, Ferrara

A curious phenomenon in a model problem, suggestive of the hydrodynamic inertial range and smallest scale of motion

John Heywood, Vancouver

Efficient multi–level methods for incompressible flow problems

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We present efficient multi–level solvers for higher order finite element discretizations of incompressible flow problems. The basic idea is to apply the accurate higher order discretization on the highest level of the multi–level hierarchy only and low order discretizations on all other levels. In this way, an accurate solution can be obtained for whose computation the efficiency of multi–level solvers for low order discretizations is exploited.

First, we investigate the convergence of multi–level solvers for finite element discretizations of an abstract mixed problem. As a basic new feature, our approach allows to use different finite element discretizations on each level of the multi–grid hierarchy. Typically, one uses different finite element spaces, which may be both conforming and nonconforming, but the use of different bilinear forms is also allowed. Thus, several levels of the multi–level algorithm may correspond to one geometrical grid (i.e., to one triangulation). Particularly, this theory covers the above–mentioned type of solvers applied to higher order finite element discretizations of the Stokes equations.

After the theoretical investigations of solvers for saddle–point problems, we shall discuss the application of the above type of solvers to the efficient solution of the Navier–Stokes equations. Both for the Stokes and the Navier–Stokes equations we shall present numerical results demonstrating the advantages of the new approach.

This is a joint work with Volker John, Gunar Matthies and Lutz Tobiska from the University of Magdeburg.

Finite Volume Equilibrium Plasma Flow Modeling

David Kolman, Prague

Some properties of a model equation of the Oseen problem

Stanislav Kračmar, Prague and Patrick Penel, Toulon

We study qualitative properties of a model equation of the Oseen problem in three-dimensional exterior domains: the existence and the uniqueness of a solution in appropriate weighted Sobolev spaces. We use the variational approach. For this we derive a variant of a weighted Friedrichs-Poincaré inequality for exterior domains. Avoiding the use of asymptotic properties of the fundamental solution of the model equation we have a possibility to apply this approach in more general situations: equations with non-constant coefficient functions.

The linear ablative Rayleigh Taylor instability: asymptotic expansion of the growth rate in time for small mixing length

Olivier Lafitte, Université de Paris Nord and CEA Saclay

with C. Cherfils, B. Helffer, P.A. Raviart

We study the modeling of the Rayleigh-Taylor instability for the ablative linear RT instability. This means that we consider the classical Rayleigh equation in which the basic density of the fluid is variable. When this density tends exponentially to the limits ρ_h and ρ_l at $\pm\infty$, it has been shown (Cherfils Raviart Lafitte MFM1) that the growth rate of the instability admits an asymptotic converging expansion in terms of the length of the mixing region ϵ. (joint work with B. Helffer)

We extend this result to arbitrary profiles and we give the number of terms of the expansion that are valid according to conditions on the density profile, less restrictive than the exponential behavior.

We turn then to a realistic profile (as the one studied by Kull and Anisimov) and we give the method to identify the growth rate in this set-up, generalizing the result obtained by Cherfils-Raviart-lafitte when the perturbation was considered as incompressible. One of the main difficulties is that the density, in this realistic set-up, goes to 0 hence the method developed above for the Rayleigh equation does not succeed.

Numerical solution of 2D and 3D incompressible viscous flows

Petr Louda, Prague
with Karel Kozel, J. Příhoda

3D Navier-Stokes Equations with Initial Data Characterized by Uniformly Large Vorticity

Alex Mahalov, A.S.U., USA

with Basil Nicolaenko

We prove existence on infinite time intervals of regular solutions to the 3D Navier-Stokes Equations for fully three-dimensional initial data in R^3 characterized by uniformly large vorticity and infinite energy; smoothness assumptions for initial data are the same as in local existence theorems. The global existence is proven using techniques of fast singular oscillating limits and the Littlewood-Paley dyadic decomposition. Infinite time regularity is obtained by bootstrapping from global regularity of the nonlinear fully 3D limit equations and strong convergence theorems.

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Asymptotic Analysis, vol. 15, No. 2, p. 103-150 (1997).

Steady flows of fluids with pressure and shear dependent viscosity subjected to the no-slip boundary conditions

Josef Málek, Prague

Interaction of fluid and elastic body - application of FEM

Pavel Moses

Processes where fluid and elastic structure interfere play crucial role in engineering. Based on FEM solvers for fluid and structure we adopt Arbitrary Lagrangian-Eulerian method to deal with interaction in case of large deformations. Some numerical results for small Reynolds numbers are presented.

Steady fall of a body in viscous fluids

Šárka Nečasová, Prague

Several remarks to interior regularity of weak solutions to the Navier-Stokes equations

Jiří Neustupa, Prague and Patrick Penel, Toulon

We will discuss

1) certain anisotropic sufficient conditions for interior regularity of a suitable weak solution to the Navier-Stokes equations,

2) the role of eigenvalues and eigenvectors in the theory of regularity of weak solutions to the Navier-Stokes equations.

Mathematical modelling of a steady flow through a 2D profile cascade

Tomáš Neustupa, Prague

with M. Feistauer

We formulate the mathematical problem which models a steady flow of a viscous incompressible fluid through a 2D profile cascade. We prove the existence of a weak solution and we discuss the possibility of a numerical solution.

Long time regularity of 3D Euler and Navier-Stokes equations with uniformly large initial vorticity in cylindrical domains

Basile Nicolaenko (A.S.U.,USA)
with C. Bardos, F. Golse, A. Mahalov

We prove existence on arbitrary long time intervals of regular solutions to the 3D Euler Equations in bounded cylindrical domains with fully 3D initial data characterized by uniformly large vorticity weakly aligned with the cylinder's axis; smoothness assumptions for initial data are the same as in local existence theorems. The long-time existence theorems are proven using techniques of fast singular oscillating limits and long-time regularity of the limit 3D equations. The bounded cylindrical geometry prevents the usual Fourier Analysis tools and requires the geometry of the Curl operator restricted to appropriate linear subspaces. Cancellations of oscillations are obtained via explicit commutation of such restricted Curl operator with the Poincaré-Sobolev wave operator, and strong convergence theorems rely on equivalent Curl norms instead of the usual Sobolev norms. Similar results are obtained for 3D Navier-Stokes equations with special stress-free boundary conditions for the cylinder.

On the existence of weak solutions to the steady compressible Navier-Stokes equations in domains with several conical exits

Antonín Novotný, Toulon

Steady flows of compressible fluids in a rigid container with upper free boundary

Mariarosaria Padula, Ferrara

with Bum Ja Jin

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We consider fluid in a smooth container whose upper is like a piece of cylinder, bounded above by a free upper surface. As basic flow we consider the rest state with a steady distribution of density in a finite cylinder under gravity, in the presence of a surface tension. Under the same assumptions, we study the existence of a steady free boundary Γ and a steady motion in Ω of an isothermal compressible viscous gas, resulting as perturbation to the rest state in correspondence of small non potential perturbations to the (large potential) gravitational force.

The mixed elliptic-hyperbolic character of the system causes severe problems in constructing effective approximation schemes. We begin linearize the problem by prescribing the unknown domain Ω, then we make use of a new iterative factorization scheme for numerical solution of the steady Poisson-Stokes equations recently introduced by Heywood and Padula. Our method is based on an iteration between the Neumann problem for a Stokes problem for the velocity, an elliptic problem on Γ for height, and a steady transport equation for the perturbation to the density. The difference of boundary condition causes a singularity at the contact line between lateral surface and the free upper surface. To avoid singularities on the contact line, we adopt weighted Sobolev spaces.

On some problems with the Eulerian-Lagrangean description of viscous fluids - a counterexample to the smoothness of solution to an equation arising in fluid mechanics

Milan Pokorný

with Stephen Montgomery-Smith

On the dimension of the attractor for a class of power-law fluids

Dalibor Pražák, Prague

Consider the equations of the incompressible fluid motion

$$\begin{aligned} &\boldsymbol{v}_t + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} - \operatorname{div} \mathcal{T} + \nabla \pi = \boldsymbol{f} \\ &\operatorname{div} \boldsymbol{v} = 0 \end{aligned}$$

where the stress tensor is given by

$$\mathcal{T} = \tilde{\mathcal{T}}(D) = \nu D + \mu |D|^{p-2} D.$$

For the spatially-periodic problem, we derive explicit estimates of the fractal dimension of the global attractor and of the exponential attractor for $p \in [2, 4)$ in dimension two and $p = 3$ (i.e., the Smagorinski model) in dimension three.

Regularity of Pressure Near Regular Points in the Navier-Stokes Equations

Zdeněk Skalák and Petr Kučera, Prague

(Poster presentation)

On the numerical investigation of complex ABL problems

I. Sládek
with L. Beneš, K. Kozel

A mathematical and numerical investigation of the flow over Giant Mountains and storage site is presented.

Mathematical model is based upon the RANS equations for an incompressible flow with an algebraic turbulence closure and given boundary conditions. Also additional transport equation for concentration of passive pollutant has been considered. The artificial compressibility method with conjunction of finite volume method and the explicit Runge-Kutta multistage scheme is used for numerical analysis.

Finite Element Solution of a Problem with Nonlinear Boundary Conditions

Petr Sváček

The present paper is devoted to the solution of partial differential equations with the Finite Element Method (FEM). The FEM is applied to a nonlinear problem and qualitative properties of numerical approximations are investigated.

We consider the numerical approximation of a problem on a two dimensional polygonal domain. We are concerned with the approximation with the aid of higher order elements, the solvability, and the convergence. Next, with the help of uniform monotonicity error estimates are shown. The effect of approximation of a nonpolygonal domain with a polygonal one is studied in the case of use of linear finite elements. The error estimates for such a situation are shown and several numerical experiments are presented.

On the asymptotic behaviour of solutions to the Navier-Stokes equations for compressible nonlinear viscous fluids

Rostislav Vodák

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The main goal of this paper is to study the asymptotic behaviour of solutions to the Navier-Stokes equations for compressible nonlinear viscous fluids on Orlicz spaces. We would like to prove the strong convergence (in Orlicz spaces) of the density to a function ρ_∞ which satisfies an appropriate rest state. The reason why we used Orlicz spaces is to obtain the convergence independently on dimension. In the first part we prove the existence of a solution of the problem $\nabla \cdot \mathbf{v} = f$, where $f \in L^\infty(\Omega)$. In the second part we test Navier-Stokes equations in pressure term with an appropriate function using the existence result from the first part to be able to study the asymptotic behaviour of the density.
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Orr-Sommerfeld problem for the wavy film flows

Ondřej Wein

with J. Tihon

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The Orr-Sommerfeld (OS) problem is analyzed in terms of the relation between complex-valued wavenumber a and celerity c. For the long-wave regular branch, Taylor series to c(a) is found to be of limited use, as its convergence radius r is too small. Wave characteristics of the inception region, including besides c(a) also the wall shear rate impedance q(a), are obtained for the maximum spatial growth rate scenario by solving the OS problem numerically. In addition to the Yih-Benjamin regular branch of c(a), which surprisingly terminates at finite wavenumber, other branches are identified. Some of the numerically identified branches correspond to analytical asymptotes: long-wave singular branches and short-wave surface boundary layers. According to the numerical simulations, only the regular branch supports instability of primary flow at low and medium Reynolds numbers.

Global special regular solutions to Navier-Stokes Equations

Wojciech Zajączkowsky, Warsaw

Existence and uniqueness of global regular solutions to Navier-Stokes equations with boundary slip conditions for either interior or exterior domains of axially symmetry is considered.The proof is done under some assumptions on smallness of either angular component of velocity or some derivatives of velocity.The proof basis on estimate for the angular component of vorticity which was proved by Ladyzhenskaya in 1968.The existence is proved in weighted Sobolev spaces.