

New counterexamples of elliptic and parabolic equations with discontinuous coefficients

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Abstract

We present elliptic and parabolic equations

$$\sum_{i,k=1}^n D_i(a_{ik}(x)D_k u) = 0$$
$$u_t - \sum_{i,k=1}^n D_i(a_{ik}(t,x)D_k u) = 0$$

in \mathbb{R}^n or $[0, T] \times \mathbb{R}^n$ having discontinuous or unbounded complex valued solutions. The coefficients are uniformly elliptic and also complex valued. This shows that the theorem of De Giorgi-Nash cannot be extended to the complex valued case. For dimension $n \geq 5$ and the elliptic case such examples have been presented in the book of Mazya-Nasarov-Plamenevskii. The counterexample in case of dimension $n \in [3, 5]$ and the parabolic case are new.