Stationary stagnation point flows in the vicinity of a 2D magnetic null point

Dieter Nickeler

Astronomical Institute, Ondřejov

in collaboration with

Marian Karlický, Astronomical Institute, Ondřejov

Miroslav Bárta, Astronomical Institute, Ondřejov

Overview

- Motivation and Introduction: Solutions of the resistive MHD equations in the vicinity of (X-type) null points, corresponding geometries and topologies of flow and field lines, ‘allowed’resistivities

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- Preliminary solutions, discussion and conclusions

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- Assumptions and basic MHD equations: First conclusions
- Preliminary solutions, discussion and conclusions
- Problems and Outlook

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$$\vec{B}(\vec{x}) = \vec{B}(\vec{x}_0) + (\vec{x} - \vec{x}_0) \cdot \vec{\nabla}\vec{B} + ...$$

- Here $\vec{x}_0 = \vec{0}$, $\vec{B}(\vec{x}_0 = \vec{0}) = \vec{0}$ therefore

$$\vec{B}(\vec{x}) = \vec{x} \cdot \vec{\nabla}\vec{B}(\vec{0}) + ...$$

Excursion: Assumptions and basic MHD equations

Assumptions

- Solving the problem in pure 2D, $\partial/\partial z = 0$

$$\vec{\mathrm{v}} = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}$$

in analogy to the magnetic field $\vec{B}$

$$\vec{\mathrm{B}} = \overleftrightarrow{B}\,\vec{x} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Motivation and introduction: possibilities...

What are the typical or possible types of dynamical systems allowing the plasma to cross the magnetic separatrices in 2D stationary resistive MHD?

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- for general 2D vector fields there are more possibilities $\Rightarrow$ resistivities can be a quadric, what are possible shapes?

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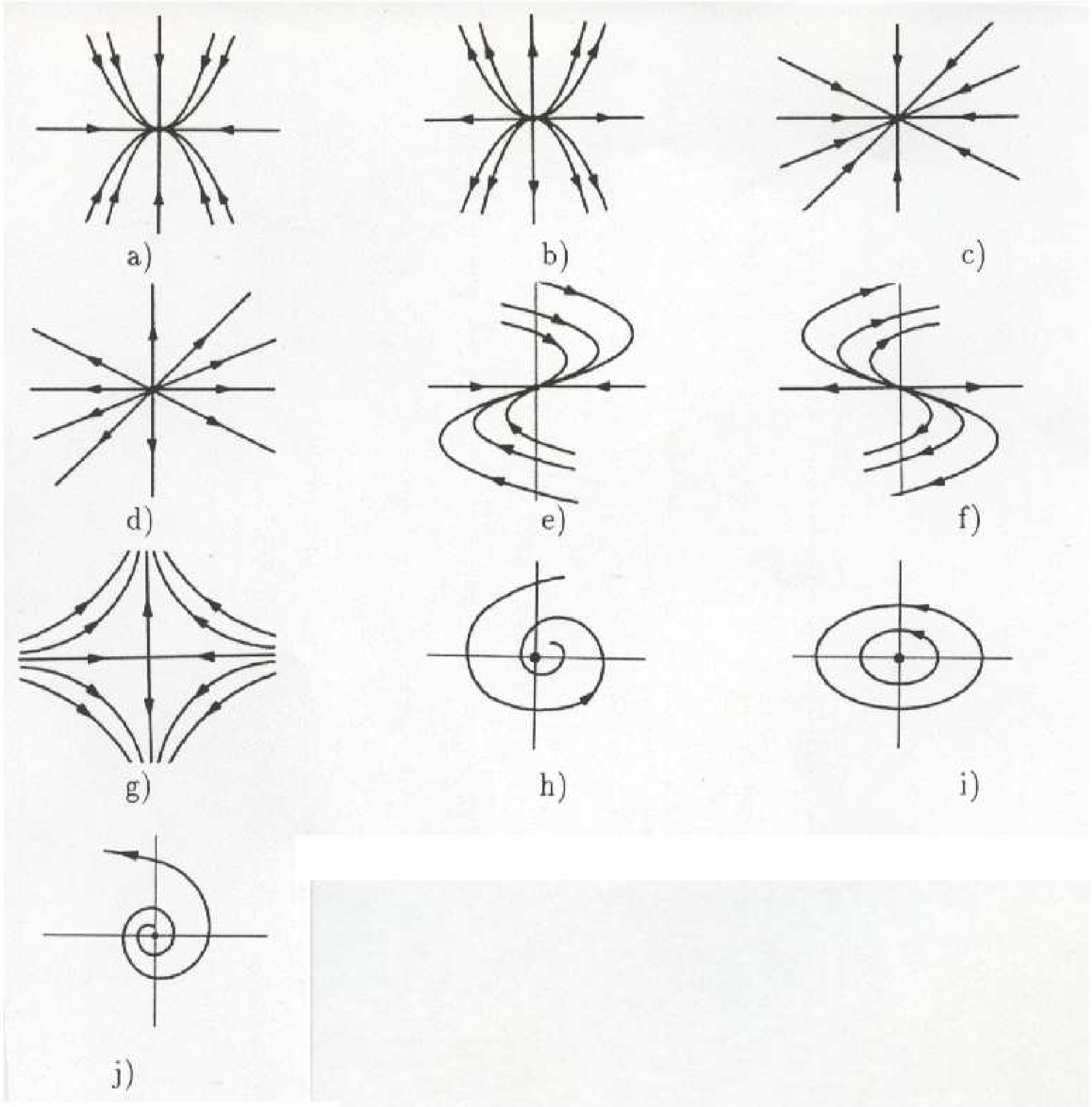
- for general 2D vector fields there are more possibilities $\Rightarrow$ resistivities can be a quadric, what are possible shapes?
- How can vector fields be classified? (Eigenvalue structure)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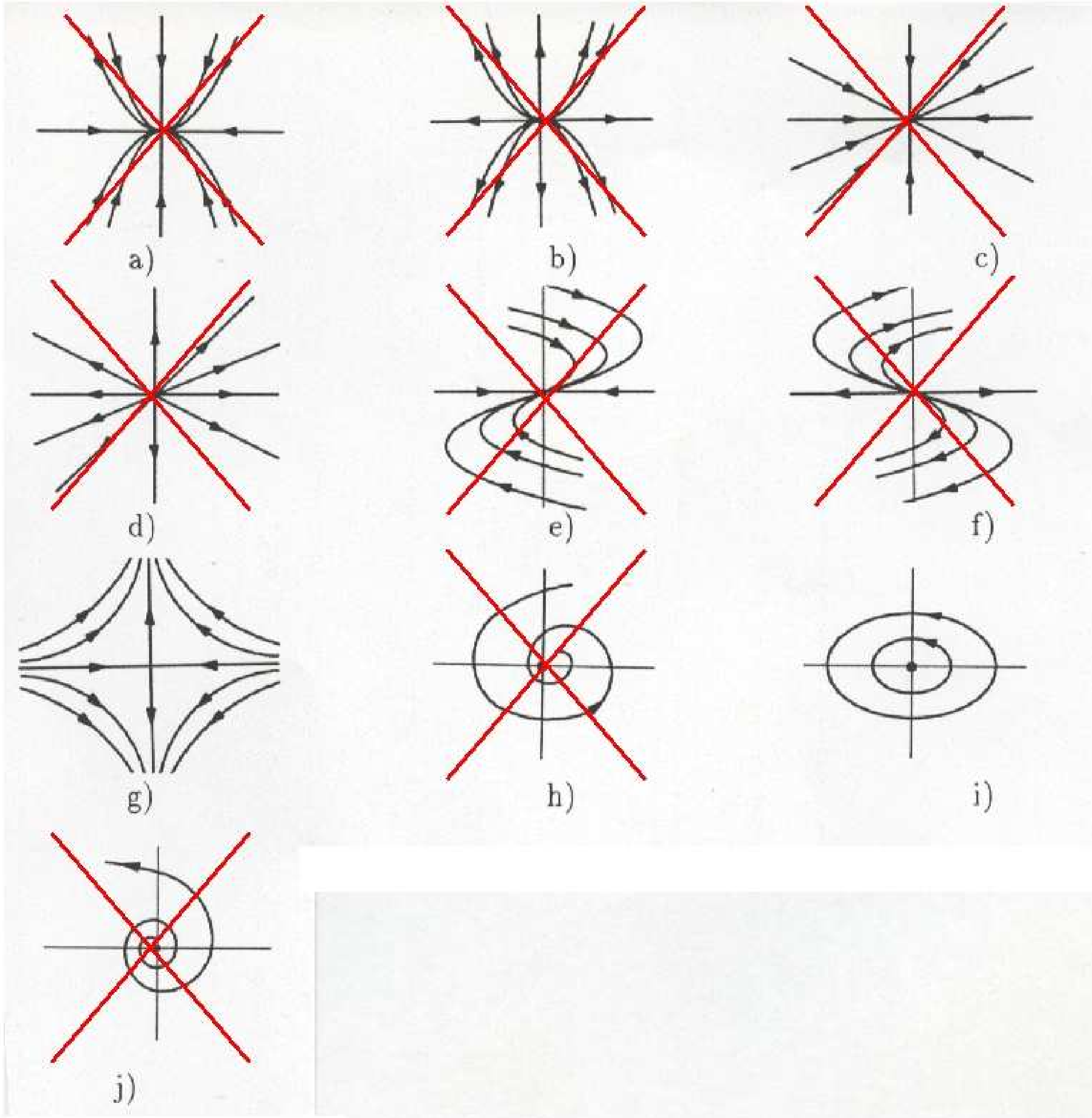
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Motivation and introduction: ...and problems

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- ...but maybe there are no ‘linear’ fields, but only higher order fields
- ...using all orders of x and y of the MHD equations or only that of order zero and one ?

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- (ii) Find restrictions that deliver reasonable resistivities.
- (iii) **Determination of the complete skeleton of RMHD solutions in 2D.**

Assumptions and basic MHD equations

Assumptions

- Solving the problem in pure 2D, $\partial / \partial z = 0$

$$\vec{\mathrm{v}} = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}$$

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$$\vec{\mathrm{B}} = \overleftrightarrow{B}\, \vec{x} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Assumptions and basic MHD equations

Basic MHD equations

$$
\begin{aligned}
\vec{\nabla}\cdot(\rho\vec{\mathrm{v}}) &= 0\,,\\
\rho(\vec{\mathrm{v}}\cdot\vec{\nabla})\vec{\mathrm{v}} &= \vec{j}\times\vec{B}-\vec{\nabla}p-\rho g\vec{e}_y\,,\; g>0\\
E_0+\mathrm{v}_x B_y-\mathrm{v}_y B_x &= \eta j_z\,,\\
\vec{\mathrm{v}}\cdot\vec{\nabla}p+\gamma p\vec{\nabla}\cdot\vec{\mathrm{v}} &= (\gamma-1)\eta j_z^2\,,
\end{aligned}
$$

where

$$\vec{B}=\vec{\nabla}\times(A\vec{e}_z)=\vec{\nabla}A\times\vec{e}_z$$

First Conclusions:

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- the electric field must be zero!

Consequently :

Ohm's law must be regarded as a definition equation for η

Assumptions and basic MHD equations

The 'standard' magnetic skeleton

Standard: $A = ax^2 + by^2$

$$a = -\frac{\mu_0}{4}\left(j_t + j_z\right) \quad b = \frac{\mu_0}{4}\left(j_t - j_z\right)$$

$$\Leftrightarrow \quad j_z = -\frac{2}{\mu_0}\left(a + b\right) \quad j_t = \frac{2}{\mu_0}\left(b - a\right).$$

$$\lambda_B \propto \pm\sqrt{j_t^2 - j_z^2}$$

Example: The complete linear approach

Resulting algebraic equation system

$$
\begin{aligned}
\rho\left(V_{11}^2+V_{12}V_{21}\right)+2p_4-2aj_z &= 0\,,\\
\rho\left(V_{11}^2+V_{12}V_{21}\right)+2p_5-2bj_z &= 0\,,\\
p_3 &= 0\,,\\
-\rho(V_{12}V_{21}+V_{11}^2)y_0+p_2+\rho g &= 0\,,\\
-\rho V_{11}^2x_0-\rho V_{12}V_{21}x_0+p_1 &= 0\,,
\end{aligned}
$$

$$
\begin{aligned}
(-V_{11}p_1-V_{21}p_2)x_0+(V_{11}p_2-V_{12}p_1)y_0 &= qE_0\,,\\
V_{21}p_2+V_{11}\left(p_1-2x_0p_4\right)-2V_{12}y_0p_4 &= 2aq(V_{11}x_0+V_{12}y_0)\,,\\
V_{12}p_1-2V_{21}x_0p_5+V_{11}\left(2y_0p_5-p_2\right) &= 2bq\left(V_{21}x_0-V_{11}y_0\right)\,,\\
2V_{12}p_4+2V_{21}p_5 &= q(-2aV_{12}-2bV_{21})\,,\\
2p_4V_{11} &= -2aqV_{11}\,,\\
-2p_5V_{11} &= 2bqV_{11}
\end{aligned}
$$

The complete linear approach

Solving the nonlinear algebraic equation system

Now we use the assumption that $V_{11} = 0$ and $x_0 = 0$.

$$
\begin{aligned}
p_2 &= -\frac{\rho g}{2} - \frac{\rho^2 g V_{12}^2}{4 b \gamma j_z} \mp \frac{\rho g}{4 b \gamma j_z} \sqrt{D}, \\
p_4 &= \frac{1}{2}\left((2a - b\gamma)\, j_z + \frac{\rho V_{12}^2}{2} \pm \frac{1}{2}\sqrt{D} \right) \\
p_5 &= \frac{1}{2}\left((2-\gamma)\, b\, j_z + \frac{\rho V_{12}^2}{2} \pm \frac{1}{2}\sqrt{D} \right), \\
y_0 &= \frac{\rho g}{2 b \gamma j_z}, \\
V_{21} &= \frac{2\gamma b\, j_z - \rho V_{12}^2 \mp \sqrt{D}}{2\rho V_{12}},
\end{aligned}
$$

The complete linear approach

Solving the nonlinear algebraic equation system

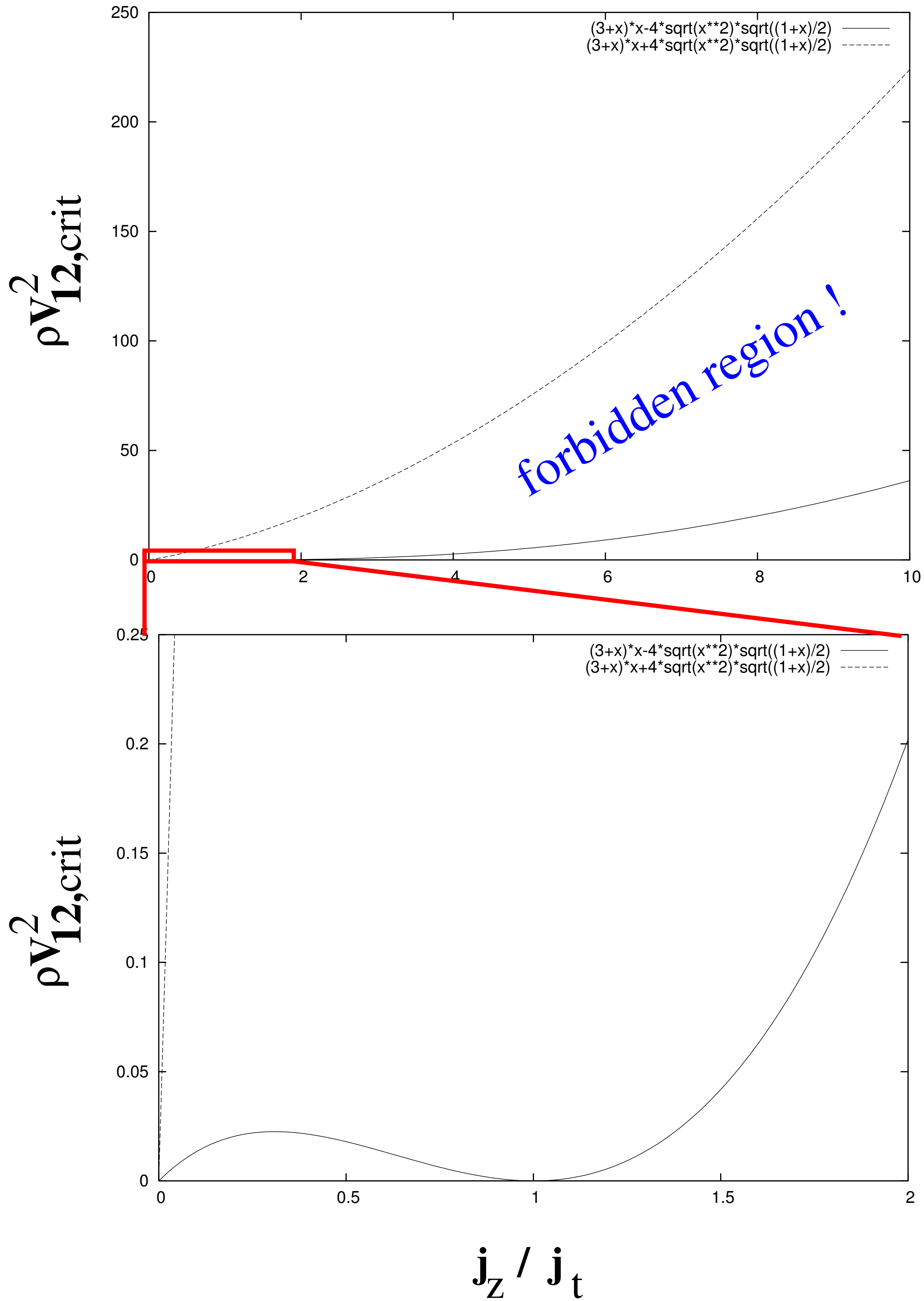
The discriminant D:

$$D = (\rho V_{12}^2)^2 + P(j_z, j_t, \gamma)\rho V_{12}^2 + Q(j_z, j_t, \gamma) \geq 0$$

Define

$$\varepsilon_1 := \rho V_{12,crit1}^2 = \frac{\mu_0}{2}(3j_t + j_z)\gamma j_z + 2\mu_0\gamma |j_z| \sqrt{\frac{j_t\,(j_t + j_z)}{2}}$$

$$\varepsilon_2 := \rho V_{12,crit2}^2 = \frac{\mu_0}{2}(3j_t + j_z)\gamma j_z - 2\mu_0\gamma |j_z| \sqrt{\frac{j_t\,(j_t + j_z)}{2}}$$


(3+x)*x-4*sqrt(x**2)*sqrt((1+x)/2)
(3+x)*x+4*sqrt(x**2)*sqrt((1+x)/2)
forbidden region !
$\rho v^2_{12,crit}$
j_z / j_t

The complete linear approach

Solving the nonlinear algebraic equation system

....but the problem is the resistivity :

$$\eta = \frac{1}{j_z}\left(2aV_{12}y_0x - (2aV_{12} + 2bV_{21})xy\right)$$

which is positive in 2 quadrants, and negative in the other 2 quadrants

Possible interpretation(s) of a negative resistivity ??

To avoid negative resistivities:

Linearization of the original set of resistive MHD equations

$$
\begin{aligned}
\rho\left(V_{11}^2+V_{12}V_{21}\right)+2p_4-2aj_z &= 0\,,\\
\rho\left(V_{11}^2+V_{12}V_{21}\right)+2p_5-2bj_z &= 0\,,\\
p_3 &= 0\,,\\
-\rho(V_{12}V_{21}+V_{11}^2)y_0+p_2+\rho g &= 0\,,\\
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\end{aligned}
$$

$$
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(-V_{11}p_1-V_{21}p_2)x_0+(V_{11}p_2-V_{12}p_1)y_0 &= qE_0\,,\\
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2p_4V_{11} &= -2aqV_{11}\,,\\
-2p_5V_{11} &= 2bqV_{11}
\end{aligned}
$$

Spatially linearized resistive MHD equations

$$\begin{aligned}
\rho\left(V_{11}^2+V_{12}V_{21}\right)+2p_4-2aj_z &= 0\,,\\
\rho\left(V_{11}^2+V_{12}V_{21}\right)+2p_5-2bj_z &= 0\,,\\
p_3 &= 0\,,\\
-\rho(V_{12}V_{21}+V_{11}^2)y_0+p_2+\rho g &= 0\,,\\
-\rho V_{11}^2x_0-\rho V_{12}V_{21}x_0+p_1 &= 0\,,
\end{aligned}$$

$$\begin{aligned}
(-V_{11}p_1-V_{21}p_2)x_0+(V_{11}p_2-V_{12}p_1)y_0 &= qE_0 \;\mathbf{= 0}\\
V_{21}p_2+V_{11}\left(p_1-2x_0p_4\right)-2V_{12}y_0p_4 &= 2aq(V_{11}x_0+V_{12}y_0)\,,\\
V_{12}p_1-2V_{21}x_0p_5+V_{11}\left(2y_0p_5-p_2\right) &= 2bq\left(V_{21}x_0-V_{11}y_0\right)\,,\\
\cancel{2V_{12}p_4+2V_{21}p_5} &= \cancel{q(-2aV_{12}-2bV_{21})}\,,\\
\cancel{2p_4V_{11}} &= \cancel{-2aqV_{11}}\,,\\
\cancel{-2p_5V_{11}} &= \cancel{2bqV_{11}}
\end{aligned}$$

Solutions of the linearized system

Neglecting gravity and setting $y_0 = 0$ delivers

$$
\begin{aligned}
p_4 &= -\frac{\rho}{2}\left(V_{11}^2 + V_{12}V_{21}\right) + a\, j_z \\
p_5 &= -\frac{\rho}{2}\left(V_{11}^2 + V_{12}V_{21}\right) + b\, j_z
\end{aligned}
$$

Normalize V_{12}, V_{21} on V_{11}, j_z on j_t

$$
V_{21} = \frac{1 + j_z}{1 - j_z} V_{12} + 2s \frac{\sqrt{1 - j_z^2}}{1 - j_z}
$$

with the restriction of $s \in [-1, 1]$.

This guarantees that $\lambda_v^2 = 1 + V_{12}V_{21} \geq 0$.

Solutions of the linearized system

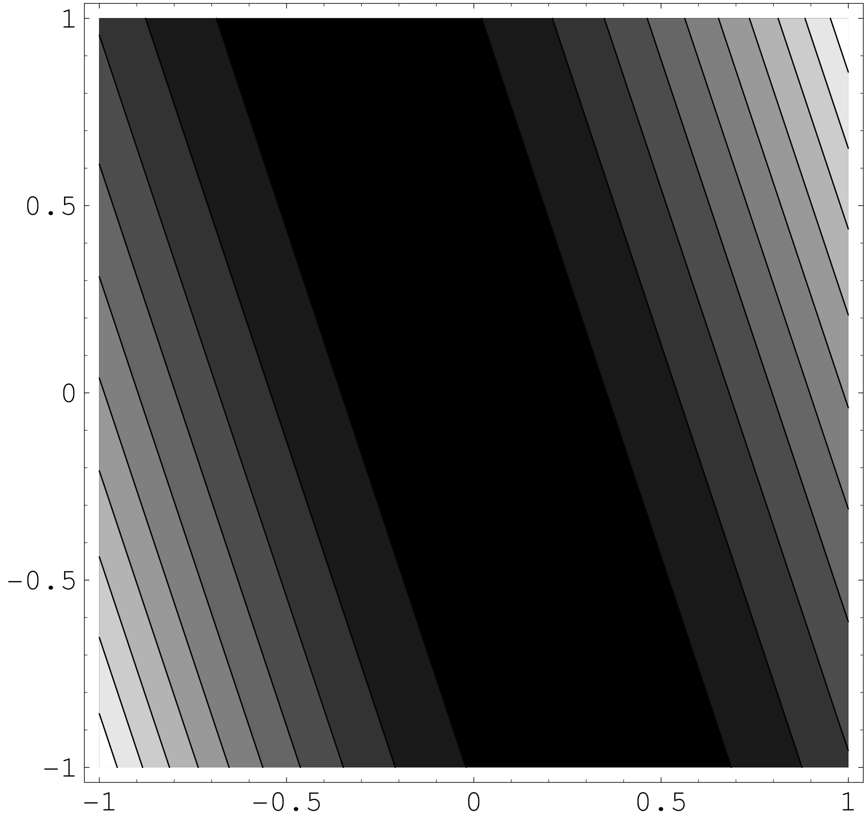
And for the resistivity it follows:

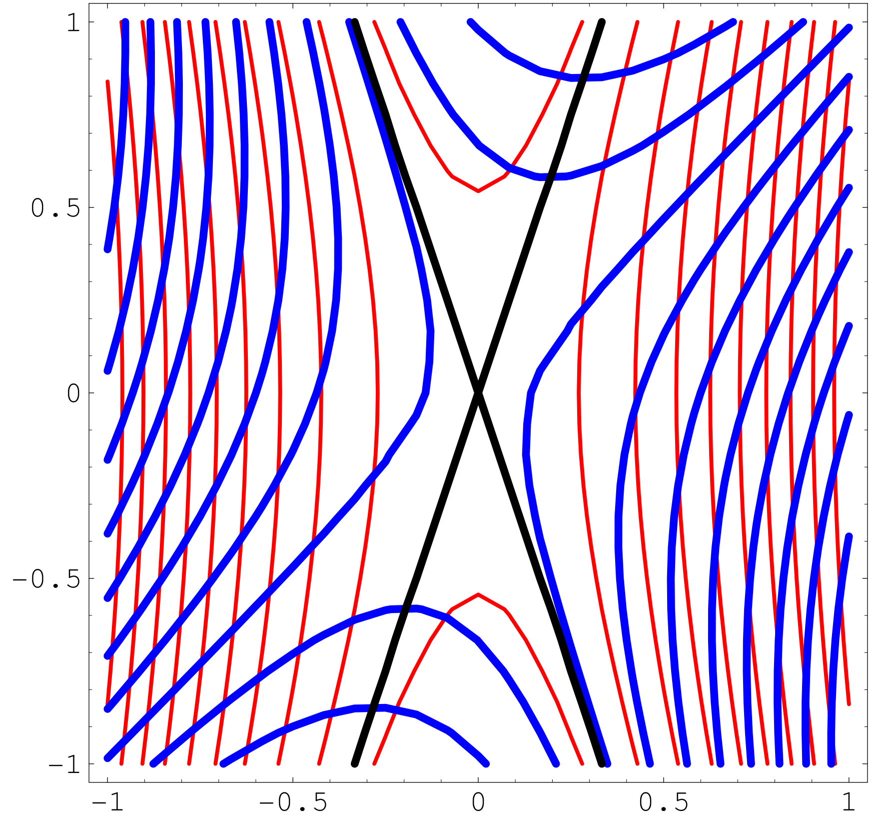
$$\eta = -2ax^2 - 2xy(aV_{12} + bV_{21}) + 2by^2$$

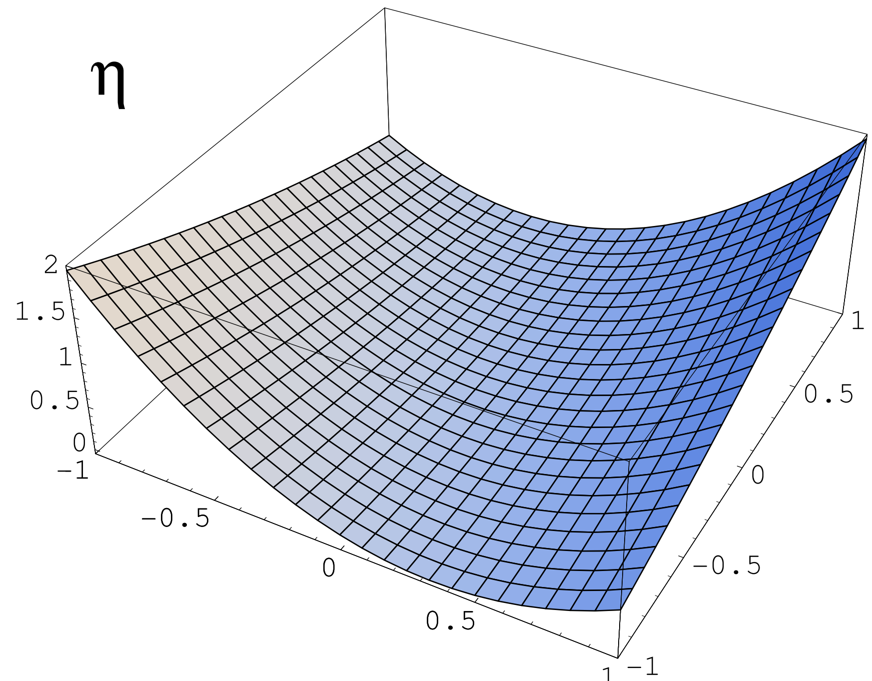
But the problem also in this case is:

No electric field !

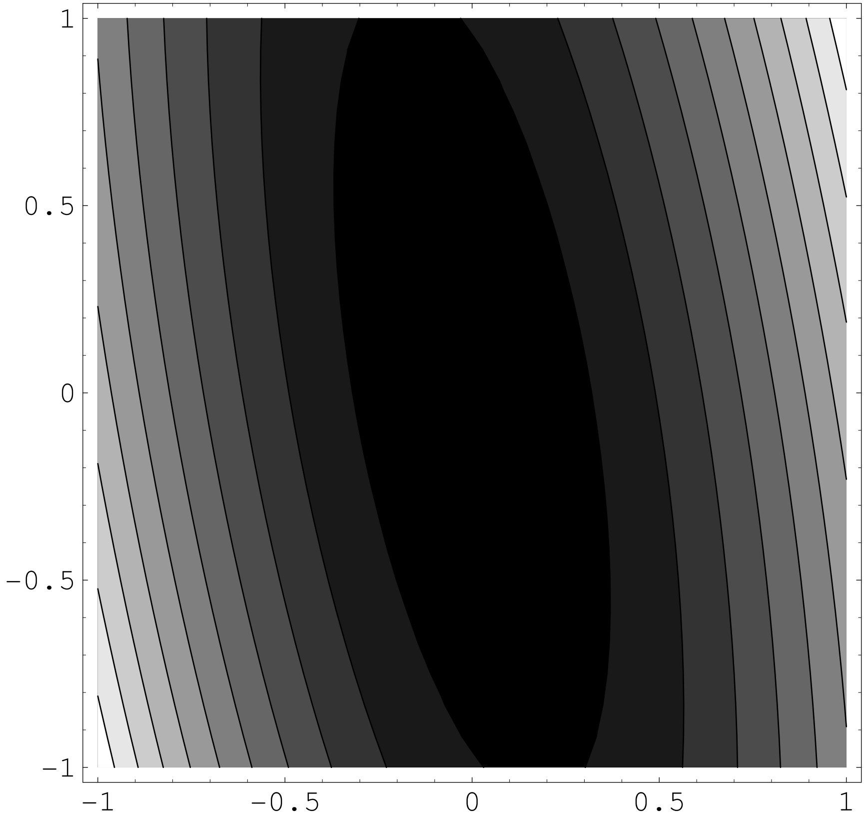
But, at least (for special values of s) the plasma can cross the separatrix.

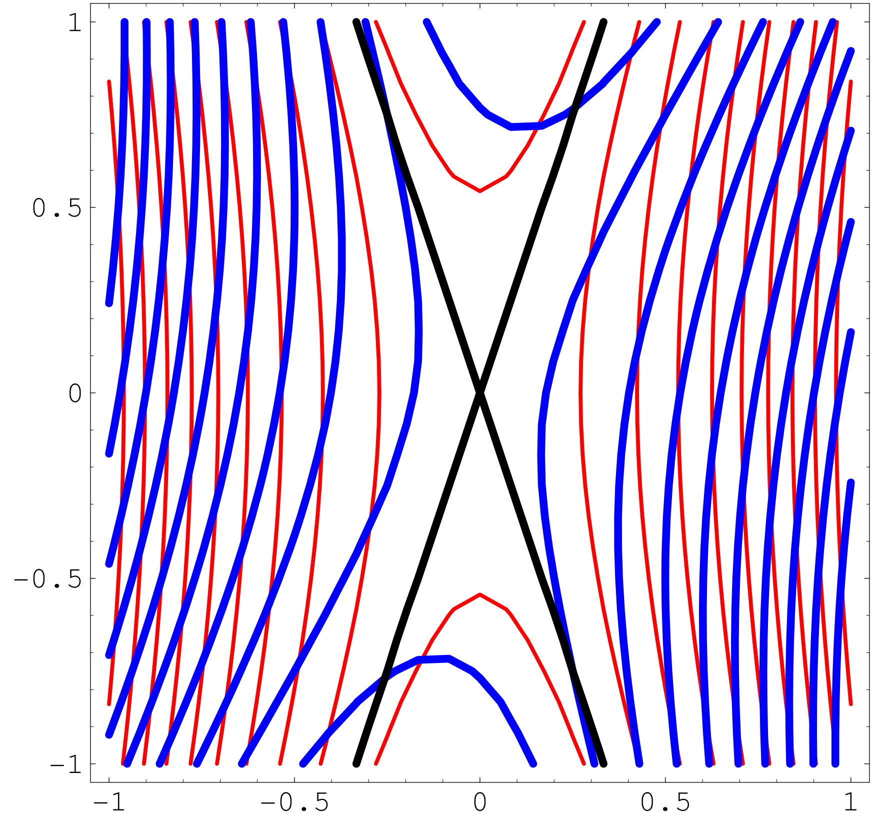


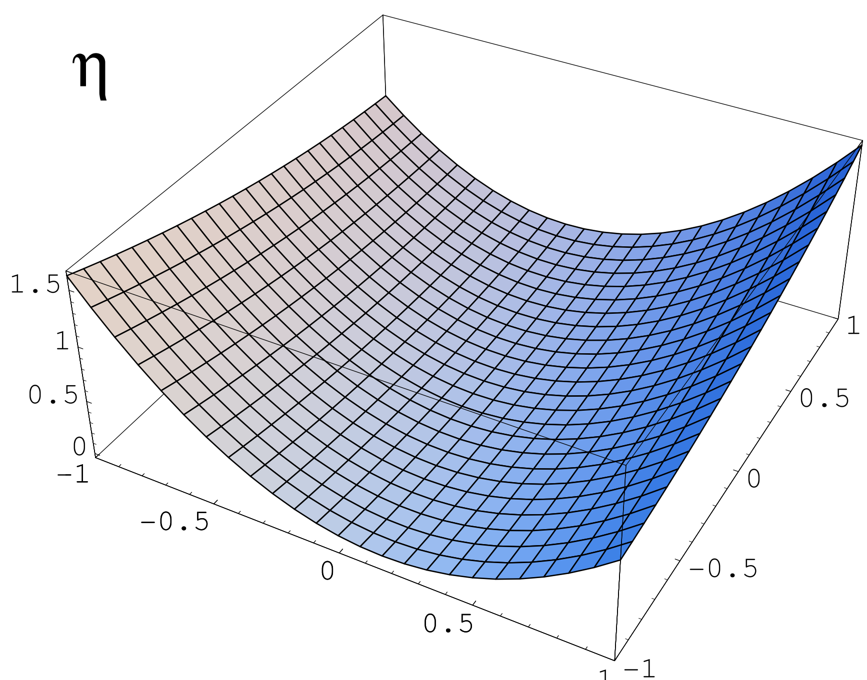




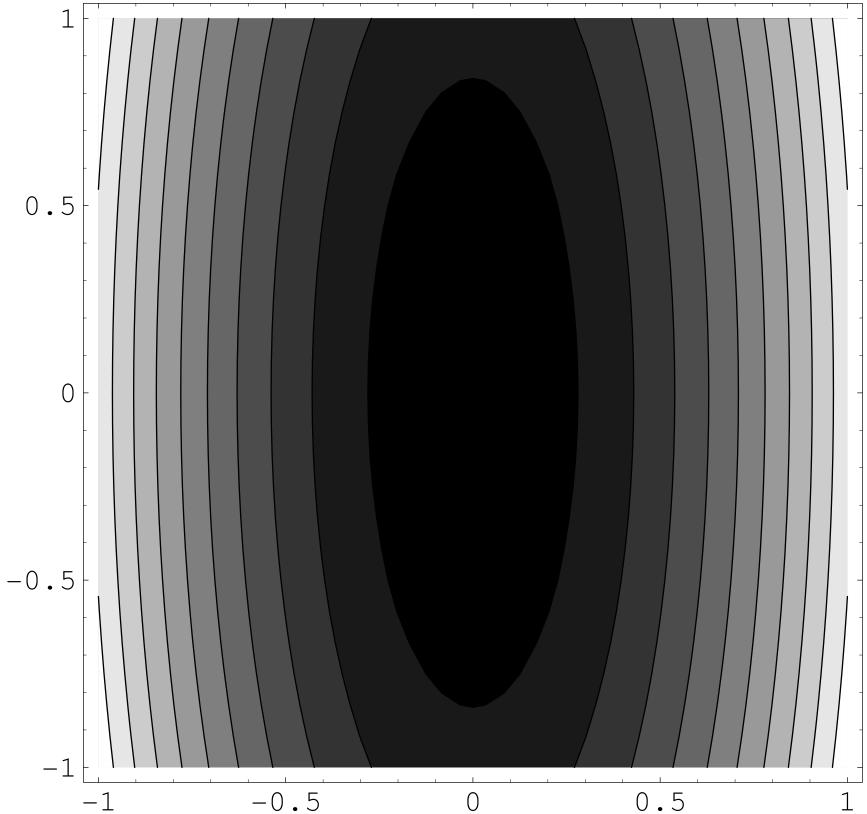
$s = -1\ , V_{12} = V_{11} = 1\ ,\ j_z = 0.8$

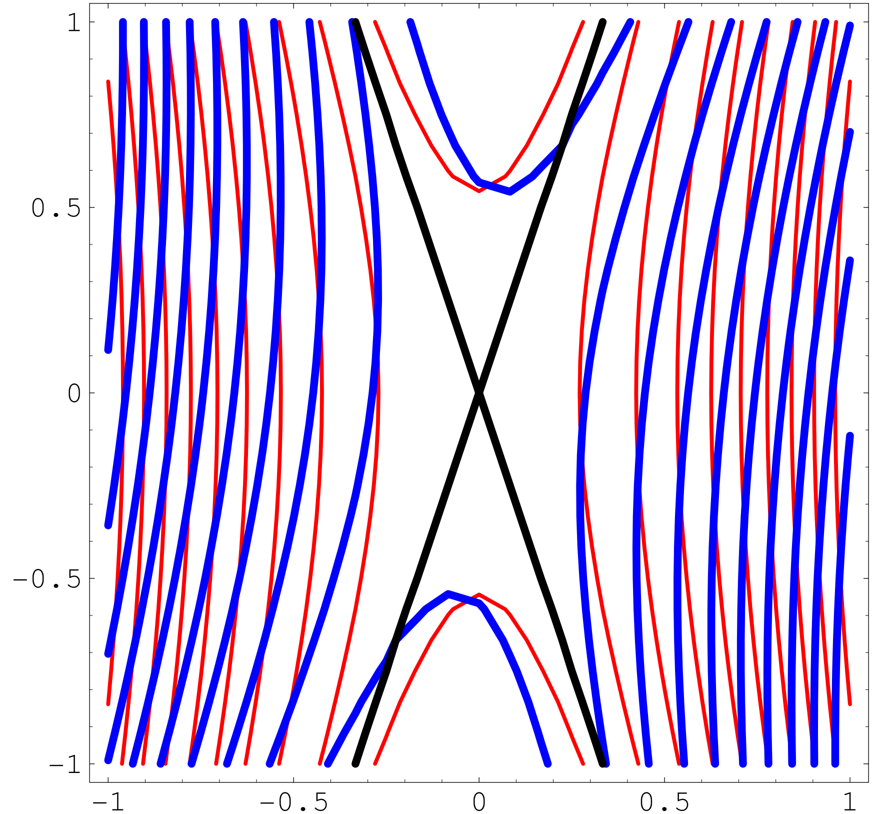


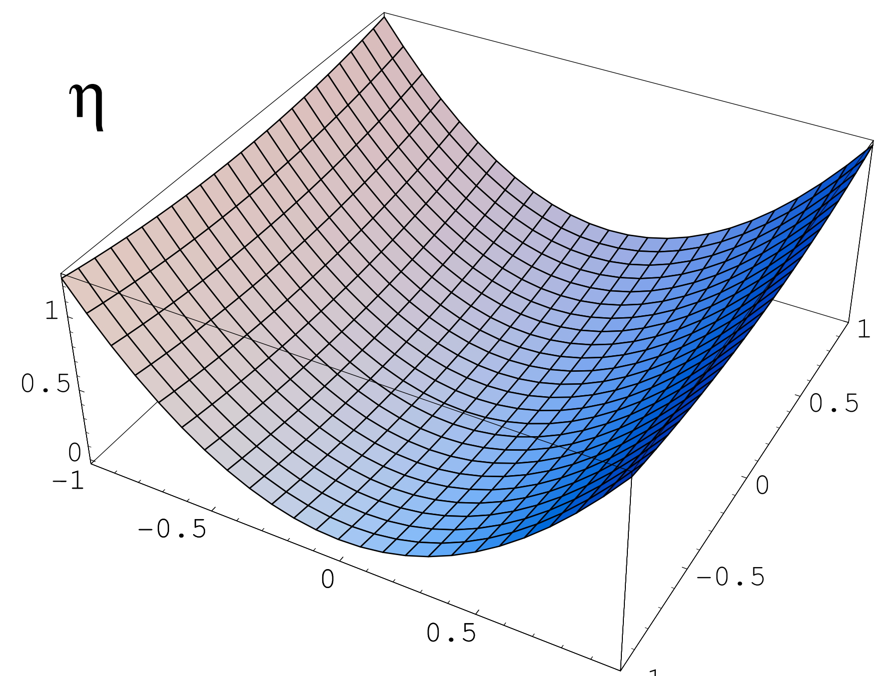




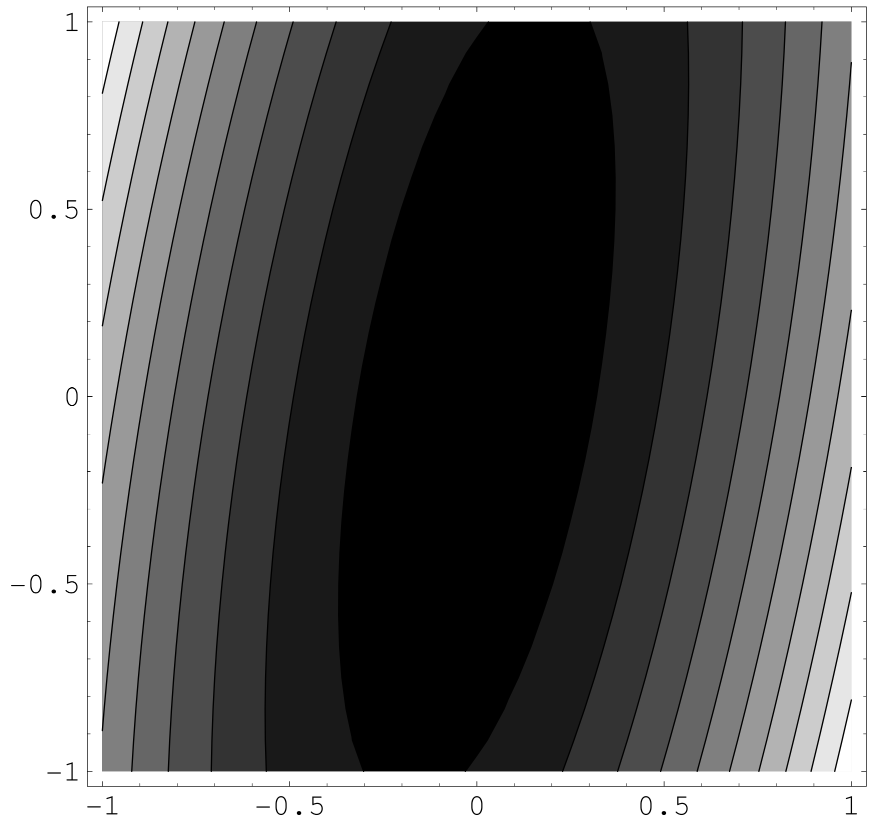
s = −0.5

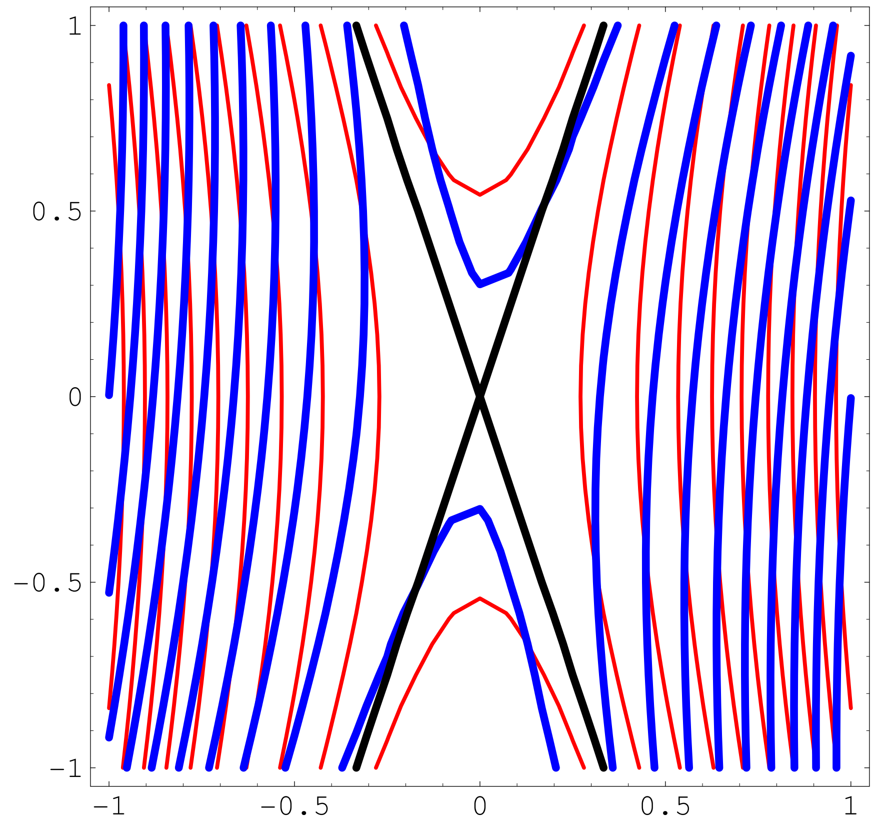


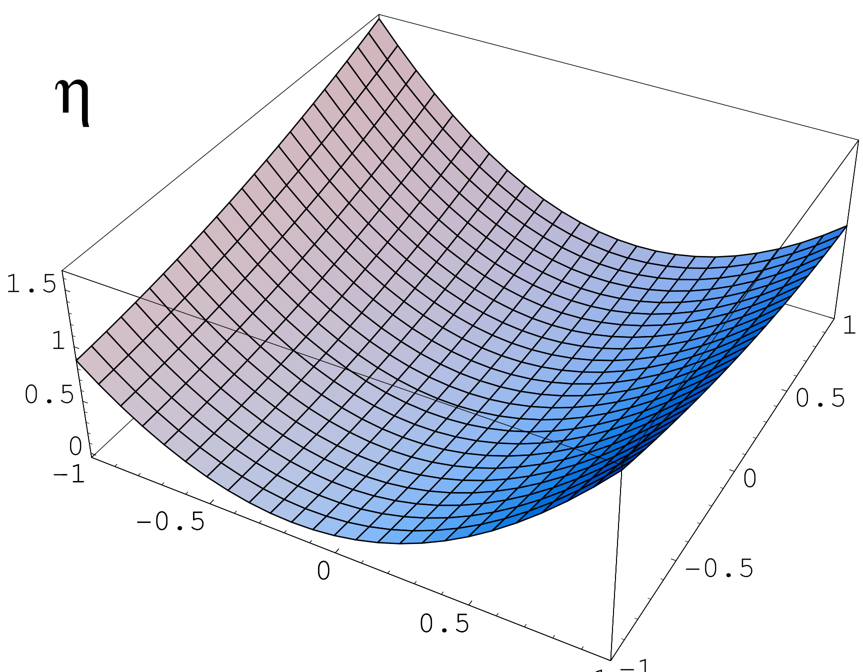




$s = 0$







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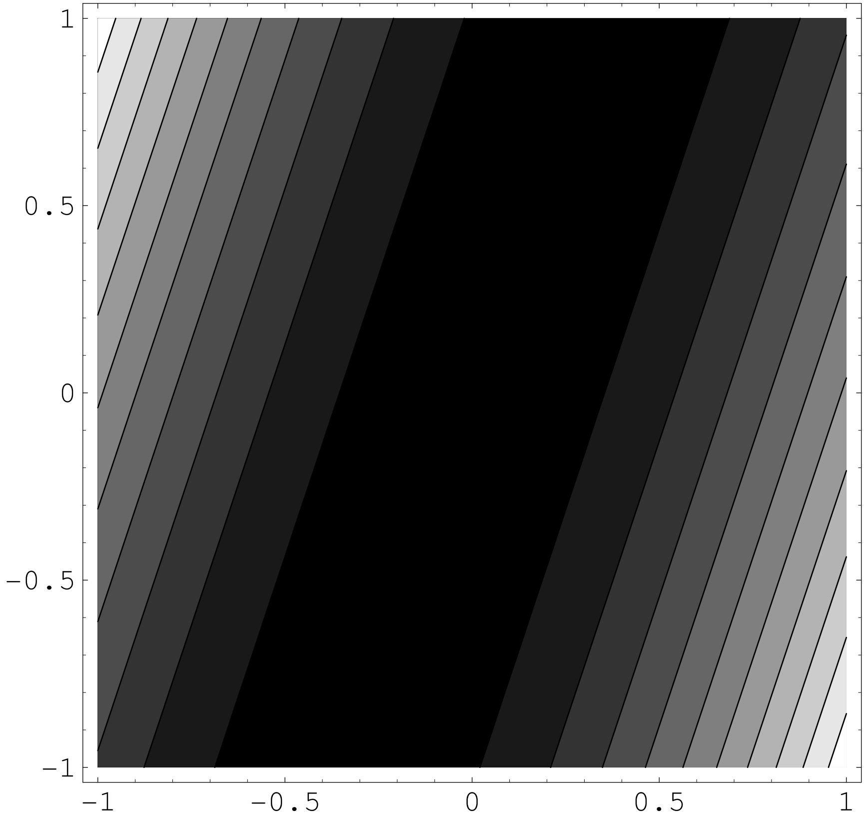

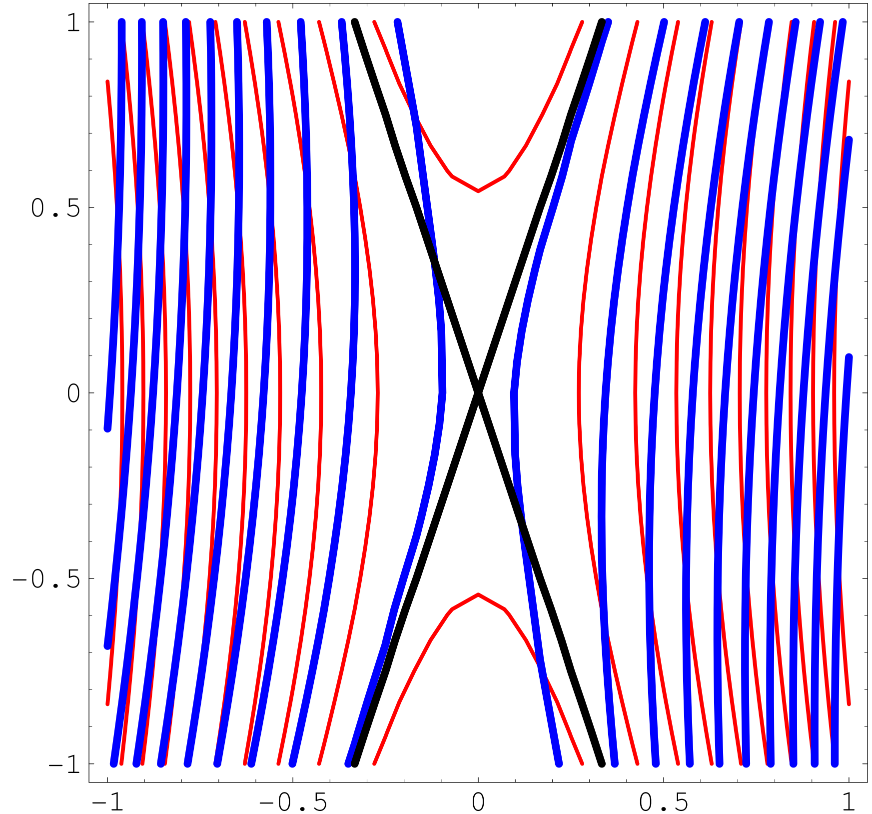

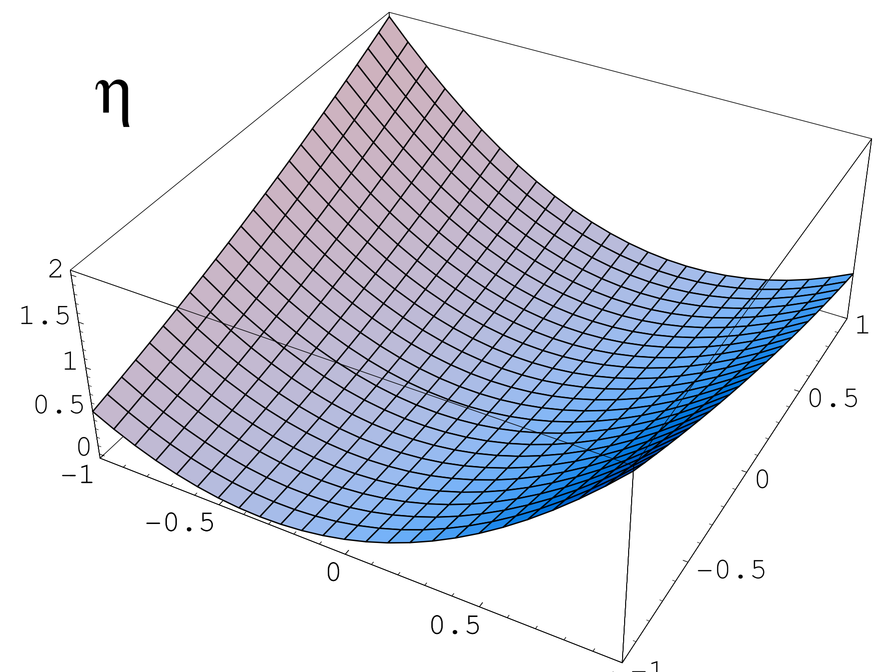

s = 1

Conclusions and outlook

- No real reconnection solutions for purely resistive stationary stagnation point flows.
- Either it is necessary to use additional dissipation terms, e.g., radiation (Steinolfson),
- or to use higher orders of x and y.
- Search for explanations for effective negative resistivity ("anti-dissipation")

The complete linear approach

Solving the nonlinear algebraic equation system

Now we use the assumption that $V_{11} = 0$. Then we rewrite the MHD equations:

$$
\begin{aligned}
\rho V_{12} V_{21} + 2 p_4 - 2 a j_z &= 0\,, \\
\rho V_{12} V_{21} + 2 p_5 - 2 b j_z &= 0\,, \\
-\rho V_{12} V_{21} y_0 + p_2 + \rho g &= 0\,, \\
-\rho V_{12} V_{21} x_0 + p_1 &= 0\,,
\end{aligned}
$$

$$
\begin{aligned}
-V_{12} y_0 p_1 - V_{21} x_0 p_2 &= q E_0\,, \\
V_{21} p_2 - 2 V_{12} y_0 p_4 &= 2 q V_{12} a y_0\,, \\
V_{12} p_1 - 2 V_{21} x_0 p_5 &= 2 q V_{21} b x_0\,, \\
V_{12} \left(p_4 + q a\right) + V_{21} \left(p_5 + q b\right) &= 0\,.
\end{aligned}
$$