

A non-arithmetical Gödel logic (Abstract.)

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Gödel fuzzy logics are many-valued logics whose set of truth degrees is a closed subset V of the real interval $[0, 1]$ containing 0 and 1 with minimum as the truth function of conjunction and Gödel truth function of implication, assigning to a pair u, v of truth values the value 1 if $u \leq v$ and the value v otherwise. (Hence disjunction is interpreted as maximum; (Gödel) negation of 0 is 1 and negation of any positive η is 1.)

Among Gödel logic a particularly interesting and important role is played by the logic with the set of truth values $V_{\downarrow} = \{0\} \cup \{\frac{1}{n} \mid n = 1, 2, \dots\}$ denoted by \mathbf{G}_{\downarrow} . \mathbf{G}_{\downarrow} was shown not axiomatizable (Π_1 -hard) already in [1].

Theorem 1 (1) The set of all formulas satisfiable in \mathbf{G}_{\downarrow} is not arithmetical.
(2) The set of all tautologies of \mathbf{G}_{\downarrow} is not arithmetical.

Let us ask for which other sets V our method of proof can be used to show that tautologies and satisfiable formulas of \mathbf{G}_V form a non-arithmetical set.

Definition 1 Let V be a closed subset of $[0, 1]$. An element $x \in V$ satisfies the condition $IRLP_V(x)$ if x is a right limit point of V isolated among the right limit points of V (If there is no danger of misunderstanding we may say that x is an isolated right limit point of V .)

Theorem 2 Let $V \subseteq [0, 1]$ be closed and let $IRLP_V(0)$. Then both the set of tautologies of \mathbf{G}_V as well as the set of all satisfiable formulas of \mathbf{G}_V are non-arithmetical.