Stellar pulsations: chaotic or quasiperiodic

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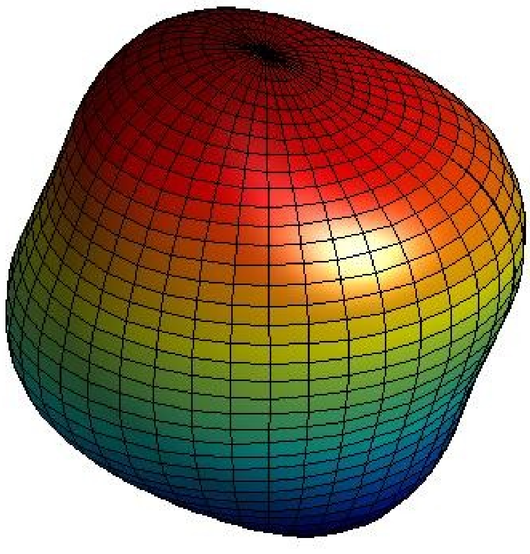
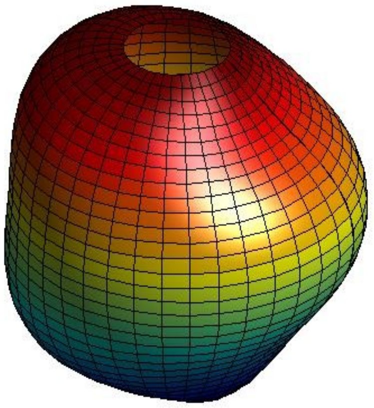


Deterministic chaos and pulsation



- Pulsating stars changing luminosity due to the radial and nonradial pulsation

- Basic mechanism of pulsation – κ mechanism

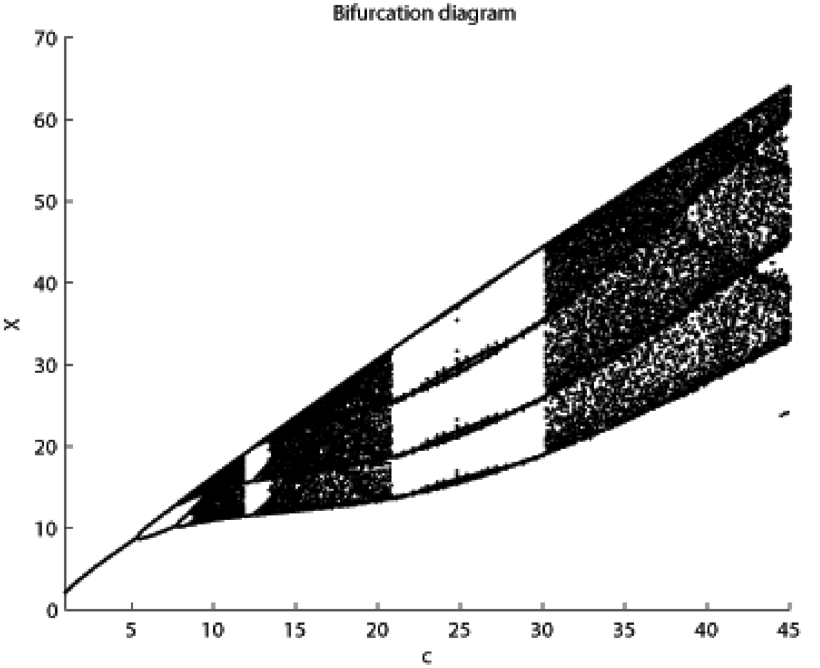
- Detection of regular or **irregular** photometric and spectral variability

- Recorded time - series are nonequidistant and very often with poor sampling and strong noise.

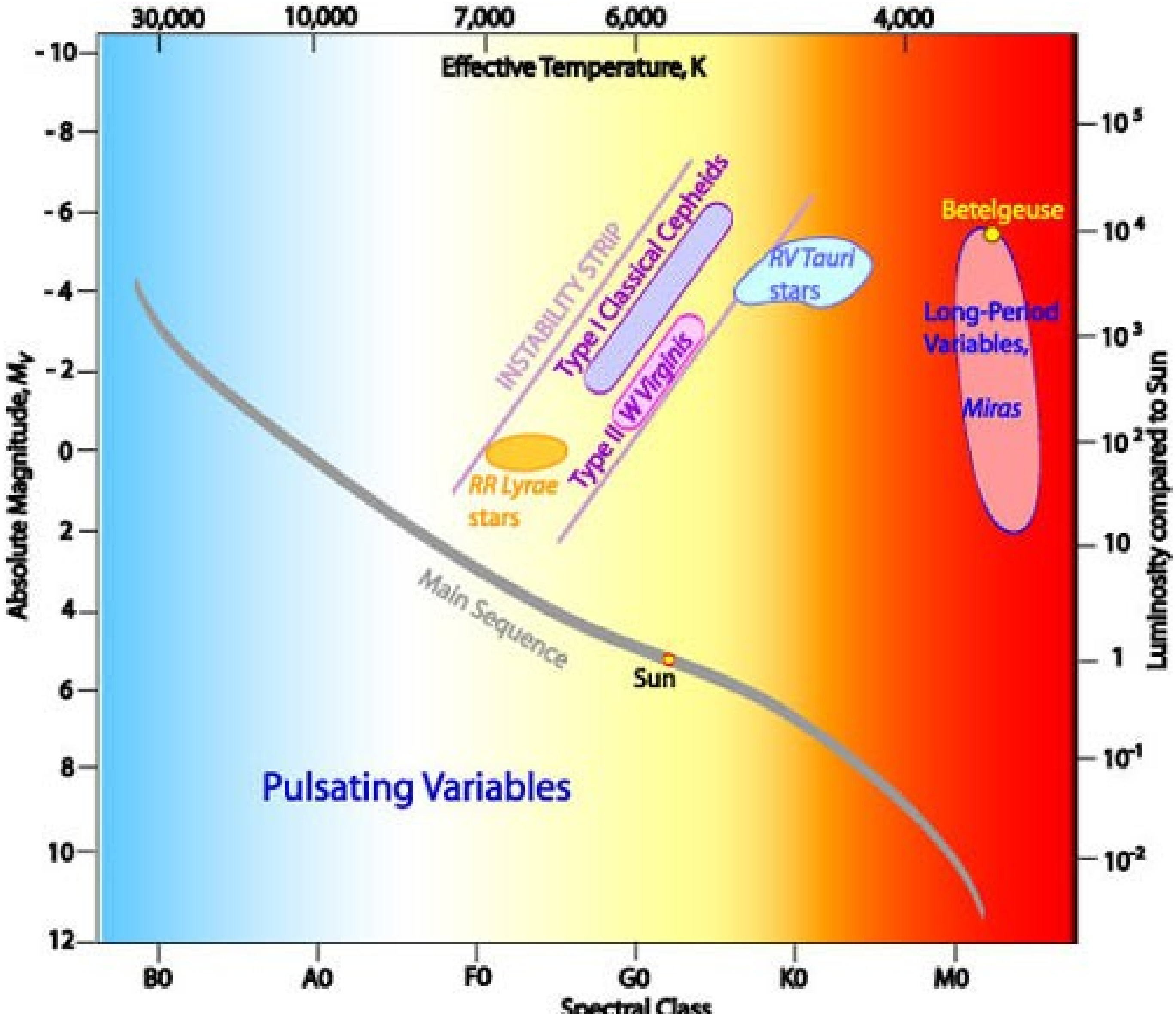
- Can be pulsations chaotic and can we find them ?

Low-dimensional deterministic chaos

- Set of coupled nonlinear ODE
- Strong sensitivity on initial conditions
- Phase portrait have fractal structure, strange attractor
- Change of driving parameters can lead to different regime



Pulsating stars



1D model of pulsations

Tanaka (1988)

Linear

$$\frac{\mathrm{d}x}{\mathrm{d}t} = y$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \alpha x + z$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -\beta \frac{\mathrm{d}x}{\mathrm{d}t}$$

Nonlinear

$$\frac{\mathrm{d}x}{\mathrm{d}t} = y$$

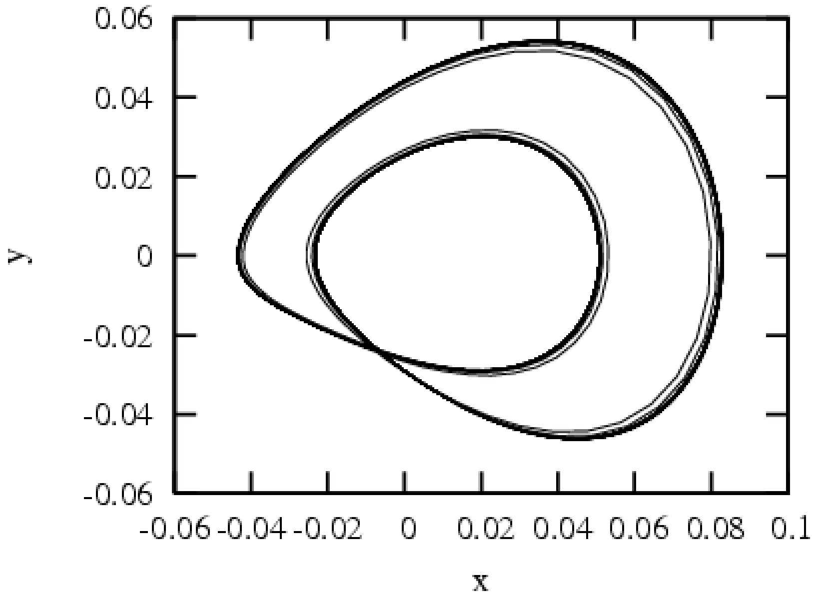
$$\frac{\mathrm{d}y}{\mathrm{d}t} = \alpha x + \mu y + z$$

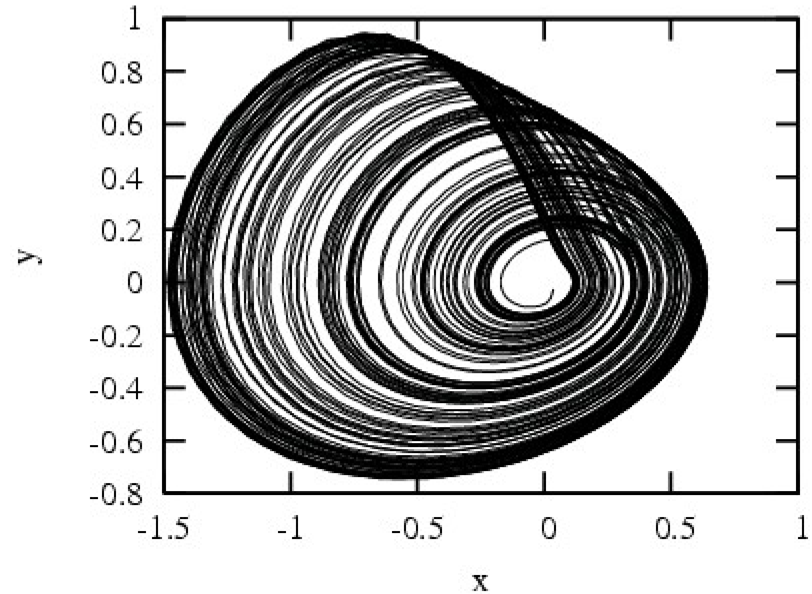
$$\frac{\mathrm{d}z}{\mathrm{d}t} = -\beta y - pz - qy + syz$$

1D model pulsations

Table 1. Parameters of the stellar pulsation model

	α	β	μ	p	q	s	σ	$\overline{x}$
periodic	-0.5	0.5	0.5	3.2	0.5	1.0	0.2	0
chaotic	- 0.5	0.5	0.5	4.0	0.5	1.0	0.3	0





Quasiperiodic behaviour

Quasiperiodic signal:

$$x_{\mathrm{quasi}}(t) = a_1 \sin(f_1 t) + a_2 \sin(f_2 t) + a_3 \sin(f_3 t)$$

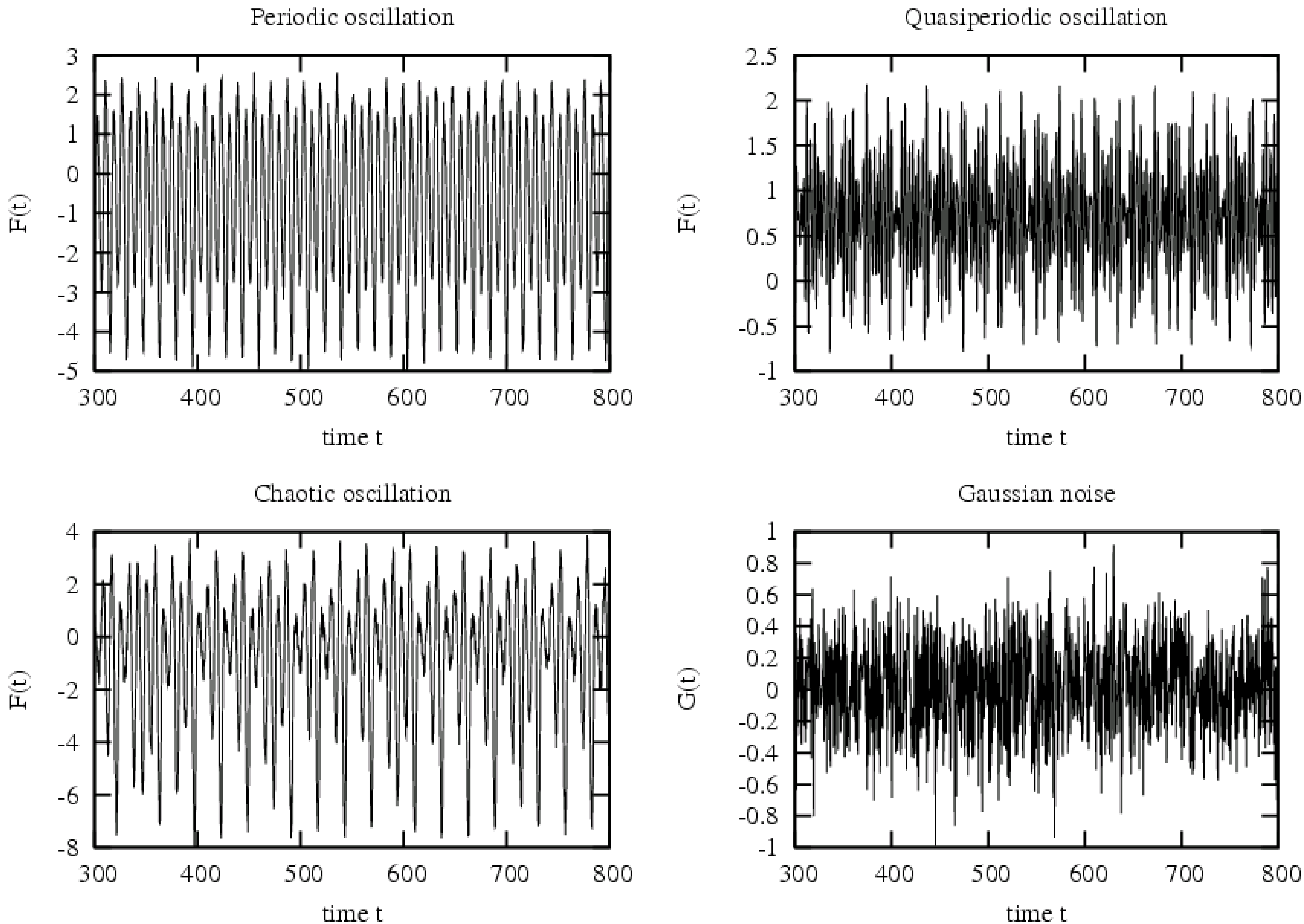
Gaussian noise:

$$G(x) = \frac{1}{\sigma} \exp\left((x - \overline{x})/\sigma^2\right)$$

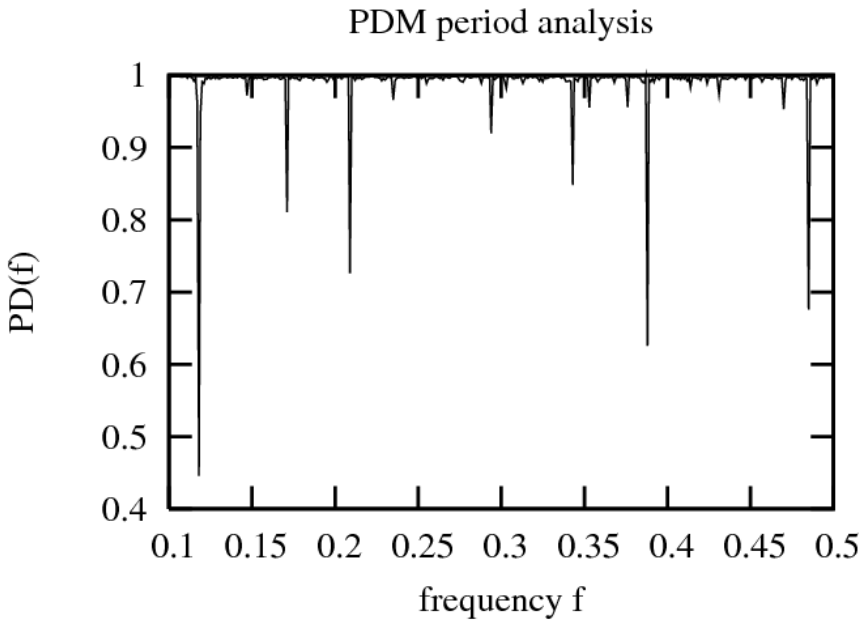
Table 2. Parameters for the quasiperiodic signal

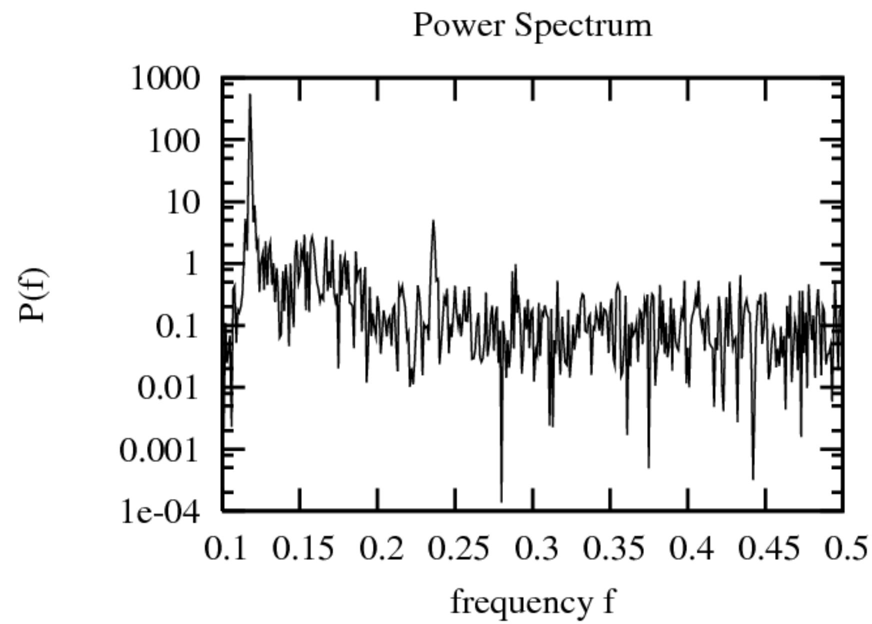
a_1	a_2	a_3	f_1	f_2	f_3	σ	$\overline{x}$
0.4	0.6	0.5	$\sqrt{5}$	$\sqrt{3}$	$\sqrt{2}$	0.1	0

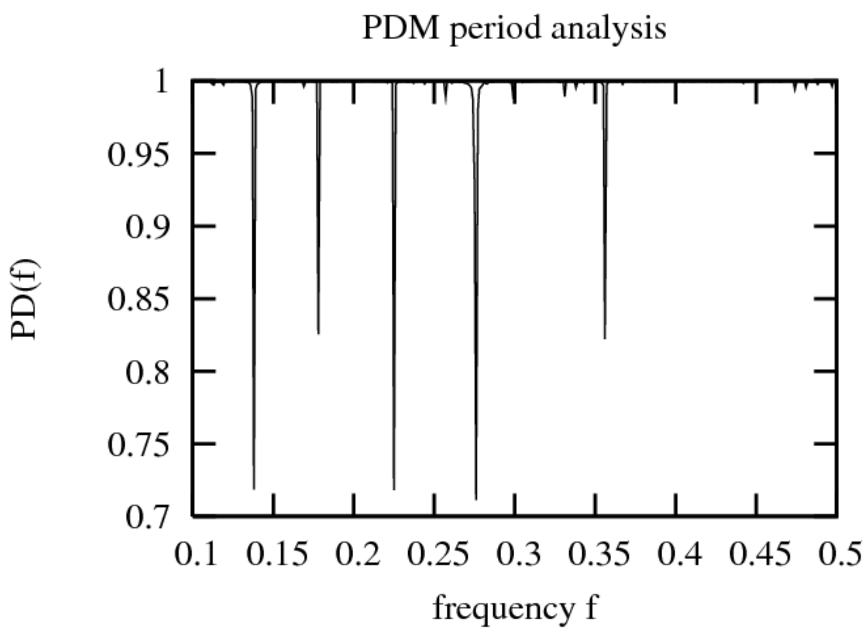
Solution of equations

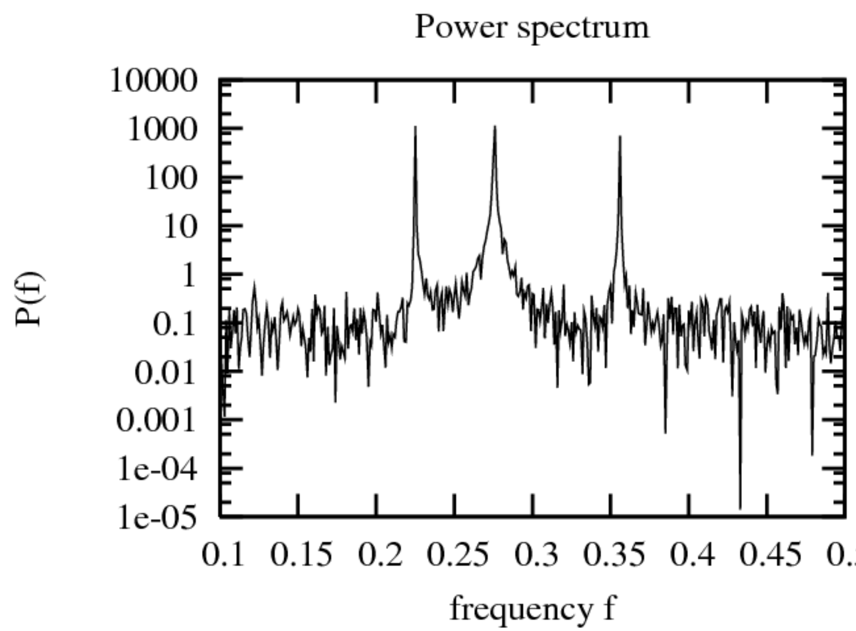


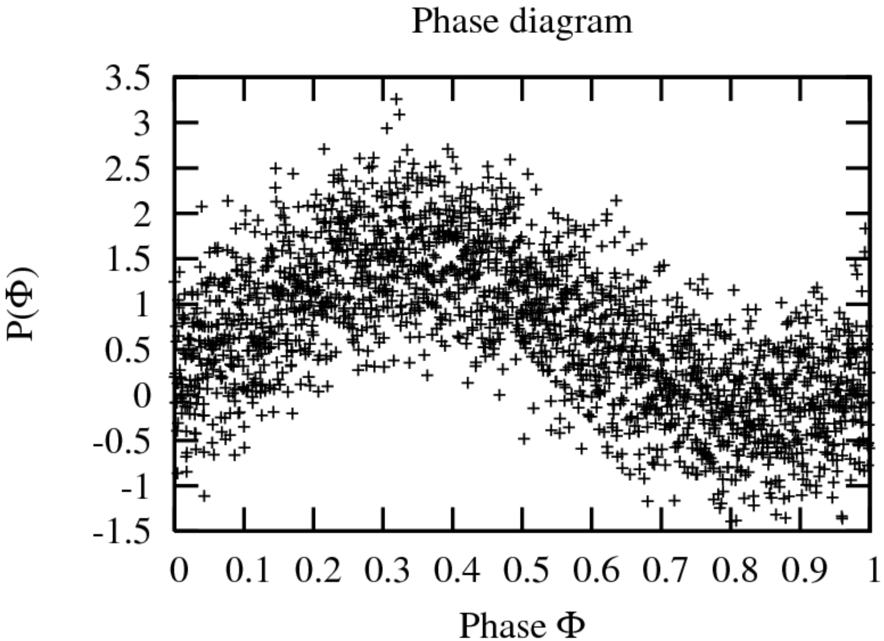
Period searches from time series

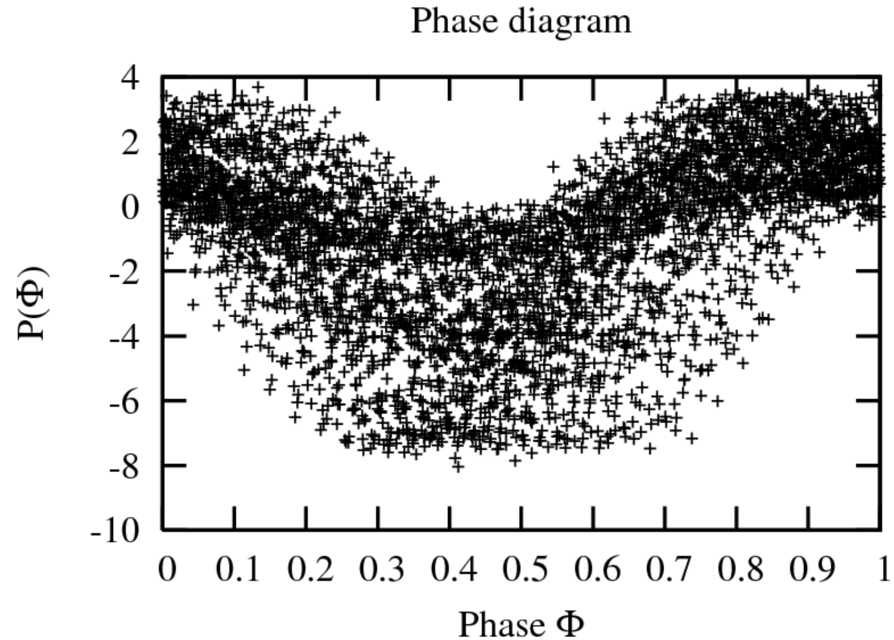


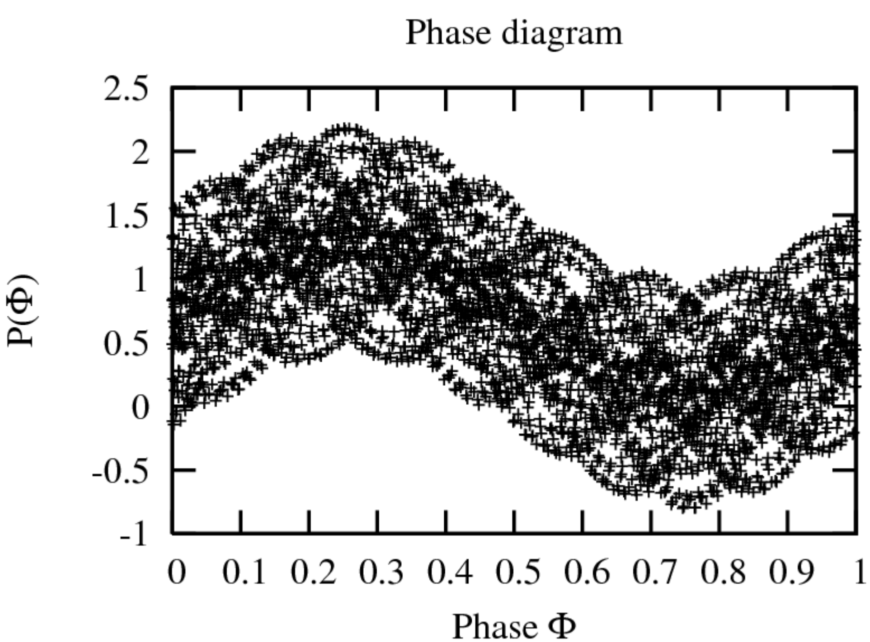


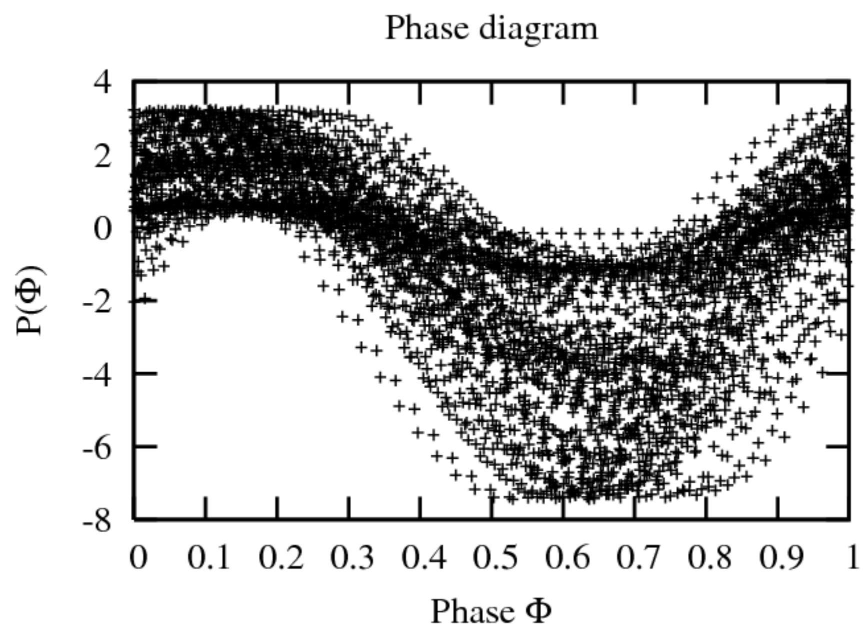





Phase diagram
P(Φ)
3.5 3 2.5 2 1.5 1 0.5 0 -0.5 -1 -1.5
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Phase Φ


Phase diagram
P(Φ)
4 2 0 -2 -4 -6 -8 -10
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Phase Φ


Phase diagram
P(Φ)
2.5 2 1.5 1 0.5 0 -0.5 -1
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Phase Φ


Phase diagram
P(Φ)
4 2 0 -2 -4 -6 -8
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Phase Φ

Reconstruction of phase portrait

- Time series (equidistant)

$$X(t) = (x_1 , x_2, x_3, \ldots, x_N)$$

- R. Takens methods of time-dealy τ

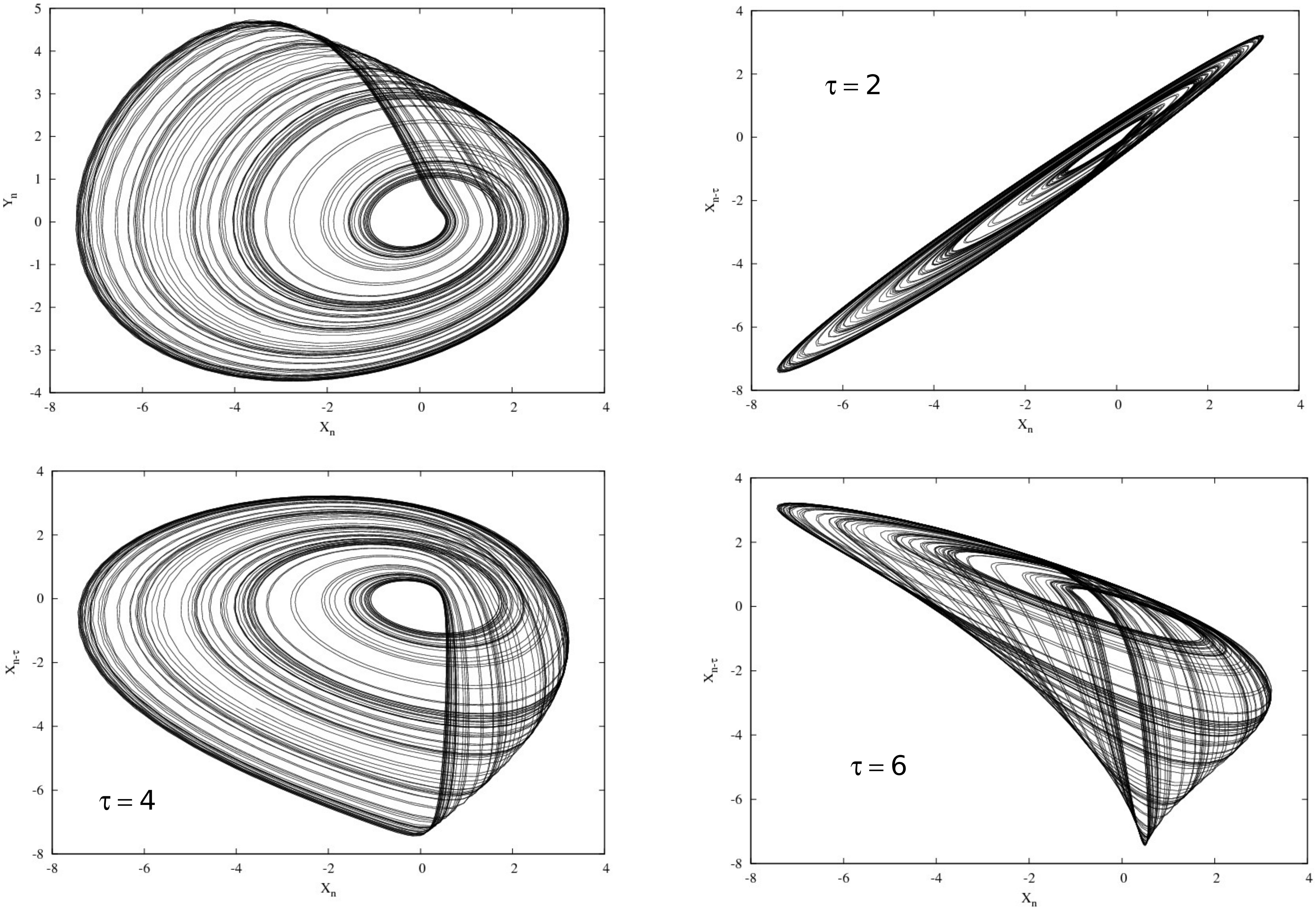
$$X_n = (x_1, x_{1+\tau} , x_{1+2\,\tau}, x_{1+\,3\,\tau\,+}, \ldots, x_{n+(m-1)\tau})$$

- Parameter of the method:

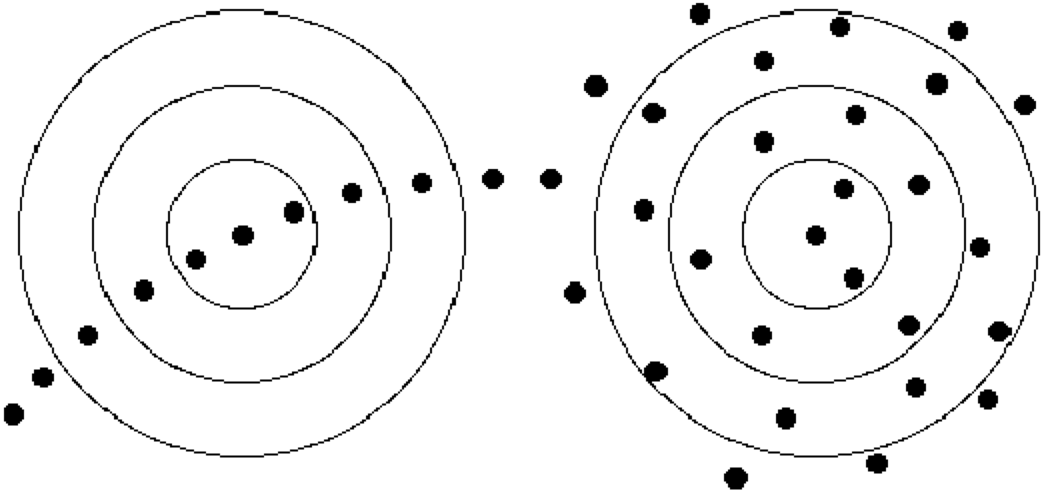
  Time delay τ
  Dimension of reconstructed pahe space *m*

- Reconstructed phase portrait have same topological invariants

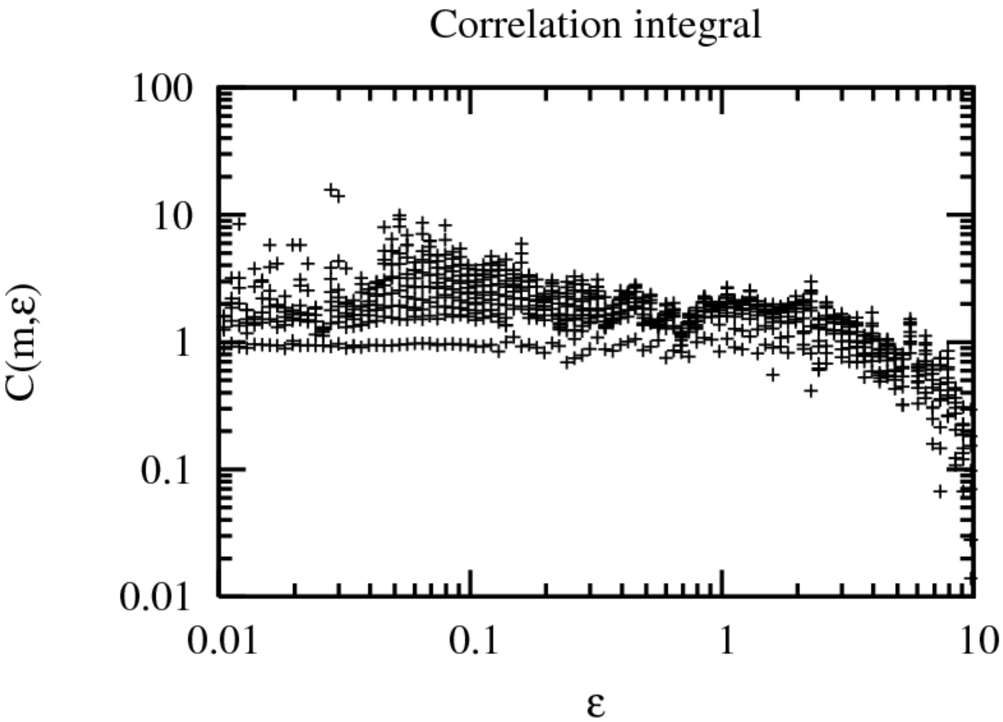
Reconstruction of phase portrait



Correlation dimension



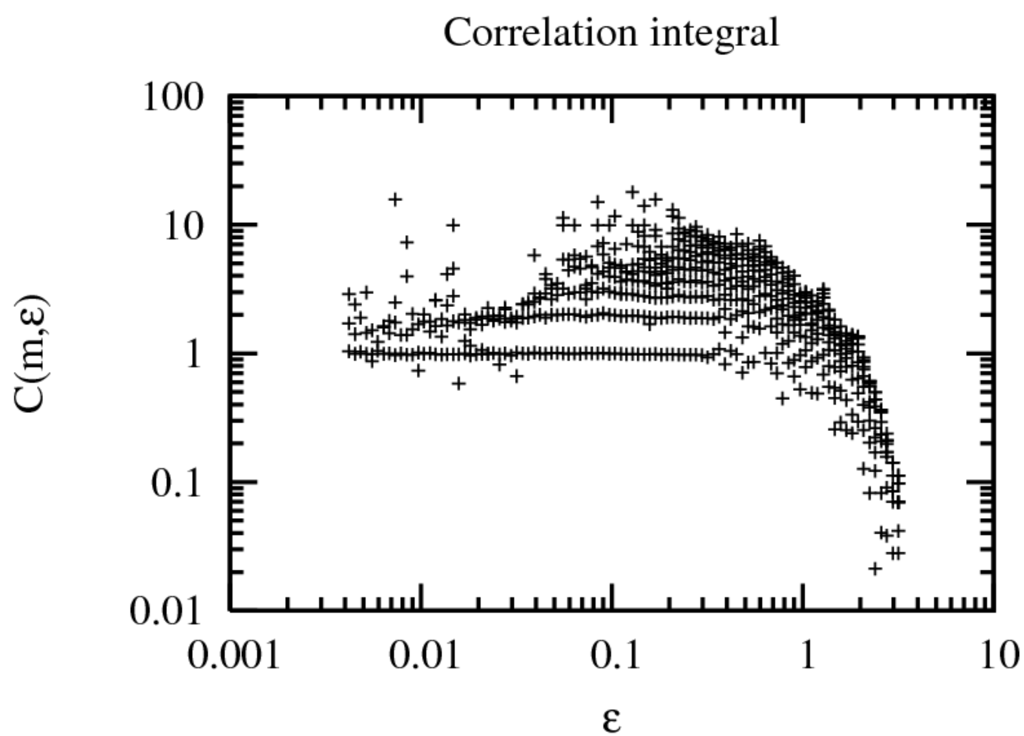
$$C(m,\epsilon) = \frac{2}{(N-n_{\min})(N-n_{\min}-1)} \sum_{i=1}^{N} \sum_{j=i+1+n_{\min}}^{N} \theta(\epsilon - |s_i - s_j|)$$



Chaotic signal

$C_{\mathsf{kor}} = 1.9 \pm 0.8$

Quasiperiodic signal



Summary

- Simple nonlinear model of stellar pulsations lead to the chaotic oscilllations

- It is difficult to distinguish between chaotic and quasiperiodic signal with using classical tools only

- When data contains important noise, quasiperiodic or chaotic character of the signal can be overlooked and simple periodic (false) solution with strongest frequency can be determined

- In order to definitely rule out presence of the chaos it is necessary to use the nonlinear time series analysis

Next step

- Chaotic behaviour does not mean inpredictable
- We can predict signal on certain time scale
- Reconstructed phase portrait can help us to derive simple physical model and find driving parameters
- With the knowledge of driving parameters and their values for different regime (chaotic or quasiperiodic) we can determine subgroups in HR diagram for such a star

First step: prediction of global behaviour

Mapping function:

$$\mathbf{F}(\mathbf{X}) = \sum_k \mathbf{C}_k \mathbf{P}_k(\mathbf{X})$$

with polynomials:

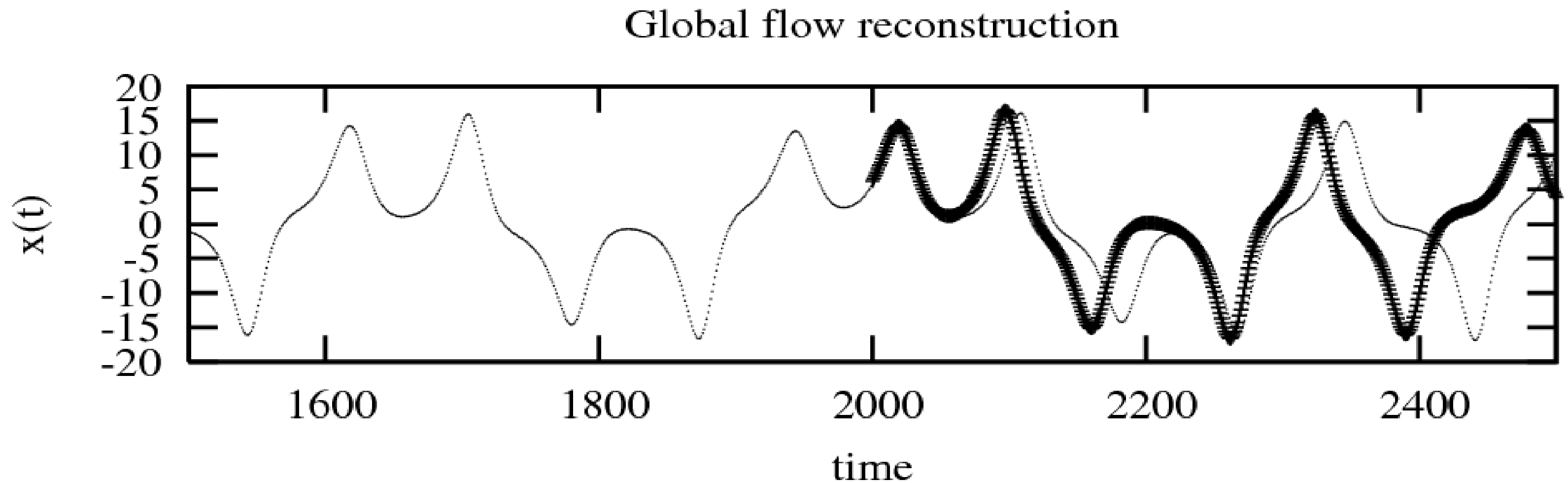
$$P_{\mathbf{k}}(\mathbf{X}) = \sum_\alpha \mathbf{A}^\alpha_{\mathbf{k}} (\mathbf{X}^{\alpha_1})^{\mathbf{k_1}} (\mathbf{X}^{\alpha_2})^{\mathbf{k_2}} \ldots \qquad \alpha = 1, 2, \cdots, d$$

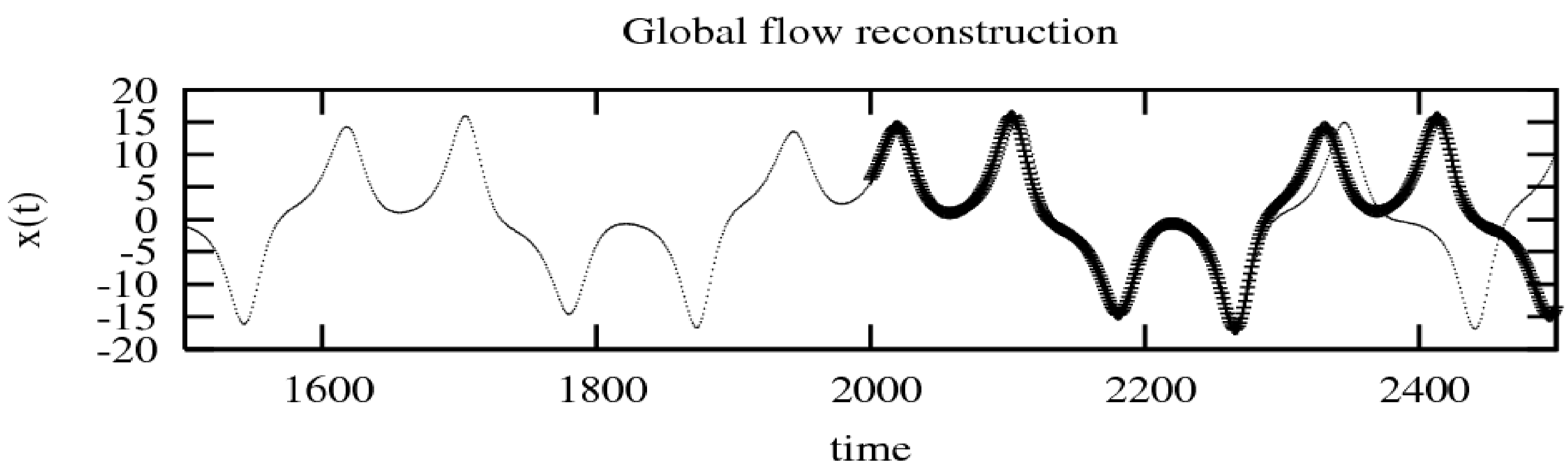
up to order

$$\sum_j k_j \leq p$$

First step: prediction of global behaviour

$$\text{Ƅ}^{ƙ}(\chi) = \sum^{\alpha} \forall^{ƙ}_{\alpha}(\chi_{\alpha 1})_{ƙ 1}(\chi_{\alpha 2})_{ƙ 2}\cdots$$





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