Point processes as a mathematical tool to describe black-hole accretion disc stochastical variability

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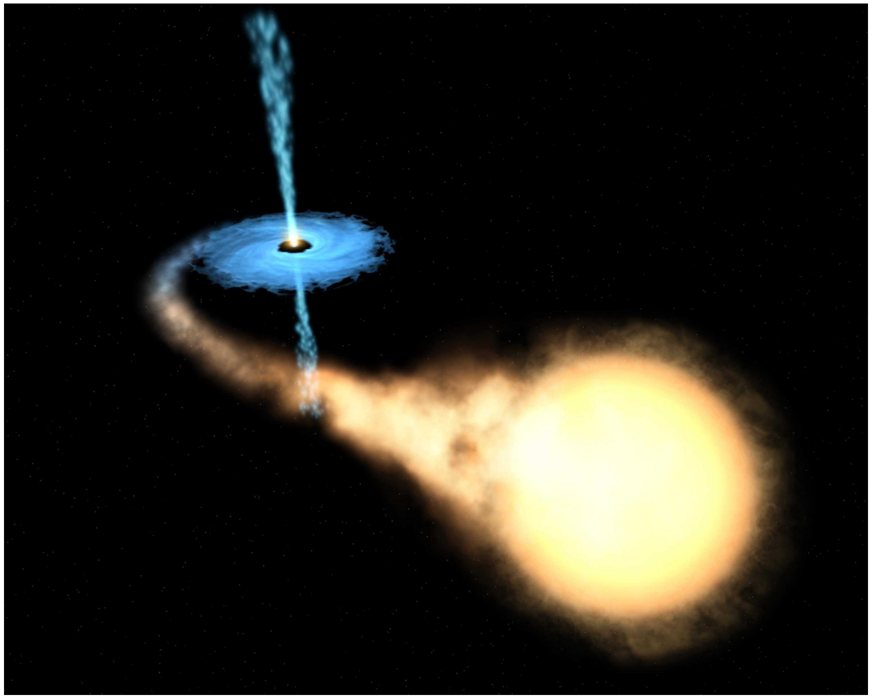
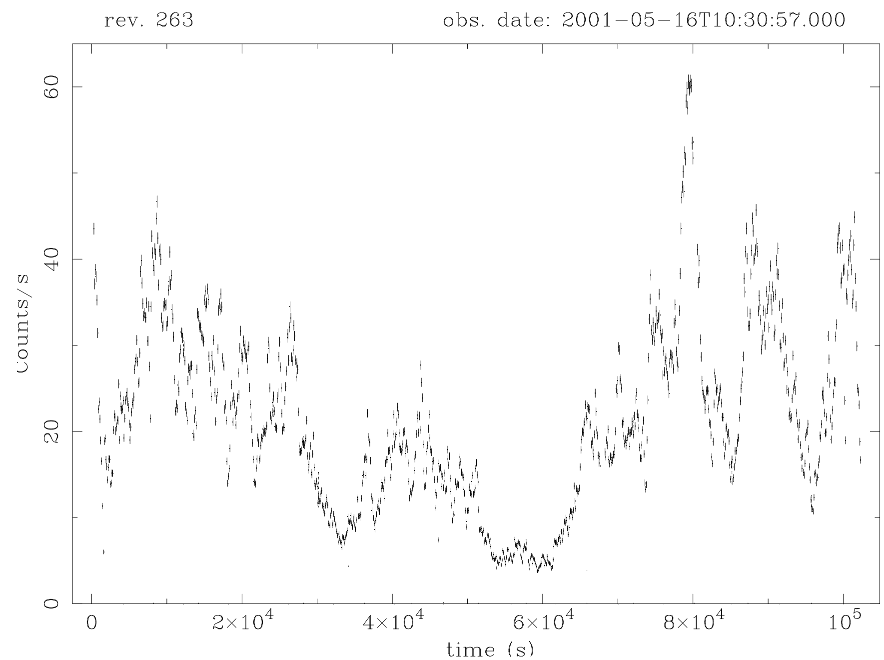
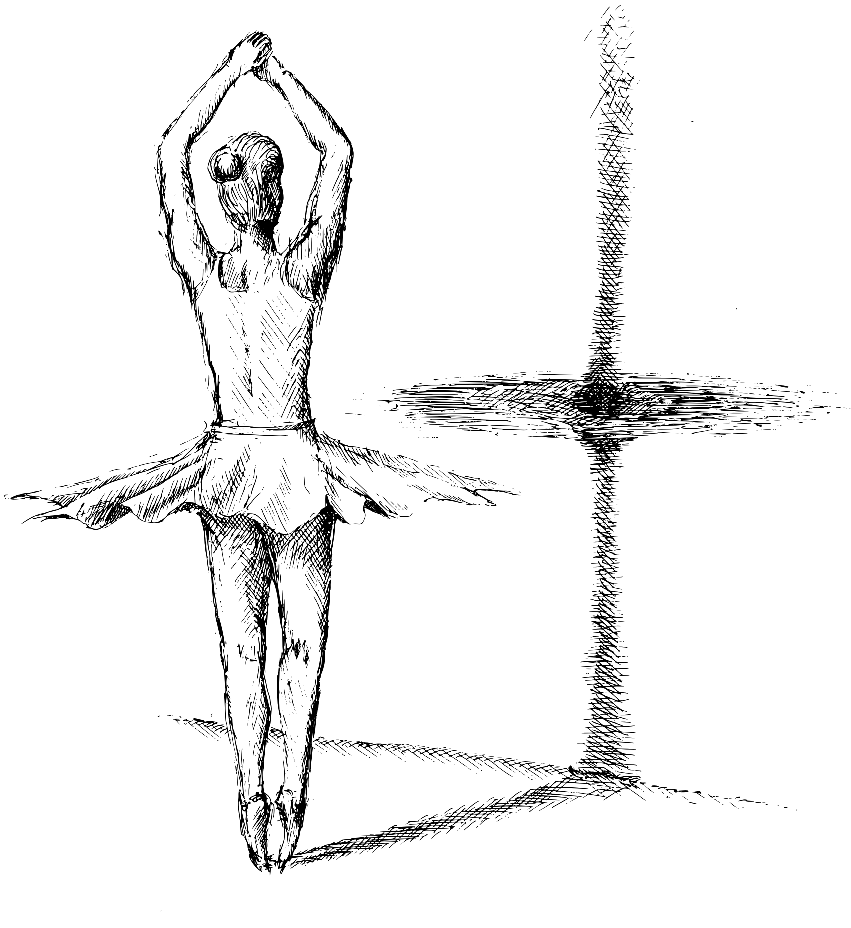
Motivation of the problem

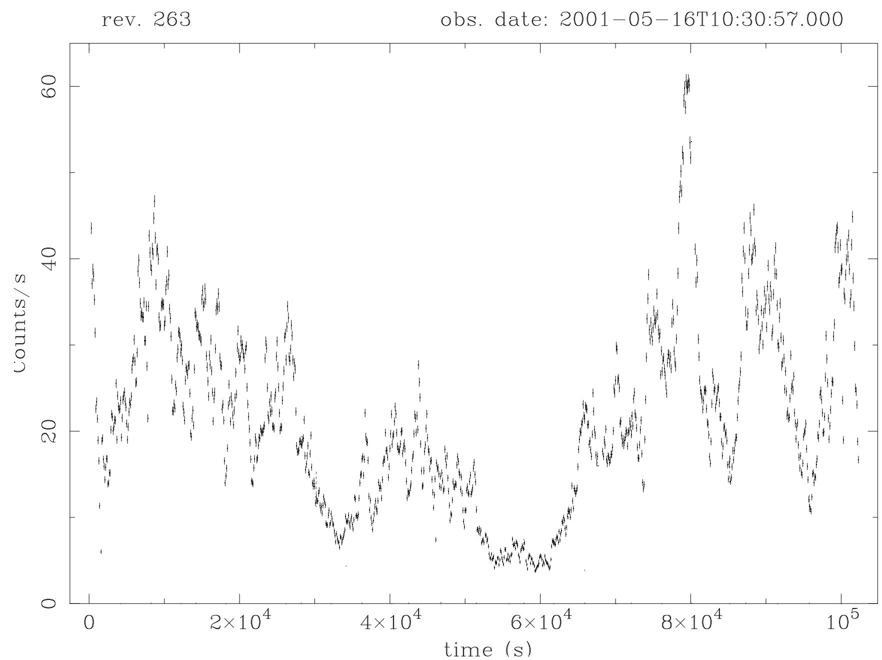




NASA; Ponti et al (2006), MNRAS

Motivation of the problem





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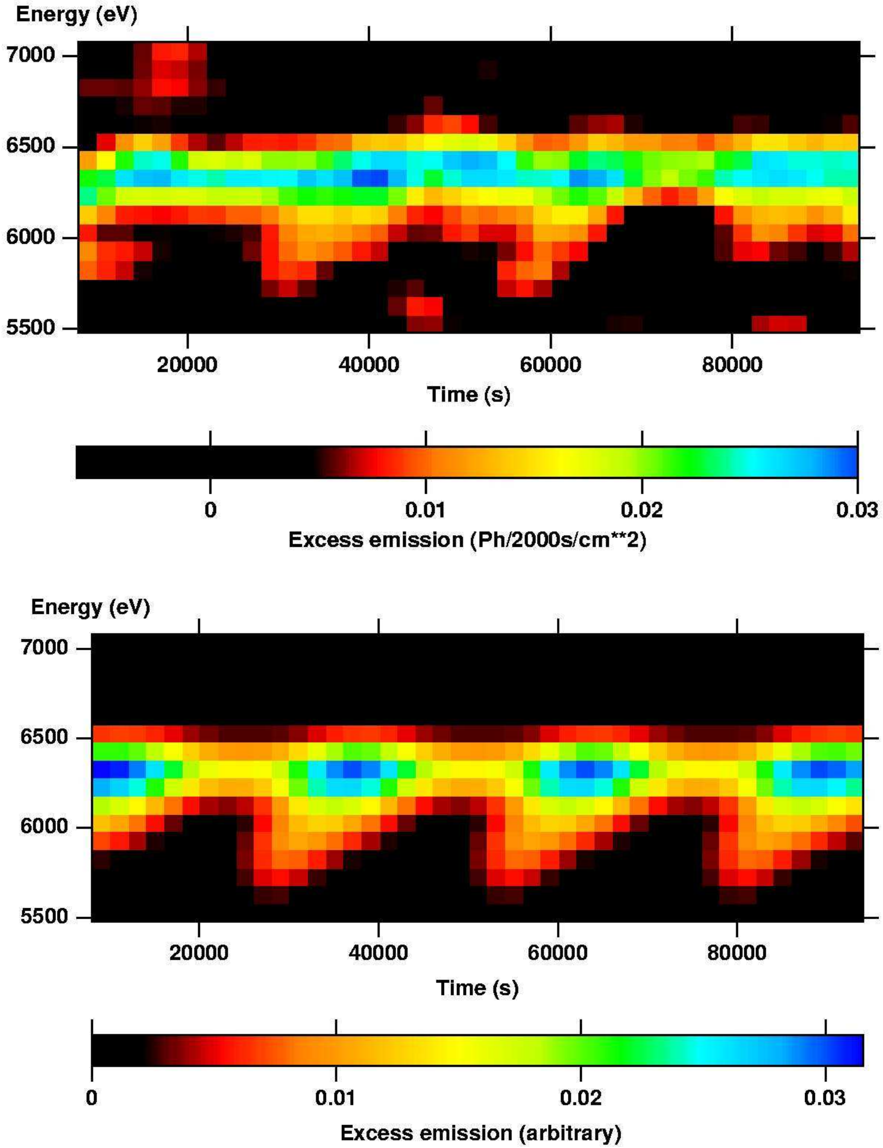
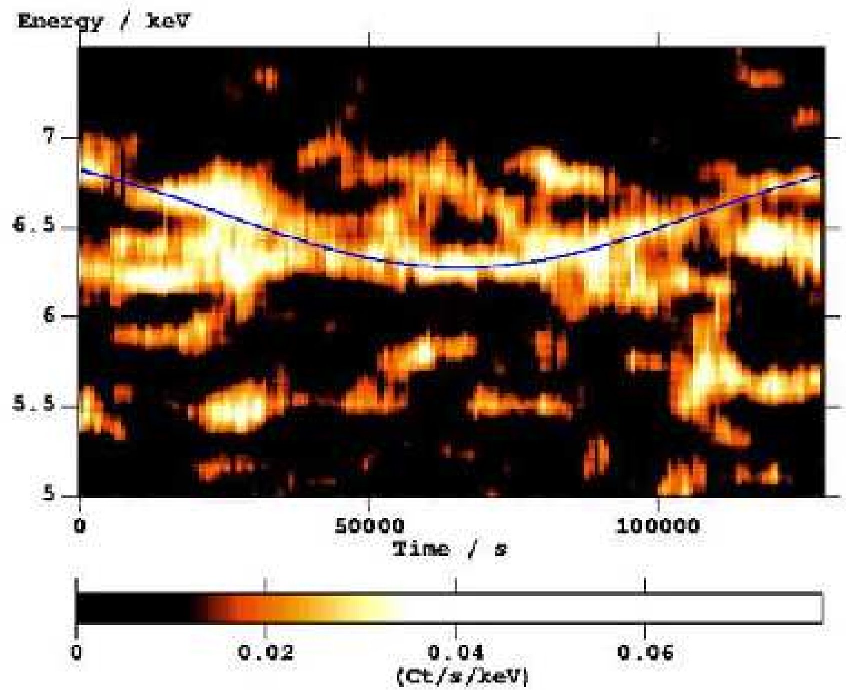




Turner et al (2005), A&A

Iwasawa et al (2004), MNRAS

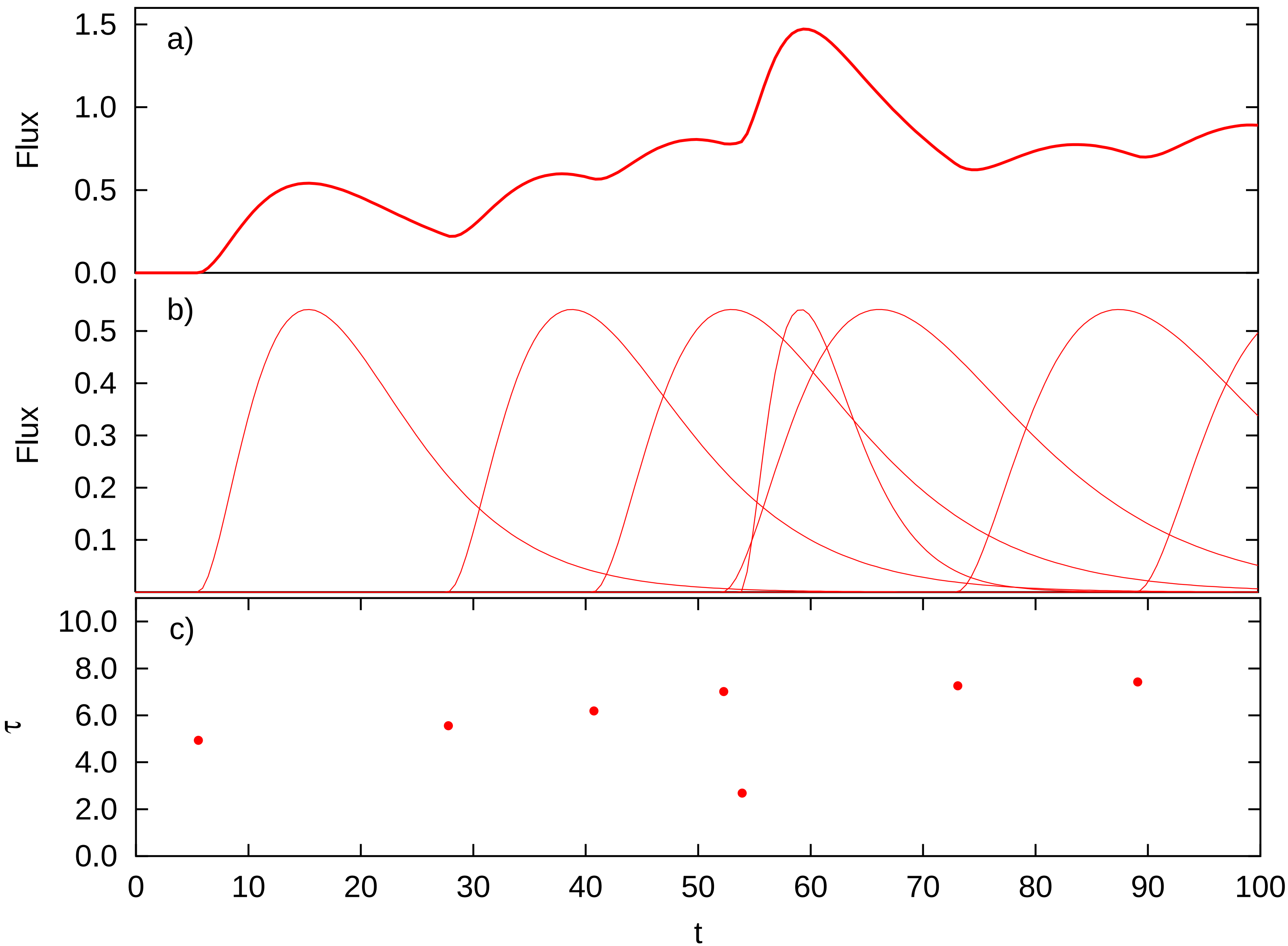
Formulation of the problem

- Spot with decaying emissivity on circular orbit.
- Each single spot is described by
  - ◇ "time and place of birth": t_j & r_j, ϕ_j
  - ◇ Other shape determining parameters: lifetime, emitted energy...
- Observed signal is modulated by relativistic effects: (redshift, gravitational lensing, time delay)

Abramowicz et. al. 1991;

Lehto 1989; Poutanen & Fabian 1999

Formulation of the problem



Power spectrum of a random process

Spectral characterisation of a stationary signal:

$$S(\omega) = \lim_{T\to\infty} \frac{1}{2T} \mathrm{E}\left[\left| \mathcal{F}_T[X(t)](\omega) \right|^2 \right],$$

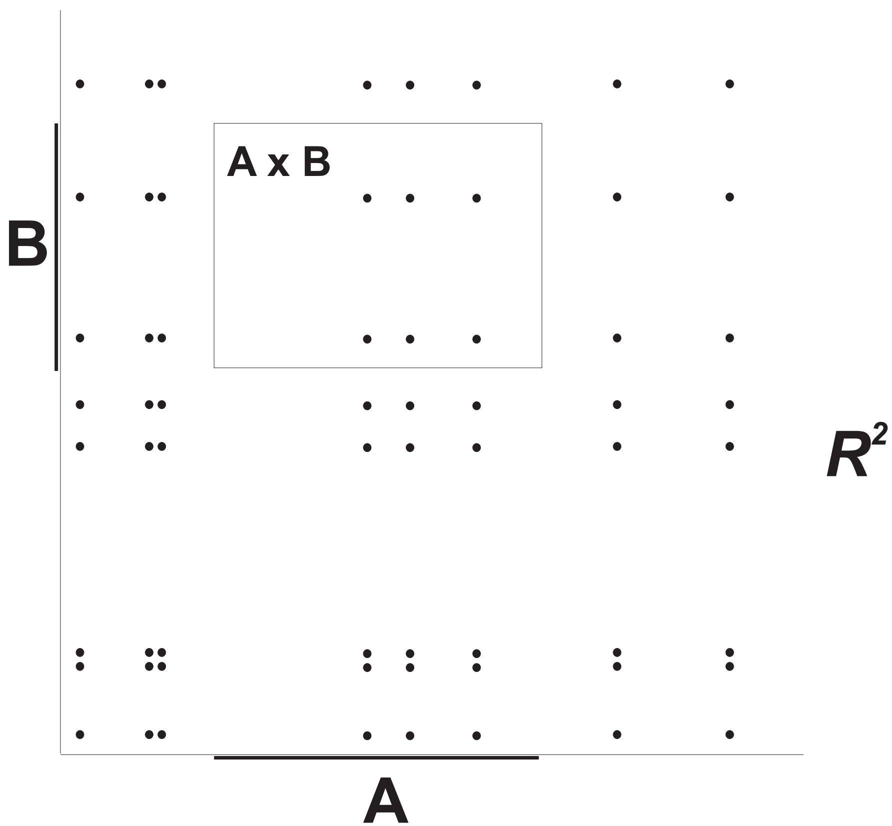
where $\mathcal{F}_T[\,]$ is the incomplete Fourier transform defined by,

$$\mathcal{F}_T[X(t)] = \int_{-T}^{T} X(t) e^{-i\omega t} \mathrm{d}t$$

. As a first step we calculate the $\mathcal{F}_T[\,]$ of

$$f(t) = \sum_j I(t - t_j - t_{0j},\, \boldsymbol{\xi}_j) F(t - t_j - t_{fj} - t_{0j},\, r_j).$$

Random point processes



Random values $N(A)$ form random point process.

Mean value and the second moment:

$$M_1(A) = \mathrm{E}\left[N(A)\right],$$

$M_1(A)$ is the mean number of points in the set A

$$N(A \times B) = N(A)N(B)$$

$$M_2(A \times B) = \mathrm{E}\left[N(A)N(B)\right],$$

$M_2(A \times B)$ is the mean number of pairs of points in the set $A \times B$

Power spectrum of a random process

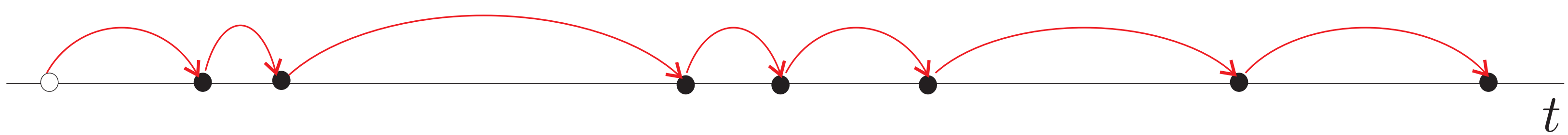
Process without relativistic effects ($F(t,r)=1$):

$$S(\omega) = m_1\,\mathrm{E}[|\mathcal{F}[I](\omega)|^2] + S_{\mathrm{P}}(\omega)\;|\mathrm{E}[\mathcal{F}[I](\omega)]|^2$$

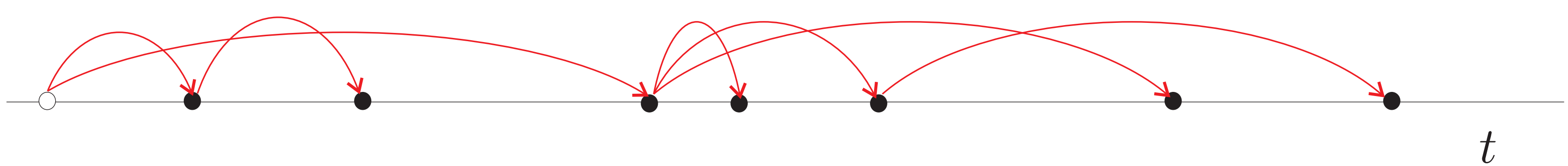
For cluster processes: $S_{\mathrm{P}}(\omega) = \lambda \tilde{m}_{[2]}(\omega, -\omega|0)$

Spontaneous centers, random clusters (avalanches).

a)



b)



Application: Marked Hawkes process

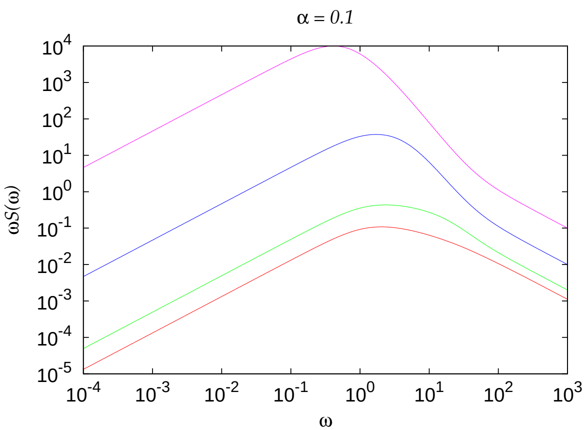
Hawkes process:

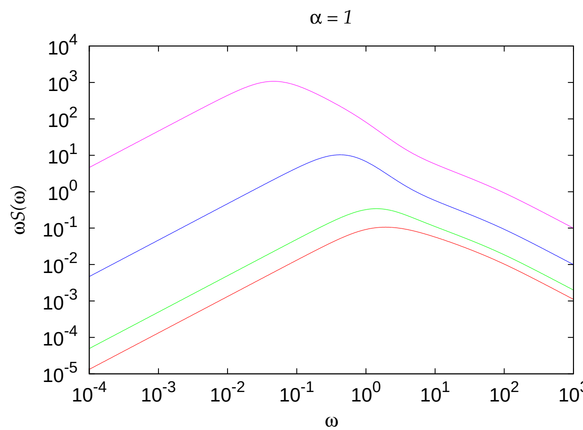
- Two types of events
- Poisson process with the intensity λ.
- Existing event with ignition time t_0 can give birth to new event
- Each point carries an i.i.d. mark $\boldsymbol{\xi}$ with distribution $\zeta(\boldsymbol{\xi})$. in time t with Poissonian intensity $\mu(t-t_0)$
- $m_1(t) = \lambda + \sum\limits_{i,\, t_i<t} \mu(t_i) = \lambda + \int\limits_{-\infty}^{t} \mu(t-x) N(\mathrm{d}x)$
- Mean number of offsprings $\nu = \int\limits_{-\infty}^{\infty} \mu(t)\mathrm{d}t$.

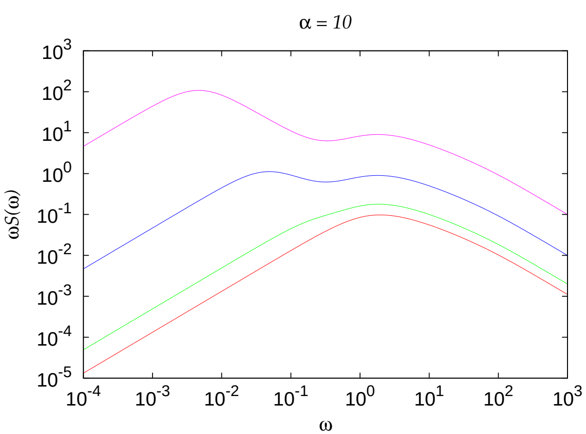
Application: Marked Hawkes process

Particular choice:

$$\mu(t) = \nu\sigma \exp(-\sigma t)\theta(t), \quad I(t,\tau) = I_0 \exp(-t/\tau), \quad \zeta(\tau) \propto \tau^{-1}$$







Modulation by relativistic effects

Formally the same formula

$$S(\omega) = m_1 \, \mathrm{E}[|\mathcal{F}[I](\omega)|^2] + S_{\mathrm{P}}(\omega) \; |\mathrm{E}[\mathcal{F}[I](\omega)]|^2$$

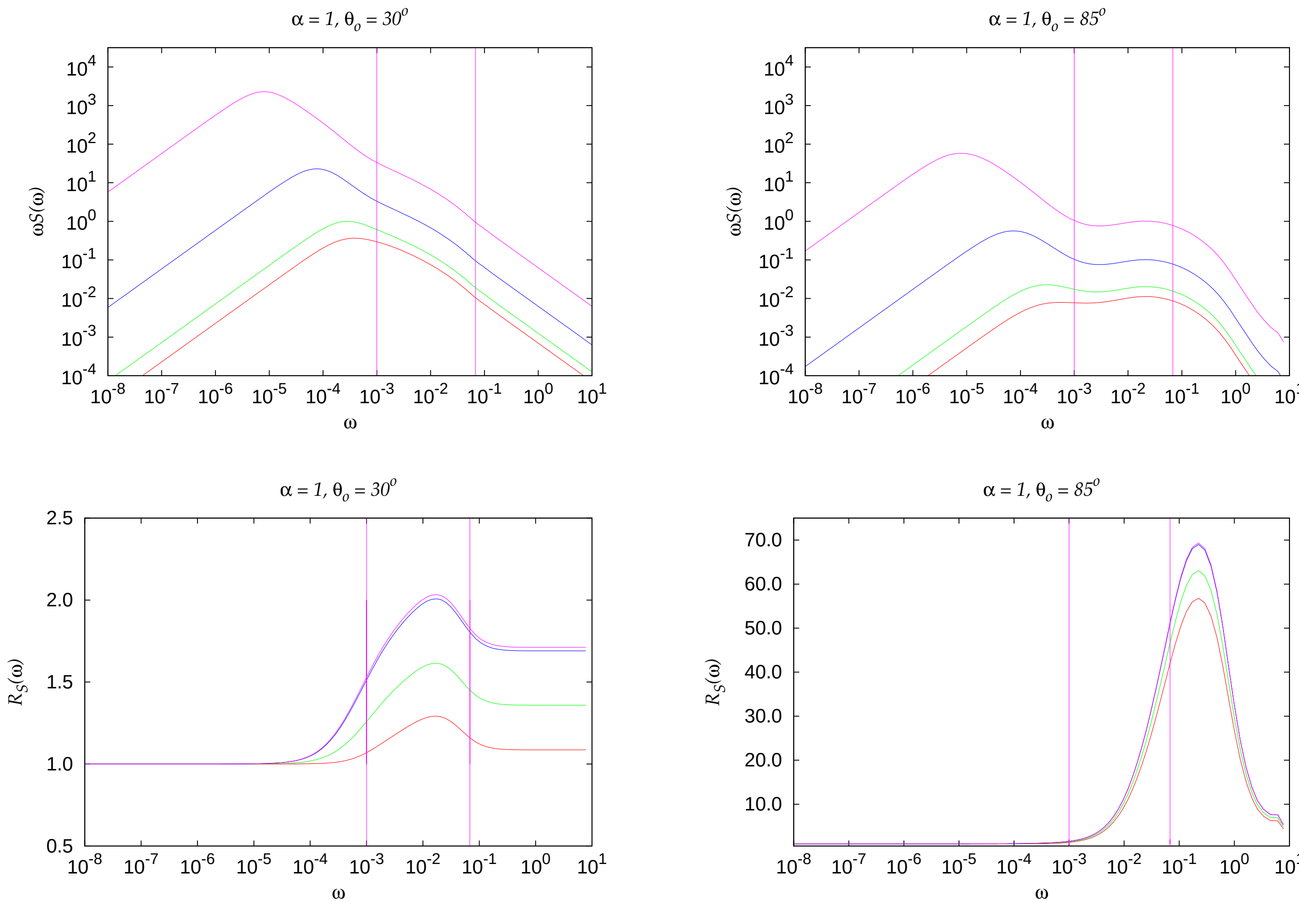
with

$$\mathcal{F}[I](\omega) \to \sum_{k=-\infty}^{\infty} c_k(r)\mathcal{F}[I](\omega - k\Omega(r))$$

and

$$F(t,r) = \sum_{k=-\infty}^{\infty} c_k(r) e^{ik\Omega(r)t}$$

Hawkes process + relativistic effects



Conclusions

The main result:
General framework with the focus on statistical approach, which permits semi-analytical determination of the power spectral density (PSD) of the resulting light curve.

See Pecháček, Karas & Czerny (2008), A&A.

Questions & Answers