The transformation of the Sylvester matrix and the calculation of the GCD of two inexact polynomials

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Abstract

Euclid's algorithm is a classical method for calculating the greatest common divisor (GCD) of two univariate polynomials. The algorithm should not be executed in a floating point environment because roundoff errors are sufficient to return an incorrect answer. If the given polynomials are inexact, only an approximate GCD can be calculated, that is, a GCD that is obtained by perturbing both polynomials slightly, such that their perturbed forms have a non-constant GCD. The calculation of an approximate GCD of two inexact polynomials is formulated as the construction of a rank deficient Sylvester resultant matrix (which will henceforth be called the Sylvester matrix) that is near the Sylvester matrix of the given inexact polynomials.

In this lecture the connection of Euclid's algorithm with the Sylvester matrix is established, the application of the method of structured total least norm for the computation of an approximate GCD of two inexact polynomials using the Sylvester matrix is described. It is shown that this yields an equality-constrained least squares problem (the LSE problem), and a review of methods for solving the LSE problem is formulated. Numerical examples is presented.

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