Extremal problems for potentials with external fields. Applications to the theory of capacities of condensers

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We shall be concerned with the well-known Gauss variational problem on the minimum of the energy in the presence of an external field, the infimum being taken over fairly general classes of signed Radon measures in a locally compact space. We shall show that, in the noncompact case, the problem is in general unsolvable, and this occurs even under extremely natural assumptions (in particular, for the Newton, Green, or Riesz kernels in an Euclidean space). Necessary and sufficient conditions for the problem to be solvable will be given. Some related extremal problems in the theory of capacities of condensers are also supposed to be discussed.