

Significance analysis of asteroid pairs

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Abstract

We studied statistical significance of asteroid pairs residing on similar heliocentric orbits with distances (approximately the relative encounter velocity between orbits) up to $d = 36$ m/s in the 5-dimensional space of osculating elements. We found candidate pairs from the Hungaria zone through the entire main belt as well as outside the main belt, one among Hildas and one in the Cybele zone. We first determined probability that the candidate pairs are just coincidental couples from the background asteroid population. Those with estimated probability < 0.3 were further investigated. In particular we computed synthetic proper elements for the relevant asteroids and used them to determine 3-dimensional distance of the members in candidate pairs. We consider small separation in the proper-element space as a signature of a real asteroid pair; conversely, cases with large separation in the proper-element space were rejected as spurious. We finally provide a list of candidate pairs that appear real, genetically related, to facilitate targeted studies, such as photometric and spectroscopic observations. As a by-product, we discovered six new compact clusters of three or more asteroids where backward orbit integrations indicate that they are young families with ages < 2 Myr.

Key words: Asteroids; Minor planets

1 Introduction

Vokrouhlický and Nesvorný (2008; hereafter VN08) revealed existence of pairs of asteroids residing on very similar heliocentric orbits. According to VN08 their compactness cannot be coincidental and they proposed that members in each of the pairs have a common origin. They suggested a few possible mechanisms of the pair formation, including rotational fission and collisional breakup of the parent body, or a gentle split of a binary pair. Working conservatively, VN08 identified 60 asteroid pairs that appeared statistically significant from their initial analysis.

The aim of this work is to estimate statistical significance of asteroid pairs in a more systematic way, in particular to establish pairs to greater relative distances than was done in VN08. The underlying scientific aim is to provide their list with information facilitating targeted studies such as photometric and spectroscopic observations.

2 Identification of candidate pairs and compact families

We searched for pairs among 342,444 orbits of asteroids from the AstDyS catalog (February 2009 release; Knežević et al. 2002, Knežević and Milani 2003) with more than one observed opposition and the semimajor axis less than 10 AU. For each couple of neighboring orbits, we computed their distance d in 5-dimensional space of osculating orbital elements $(a, e, i, \varpi, \Omega)$ defined as a positive-definite quadratic form:

$$\left(\frac{d}{na}\right)^2 = k_a \left(\frac{\delta a}{a}\right)^2 + k_e (\delta e)^2 + k_i (\delta \sin i)^2 + k_\Omega (\delta \Omega)^2 + k_\varpi (\delta \varpi)^2, \quad (1)$$

where n and a is the mean motion and semimajor axis of either of the two asteroids and $(\delta a, \delta e, \delta \sin i, \delta \varpi, \delta \Omega)$ is the separation vector of their orbital elements.¹

Following Zappalà et al. (1990), we used $k_a = 5/4$ and $k_e = k_i = 2$. The values of k_ϖ and k_Ω were estimated from a magnitude of differential precession that members of a pair could accumulate during a presumably young pair's age (assuming the initial dispersion in the secular angles was small). For an asteroid pair at $a \simeq 2.5$ AU and with an age of ≤ 250 ky, we note $k_\varpi = k_\Omega \simeq 3.5 \times 10^{-5}$ imply that $k_\Omega (\delta \Omega)^2$ and $k_\varpi (\delta \varpi)^2$ terms have about the

¹ Note that the metrics (1) uses ill-defined variables for orbits with very small eccentricities and inclinations. In this way, a presumably small number of candidate pairs in this zone of orbital element space may escape our attention, but we do not cure this issue in this paper.

same contribution in d as the corresponding terms in (a, e, i) (assuming pair-components dispersion in these elements corresponds to the escape velocity of the typical multi-kilometer parent body).

We performed our analyses presented in the following sections for three different values for k_{ϖ} and k_{Ω} , namely 10^{-5} , 3×10^{-5} , and 10^{-4} . We found the best consistency between estimated probabilities and long term dynamics of candidate pairs for the largest value $k_{\varpi} = k_{\Omega} = 10^{-4}$. Therefore we adopted these values for the analyses presented in this paper.

We point out that the metrics with $k_{\varpi} = k_{\Omega} = 10^{-4}$ differs from the one used in VN08 where the value of 10^{-5} was used for the coefficients in the secular angles. While distances d between members in real pairs changed only moderately (which is due to their typically small separations in the secular angles and therefore a lower sensitivity to a choice of the value of the coefficients), distances between members of coincidental pairs usually increased substantially as their secular angles typically differed more than in real pairs; the distances for coincidental pairs in the modified metrics are greater by a factor between 1 and $\sqrt{10}$ than their distances in the metrics of VN08.

VN08 used a conservative limit $d = 10$ m/s for the pair identification. Their choice was based on behavior of the cumulative distribution $N(< d)$ function in the asteroid belt population, notably a change between two distinct regimes $N(< d) \propto d^5$ for large d values to $N(< d) \propto d^2$ for small d values. While the former corresponds to a uniform distribution of points in a 5-dimensional space, the latter reveals an excess of nearby orbits. While setting a general scheme, VN08 did not attempt searching putative asteroid pairs beyond their 10 m/s limit of distance.

In a search for wider pairs, we conducted analysis for higher limits on d in this paper. Indeed, we found numerous significant pairs with $d > 10$ m/s, and the widest pairs that we were able to identify have $d \sim 35$ m/s. After the test runs, we used an upper limit on d of 36 m/s for analyses presented below.

We found 558 candidate pairs with $d < 36$ m/s among the multiple-opposition orbits. We exclude poorly constrained single-opposition orbits from our analysis because the formal uncertainty of distance d in pairs involving single-opposition asteroids were often comparable to or not much less than the distance itself. For pairs involving multiple-opposition asteroids, the uncertainty $\sigma(d)$ (the $1\text{-}\sigma$ uncertainty of d propagated from the covariance matrices of the orbits of the pair asteroids) was in a vast majority of cases much less than the distance itself, therefore allowing a reliable analysis of their significance.

Among the candidate pairs, there is a mixture of real, genetically related couples of asteroids and coincidental pairs of genetically unrelated asteroids from background population that were transported to similar orbits through their dynamical or collisional evolution. To discriminate asteroid couples in these two categories, we developed the following approach. First, we analysed a population of asteroids surrounding each candidate pair, specifically, we

Table 1

Osculating elements ranges of populous asteroid families excluded from pair significance analysis in this work. Note that the Koronis family harbors the moderately young Karin family.

| Family | a (AU) | e | i (deg) |
|----------|-------------|-----------|-----------|
| Aeolia | 2.738–2.744 | 0.12–0.21 | 2.2– 4.7 |
| Eos | 2.96 –3.08 | 0.02–0.13 | 8.4–12.0 |
| Koronis | 2.83 –2.95 | 0.00–0.11 | 0.8– 3.5 |
| Massalia | 2.38 –2.44 | 0.11–0.21 | 0.5– 2.6 |
| Nysa | 2.34 –2.47 | 0.11–0.24 | 1.1– 3.9 |
| Veritas | 3.16 –3.18 | 0.02–0.10 | 8.0–10.6 |

determined whether the surrounding population distribution is consistent with uniform distribution in the 5-dimensional space of osculating elements. Second, for pairs that passed the uniform background population distribution test, we estimated probability that the given pair of orbits is coincidental or not. Mathematical details of our procedure are described in Sections 3 and 4, and results are commented in Sect. 5.

2.1 Pairs in families excluded

Some of the candidate pairs were located in very young or populated asteroid families. VN08 showed that pairs identified in recently formed families Datura, Emilkowalski, Iannini, Lucascavin, and 1992 YC2 (e.g., Nesvorný et al. 2003, Nesvorný et al. 2006, Nesvorný and Vokrouhlický 2006), are probably fragments launched by impacts onto very similar orbits and thus they do not require a special formation mechanism. As such we drop them from our analysis in Sect. 3 and following. (These recent families have been, however, clearly detected as clusters of three or more asteroids by our pairs finding program; see below also for comments on a detection of other, new asteroid clusters.)

Pairs found in large and old families may be real and formed long after the family origin. However, a statistical estimate of significance of a candidate pair located in a populated family would require a detailed model of the distribution of asteroids in the family which is often non-uniform. We do not have available such a model for any of the existing families. We thus dropped from our work analysis of asteroid candidate pairs in six large families that have big local deviations from uniform orbit distribution. As a guidance to localize borders of the six families, we first downloaded family-identification files from the PDS website² (Mothé-Diniz et al. 2005) and translated the families' locations in the proper element space to the space of osculating orbital elements. Table 1

² <http://www.psi.edu/pds/resource/mothefam.html>

Table 2

Members of Irvine family. Absolute magnitude H estimates and five osculating orbital elements for the epoch MJD 54900 from the AstOrb catalogue are given.

| Designation | H | a (AU) | e | i (deg) | Ω (deg) | ω (deg) |
|-------------------|------|-------------|------------|--------------|-------------------|-------------------|
| 6825 Irvine | 13.9 | 2.16766795 | 0.01449400 | 5.406520 | 6.856330 | 310.272305 |
| 143797 2003 WA112 | 16.4 | 2.16718955 | 0.01473914 | 5.429350 | 8.163460 | 328.810703 |
| 180233 2003 UU192 | 16.9 | 2.16700477 | 0.01487181 | 5.418685 | 8.891104 | 323.610355 |
| 2005 UL291 | 17.3 | 2.16699552 | 0.01456923 | 5.417964 | 11.025187 | 324.850882 |

Table 3

Members of the 2000 GP36 family. Absolute magnitude H estimates and five osculating orbital elements for the epoch MJD 54900 from the AstOrb catalogue are given. Object denoted with a star is single opposition only.

| Designation | H | a (AU) | e | i (deg) | Ω (deg) | ω (deg) |
|-----------------|------|-------------|------------|--------------|-------------------|-------------------|
| 81337 2000 GP36 | 15.1 | 2.60883193 | 0.11883012 | 13.649054 | 10.369383 | 198.348565 |
| 2001 UR193 | 15.6 | 2.61185823 | 0.11821001 | 13.604727 | 9.573822 | 200.805972 |
| 2003 FK6 | 16.2 | 2.60987513 | 0.11761935 | 13.663285 | 11.953908 | 197.455703 |
| 2008 GW33* | 18.1 | 2.61163693 | 0.11680772 | 13.711007 | 13.121635 | 196.014329 |

gives the derived ranges of the six asteroid families in osculating (a, e, i) .

We also tested cases where one asteroid was involved in more than one apparent pair, i.e., being a cluster of three or more asteroids rather than a solitary pair. We found seven such clusters. Five of them were identified as the known compact families Datura, Emilkowalski, Iannini, Lucascavin, and 1992 YC2 that were found earlier by Nesvorný et al. (2003), Nesvorný et al. (2006), and Nesvorný and Vokrouhlický (2006). Two clusters are new. Backward orbit integrations showed that they both are very young families.

The first one has the brightest member (6825) Irvine, and we list identified members of the family in Table 2. Backtracking of their nominal orbits with clones suggests the most likely age between 1.4 and 1.8 Myr for the family. Orbital elements of the members of the family show a correlated offset that is due to effects of a secular resonance, most likely ν_6 . An effect of the size-correlated offset on distances between members of the cluster is that the three smaller members form a very compact group with distances ~ 15 m/s one from each other, while the largest member (6825) Irvine is somewhat displaced with $d \sim 60$ m/s from the three smaller members. The three small objects thus

Table 4

Members of the Rampo family. Absolute magnitude H estimates and five osculating orbital elements for the epoch MJD 54900 from the AstOrb catalogue are given. Object denoted with a star is single opposition only.

| Designation | H | a (AU) | e | i (deg) | Ω (deg) | ω (deg) |
|-------------|------|-------------|------------|--------------|-------------------|-------------------|
| 10321 Rampo | 14.4 | 2.32952497 | 0.09429812 | 6.058556 | 53.984982 | 278.732560 |
| 2006 UA169* | 18.2 | 2.32822737 | 0.09472504 | 6.079391 | 58.541895 | 271.906463 |
| 2007 UM101 | 17.2 | 2.32937162 | 0.09475521 | 6.050718 | 53.312986 | 280.347945 |

Table 5

Members of the 1998 HR37 family. Absolute magnitude H estimates and five osculating orbital elements for the epoch MJD 54900 from the AstOrb catalogue are given. Objects denoted with a star are single opposition only.

| Designation | H | a (AU) | e | i (deg) | Ω (deg) | ω (deg) |
|-----------------|------|-------------|------------|--------------|-------------------|-------------------|
| 39991 1998 HR37 | 14.1 | 2.44428599 | 0.16160551 | 3.430221 | 334.241401 | 98.231774 |
| 2005 UU94* | 18.5 | 2.44617318 | 0.15800130 | 3.423584 | 334.228120 | 97.903022 |
| 2006 BR54 | 17.9 | 2.44493903 | 0.15923738 | 3.430668 | 334.528117 | 97.469210 |
| 2006 YE19* | 18.2 | 2.44646339 | 0.16075969 | 3.416523 | 334.480679 | 96.398549 |
| 2008 YV80 | 17.3 | 2.44458581 | 0.16090795 | 3.430890 | 334.179068 | 98.445955 |

represent a cluster of fragments that were initially ejected with comparable velocity vectors. The second new family has the brightest asteroid (81337) 2000 GP36 and its members are listed in Table 3. Backtracking of their nominal orbits suggests an age about 0.7 Myr. The cluster is harbored in the Eunomia family.

During additional experiments analysing orbit similarities to greater distances and including single-opposition orbit asteroids, we have found four more clusters where our backward orbit integrations confirmed (or suggested, in one case) that they are young families too. The first one with the brightest member (10321) Rampo appears to have an age between 0.5 and 1.1 Myr, from a simple backtracking of nominal orbits. Its members are listed in Table 4. The second one has the brightest member (39991) 1998 HR37 and it has a core of a compact group ($d < 20$ m/s) of three asteroids and two other slightly more distant members; they are listed in Table 5. The cluster lies in the Nysa family that we have excluded from our regular analyses (see above). The third cluster, around asteroid (57738) 2001 UZ160, is adjacent to the J3/1 mean motion resonance and orbits of its members are affected by the $g + g_5 - 2g_6$ secular resonance. A simple backtracking of the nominal orbits suggests an age

Table 6

Members of the 2001 UZ160 family. Absolute magnitude H estimates and five osculating orbital elements for the epoch MJD 54900 from the AstOrb catalogue are given. Objects denoted with a star are single opposition only.

| Designation | H | a (AU) | e | i (deg) | Ω (deg) | ω (deg) |
|------------------|------|-------------|------------|--------------|-------------------|-------------------|
| 57738 2001 UZ160 | 15.3 | 2.56350387 | 0.18300953 | 4.323400 | 104.967993 | 181.295028 |
| 2001 NH14 | 17.1 | 2.56498589 | 0.17944925 | 4.312641 | 105.256929 | 181.961396 |
| 2008 HQ46* | 17.7 | 2.56344052 | 0.18457031 | 4.321620 | 105.338168 | 182.335050 |
| 2008 JK37* | 17.5 | 2.56348204 | 0.18273519 | 4.326637 | 104.339889 | 181.623919 |
| 18777 Hobson? | 15.1 | 2.56097740 | 0.18508555 | 4.323817 | 105.509410 | 179.721618 |

less than 0.5 Myr. Members of the family are given in Table 6. There is also a nearby asteroid (18777) Hobson, another possible member of the cluster, but its association needs to be verified with a more detailed backward integration of the multi-opposition orbits. The similarity of the sizes of Hobson and 2001 UZ160 would make the family unusual. The 2001 UZ160 cluster was not found in our regular analysis runs as it contains no pair of > 1 -opposition asteroids with d less than the limit of 36 m/s in the current orbit catalog. The last cluster, around asteroid (119401) 2001 TY50, is a possible young family too. Initial backward integration of their nominal orbits shows a convergence of the three members of the cluster in Ω , but not in ϖ , so a more detailed analysis or more accurate orbits are needed to confirm the cluster as being a real family.

3 Background asteroid population density

To estimate a probability whether a small distance between two orbits from the asteroid population is coincidental or not, we first need to determine density of asteroids in the background population around a position of a candidate pair in the 5-dimensional space of osculating elements. To do so, we constructed three boxes centered around each of the candidate pairs. The largest box had a width of $\Delta\varpi = \Delta\Omega = 2\pi$ in ϖ and Ω (reflecting the assumption that no variation of asteroid density in the two angular elements was expected). The width of the box in a , e , and $\sin i$ was set as $\Delta a = \sqrt{10^{-5}/k_a} a \Delta\Omega$, $\Delta e = \sqrt{10^{-5}/k_e} \Delta\Omega$, and $\Delta \sin i = \sqrt{10^{-5}/k_i} \Delta\Omega$. For the adopted coefficients (see Sect. 2), the widths were $\Delta a \doteq 0.0178a$ and $\Delta e = \Delta \sin i \doteq 0.0140$. The other two nested boxes, a middle and a small one, respectively, had widths 1/2 and 1/4 of the largest box in each of the five elements. Volume of these two smaller boxes was $V_{1/2} \doteq V_1/2^5 = V_1/32$ and $V_{1/4} \doteq V_1/4^5 = V_1/1024$, where V_1 is the volume of the largest box. All boxes were centered on the orbit

of the candidate pair.

Next we determined number of orbits N_1 , $N_{1/2}$ and $N_{1/4}$ in the three nested boxes around each candidate pair. Only asteroids with absolute magnitudes $H < H_2 + 0.5$, where H_2 is the absolute magnitude of the fainter member of the asteroid pair, were counted. It was because only orbits of asteroids of size (brightness) similar to or greater than the members of the pair are relevant for statistical estimates below. The $+0.5$ mag buffer with respect to H_2 was chosen because an error of ± 0.5 mag is about a typical uncertainty of H estimates from astrometric measurements.

Using the data in the largest box, the local number density of orbits around a candidate pair was estimated as $\eta = N_1/V_1$. Our working hypothesis, necessary for analysis in Sect. 4, is that of a uniform (random) distribution of orbits in the pair neighborhood. A basic test of this assumption was performed by comparing the observed asteroid counts $N_{1/2}$ and $N_{1/4}$ in the smaller boxes with the expected values $\eta V_{1/2}$ and $\eta V_{1/4}$. This is done as follows.

For a uniform distribution of orbits in the box, the probability P_b that there are N_b or more orbits in the box is given by the binominal distribution,³

$$P_b = 1 - \sum_{i=0}^{(N_b-2)-1} \binom{N_1-2}{i} p_v^i (1-p_v)^{N_1-2-i}, \quad (2)$$

where $b = 1/2$ and $1/4$ for the two nested boxes, and $p_v = b^5$ is their volume ratio to the largest box. In the above formula, the numbers entering the probability computation are $(N_b - 2)$ and $(N_1 - 2)$; two are subtracted from the total numbers because we estimate the probability of a number of orbits surrounding the specific pair. If the probability P_b is low, the observed number N_b in the box is inconsistent with the assumed uniform distribution of orbits in the largest box from which the number density η was estimated. In this case, the smaller box contains excess number of orbits.

The above test of consistency of the orbit counts in the pair-surrounding boxes with assumed uniform number distribution of orbits in the largest box was performed for each candidate pair. When $P_{1/2}$ was found to be less than a certain level, the candidate pair did not pass the test for uniform distribution of orbits in the surrounding population and we dropped it from further considerations. The excess number of orbits observed in the smaller box around such candidate pair may be due to rapid changes of the density of orbits in the range around the position of the pair, but it may also be due to a concentration of orbits near the position of the pair. In the latter case, there may be a relation between the pair and the concentration of orbits around it, however, we did not study such possibilities.

³ For large N_1 (i.e., for pairs residing in ranges with high density of orbits), the binominal distribution is approximated with the Poisson distribution. Formula (2) then becomes $P_b = 1 - \sum_{i=0}^{(N_b-2)-1} e^{-\lambda} \frac{\lambda^i}{i!}$, where $\lambda = (N_1 - 2)p_v$.

We chose following limiting levels for $P_{1/2}$ below which we considered that a candidate pair did not pass the test for uniform background asteroid distribution: (i) 0.01 for pairs with $d < 10$ m/s, and (ii) 0.05 for pairs with greater d . The reason for being less conservative with the closer pairs was that further analyses were typically less sensitive to non-uniformities in the distribution of background asteroids for the closer pairs.

We have to point out that some candidate pairs that did not pass the test for uniform background distribution, and thus not considered in the remaining part of this paper, may still be real pairs. We just cannot use the method presented in this paper to estimate a probability that the similarity of their orbits is a fluke.

An example of a real pair that did not pass the test for uniform background distribution is the pair 2002 PU155 and 2006 UT69 (backward integration of their orbits suggests good convergence of these two orbits in the past). For this case we have $N_1 = 174$ and $N_{1/2} = 18$, giving $P_{1/2} = 0.00015$. In other words, it is unlikely that the observed number of 18 orbits (including the two orbits of the pair) in the middle box is just a random fluctuation of a uniform distribution of orbits in the largest box surrounding the pair. Whether the apparently non-uniform distribution of asteroids in the range around the pair is due to rapid changes of density unrelated to the pair, or if the pair and the apparent nearby concentration of orbits are related, remains to be seen from future studies.

4 Number of coincidental pairs

A probability to find two orbits from a population with the number density η within a volume V is given by the Poisson statistics

$$p_2(V) = \frac{\nu^2}{2} e^{-\nu} , \quad (3)$$

where $\nu = \eta V$. If there are M boxes of the volume V , the expected number of pairs found is

$$P_2(V) = M p_2(V) = M \frac{\nu^2}{2} e^{-\nu} . \quad (4)$$

Since

$$M = \frac{N}{\nu} , \quad (5)$$

where N is a number of orbits within the total volume of M boxes, we obtain

$$P_2(V) = \frac{N\nu}{2}e^{-\nu}. \quad (6)$$

Using the radius d of a hypersphere of volume V in the 5-dimensional space given by following formula

$$d = \left(\frac{8}{15}\pi^2\frac{1}{V}\right)^{-5}, \quad (7)$$

and the radius R_0 of a hypersphere with specific volume (note R_0 is a characteristic distance of objects for the observed density η)

$$R_0 = \left(\frac{8}{15}\pi^2\eta\right)^{-5}, \quad (8)$$

we obtain

$$\nu = \eta V = \left(\frac{d}{R_0}\right)^5. \quad (9)$$

Therefore the expected number of pairs with distances $\leq d$ in the population is given by

$$P_2(V) = P_2(d) = \frac{N}{2} \left(\frac{d}{R_0}\right)^5 e^{-\left(\frac{d}{R_0}\right)^5}. \quad (10)$$

We call the ratio d/R_0 to be a normalized distance of orbits. In our analyses below, d/R_0 is always small, and the exponential term on the right side of Eq. (10) is close to unity and may be dropped in the first approximation. With the formalism, the above formula can be used to estimate the expected number of pairs with normalized distances even in a population where the number density η is not constant. (The assumption of locally uniform number density needed for the estimation of η in Sect. 3 does not need to be held globally in the whole asteroid population.)

5 Statistically significant pairs

A given pair with normalized distance d/R_0 is considered to be statistically significant if it has a low probability to be a chance coincidence of two independent orbits, i.e., if the expected number of pairs with normalized distances $\leq d/R_0$ is much less than an actual number of such pairs found in the population.

For each candidate pair with the distance d and the radius of specific volume R_0 , we considered a number of pairs (N_p) in the volume V centered at d/R_0 and the total number of asteroids in the population (N).⁴ For the countings, we used the same criteria as in the pair selection procedure above. Specifically, we counted asteroids with absolute magnitudes less than $H_2 + 0.5$ that are not members of families listed in Table 1. The population and pairs countings were restricted to an orbital zone of the pair, to suppress biases caused by different observational completeness levels in different zones. Specifically, we divided the orbital space into six zones (where at least one pair was found) as follows:

- Hungaria, $1.78 < a < 2.00$ AU, $16^\circ < i < 34^\circ$, $e < 0.18$,
- Inner main belt, $2.00 < a < 2.50$ AU,
- Central main belt, $2.50 < a < 2.82$ AU,
- Outer main belt, $2.82 < a < 3.27$ AU,
- Cybele, $3.27 < a < 3.70$ AU, $i < 25^\circ$, $e < 0.30$,
- Hilda, $3.74 < a < 4.02$ AU.

In electronic file available on <http://www.asu.cas.cz/~asteroid/astpairs.htm>, we list for each pair the number of pairs with normalized distances $\leq d/R_0$ found (N_p) and the expected number of pairs P_2 in the population of the pair's zone. When the ratio P_2/N_p is much less than 1, we consider the pair to be statistically significant, i.e., having low probability to be a chance orbital coincidence of two genetically unrelated asteroids.

Obviously, the most statistically significant pairs are the closest ones (with small d) or those that lie in low-density regions of the orbital space (with large R_0). On the other hand, we expect a certain contamination by coincidental pairs among somewhat less significant pairs with $P_2/N_p \sim 0.1$ and larger.

In Table 7, we list pairs with $P_2/N_p < 0.3$. The first and second columns give designations of the brighter and fainter members of the asteroid pair. The third column gives the distance in the 5-dimensional space of osculating orbital elements. The fourth column gives a distance in the space of proper orbital elements (see below). The next three columns contain the osculating semi-major axis, eccentricity, and inclination. The eighth and ninth columns give the absolute magnitude of the brighter member of the pair and the absolute magnitude difference between the two asteroids, respectively.⁵ The tenth column gives the computed probability of that the pair is a coincidental pair of two unrelated background asteroids. The last column contains a remark on a long-term dynamics of the pair (see below).

⁴ The volume between hyperspheres with radii $f_V^{-1}d/R_0$ and $f_V d/R_0$, where $f_V = [(1 + \sqrt{5})/2]^{1/5}$ is equal to the volume V of the hypersphere with radius d/R_0 . We determined the number of pairs in the equivalent volume between the two hyperspheres rather than in the hypersphere with radius d/R_0 .

⁵ The absolute magnitudes in the AstDyS catalog were derived from magnitude estimates from astrometric observations, and they are typically uncertain by ± 0.5 mag.

5.1 Distances in proper elements

To further examine the relation between the pair orbits and to help discriminating between real and coincidental pairs, we determined proper orbital elements (a_{prop} , e_{prop} , i_{prop}) of the asteroids in the candidate pairs (one Hilda pair and one Cybele pair were not studied in this section). In the first step, we performed numerical integration of the planetary system and selected asteroids (using their nominal, best-fit orbits) over a timespan of 5 Myr. We used asteroid initial data from AstDyS site for MJD54800 epoch and propagated JPL405 planetary orbits to the same epoch. Integration was performed with dense-enough output sampling, 100 yr in our case. We then applied methods similar to Knežević et al. (2002) and Knežević and Milani (2003) to determine synthetic proper orbital elements for the candidate asteroids. In particular, we Fourier-analysed secular frequencies in non-singular eccentricity and inclination vectors of planets. Signal with these frequencies was then filtered out from the corresponding non-singular elements of asteroid orbits; the remaining signal should very well correspond to the contribution of the proper term (either eccentricity or inclination). The numerically-determined mean value, and the corresponding standard deviation, of the amplitude of non-singular, post-filtered eccentricity and inclination vectors were thus considered as the best available proxy of the proper e_{prop} and i_{prop} values over the time interval of 5 Myr. The proper semimajor axis a_{prop} value was determined simply as numerically-determined mean value, and the corresponding standard deviation, of the osculating semimajor axis. In most cases of the numbered asteroids, for which the AstDyS site also provides synthetic proper orbital elements, our values and those of Knežević and Milani agreed very well. Small deviations were observed in cases of orbits residing very close or inside weak mean motion or secular resonances, for which the AstDyS site provides more stable values. These cases were, however, inspected individually and we decided how reliable is the proper-element computation for our purposes.

Having determined synthetic proper orbital elements for all asteroids of interest, we could then compute their traditional 3-dimensional distance d_{prop} in the space of proper elements (e.g., Zappalà et al. 1990); d_{prop} is basically given by formula (1) with the first three terms on the right hand side and proper elements used instead of the osculating elements. We give d_{prop} for asteroids in each of the candidate pairs in the fourth column of Table 7.

Pairs with low chance of being random coincidence, as revealed by small probabilities P_2/N_p , have systematically small value of d_{prop} . This is expected and consistent with a separation at very low relative velocity at origin as suggested by VN08. As the value of probability P_2/N_p increases, reflecting higher chance of fluke in having close-by orbits, we obtain large value of d_{prop} more frequently. The large d_{prop} may be a tracer of a coincidental asteroid pair, but before we can conclude so we need to check that proper elements have been determined reliably enough. Note that candidate pairs that happen to reside near (or inside) mean motion and/or secular resonances may have uncertain proper elements. In this case the nominal orbits used in our analysis

may be incidentally distant, while the overlap of proper-element distributions determined from admissible asteroid clones may correspond to much smaller distance. Since we do not intend to perform an in-depth analysis of such cases, we took a simple procedure.

We numerically integrated planetary system with selected asteroids for another 5 Myr and determined synthetic proper elements of asteroid orbits anew from this second interval of time. If the pair distances d_{prop} in the space of proper orbital elements determined from the first and second interval of 5 Myr differed by more than 3 m/s, we flagged the pair as having apparently unstable orbits (the “v” flag in the fourth column of Table 7). An unflagged (apparently long-term stable orbits) pair was considered consistent with being a real pair with its components separated in a gentle way if its d_{prop} was less than 15 m/s. The chosen $d_{\text{prop}} \simeq 15$ m/s threshold value is somewhat arbitrary, but we consider it plausible as it is a few times the escape velocity from a few km-sized asteroid (the typical size of brighter members of asteroid pairs in the sample). A pair with unflagged (stable) d_{prop} greater than 15 m/s was, on the other hand, considered spurious, as high relative velocities of pair members are not expected from the separation mechanisms, and such pairs might be coincidental couples of unrelated asteroids. Pairs with $d_{\text{prop}} > 15$ m/s, but apparently unstable orbits (with the “v” flag), were set for a more detailed check of their orbit evolution over the total 10-Myr interval. Usually, we were able to recognize dynamical causes for the long-term orbital irregularities (usually jumps of the semimajor axis over a weak mean-motion or secular resonance). Such pairs are considered as orbitally unproblematic despite their higher d_{prop} values and considered being real couples. Few examples are in order to see our procedure.

9068 and 2002 OP28 – The first pair with noticeably large d_{prop} , yet very small value of P_2/N_p probability, is the Hungaria-population pair (9068) 1993 OD and 2002 OP28. Detailed inspection of their orbits reveals that both asteroids undergo frequent close approaches to Mars (well within its Hill sphere) that makes their semimajor axis values random walk with a typical sudden increment of $\sim 10^{-3}$ AU at each encounter. This explains the elevated d_{prop} for this pair⁶ and we consider this couple a real pair of asteroids. We also note that the ability to closely approach Mars for this case is exceptional even for asteroid pairs in the Hungaria population. While direct tidal disruption of the parent body in the Mars’ gravity field is an unlikely formation mechanism for this pair, tidal split of weakly-bound binary would be a possible process additional to those mentioned in Sect. 1. However, a more detailed analysis of this interesting pair requires more astrometric data to improve orbit determination for both components.

56232 and 115978 – Another interesting situation is presented by the pair (56232) 1999 JM31 and (115978) 2003 WQ56. This case is again characterized

⁶ In fact, d_{prop} computed from the second 5-Myr long time interval increases to ~ 2.6 km/s. This witnesses significant divergence of the two orbits triggered by continuous Mars encounters.

by (i) small P_2/N_p probability and (ii) significantly elevated d_{prop} value. A difference between $d_{\text{prop}} \simeq 40.3$ m/s computed from the first 5-Myr interval with $d_{\text{prop}} \simeq 68.1$ m/s computed from the second 5-Myr interval suggests orbital instability. A detailed scrutiny shows that: (i) semimajor axes of both components in this candidate pair exhibit random jumps over a weak mean motion resonance of $\sim 10^{-3}$ AU width (most likely the M11/19 exterior resonance with Mars; e.g., Morbidelli and Nesvorný 1999), and (ii) the eccentricity and inclination values are strongly affected by z_2 secular resonance (e.g., Knežević et al. 2002). This latter resonance causes inclination and eccentricity values oscillate with a very long period (comparable to 10 Myr). The 5-Myr time intervals that serve for computation of the proper e_{prop} and i_{prop} are thus insufficiently long and result in large uncertainty in their values. Therefore we propose 56232 and 115978 is a real asteroid pair with common origin. To further probe this conclusion we numerically backtracked orbits of 2000 geometric and Yarkovsky clones of both asteroids using technique described in Vokrouhlický and Nesvorný (2009). We note that the closest clones can approach to 750 km (about the Hill sphere of mutual gravitational influence of the two components) for epochs earlier than ~ 60 ky before present, confirming thus a possibility of their common origin. However, both orbits are much too uncertain to draw more conclusions.

117025 and 169718 – Similarly to the previous example, this candidate pair has (i) small P_2/N_p probability and (ii) significantly elevated d_{prop} value. The difference though is that here we do not see any orbital instability on the monitored 10-My interval. Indeed, the d_{prop} values computed from the two consecutive 5-My long intervals of time differ by less than 1.5 m/s. As a result we are thus led to conclude that we deal with random fluke in orbital distribution of asteroids (despite its probability of $\sim 2\%$, we note flukes exist).

2000 SM320 and 2008 TN44 – This example illustrates that real asteroid pairs can be found at significantly larger d distances than considered by VN08. In this case $d \simeq 24.7$ m/s, yet $d_{\text{prop}} \simeq 1.38$ m/s a very small value. While the two orbits are presently too uncertain to explore their past evolution, the small d_{prop} value is suggestive of a real pair.

34380 and 2006 TE23 – Our final example has again, as in the previous case, a somewhat elevated d value and a small d_{prop} value, hinting a real pair. In this case though, we have also moderate $P_2/N_p \simeq 0.27$ probability of a coincidental association. To see the case, we thus integrated backward orbits of the two asteroids together with their clones and noticed a possibility of close approaches at the mutual Hill sphere distance for epochs larger than 100 ky before present. Indeed, the formation mechanism of the asteroid pairs operates uniformly in time and space such that we should expect pairs with a continuum of ages. The older ones, likely the case of 34380 and 2006 TE23 pair, would thus show larger separations in d . Moreover, some of them are also expected to be located in denser locations of the asteroid belt where P_2/N_p is found higher.

Performing similar analysis when needed we give a remark on the long-term

dynamics of the pairs in the last column of Table 7. In particular the note “ok” specifies pairs with $d_{\text{prop}} < 15$ m/s and/or those for which our checks of long-term orbital evolutions showed dynamical causes for their increased d_{prop} values. We sometimes mention specific dynamical causes of the apparent long-term orbital instabilities, such as (weak) mean-motion resonance for interaction with a mean motion resonance, z_2 for the appropriate weak secular resonance, or close encounters with Mars in the above discussed case of 9068 – 2002 OP28 pair. Pairs that have $d_{\text{prop}} > 15$ m/s and that appear stable are marked as spurious and they may be random flukes in the asteroid orbital distribution.

An interesting statistics indicating different physical properties of the groups of ok and spurious pairs is a distribution of absolute magnitude differences between pair members, see Fig. 1. The ΔH distribution for real pairs is wide, apparently reflecting a wide size distribution of real pairs, modified by selection effects (incompleteness of the observed asteroids population) of surveys as well as our pair significance estimation method, that both are probably responsible for most or all of the decrease of the number with increasing ΔH in bins > 1.5 mag. The distribution for spurious pairs is quite different, with a majority of spurious pairs having $\Delta H < 1$ (median ΔH in the “spurious pairs” sample is 0.69), i.e., coincidental pairs mostly consist of nearly equal brightness asteroids. It is consistent with expectation; coincidental pairs mostly occur between asteroids that are most numerous in the sample population, thus they tend to be similar size.

6 Concluding remarks

Studies of most significant pairs (with lowest P_2/N_p , low d_{prop} and/or an “ok” long-term dynamics in Table 7) should bring the best data sample for understanding their characteristics. This is because we believe such sample is the least contaminated by spurious coincidental pairs. We plan to use this information in a photometric survey of the paired asteroids.

Two additional considerations are worth a note. Working conservatively, we discarded all candidate pairs for which the asteroid density in the 5-dimensional space of osculating orbital elements was not uniform enough (Sect. 3) and those located in certain asteroid families (Sect. 2). With that, our analysis certainly misses a number of interesting pairs for which we simply cannot estimate probabilities of being coincidental pairs. For instance, we had to discard the best studied case of (6070) Rheinland and (54827) 2001 NQ8 (VN08, Vokrouhlický and Nesvorný 2009) that resides at the outskirts of the Nysa-Plana clan. The best way for these situations, and in general, is to perform backward integrations of the asteroids in the pair to see if their orbits converge enough. Most often though the orbits are uncertain and a huge number of equivalent clones needs to be propagated along with the nominal solutions. This is computationally expensive and anyway does not reveal deterministic conclusions. Some of

these candidate cases should be, however, studied individually elsewhere.

A question also remains whether real, genetically related asteroid pairs were all formed with low relative velocities. If some of them formed by more energetic mechanism(s) that gave them a significant impulse, then their d_{prop} may not be as low as proposed in VN08. If true, some pairs dubbed “spurious” in Table 7 may still be real ones, just formed or affected by another mechanism than the low- d_{prop} pairs (for instance, medium-size asteroid families formed by super-catastrophic collisions). Studies of some pairs with low P_2/N_p but the “spurious” remark could be therefore useful as well.

Table 7: Asteroid pairs

| | Pair | d (m/s) | d_{prop} (m/s) | a (AU) | e | i (deg) | H_1 | ΔH | P_2/N_p | l.t.d. |
|-----------|-----------|-----------|-------------------------|----------|--------|-----------|-------|------------|-----------|----------|
| 63440 | 2004TV14 | 0.52 | 0.21 | 1.9379 | 0.0885 | 19.987 | 14.90 | 2.33 | 0.0000 | ok |
| 21436 | 2003YK39 | 0.98 | 5.88 | 2.1875 | 0.0844 | 3.736 | 15.02 | 3.27 | 0.0000 | ok |
| 76111 | 2005JY103 | 1.64 | 0.50 | 2.7166 | 0.0483 | 6.925 | 14.54 | 1.84 | 0.0000 | ok |
| 38707 | 32957 | 1.75 | 1.01 | 2.2782 | 0.1160 | 5.925 | 14.86 | 1.11 | 0.0000 | ok |
| 106700 | 2007UV | 4.44 | 0.61v | 1.9095 | 0.0613 | 23.095 | 16.29 | 0.78 | 0.0000 | ok |
| 88259 | 1999VA117 | 7.03 | 0.61v | 1.9352 | 0.0647 | 20.182 | 14.82 | 2.16 | 0.0000 | ok |
| 23998 | 205383 | 7.54 | 3.25 | 1.9346 | 0.0530 | 18.117 | 15.24 | 1.29 | 0.0000 | ok |
| 2007AQ6 | 2002SF64 | 8.25 | 12.92v | 1.9032 | 0.1076 | 19.034 | 17.34 | 1.07 | 0.0000 | ok |
| 84203 | 2000SS4 | 8.33 | 3.46 | 1.9611 | 0.0911 | 19.995 | 15.58 | 1.02 | 0.0000 | ok |
| 195479 | 2008WK70 | 4.06 | 0.28 | 2.5556 | 0.0460 | 1.469 | 16.18 | 0.94 | 0.0001 | ok |
| 25884 | 48527 | 11.88 | 7.49v | 1.9542 | 0.0803 | 21.561 | 14.31 | 1.40 | 0.0001 | ok |
| 9068 | 2002OP28 | 16.75 | 34.15v | 1.8204 | 0.1498 | 20.298 | 13.45 | 4.10 | 0.0001 | ok (a) |
| 4765 | 2001XO105 | 11.00 | 3.49v | 1.9454 | 0.0603 | 23.707 | 13.54 | 3.80 | 0.0002 | ok |
| 21930 | 22647 | 17.48 | – | 3.9545 | 0.2520 | 2.324 | 12.79 | 0.37 | 0.0002 | |
| 17198 | 2004FC126 | 4.91 | 0.93 | 2.2795 | 0.1027 | 3.288 | 14.91 | 2.57 | 0.0005 | ok |
| 180906 | 2003YR67 | 4.98 | 0.25 | 2.2362 | 0.1159 | 3.843 | 17.37 | 0.24 | 0.0005 | ok |
| 2002RJ126 | 2008TS51 | 6.56 | 9.46v | 2.2047 | 0.2299 | 5.773 | 17.41 | 0.84 | 0.0005 | ok |
| 38184 | 2008GR90 | 4.58 | 1.25 | 2.3269 | 0.2341 | 1.777 | 14.99 | 2.20 | 0.0006 | ok |
| 52773 | 2001HU24 | 7.82 | 2.07 | 2.2184 | 0.2035 | 3.661 | 15.34 | 2.15 | 0.0013 | ok |
| 56232 | 115978 | 9.85 | 40.31v | 2.1926 | 0.2025 | 2.936 | 15.06 | 1.13 | 0.0013 | ok (b) |
| 70511 | 2007TC334 | 7.48 | 1.60 | 2.4009 | 0.1943 | 4.404 | 15.13 | 3.31 | 0.0014 | ok |
| 13653 | 113029 | 7.85 | 30.57v | 2.1865 | 0.1670 | 4.599 | 15.38 | 0.99 | 0.0015 | ok (b) |
| 184300 | 2001UU227 | 12.44 | 4.35v | 2.1240 | 0.1809 | 2.739 | 17.62 | 1.19 | 0.0015 | ok |
| 99052 | 2006KM53 | 5.37 | 10.20v | 2.7313 | 0.0867 | 9.614 | 15.10 | 1.39 | 0.0016 | ok |
| 2110 | 44612 | 14.81 | 3.36 | 2.1982 | 0.1775 | 1.129 | 13.13 | 2.22 | 0.0019 | ok |
| 189994 | 2004RJ294 | 8.17 | 2.16 | 2.2667 | 0.0981 | 4.239 | 16.69 | 1.50 | 0.0020 | ok |
| 112249 | 2006GR49 | 8.29 | 1.10 | 2.2370 | 0.0863 | 5.599 | 16.02 | 0.75 | 0.0024 | ok |
| 5026 | 2005WW113 | 8.97 | 13.99v | 2.3769 | 0.2424 | 4.291 | 13.68 | 4.09 | 0.0029 | ok |
| 15107 | 2006AL54 | 7.71 | 2.07 | 2.2721 | 0.1761 | 4.591 | 14.27 | 2.55 | 0.0039 | ok |
| 92336 | 143662 | 19.26 | 2.82v | 1.9566 | 0.0879 | 22.438 | 15.30 | 1.10 | 0.0039 | ok |
| 11842 | 2002VH3 | 9.76 | 0.71 | 2.2511 | 0.0940 | 3.691 | 13.89 | 2.56 | 0.0042 | ok |
| 2000SP31 | 2007TN127 | 7.88 | 1.66 | 2.4054 | 0.1316 | 6.077 | 16.45 | 1.41 | 0.0047 | ok |
| 10484 | 44645 | 12.97 | 0.43 | 2.3205 | 0.0791 | 5.726 | 13.72 | 0.92 | 0.0057 | ok |
| 38395 | 141513 | 28.42 | – | 3.5082 | 0.0664 | 10.293 | 13.01 | 1.41 | 0.0061 | |
| 80218 | 2002ES90 | 19.27 | 1.34 | 2.2189 | 0.0274 | 1.796 | 16.50 | 0.03 | 0.0098 | ok |
| 17288 | 203489 | 11.74 | 1.26 | 2.2870 | 0.1802 | 4.097 | 14.11 | 2.13 | 0.0104 | ok |
| 2003AN55 | 2003UW156 | 11.91 | 11.23v | 2.3613 | 0.2140 | 5.830 | 16.22 | 1.34 | 0.0114 | ok |
| 1979 | 13732 | 17.82 | 9.02 | 2.3740 | 0.1000 | 6.046 | 13.44 | 0.66 | 0.0118 | ok |
| 26416 | 2007WO58 | 16.64 | 0.50 | 2.3434 | 0.0505 | 4.532 | 14.04 | 2.53 | 0.0119 | ok |
| 40366 | 78024 | 13.94 | 27.22v | 2.1760 | 0.1808 | 2.490 | 15.66 | 1.20 | 0.0158 | ok (b) |
| 51609 | 1999TE221 | 11.72 | 1.49 | 2.3090 | 0.1540 | 5.641 | 15.01 | 1.52 | 0.0167 | ok |
| 29358 | 40485 | 15.87 | 10.78v | 2.2779 | 0.1450 | 2.666 | 15.01 | 0.43 | 0.0173 | ok |
| 69142 | 127502 | 30.12 | 4.81v | 1.9640 | 0.0514 | 22.797 | 15.27 | 1.03 | 0.0174 | ok |
| 117025 | 169718 | 10.84 | 53.53 | 2.6683 | 0.0909 | 15.638 | 15.01 | 0.69 | 0.0200 | spurious |
| 6412 | 24903 | 28.99 | 65.54 | 2.3575 | 0.0479 | 6.835 | 13.38 | 0.21 | 0.0234 | spurious |
| 34081 | 33560 | 25.30 | 72.78 | 2.3344 | 0.1056 | 7.109 | 14.22 | 0.05 | 0.0304 | spurious |

Table 7: Asteroid pairs

| | Pair | d (m/s) | d_{prop} (m/s) | a (AU) | e | i (deg) | H_1 | ΔH | P_2/N_p | l.t.d. |
|-----------|-----------|-----------|-------------------------|----------|--------|-----------|-------|------------|-----------|-----------|
| 138938 | 2003SL308 | 19.81 | 185.93v | 2.1794 | 0.1105 | 5.592 | 16.03 | 1.07 | 0.0375 | ok (b) |
| 54041 | 2002TO134 | 14.25 | 0.56 | 2.3234 | 0.1316 | 5.515 | 14.43 | 1.86 | 0.0425 | ok |
| 19289 | 2006YY40 | 25.72 | 6.31v | 2.1197 | 0.1262 | 1.632 | 15.25 | 2.33 | 0.0444 | ok |
| 31569 | 21321 | 25.42 | 141.83 | 2.3143 | 0.1223 | 6.405 | 14.00 | 0.24 | 0.0510 | spurious |
| 101065 | 2002PY103 | 22.53 | 1.34 | 2.3782 | 0.2224 | 8.513 | 15.73 | 1.80 | 0.0543 | ok |
| 88604 | 60546 | 17.09 | 1.61 | 2.6667 | 0.1085 | 11.779 | 13.10 | 1.31 | 0.0548 | ok |
| 90757 | 170236 | 19.49 | 30.93 | 2.3677 | 0.1542 | 4.606 | 15.84 | 0.39 | 0.0553 | spurious |
| 2005UJ157 | 2001OT77 | 28.53 | 356.73v | 2.2967 | 0.2801 | 5.908 | 16.29 | 1.37 | 0.0564 | spurious |
| 92652 | 194083 | 24.88 | 0.15 | 2.3408 | 0.0744 | 2.231 | 15.11 | 1.53 | 0.0570 | ok |
| 103483 | 179863 | 20.90 | 38.79v | 2.3638 | 0.1502 | 4.088 | 15.76 | 0.01 | 0.0592 | spurious |
| 7343 | 154634 | 18.95 | 19.91v | 2.1921 | 0.1397 | 3.959 | 13.82 | 2.94 | 0.0593 | ok (b, c) |
| 74043 | 111919 | 26.52 | 93.70v | 2.2980 | 0.0599 | 8.291 | 15.30 | 0.69 | 0.0595 | spurious |
| 22280 | 2005YN176 | 15.03 | 2.80v | 3.0387 | 0.1970 | 0.565 | 13.70 | 2.80 | 0.0601 | ok |
| 52852 | 2003SC7 | 19.44 | 1.51v | 2.2629 | 0.1113 | 7.264 | 14.57 | 1.99 | 0.0601 | ok |
| 60744 | 2002MH3 | 27.05 | 8.18 | 2.3084 | 0.2381 | 7.613 | 14.69 | 1.07 | 0.0620 | ok |
| 150202 | 2007VC55 | 17.51 | 33.42v | 2.3183 | 0.1591 | 4.370 | 16.46 | 0.57 | 0.0659 | spurious |
| 157576 | 120638 | 19.51 | 34.68 | 2.3456 | 0.0880 | 4.614 | 16.55 | 0.19 | 0.0668 | spurious |
| 106598 | 2000AH207 | 20.92 | 299.01v | 2.2207 | 0.0598 | 6.031 | 15.24 | 1.37 | 0.0679 | ok (c) |
| 10123 | 117306 | 23.92 | 20.04v | 2.2696 | 0.2057 | 4.053 | 14.04 | 2.23 | 0.0705 | ok (d) |
| 31507 | 98212 | 23.27 | 51.93 | 2.2765 | 0.1005 | 5.066 | 14.82 | 0.55 | 0.0716 | spurious |
| 52478 | 2003QA18 | 15.17 | 4.51 | 2.2100 | 0.1577 | 3.747 | 15.38 | 1.71 | 0.0727 | ok |
| 2000SM320 | 2008TN44 | 24.65 | 1.38 | 2.5993 | 0.1557 | 17.555 | 16.87 | 0.74 | 0.0752 | ok |
| 139537 | 2001SR218 | 21.25 | 6.61v | 2.6849 | 0.2791 | 7.040 | 14.84 | 1.52 | 0.0862 | ok |
| 194561 | 2008VR13 | 21.97 | 15.41v | 2.4035 | 0.2075 | 6.175 | 16.27 | 0.83 | 0.0866 | spurious |
| ... | | | | | | | | | | |
| 2005QC62 | 2005VC55 | 19.53 | 11.65v | 2.7644 | 0.1117 | 3.677 | 16.75 | 0.19 | 0.1462 | ok |
| 51161 | 2001BN16 | 27.36 | 391.66v | 2.6017 | 0.1734 | 5.839 | 15.00 | 0.88 | 0.1559 | ok (e) |
| 118645 | 2007DK79 | 17.80 | 6.87v | 2.3316 | 0.1700 | 2.919 | 16.14 | 1.53 | 0.1735 | ok |
| 13046 | 2003QS31 | 28.66 | 1.59v | 2.5514 | 0.2629 | 2.956 | 14.71 | 2.45 | 0.1908 | ok |
| 2003UQ164 | 2003SB338 | 26.29 | 4.86 | 2.5890 | 0.1875 | 9.262 | 16.01 | 0.89 | 0.1974 | ok |
| 76148 | 56048 | 32.27 | 0.98 | 2.3892 | 0.0764 | 7.992 | 14.82 | 0.17 | 0.2137 | ok |
| 128637 | 2001WR54 | 35.10 | 1.65 | 2.6596 | 0.2169 | 1.913 | 15.71 | 1.05 | 0.2236 | ok |
| 42946 | 165548 | 21.25 | 35.23v | 2.5676 | 0.0734 | 4.686 | 13.52 | 1.95 | 0.2252 | ok (b) |
| 64092 | 130179 | 35.26 | 9.27v | 2.7068 | 0.1898 | 4.350 | 14.60 | 1.15 | 0.2318 | ok |
| 101703 | 142694 | 33.90 | 13.82v | 2.5427 | 0.1288 | 2.577 | 15.07 | 1.94 | 0.2381 | ok |
| 2003SG111 | 2001AV49 | 34.34 | 3.49 | 2.6445 | 0.2907 | 10.676 | 16.25 | 0.04 | 0.2394 | ok |
| 2000YY25 | 2001BD4 | 30.58 | 12.90 | 2.5788 | 0.1595 | 10.677 | 16.01 | 0.10 | 0.2435 | ok |
| 40837 | 2005UL431 | 34.07 | 2.74 | 2.6770 | 0.0608 | 3.239 | 14.67 | 2.26 | 0.2626 | ok |
| 34380 | 2006TE23 | 28.71 | 3.35 | 2.6582 | 0.1022 | 2.246 | 14.88 | 1.91 | 0.2661 | ok |
| 165389 | 2001VN61 | 19.73 | 1.91 | 2.3087 | 0.1114 | 6.695 | 16.30 | 0.27 | 0.2872 | ok |

Notes: (a) the computation of proper elements was corrupted by perturbations from Mars, (b) by irregular jumps over a weak mean-motion resonance, (c) by interaction with the z_2 secular resonance, (d) by irregular jumps over the 7:2 mean-motion resonance with Jupiter, (e) by interaction with the $g + g_5 - 2g_6$ secular resonance. Spurious pairs with $P_2/N_p > 0.1$ are not listed, they are available in the file mentioned in Sect. 5.

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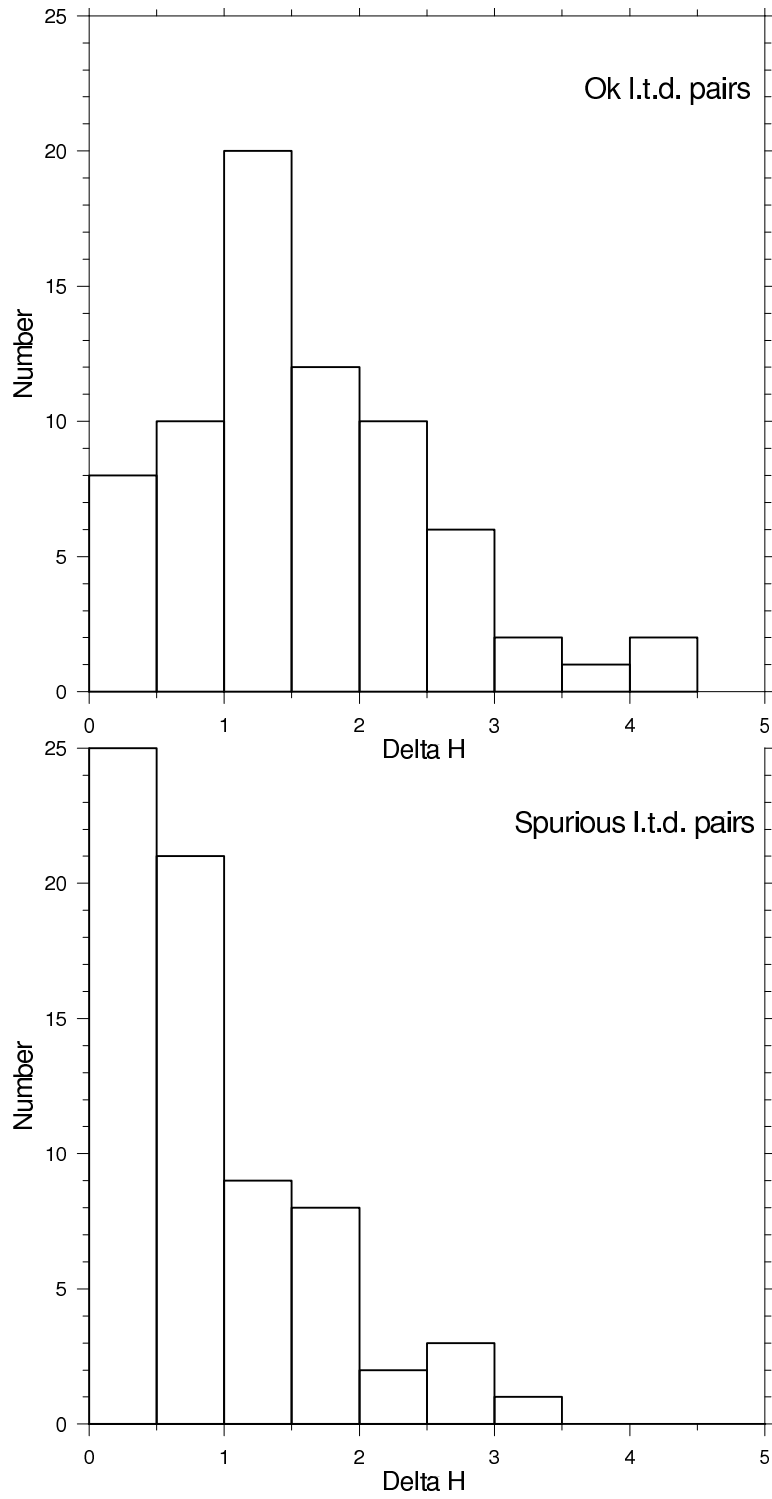


Fig. 1. Distributions of differences of absolute magnitudes $\Delta H = H_2 - H_1$ of ok pairs (upper panel) and spurious pairs (lower panel).