WIND LINE PROFILE USING MONTE CARLO RADIATION TRANSFER

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Radiative (Magneto) Hydrodynamic seminar
Ondřejov 28.01.2010

Basic properties

- HOT STARS - spectral types O, B and A; $T_{\rm eff} > 10000$ K
- STELLAR WIND - escape of particles from star (strongest winds from massive luminous hot stars)
- MASS LOSS RATE and TERMINAL VELOCITY - for hot-stars $\dot{M}$ up to 10^{-6} $M_{\odot}$ *year*$^{-1}$ and V_{∞} up to $\approx$ 3000 *km s*$^{-1}$

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- evolution of massive stars
- energy and momentum input into interstellar medium (ISM)
- enrichment of ISM with heavier elements (metals)

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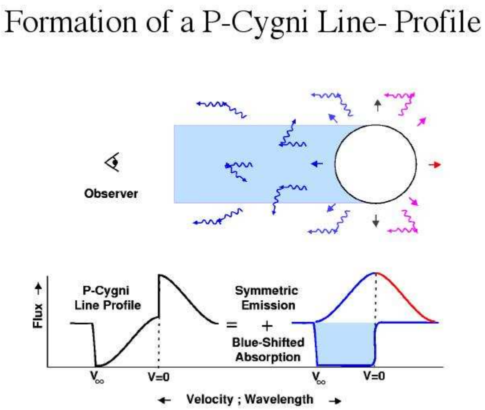
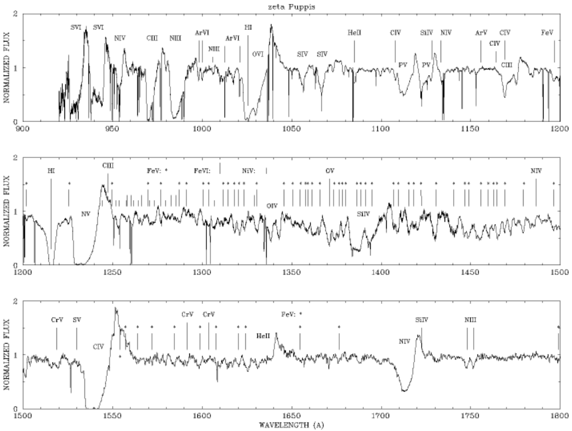
- P CYGNI PROFILE - profile of line which is created in differentially expanding medium

P-Cygni profile



by Stan Owocki

Wind lines of ζ Puppis (Pauldrach et al., 1994)

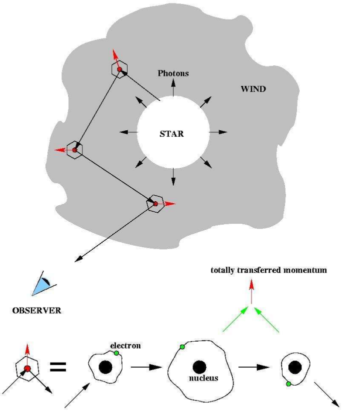


Merged spectrum of Copernicus and IUE UV high-resolution observations of the O4I(f) supergiant ζ Puppis

(900-1500: Morton & Underhill1977; 1500-1800: Walborn et al.1985)

Basic properties

The principle of radiatively driven winds



- LINE RADIATION DRIVEN WIND - (Lucy & Solomon, 1970)

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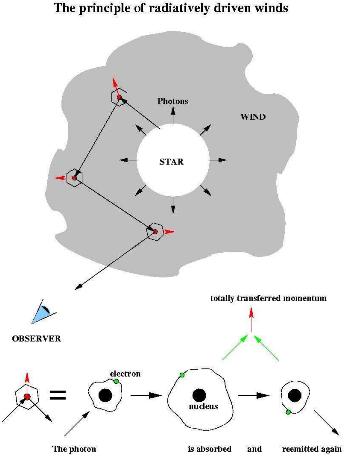

- LINE RADIATION DRIVEN WIND - (Lucy & Solomon, 1970)
- CAK MODEL - the first hydrodynamical solution of the line driven wind (Castor, Abbott & Klein, 1975)
- STANDARD WIND MODEL ASSUMPTIONS - stationary, homogeneous and spherically symmetric wind
- Hot-star winds NEITHER SMOOTH NOR STATIONARY – there is CLUMPING

Clumping in hot-star winds

- CLUMPS - regions with different density than the surrounding wind matter)
  - Discrete Absorption Components (DAC) - from observations (e.g. Prinja & Howarth 1986)
  - Line Profiles Variations (LPVs) - evidence of clumps (e.g. Lépine & Moffat 1999; Lépine et al. 1999)
- Stability analysis of stellar winds (Lucy & Solomon 1970; MacGregor et al. 1979; Abbott 1980; Carlberg 1980; Owocki & Rybicki 1984; Owocki & Puls 2002, Krtička & Kubát 2002, and other papers).
- Hydrodynamical simulations (Owocki et al. 1988; Feldmeier 1995, 2003; Dessart & Owocki 2003, 2005; Votruba et al. 2007, and other papers)

How do clumps create?

Small perturbation of driving force tends to grow and steepens into shocks - SHOCK COMPRESSION

Motivation

- Modeling expanding atmosphere is a difficult task
- Stellar winds are usually described in spherical symmetry
- In moving atmosphere due to Doppler shift, opacity and emissivity are not isotropic
- We are able to model 1D smooth wind
- Solve GENERAL RADIATION TRANSFER EQUATION IN 3D for non-smooth wind (INCLUDE CLUMPS)

$$\frac{1}{c}\frac{\partial I}{\partial t} + \vec{n}\cdot\nabla I = \eta - \chi I$$

Wind model

Toy model

- Density $\rho(r)$, temperature $T(r)$ and velocity $v(r)$ structure from the model of the star with $\mathrm{R} = 9.9\,\mathrm{R}_{\odot}$, $\mathrm{M} = 32\,\mathrm{M}_{\odot}$, $\mathrm{L} = 1.74{\cdot}10^{5}\,\mathrm{L}_{\odot}$ ($T_{\mathrm{eff}} = 37500\,\mathrm{K}$), $\dot{\mathrm{M}} = 2.8{\cdot}10^{-7}\,\mathrm{M}_{\odot}/\mathrm{yr}$ and $\mathrm{v}_{\infty} = 3270 km\ s^{-1}$ (Krtička et al., 2009; Krtička & Kubát, 2004)
- Flux at lower boundary of the wind - static spherically symmetric NLTE model atmosphere code of Kubát (2003)

Our adopted wind model consists:

- 90 depth points (89 zones), similarly to Lucy & Abbott (1993)
- Density $\rho(r)$ is taken to be constant within a zone and equal to the value at the lower radius of the zone
- Radial velocity $v(r)$ is linearly interpolated inside the zone

Wind model

Assumptions

- All electrons in the wind come from HYDROGEN ionization
- The opacity of the medium consists of only two processes, LINE SCATTERING under Sobolev approximation, and the ELECTRON SCATTERING
- Ionization and excitation – LTE

Monte Carlo Radiation Transfer (MCRT)

- MONTE CARLO METHOD - quantity $\varepsilon \in [\varepsilon_1, \varepsilon_2]$ can be sampled from a probability distribution function (PDF) P_ε using uniformly distributed RANDOM NUMBERS R in the interval $\langle 0; 1)$

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Advantage

- Quite simple compared to other radiative transfer techniques
- Relatively easy to develop and less likely to suffer from numerical problems (Auer 2003)
- Easy to extend to multi-dimensional problems
- Easy to parallelize (photons can propagate independently of each other)

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Disadvantage

- Enormous need of computational power to obtain results with sufficient signal-to-noise ratio
- Requires a large number of photons to be tracked

Monte Carlo Radiation Transfer (MCRT)

1. The Cumulative Distribution Method – if an analytical solution of $P(\mathrm{x})$ for x_0 is possible

$$\xi = \int_{\mathrm{a}}^{\mathrm{x}_0} P(\mathrm{x})\, d\mathrm{x} = \psi(\mathrm{x}_0)$$

$$P(\mathrm{x}) \leqslant 1; \quad \int_{\mathrm{a}}^{\mathrm{b}} P(\mathrm{x})\, d\mathrm{x} = 1$$

x_0 – parameter we wish to obtain
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2. The Accept/Reject Method – works for any PDF if we know the maximum value

- Pick x_1 in range $[a, b]$: $\mathrm{x}_1 = a + \xi_x(b - a)$, calculate $P(\mathrm{x}_1)$
- Pick y_1 in range $[0, P_{max}]$: $\mathrm{y}_1 = \xi_y P_{max}$
- If $\mathrm{y}_1 > P(\mathrm{x}_1)$, reject x_1; if $\mathrm{y}_1 < P(\mathrm{x}_1)$, accept x_1

Random number generator (RNG)

- Computational RNGs produce sequences of PSEUDO RANDOM NUMBERS (PRN), (Press et al., 1992)
- PRNs are determined by the NUMERICAL ALGORITHM in use and an INITIAL SEED
- RNGs have to provide sufficiently long sequences of independent RN, and also should be fast and efficient

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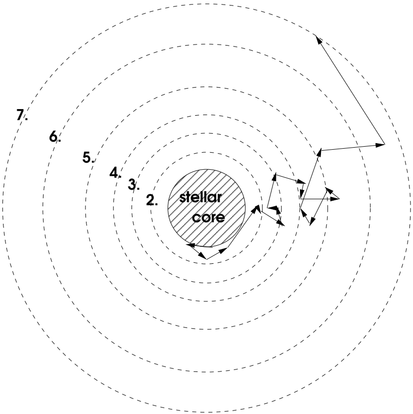
- In our Monte Carlo scheme we used a UNIFORM RNG (Pang 1977)
- Multiplicative congruential algorithm (Lehmer 1951)

$$x_{i+1} = (ax_i + c) \mod m; \quad m = 2^{31} - 1; \quad a = 7^5; \quad c = 0$$

Minimal Standard generator (Park & Miller 1988)
x_i – sequence of pseudo random values
$m > 0$ – the "modulus" (the sequence repeats itself after $m - 1$ values)
$0 < a < m$ – the "multiplier"
$0 \leq c < m$ – the "increment"
$0 \leq x_0 < m$ – the "seed" or "start value"

MCRT schame

- The basic concept of a MCRT code is to TRACK PHOTONS
- Photon path and its interaction are simulated by sampling randomly from PDF
- Emit photon (pick random starting frequency and direction)
- Photon travels some distance (pick random optical depth)
- Something happens ... electron scattering or line scattering ... (pick random isotropic direction)
- If photon exit the medium, capture it



Photon path inside wind

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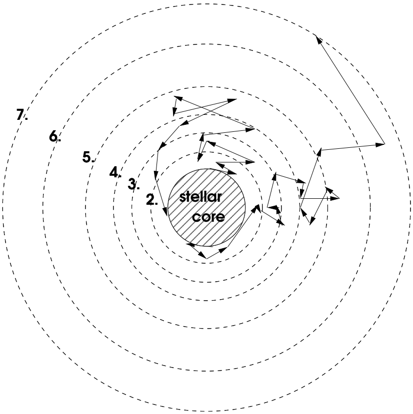

Photon path inside wind

Creation of photon

- Photons are sent from the stellar surface ($R_\star = 1$) outwards
- Frequency of newly created photons is determined using the emergent flux distribution from the static hydrogen-helium photosphere (Kubát, 2003), with a help of the accept/reject method
- The direction of the photon is randomly chosen (FLUX in any direction of emission is ISOTROPIC)

$$F_\nu = \int I_\nu \cos\theta d\Omega; \quad \xi = 2\int_0^\mu \mu' \, d\mu'; \quad \xi = \frac{1}{2\pi}\int_0^\phi d\phi \Rightarrow$$

$$\mu = \cos\theta = \sqrt{\xi}; \quad \phi = 2\pi\xi$$

- Initial photon direction

$$s_x = \sin\theta\cos\phi; \quad s_y = \sin\theta\sin\phi; \quad s_z = \cos\theta$$

Optical depth calculation

- The optical depth is randomly chosen

$$P(\tau) = e^{-\tau}; \quad \xi = \int_0^{\tau_\xi} e^{-\tau}\, d\tau = 1 - e^{-\tau_\xi} \Rightarrow$$

$$\tau_\xi = -\log \xi$$

- The actual optical depth is calculated by summing opacity contribution along photon path

$$\tau = \int_0^L \chi\, ds$$

- New photon's position is then updated according to

$$\mathrm{x} = \mathrm{x} + \mathrm{L} \sin\theta \cos\theta; \quad \mathrm{y} = \mathrm{y} + \mathrm{L} \sin\theta \sin\phi$$

$$\mathrm{z} = \mathrm{z} + \mathrm{L} \cos\theta$$

Optical depth calculation

- SCATTERING ON FREE ELECTRONS – this process acts on all photons with any frequency in the same way ($\chi_e = n_e \sigma_e$)
- RESONANCE LINE SCATTERING – this process needs that the frequency of a photon meets a Doppler shifted frequency of a line of a scattering atom
- The condition that the line scattering may happen is

$$\nu_{\mathrm{line}} = \nu_{\mathrm{obs}} \left(1 - \frac{\vec{s} \cdot \vec{v}(r)}{c} \right)$$

- The optical depth of the electron scattering τ_{elsc} is calculated as

$$\tau_{\mathrm{elsc}} = \int_0^{L'} n_{\mathrm{e}}(r)\, \sigma_{\mathrm{e}}\, ds; \quad \sigma_{\mathrm{e}} = 6.6516 \cdot 10^{-25} cm^2$$

Electron number density calculation

- Electron number density from given density ρ

$$\rho = n_e m_e + N\bar{m}; \quad n_e = N \sum_a \alpha_a \sum_{j=0}^{J_a} j\, f_{j,a}; \quad n_e = n_e^0 + \delta n_e$$

$f_{j,a} = \frac{N_{j,a}}{N_a}$ – ionization fraction

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- Saha distribution (ionization state of a gas in LTE)

$$\left[\frac{N_{j,a}}{N_{j+1,a}}\right]_{LTE} = 2n_e \left(\frac{h^2}{2\pi m_e kT}\right)^{3/2} \frac{U_{j,a}(T)}{U_{j+1,a}(T)} e^{-(E_{j,a}-E_{j+1,a})/kT}$$

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Partition function

$$U_{j,a}(T) = \sum_i g_{i,j,a}\, e^{-E_{i,j,a}/kT}$$

- The Boltzmann excitation distribution (excitation state of a gas in LTE)

$$\left[\frac{n_{i,j,a}}{n_{l,j,a}}\right]_{LTE} = \frac{g_{i,j,a}}{g_{l,j,a}} e^{-(E_{i,j,a}-E_{l,j,a})/kT}$$

Opacity calculation

- Optical depth for line scattering

$$\tau_{\mathrm{line}} = \frac{\pi e^2}{m_{\mathrm{e}} c} f_{\mathrm{line}} n_{i,j,a} \frac{c}{\nu_0} \left[\mu^2 \frac{\mathrm{d}v(r)}{\mathrm{d}r} + \left(1 - \mu^2\right) \frac{v(r)}{r} \right]^{-1}$$

Sobolev approximation

$$\tau = \frac{\chi_0(r)c}{\nu_0} \frac{1}{\left|\vec{s} \cdot \nabla(\vec{v} \cdot \vec{s})\right|} = \frac{\chi_0(r)c}{\nu_0} \frac{1}{\left|\frac{dv_s}{ds}\right|}$$

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- Total optical depth

$$\tau_{\rm line} + \tau_{\rm elsc} > \tau_\xi,$$

(photon terminates its travel in the layer where this condition was first fulfilled)

Scattering events

- After a scattering (either line or electron) photon obtains new direction, chosen randomly for the case of ISOTROPIC SCATTERING (Wood et al., 2004)

$$d\Omega = \sin\theta \, d\theta \, d\phi; \Rightarrow \quad P(\theta) = \frac{1}{2}\sin\theta; \quad P(\phi) = \frac{1}{2\pi}$$

$$\xi = \int_0^\theta P(\theta)\, d\theta = \frac{1}{2}\int_0^\theta \sin\theta \, d\theta = \frac{1}{2}(\cos\theta - 1) \Rightarrow \theta = \cos^{-1}(2\xi - 1)$$

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- For the case of line scattering, photon obtains a Doppler shifted frequency (in the observer frame)

$$\nu_{\mathrm{obs,new}} = \nu_{\mathrm{line}} \left(1 - \frac{\vec{S_{new}} \cdot \vec{v}(r)}{c}\right)^{-1}$$

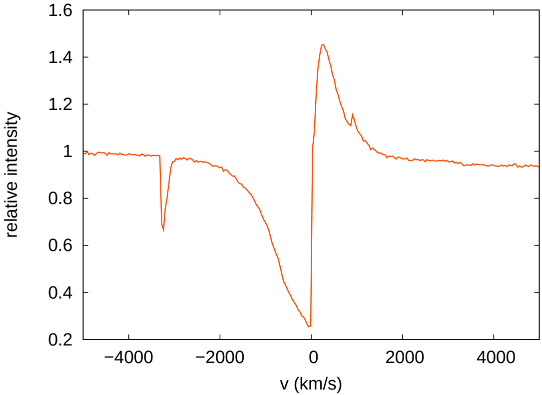
- New optical depth randomly is chosen again

Binning

- Photon can escape from the wind region either back to the stellar surface or towards the observer
- We define a frequency grid, which determines frequency intervals - for determining emergent flux
- Each escaping photon is then counted according to its frequency to the proper frequency interval

Working line profile

The profile of the Hα line
(the flux is expressed as relative intensity with respect to local continuum)



Further work

- Improve line profile
- Full line spectrum
- More continuum opacity sources (bound-free and free-free transitions)
- Extension to 3D to handle inhomogeneous (clumped) stellar wind
- NLTE efects

Radiative (Magneto) Hydrodynamic seminar
Ondřejov 28.01.2010

THANK YOU FOR YOUR ATTENTION!