RADIATIVE TRANSFER IN AXIAL SYMMETRY

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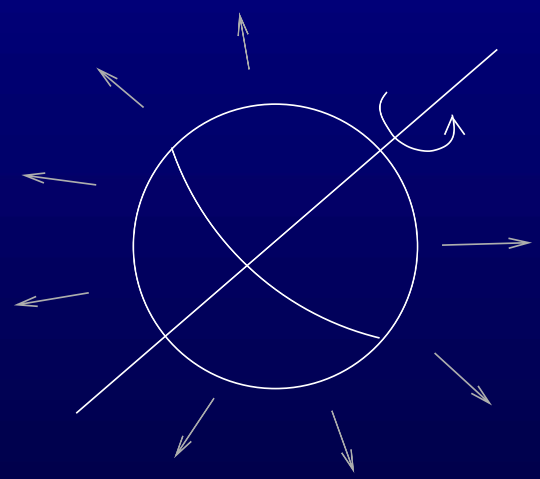
outline

- why axial symmetry?
- description of the method
- examples
  - limb darkening
  - stellar rotation
  - stellar wind
  - accretion disc



WHY AXIAL SYMMETRY?

- stellar rotation
  - rapidly rotating stars – far from spherical symmetry (α Eri – $R_{pole}/R_{equator} \sim 1/2$)
  - limb darkening and gravity darkening naturally included
- possible to include the anisotropy of stellar wind
- axially symmetric planetary nebulae and accretion discs (without a hot spot)

DESCRIPTION OF THE METHOD

– axial symmetry

– LTE (NLTE)

– hydrogen

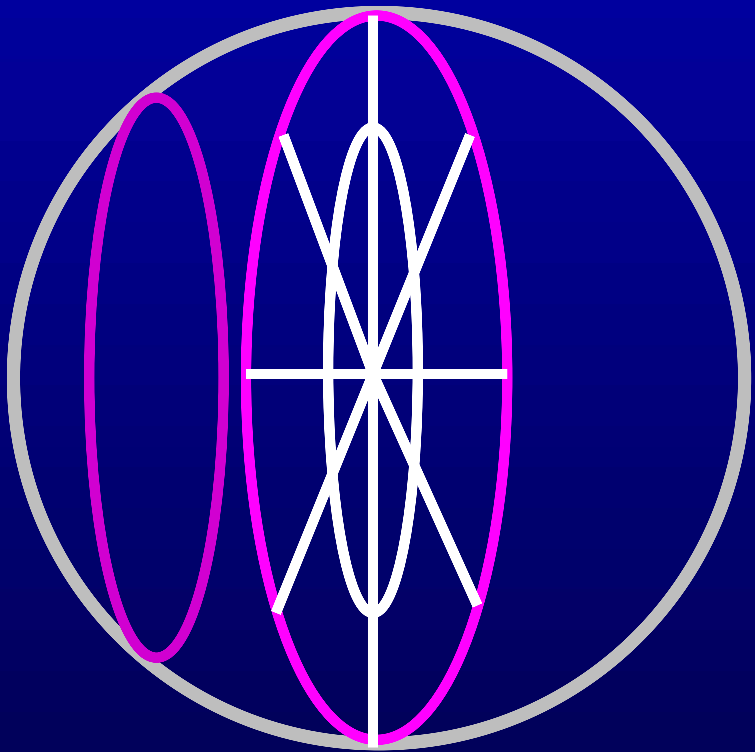
– input – $n_e(r,\theta)$, $T(r,\theta)$, v(r,θ)

– output – line profile, intensity map

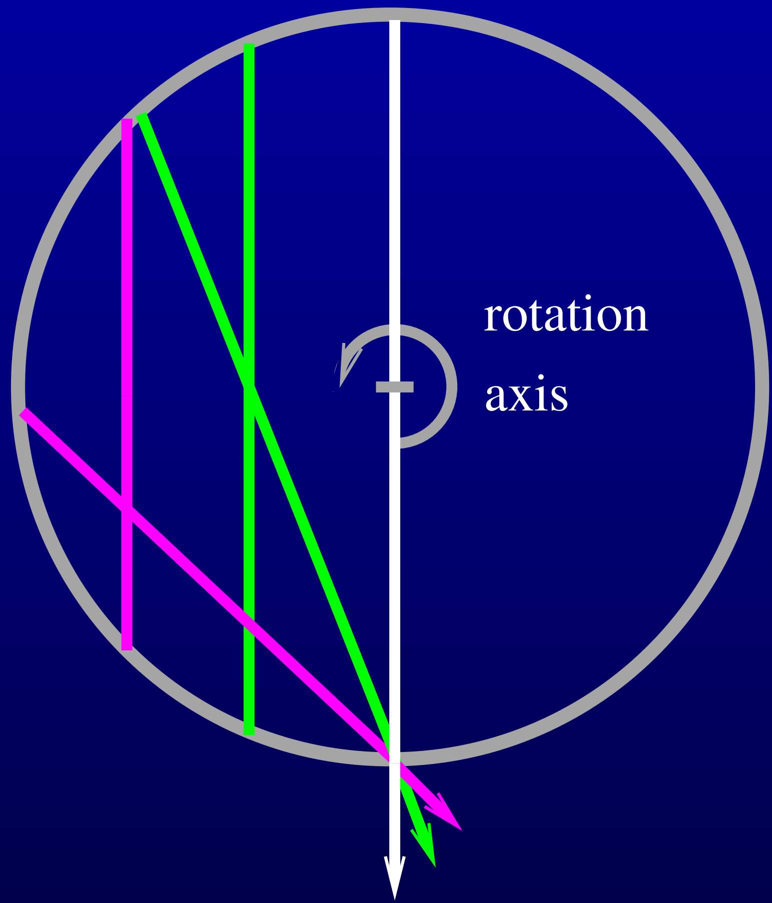
basic idea

- solution of the radiative transfer equation in separated planes
- polar coordinates in every plane
- “extended” short characteristic – combination of the short and long characteristics
- velocity field – Lorentz invariant of RTE

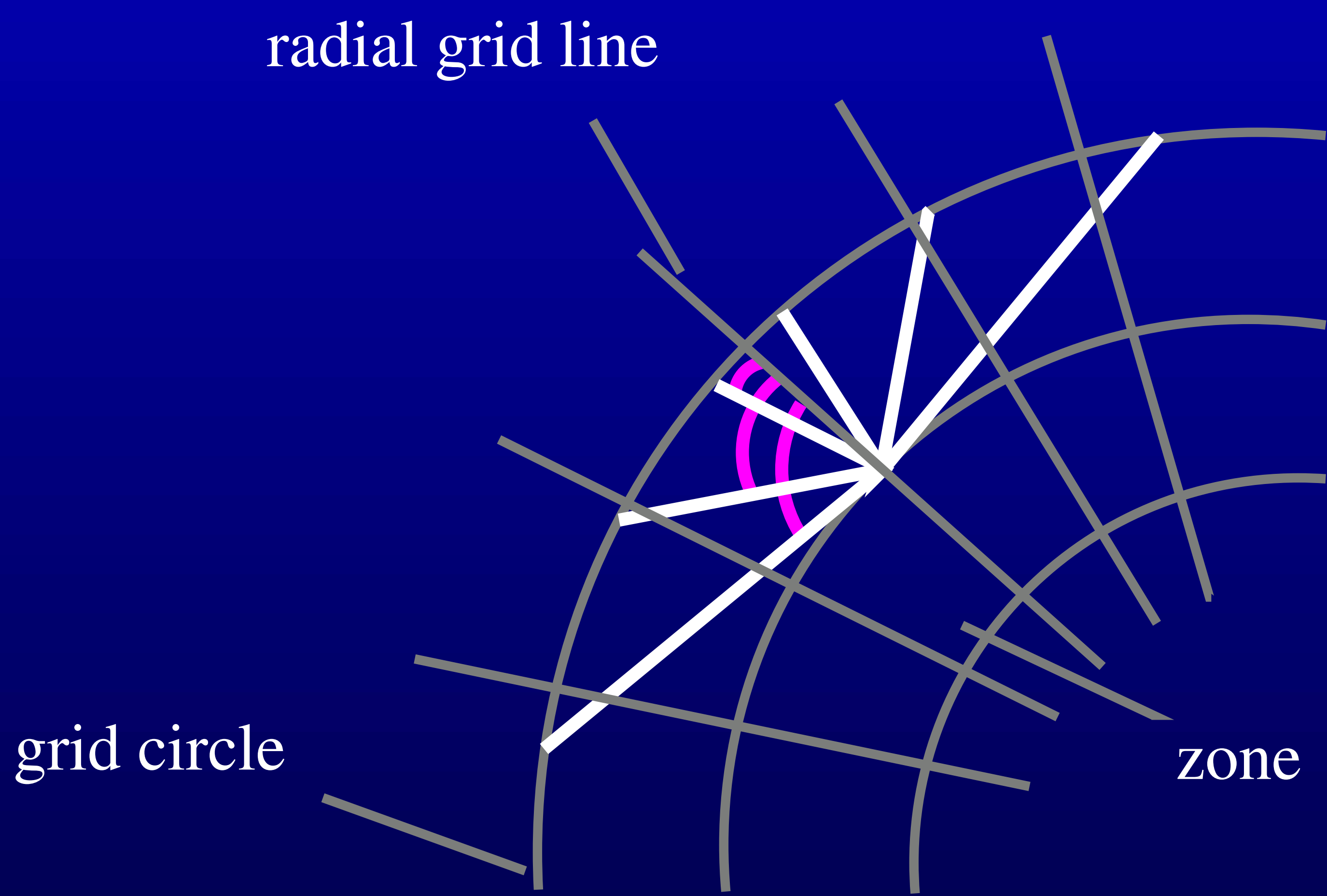
longitudinal planes



whole radiation field



upper boundary condition:



integration

$$I_{(B)} = I_{(A)} e^{-\Delta\tau_{(AB)}} + \int_0^{\Delta\tau_{(AB)}} S(t) e^{[-(\Delta\tau_{(AB)} - t)]} dt$$

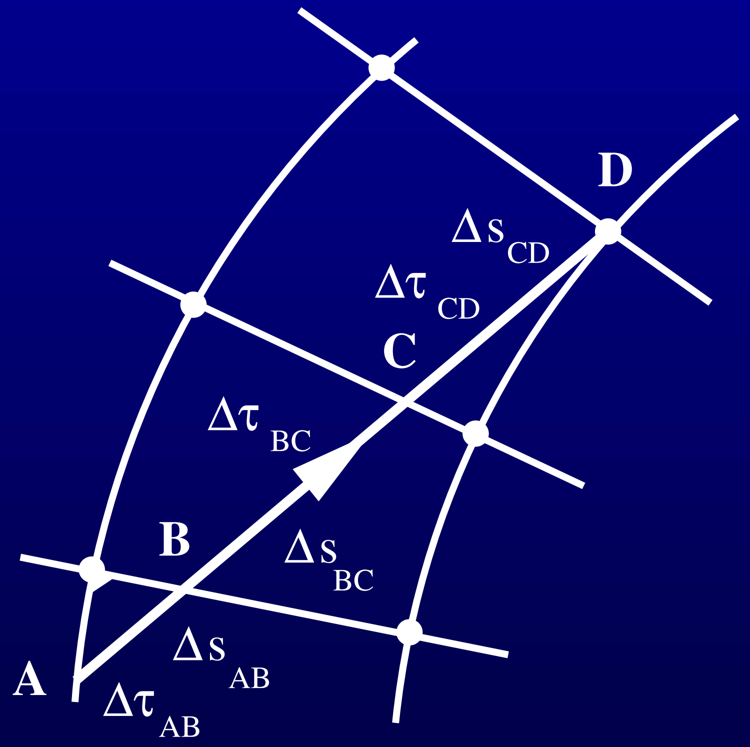
S(t):

bound-bound

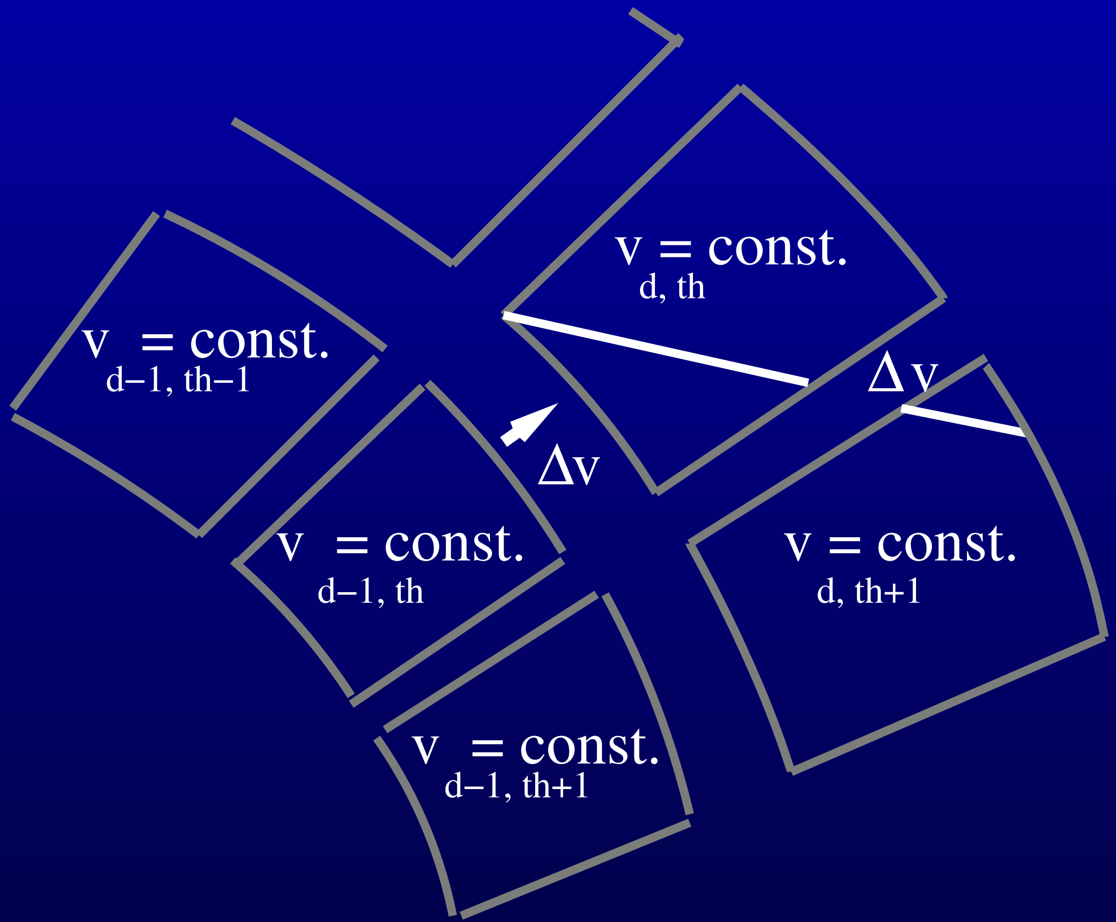
bound-free

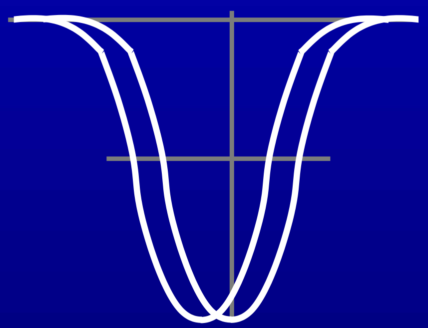
free-free

Thomson scattering

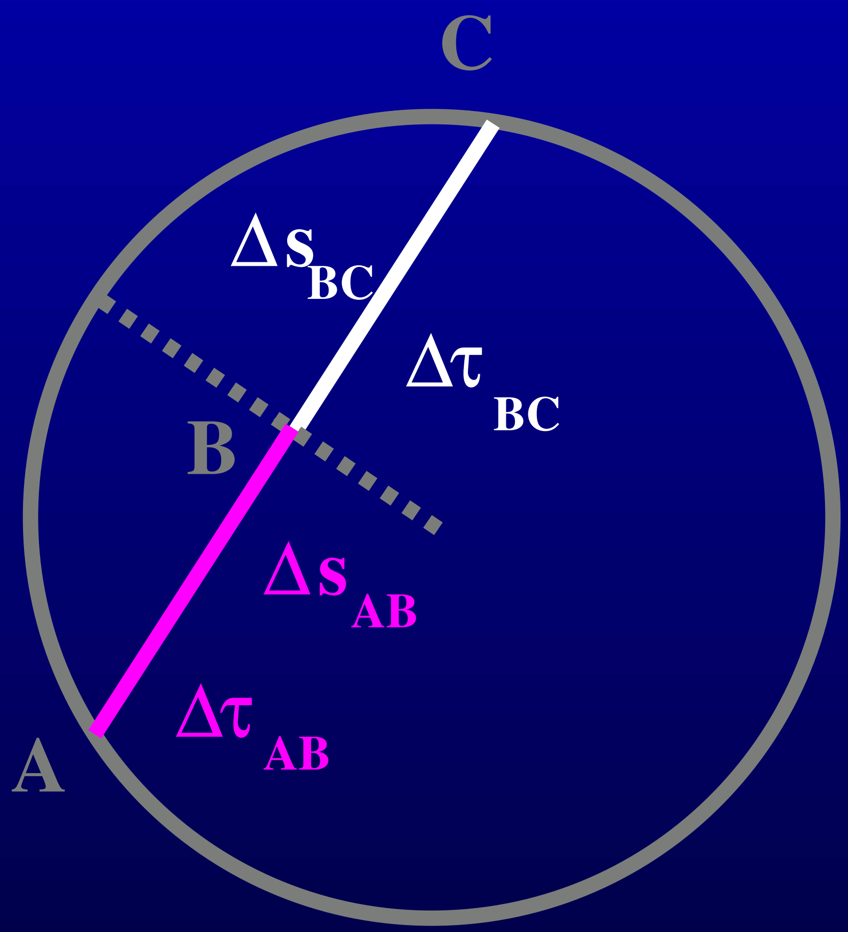


velocity field

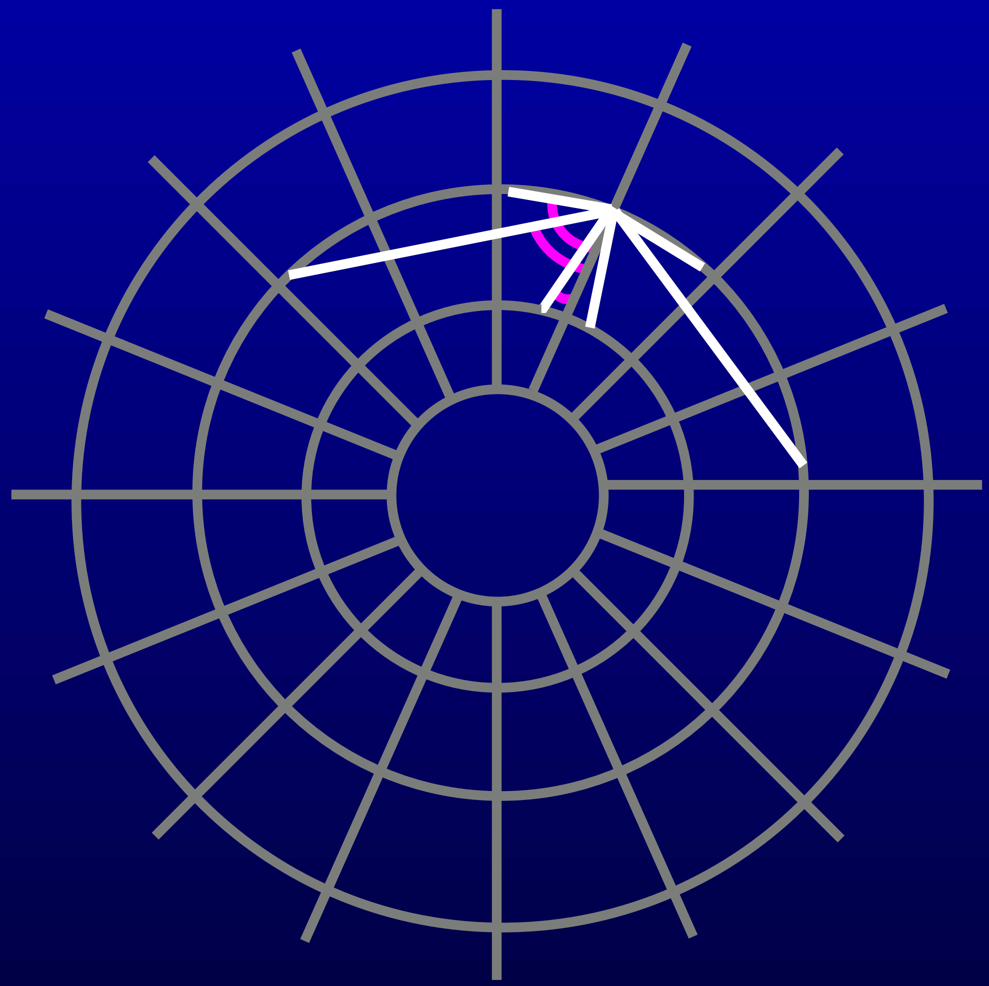




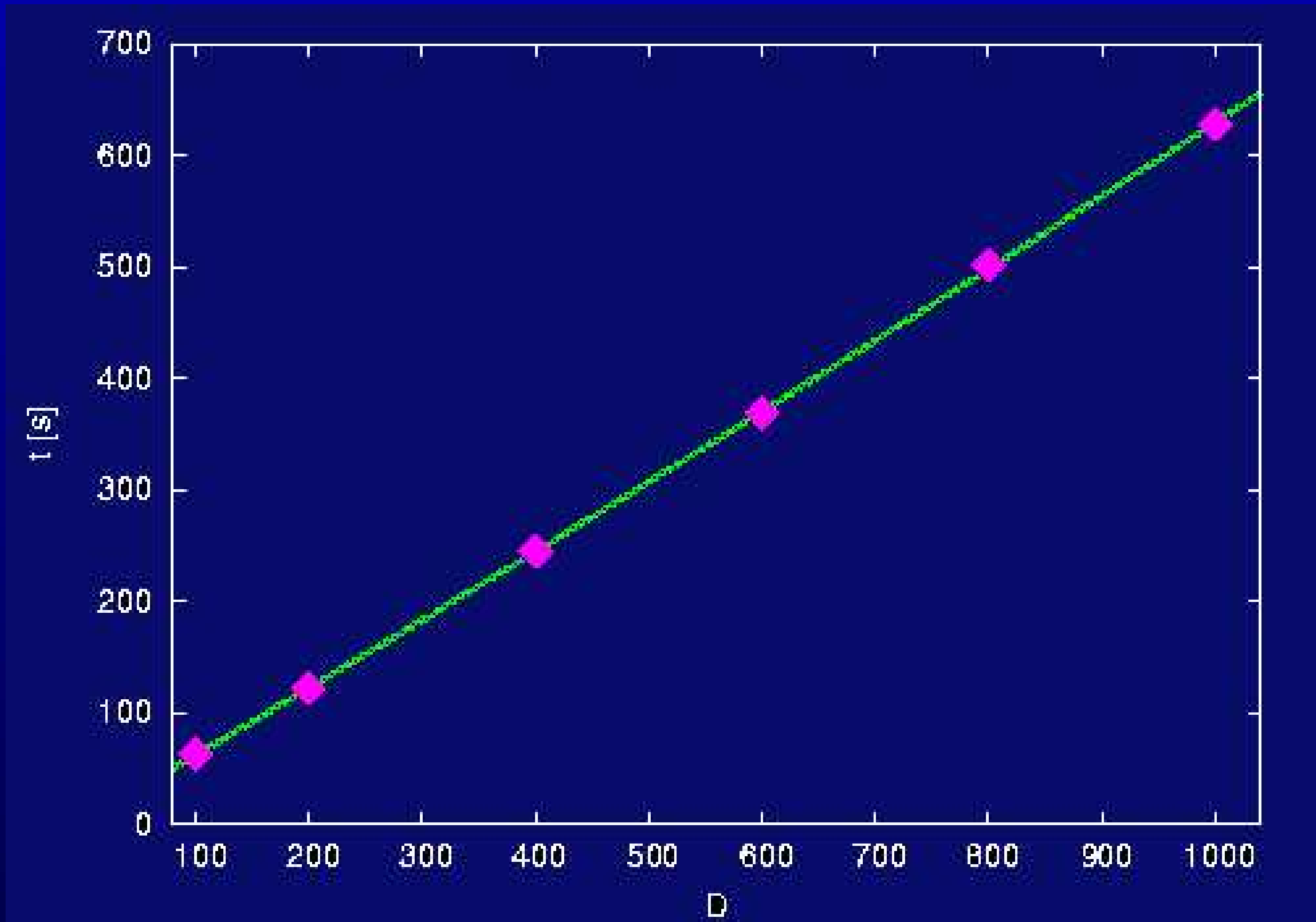
solution in the central region:



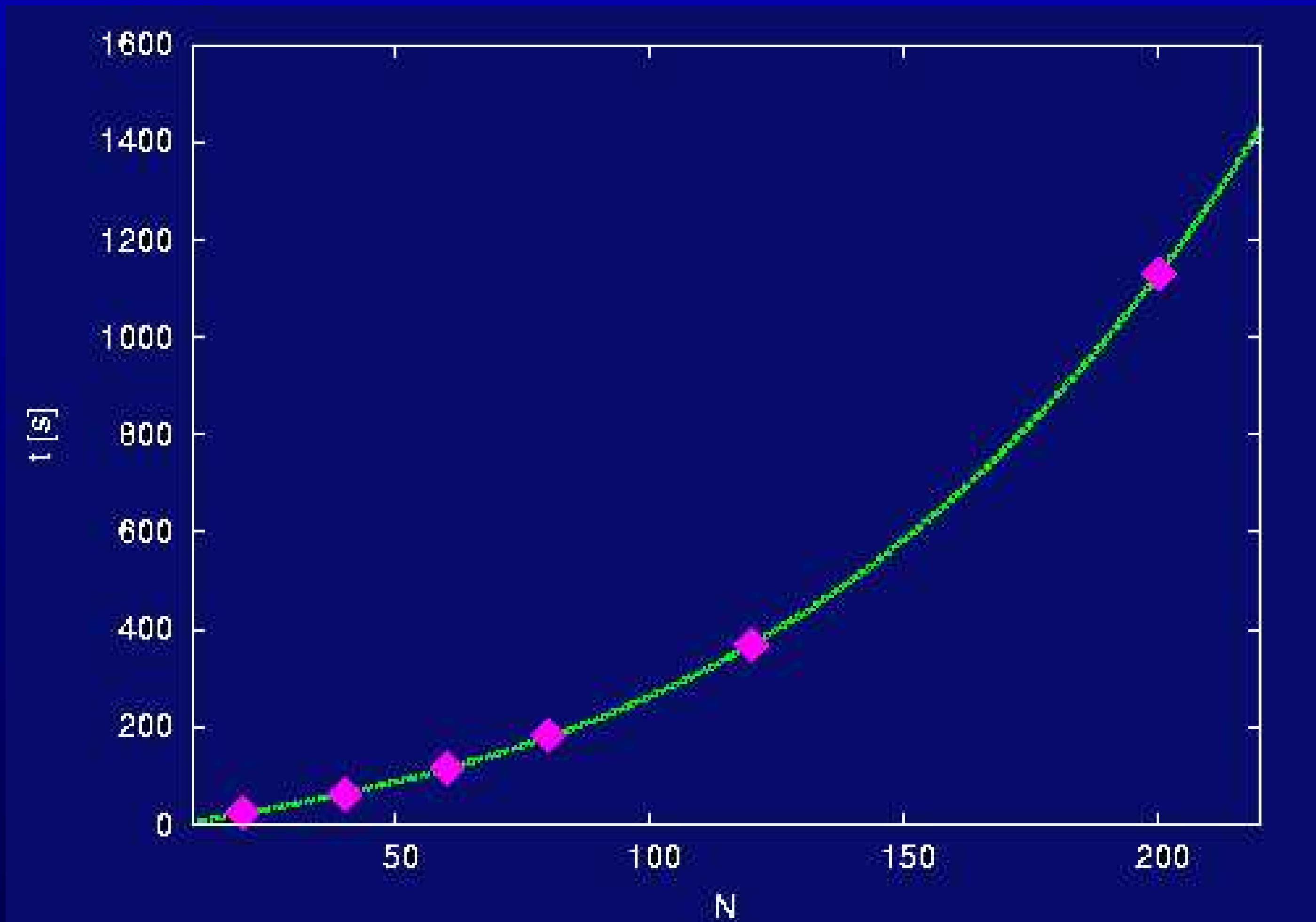
lower boundary condition:



computing time $= f$ (number of depth points)



computing time $= f$ (number of frequency points)



advantages

- better description of the global character of the radiation field than by the short characteristic method
- not so time consuming as for the long characteristic method
- arbitrary velocity field

disadvantages

- high velocity gradients $\Longrightarrow$ finer grid is necessary

usage of this method

- limb darkening
- stellar rotation – gravity darkening, differential rotation
- stellar wind – polar and equatorial region together
- accretion discs – hot corona, jet, central object, boundary region, optically thin or thick disc
- axially symmetric planetary nebulae

EXAMPLES

- a main sequence star

  $T_e = 17\,000K,\, log\ g = 4.12,\, R_* = 3.26R_\odot$

- the lower boundary condition –

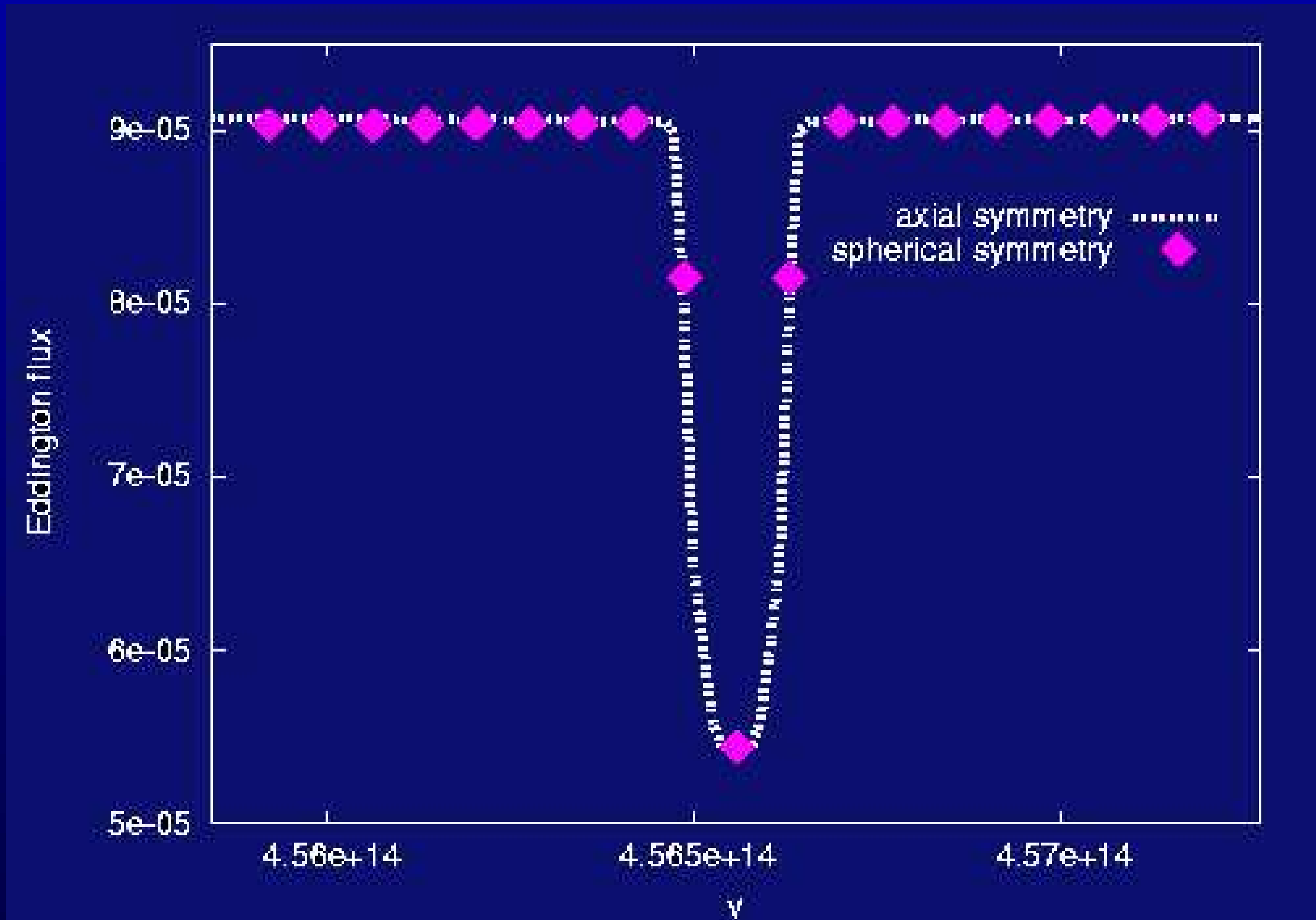
  the diffusion approximation

- the upper boundary condition –

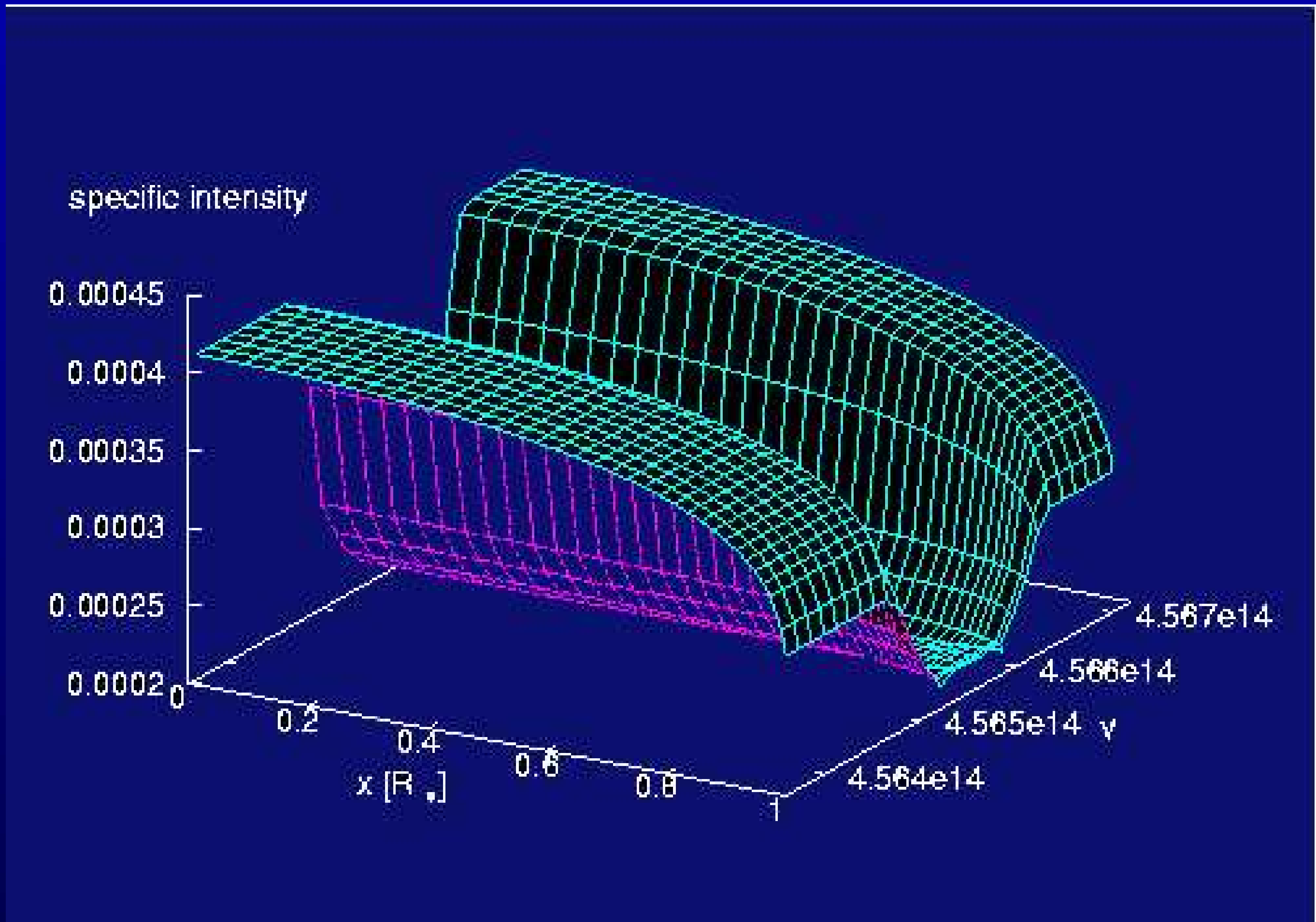
  no incoming radiation

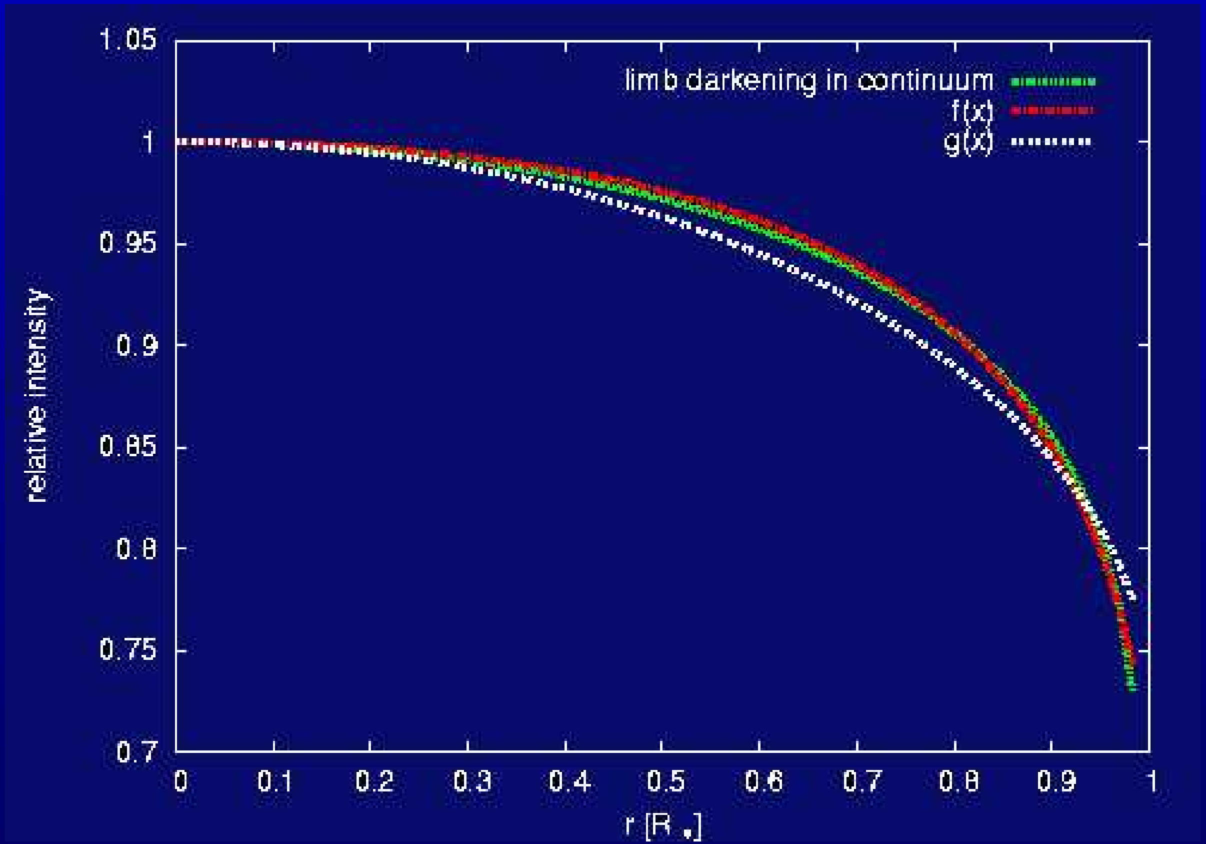
axial symmetry × spherical symmetry

(Kubát, 1994, A&A, 287, 179)



limb darkening



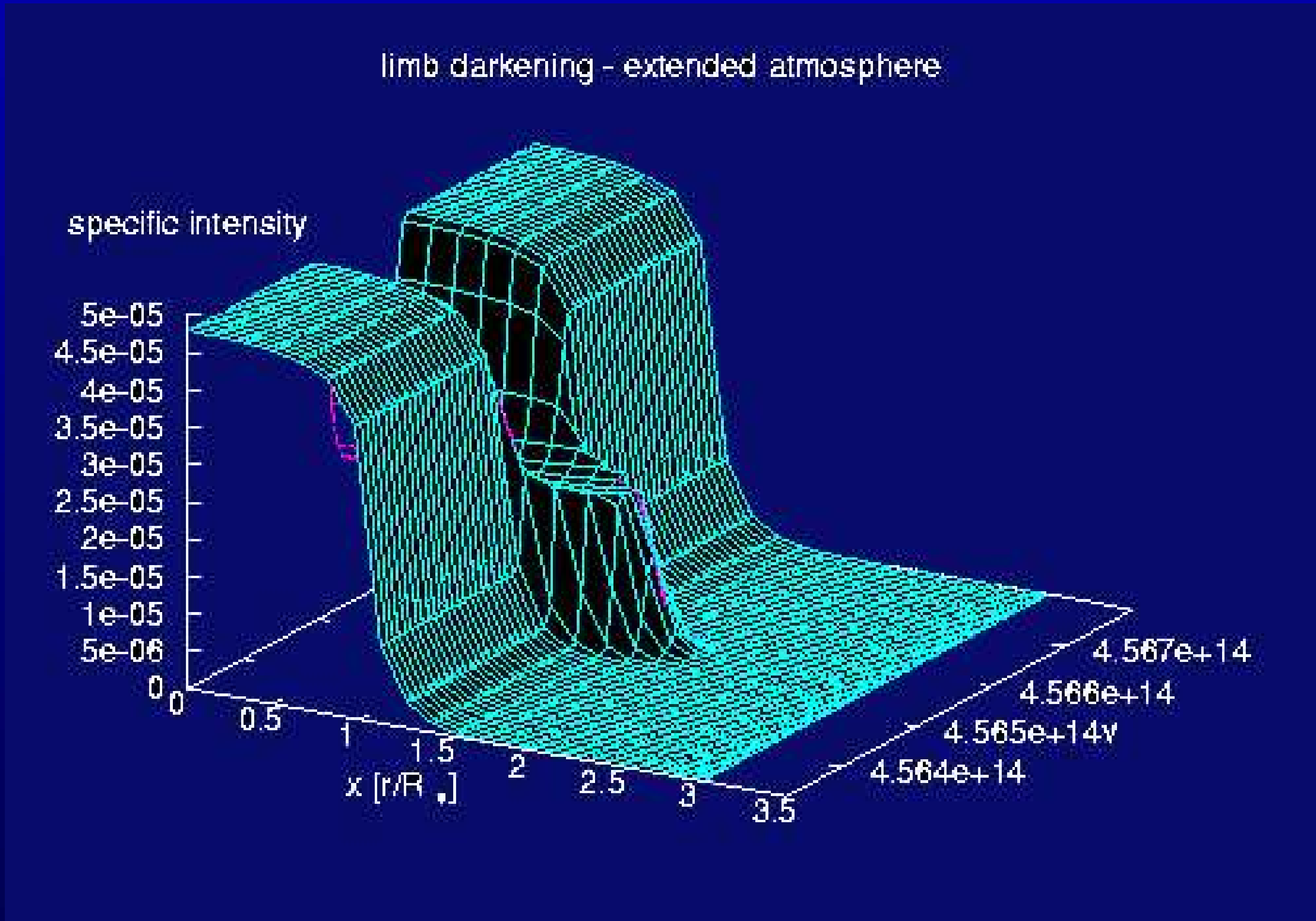


$I(x) = 1.0 - a - b + a(1.0 - r^2)^{1/2} + b(1.0 - r^2)$ $a = 0.55 \pm 0.02$

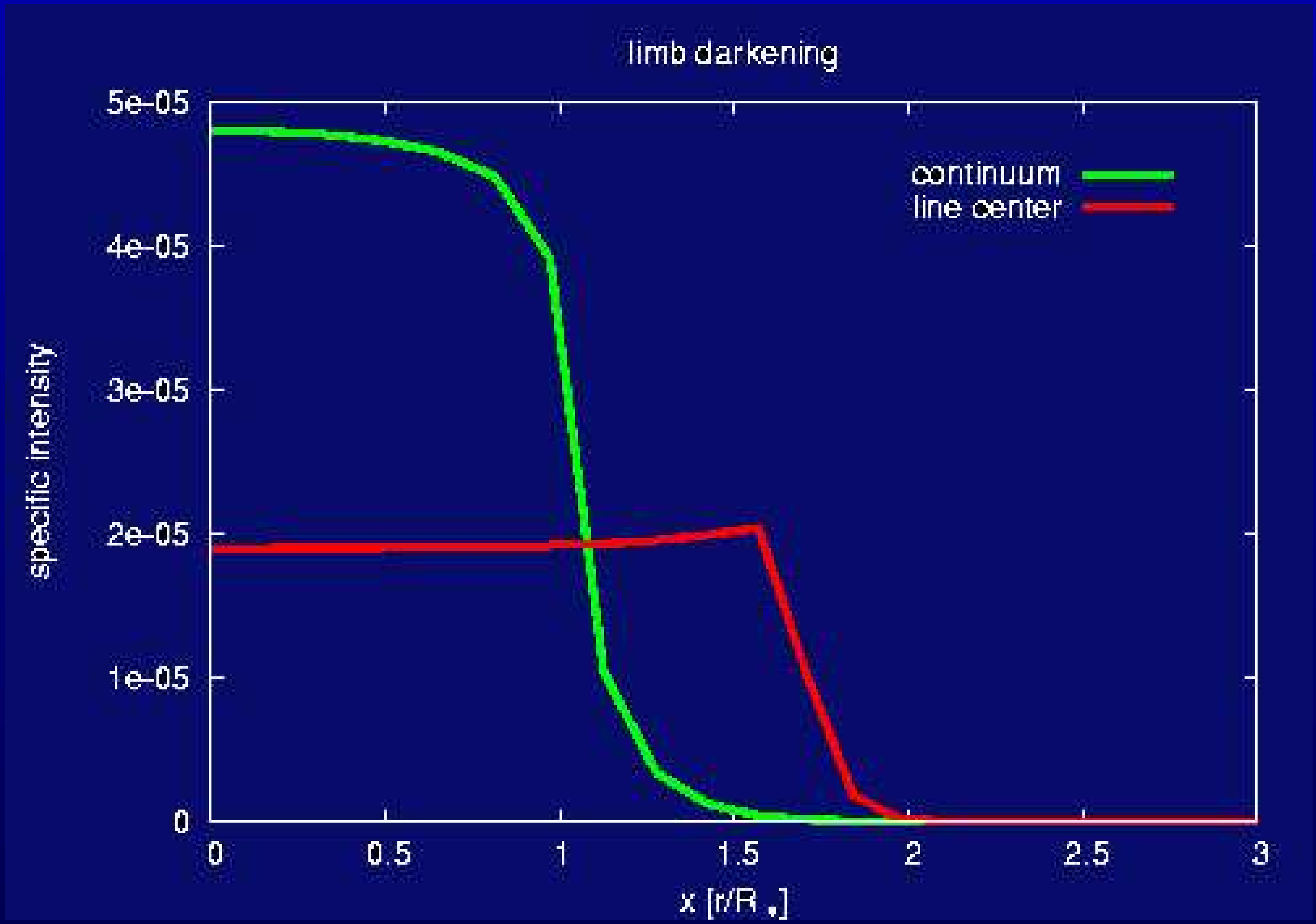
$b = -0.20 \pm 0.01$

$I(x) = (1.0 - \epsilon) + \epsilon(1.0 - r^2)^{1/2}$ $\epsilon = 0.277 \pm 0.008$

extended atmosphere



limb darkening for an extended atmosphere



stellar wind

– velocity law for stellar wind

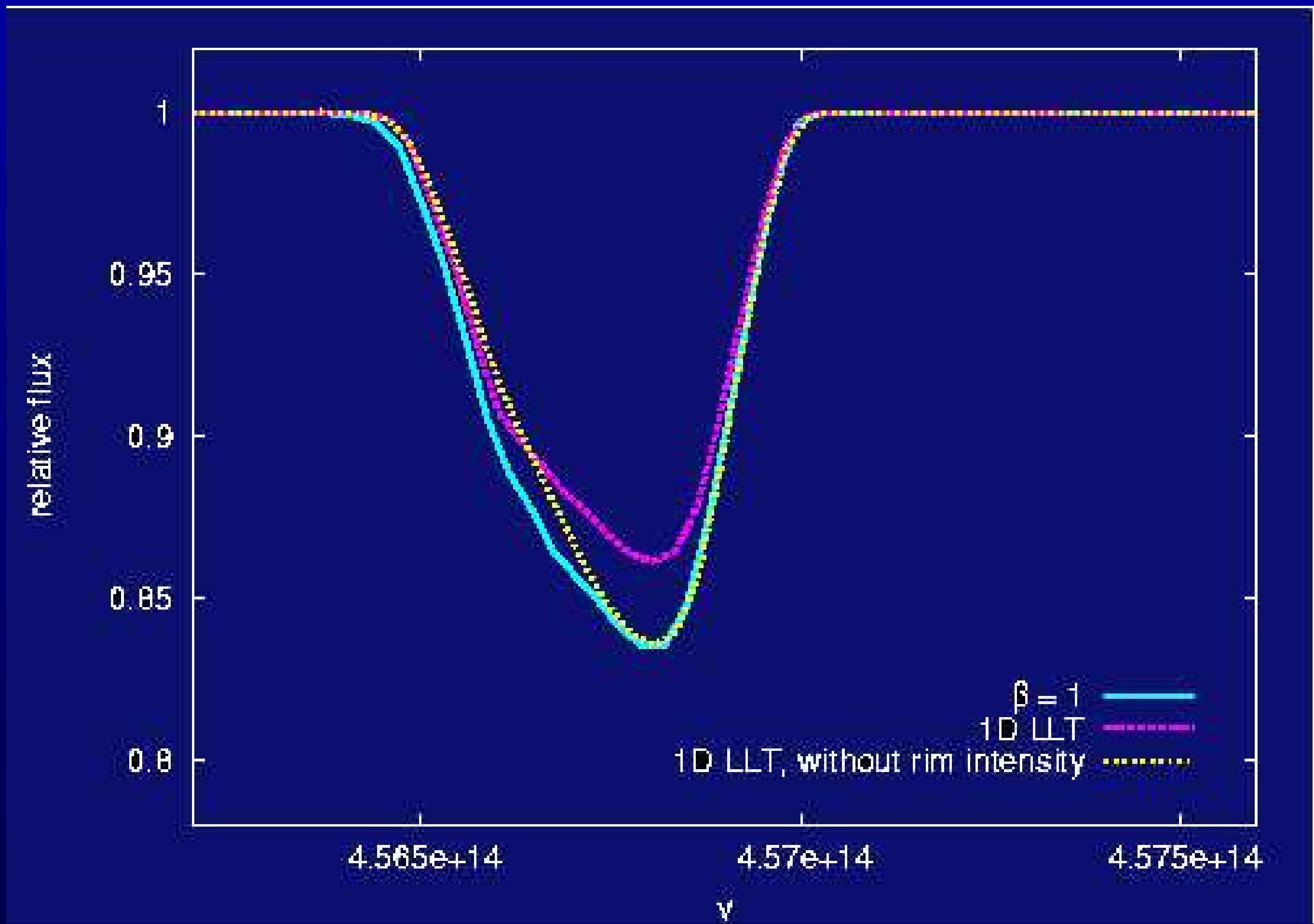
$$v(r) = v_\infty \left\{ 1 - \left[1 - \left(\frac{v_R}{v_\infty} \right)^{\frac{1}{\beta}} \right] \frac{R}{r} \right\}^\beta$$

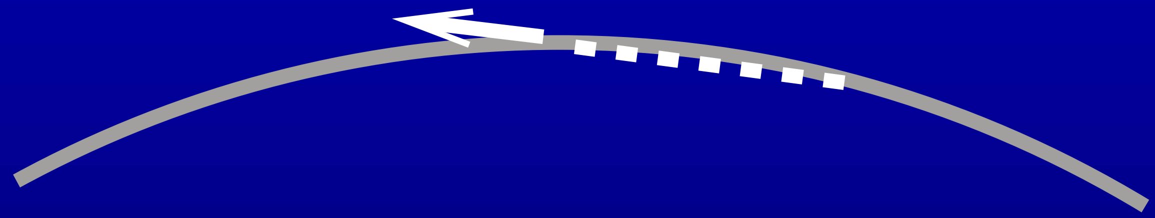
$$v_R = 200\,\mathrm{km} \cdot \mathrm{s}^{-1}, v_\infty = 2000\,\mathrm{km} \cdot \mathrm{s}^{-1}, \beta = 1$$

– decelerating velocity field – linear dependence of the velocity on the radial distance in logarithmic scale

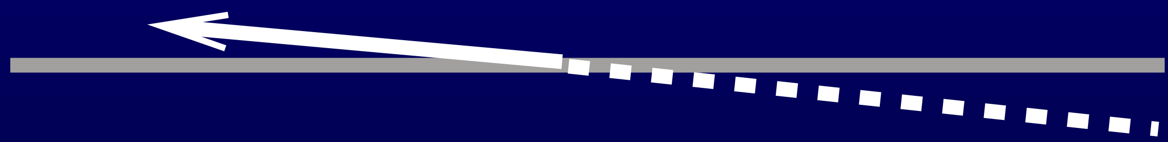
axial symmetry × plane parallel geometry

(Korčáková, D. & Kubát J., 2003, A&A, 401, 419)



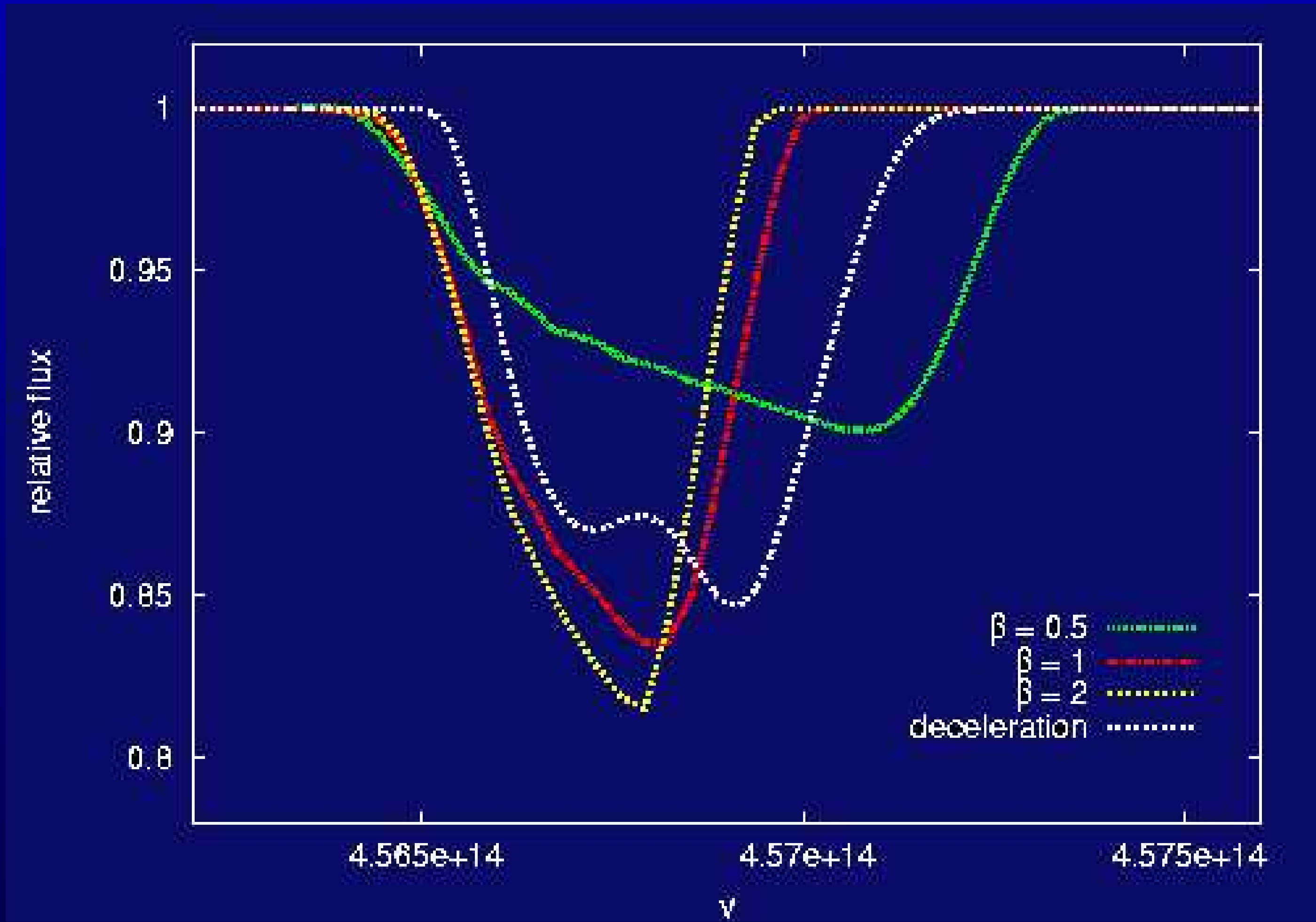


spherical symmetry



plane–parallel

stellar wind – beta law and decelerating velocity field



stellar rotation

- hydrostatic model
- rotation velocity

$$v_{rot}(r) = v_{rot}(R)\left(\frac{r}{R}\right)^{-j} \quad \text{where} \quad j = 1$$

- differential rotation

$$\omega(\theta) = \omega_{equator}\,(1\, -\, \alpha\, cos^2\theta)\, sin\,\theta \quad \text{where} \quad \alpha = 0.6$$

- gravity darkening

von Zeipel theorem $\Longrightarrow$ $T_{eff} \sim g^{\beta} \qquad \beta = 1/4$

gravitation potential

$$\phi = \frac{G\,M}{r} + \frac{1}{2}\,\omega^2_{equator}(\,1 - \,\alpha\,cos^2\theta)\,r^2$$

– density distribution due to the variation of the stellar radius

$$\frac{G\,M}{r} + \frac{1}{2}\,\omega^2_{equator}(\,1 - \,\alpha\,cos^2\theta)\,r^2 = \frac{G\,M}{r_{pol}}$$

- 4Her

  $T_{eff} = 12500K,\ R = 2.6\,R_{\odot},\ M = 3.2\,M_{\odot},$

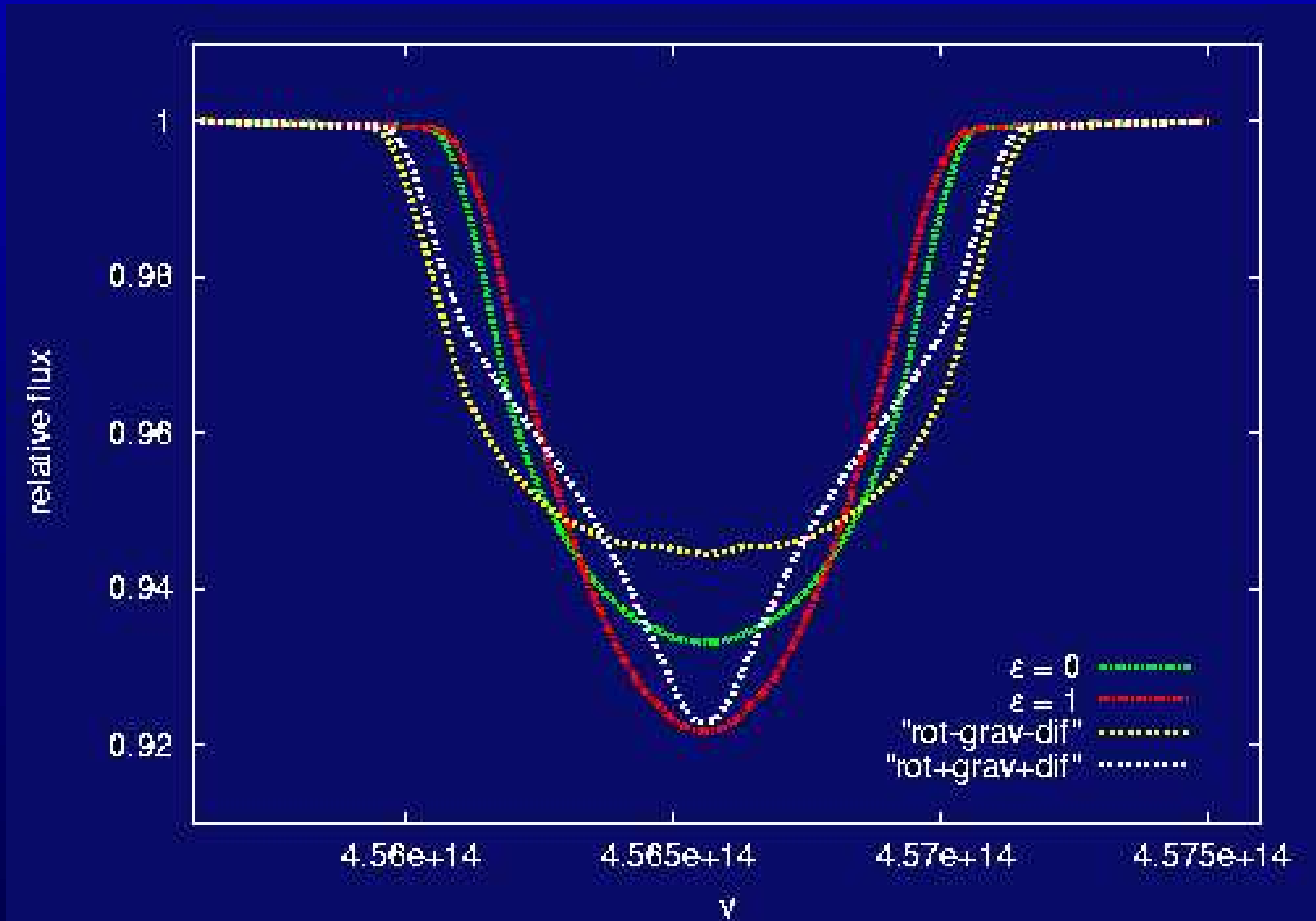
  $log\,g = 4.11,\ \mathrm{v}_{rot} = 0.8\mathrm{v}_{crit}$

- ψ Per

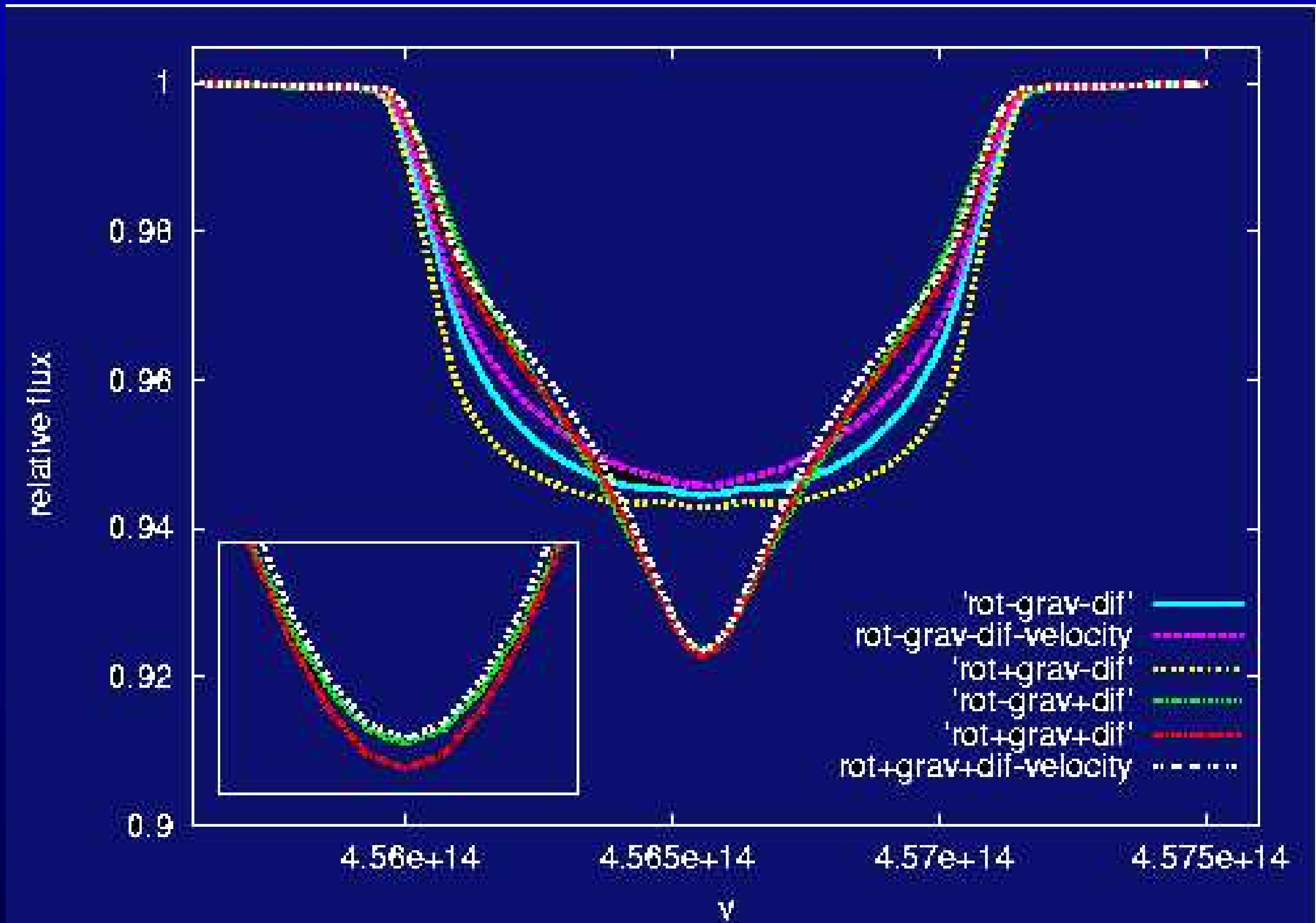
  $T_{eff} = 14500K,\ R = 6.0\,R_{\odot},\ M = 4.1\,M_{\odot},$

  $log\,g = 3.49,\ \mathrm{v}_{rot} = 0.8\mathrm{v}_{crit}$

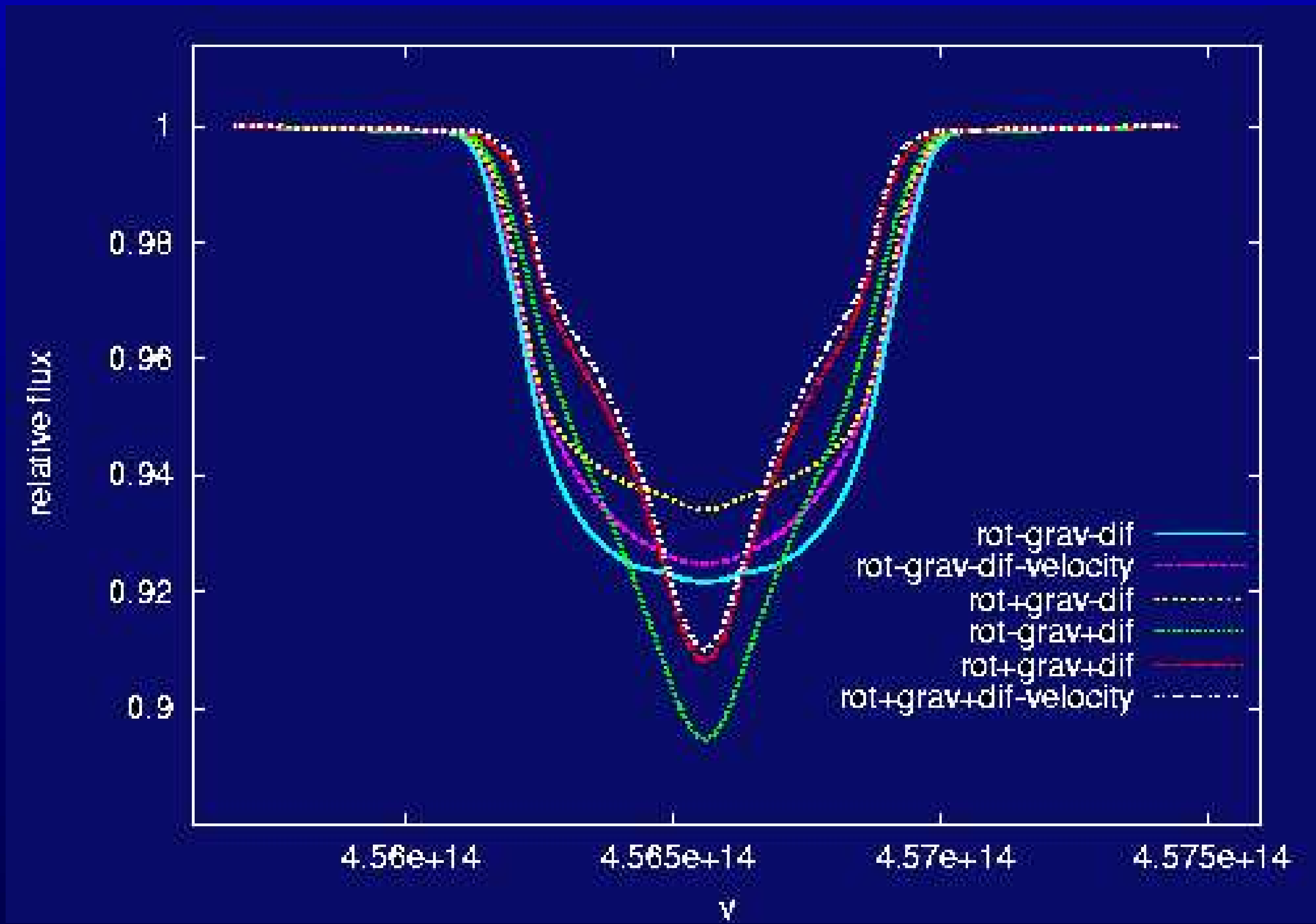
4 Her convolution, $\text{v}_{rot} = 0.8\text{v}_{crit}$



4 Her $\mathrm{v}_{rot} = 0.8\mathrm{v}_{crit}$



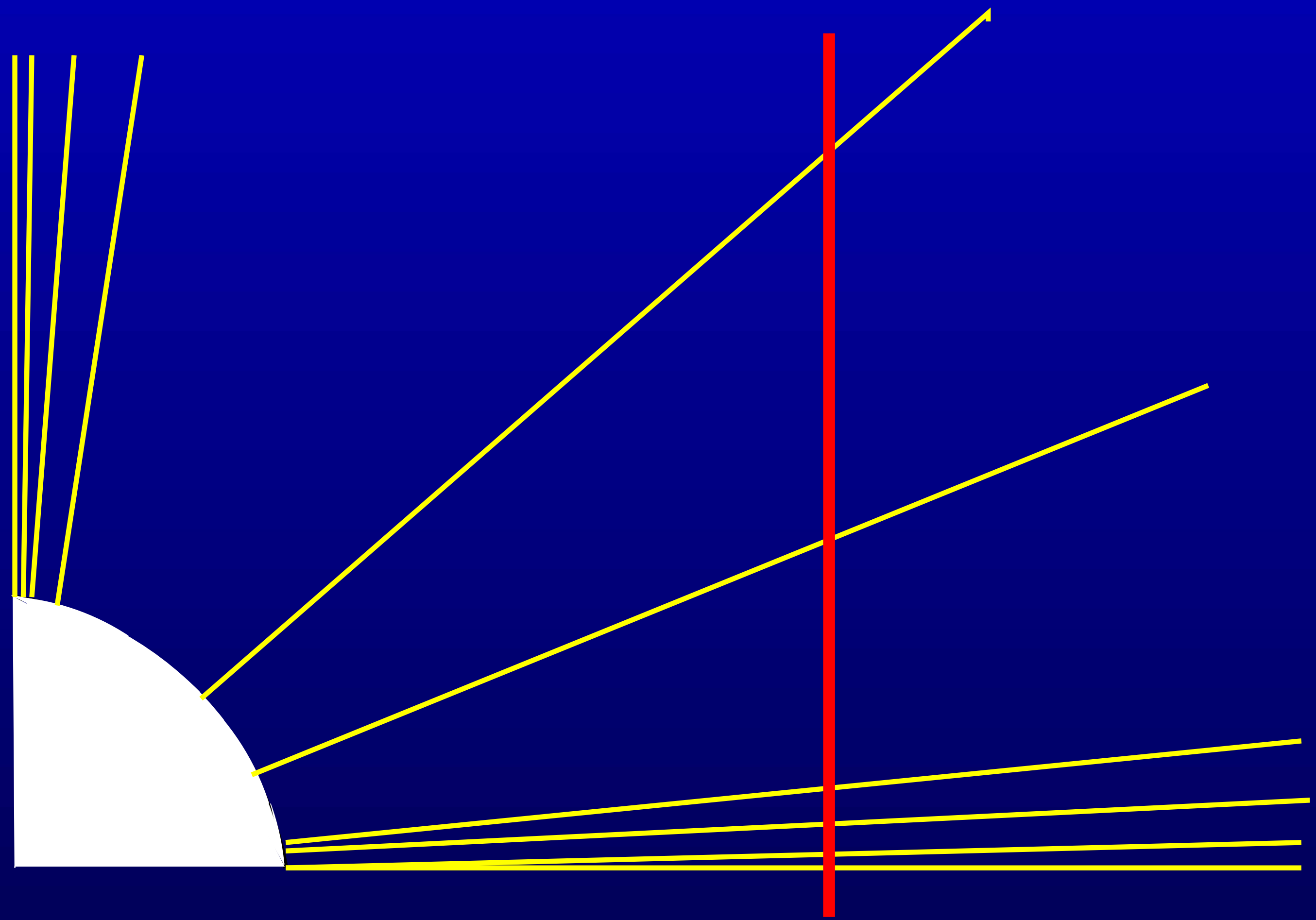
ψ Per $\quad v_{rot} = 0.8 v_{crit}$

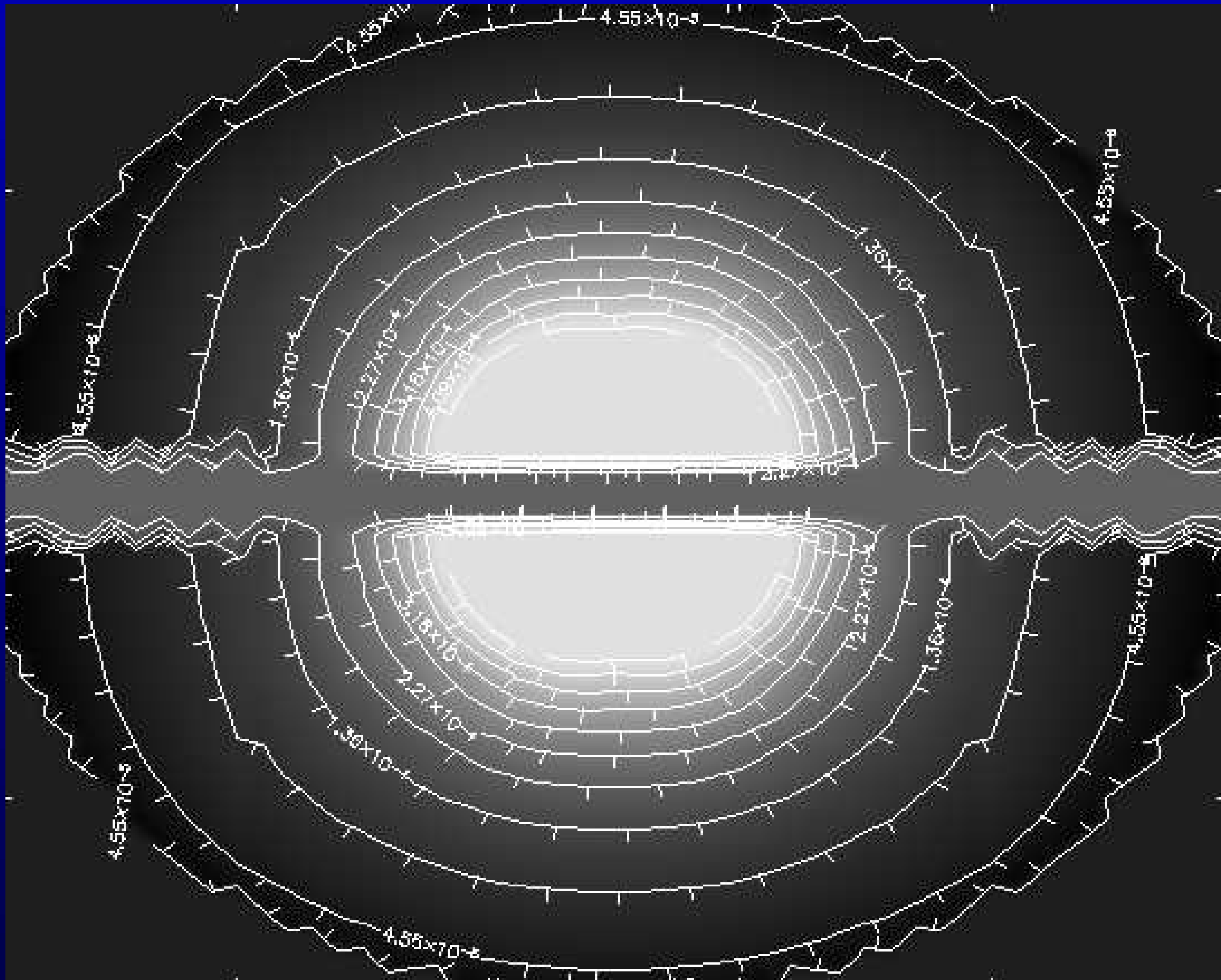


accretion disc

HT Cas – cataclysmic variable

- white dwarf
- optically thick disc
- hot corona
- wind in the polar region
- boundary region




4.55×10-5
1.36×10-4
2.27×10-4
1.36×10-4
2.27×10-4
1.36×10-4
4.55×10-5

CONCLUDING REMARKS

- method
  - axial symmetry
  - solution of the radiative transfer equation in separated planes
  - “extended” short characteristic
  - velocity – Lorentz transformation
- limb darkening

- stellar rotation
  - differential rotation
  - gravity darkening
- stellar wind
  - photospheric region + wind region
  - polar and equatorial wind, A, Be, B[e] stars
- discs
  - central object + boundary region + hot corona + polar wind + optically thick or thin disc