Stability of bound states in the light-front Yukawa model

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We study the stability of the bound states relative to the cutoff in the system of two relativistic fermions interacting by scalar exchange (Yukawa model). The problem is solved in the framework of the explicitly covariant light-front dynamics, see for review [1]. In ordinary light-front dynamics [2] this problem was studied in [3]. Though the forms of equations are different in both approaches, our equations are strictly equivalent to the ones solved in [3].

The equations are solved on the finite interval of the relative momenta $0 \le k \le k_{max}$. The dependence of the binding energy on the cutoff $k_{max} \to \infty$ is investigated. This dependence is determined by the asymptotical behavior of the interaction kernel. It turned out that for $J = 0$ state the attractive kernel has the same asymptotical behavior in the momentum space like the kernel for the non-relativistic potential $-\alpha'/r^2$. As well known, in this potential the bound states are unstable relative to the cutoff (the particle "falls" on the center) for the coupling constant above some critical value. Therefore the critical value α_c of the coupling constant $\alpha = g^2/(4\pi)$ exists also in the Yukawa model. We calculate this value from a special eigenvalue equation for it which gives $\alpha_c \approx 3.72$. In this way, we conclude that for $\alpha < \alpha_c$ the bound states for $J = 0$ are stable without any cutoff regularization. In this point our conclusion differs from the one settled in [3]. For $\alpha > \alpha_c$ the binding energies are unstable (cutoff dependent).

The numerical calculations of the binding energy v.s. $k_{max} \le 600$ GeV (for the fermion mass $m = 1$ GeV) clearly distinguish two different regimes, convergent and divergent, for α below and above the critical value respectively.

We study also the asymptotics of the wave function $\psi(k)$ at $k \to \infty$. For $\alpha < \alpha_c$ it has the form $\psi(k) \propto 1/k^{2+\beta(\alpha)}$ with $0 < \beta(\alpha) < 1$. The value $\beta(\alpha)$ v.s. α is also calculated theoretically. Note that $\beta(\alpha_c) = 0$. The exponential fit of numerical solution results in the value of $\beta(\alpha)$ coinciding with the calculated one.

The brief publication of the results can be found in [4].

References

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