Institute of Thermomechanics ASCR

ŠÍŘENÍ MECHANICKÝCH VLN V KRYSTALECH, V SÍTÍCH KONEČNÝCH PRVKŮ A VZNIK FREKVENČNÍCH OKEN

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Ústav termomechaniky

Akademie věd České republiky

Praha

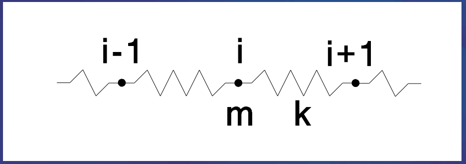
Contents

- Dispersion diagrams (overview)
- Quadratic finite elements
    - spatial discretization error
    - time discretization error
    - mass lumping for explicit schemes
- Numerical experiments
- Outlook

Dispersion curves

After Newton, Kelvin, Born . . .

$$m\ddot{u}_i = k(u_{i-1} - 2u_i + u_{i+1})$$



solution form

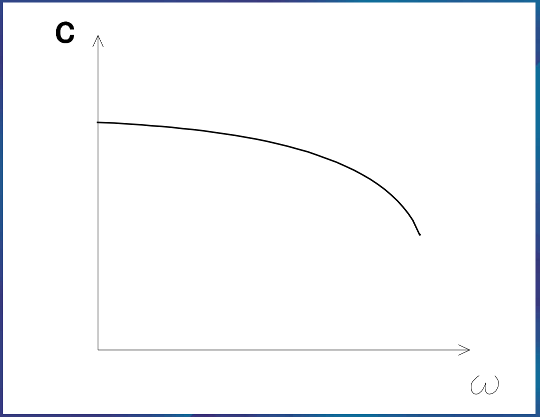
$$u_i = \hat{u}\sin K(x_i - ct)$$

wave number

$$K = \frac{2\pi}{\Lambda} = \frac{\omega}{c}$$

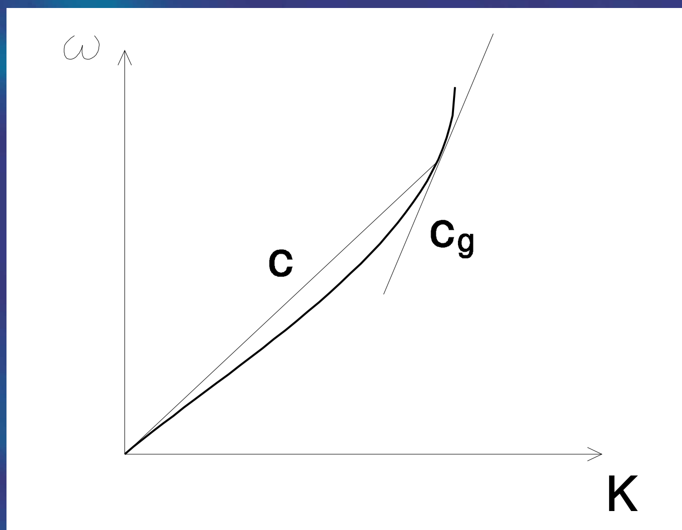
solvability condition

$$c = \text{function}(\omega)$$



Propagation of wave packets

Definition of group speeds is essential for higher order elements.



phase velocity

$$c = \frac{\omega}{K}$$

group velocity

$$c = \frac{\mathrm{d}\omega}{\mathrm{d}K}$$

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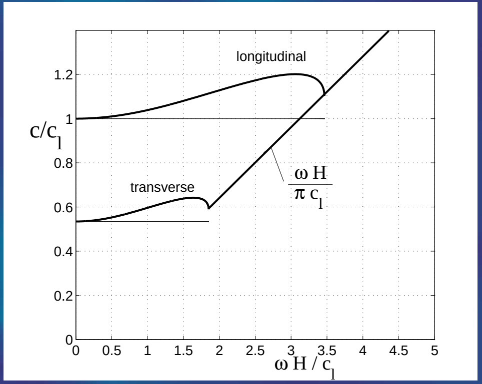
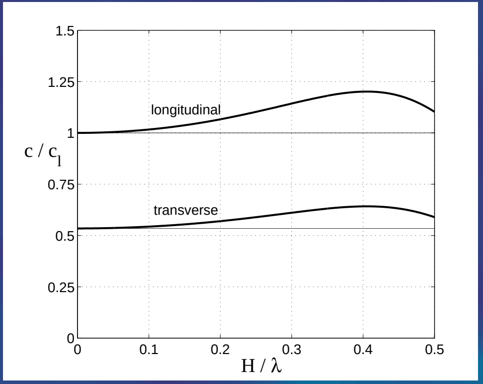
Three dimensional lattices

Brillouin, L.: *Wave Propagation in Periodic Structures.*
Dover Publications, Inc., New York 1953.

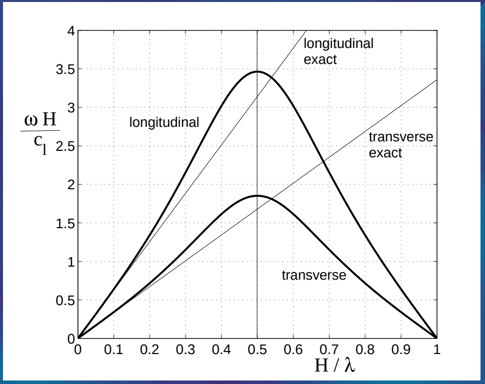
c-ω plot



c-H plot



ω-K plot

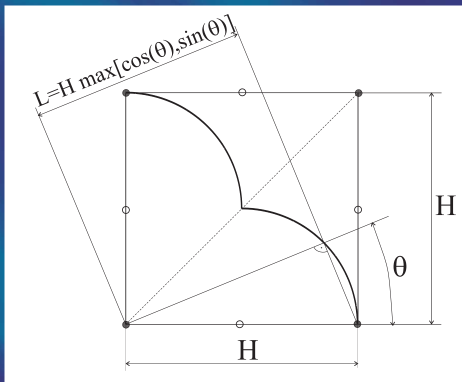


(Pictures shown from tri-linear FE analysis.)

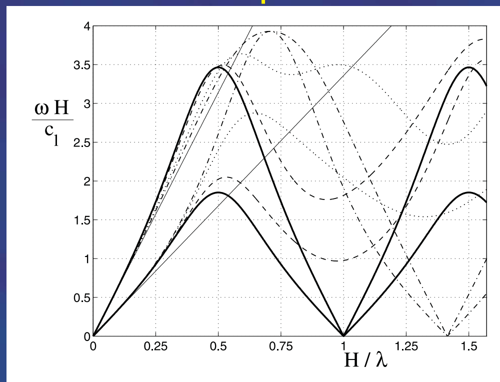
Effect of propagation angle

Characteristic length of element $L = f(H, \theta)$ defined.

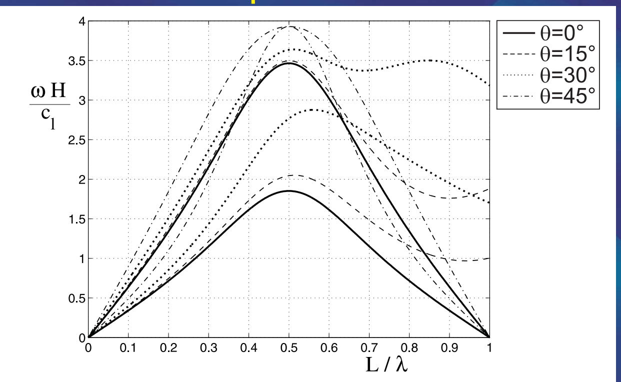
definition of L



ω-H plot



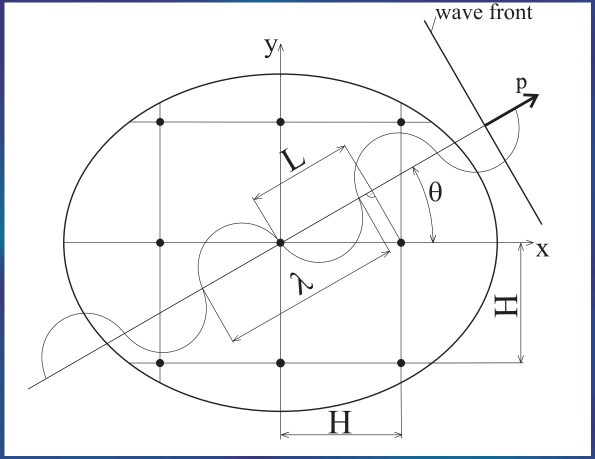
ω-L plot

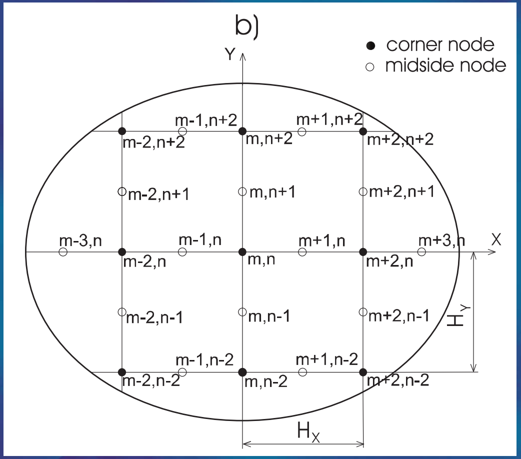


The worst case $\theta = 0$ when $L = H$ is treated further on.

Finite element method

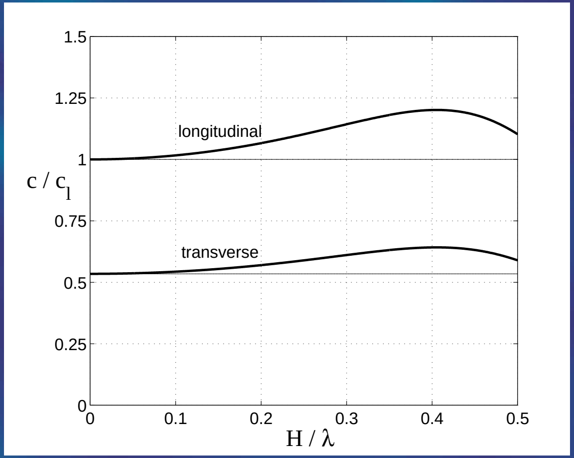
- Belytschko, T., Mullen, R.: On dispersive properties of finite element solutions, In: *Modern Problems in Elastic Wave Propagation.* Wiley 1978.
- Abboud, N.N., Pinsky, P.M.: Finite element dispersion analysis for the three-dimensional second-order scalar wave equation. Int. J. Num. Meth. Engrg., **35**, pp. 1183–1218, 1992.



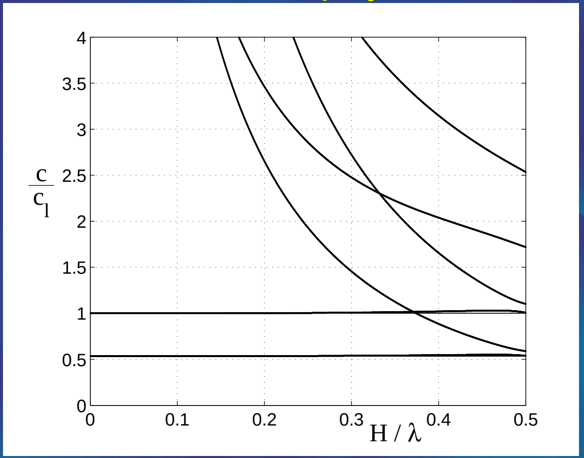


Linear versus quadratic elements

linear



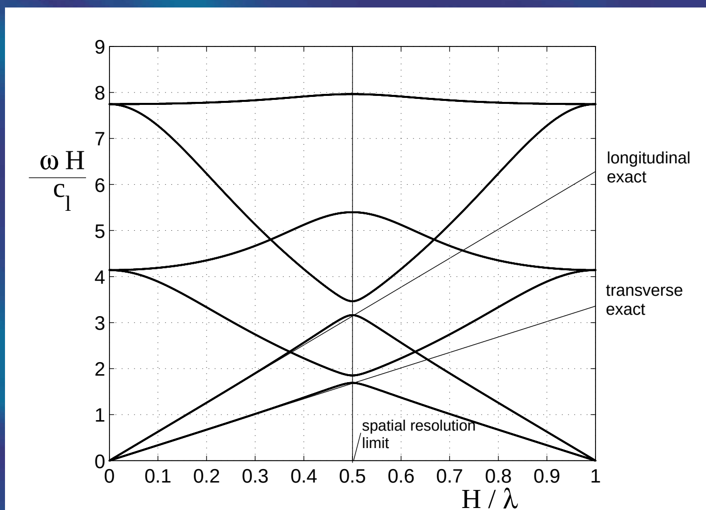
serendipity

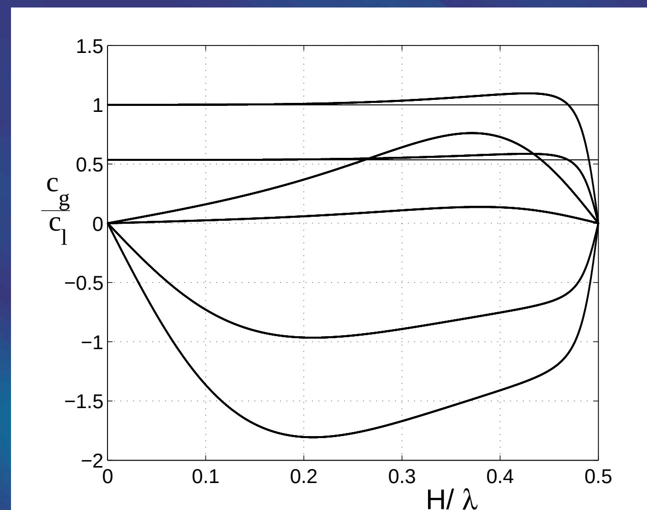


Accuracy of quadratic finite elements is by far better. There are, however, four spurious branches called *the optical modes.* The optical modes are not eigenvectors so that they do not affect numerical stability.

Group velocity

- Lamb, H.: On group-velocity. Proc. Lond. Math. Soc., ser. 2, **1**, pp. 473–479, 1904.
- Mandel'shtam, L.I.: Group velocity in a crystal lattice. Zhurn. Eksp. Teor. Fiz., **15**, pp. 475–478, 1945 (in Russian).

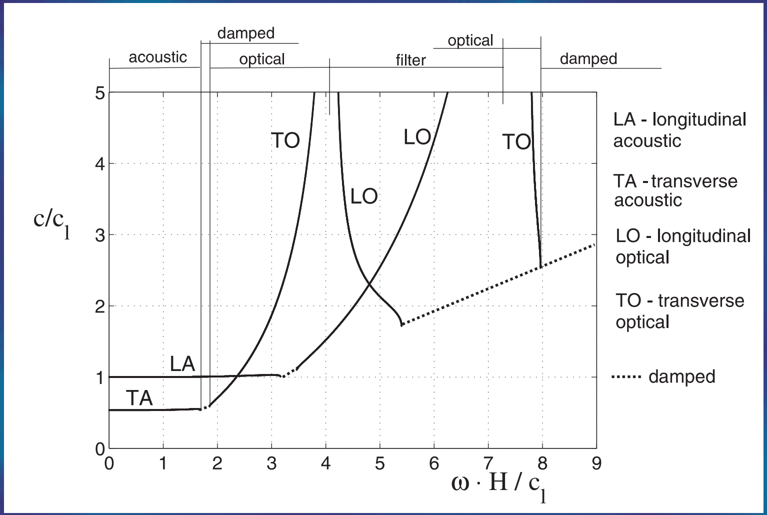




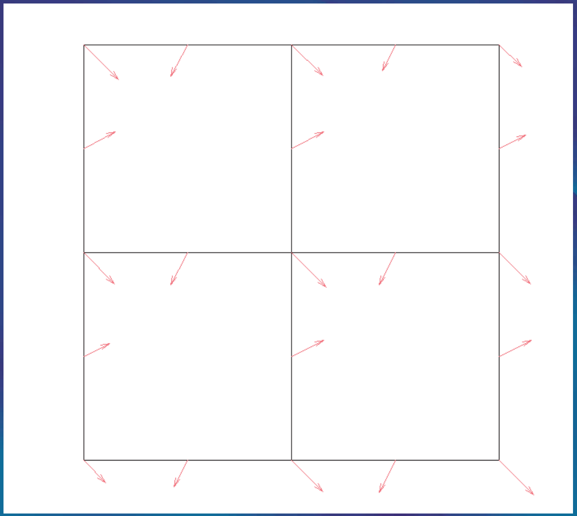
Group velocities $c_g = \mathrm{d}\omega/\mathrm{d}K$ are finite! Negative speed observed!

Optical modes and band filters

c-ω spectrum



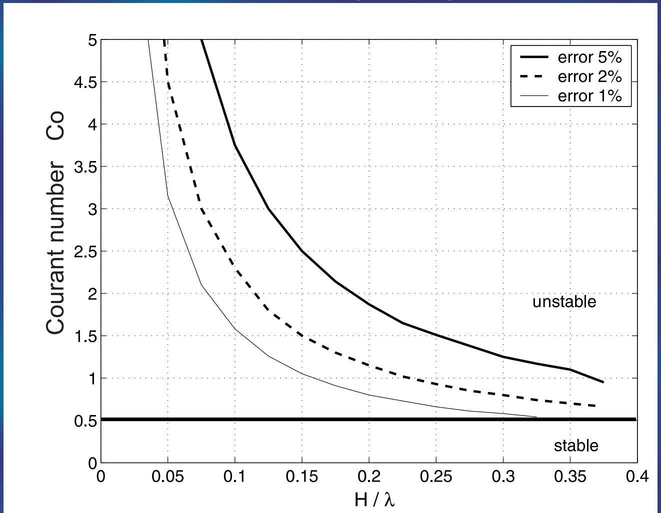
Optical mode 3



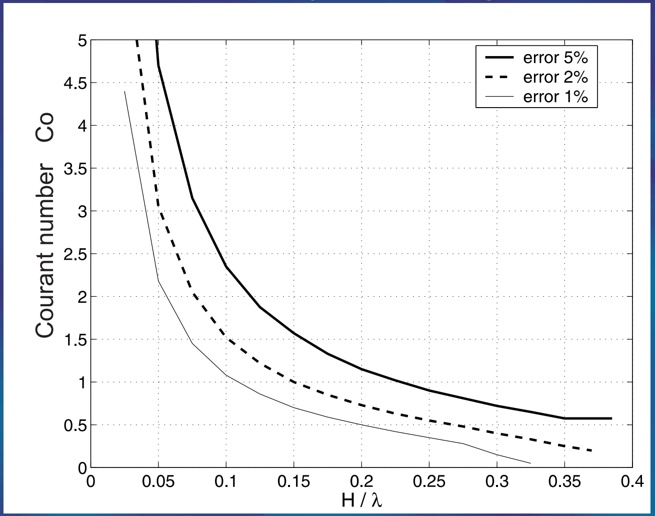
Effect of time integration

Dimensionless Courant number Co$=(c_l \Delta t)/H$

Explicit (CDF)



Implicit (Newmark)

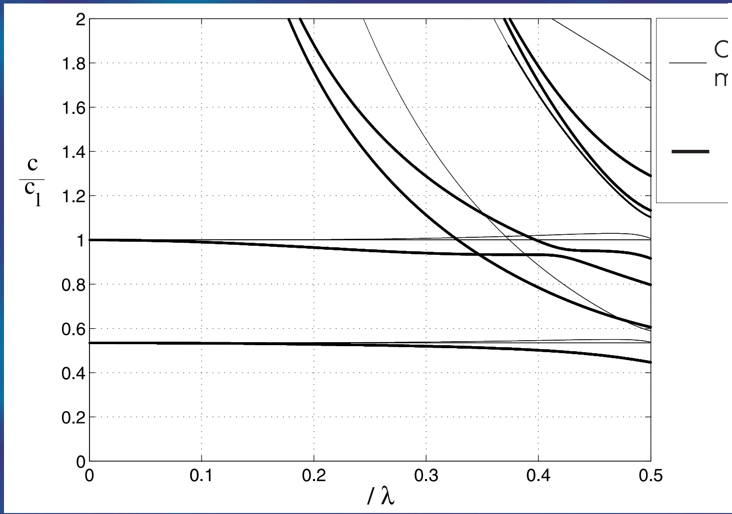


c_l-dispersion analysis now includes spatial *and* time discretization.

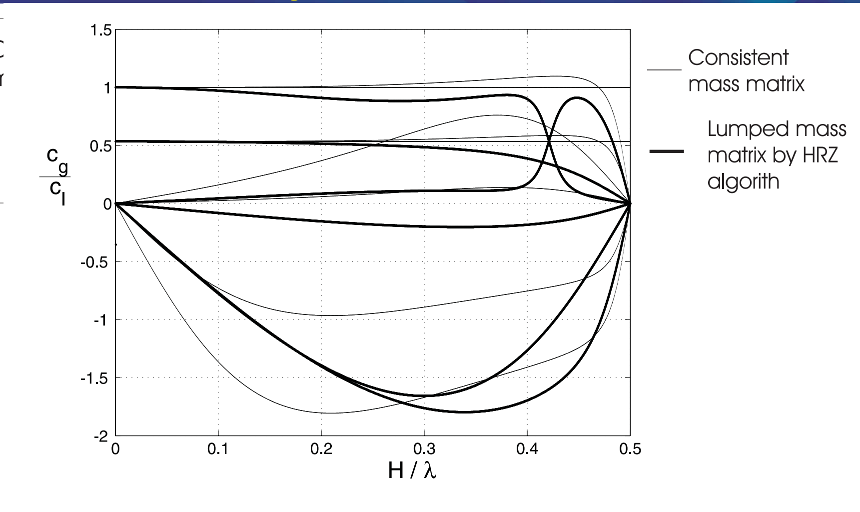
Mass matrix diagonalization

Hinton-Rock-Zienkiewicz lumping scheme used for serendipity elements.

c-H plot



c_g-H plot

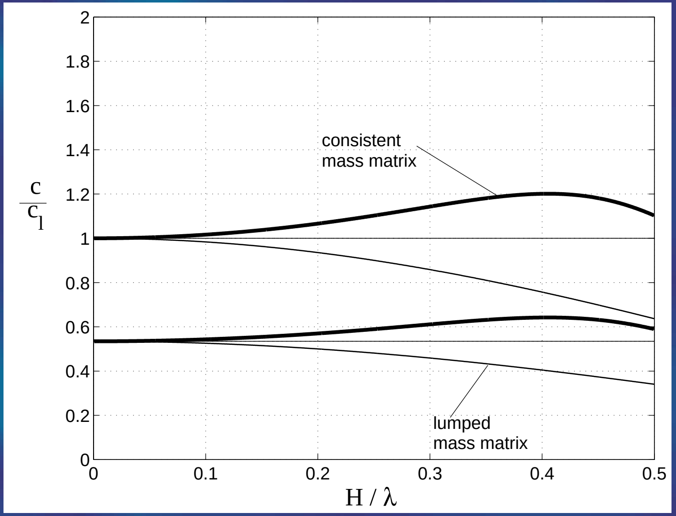


Favourable properties of serendipity elements with consistent mass matrix were spoiled. Transversal wave can overtake the longitudinal wave!

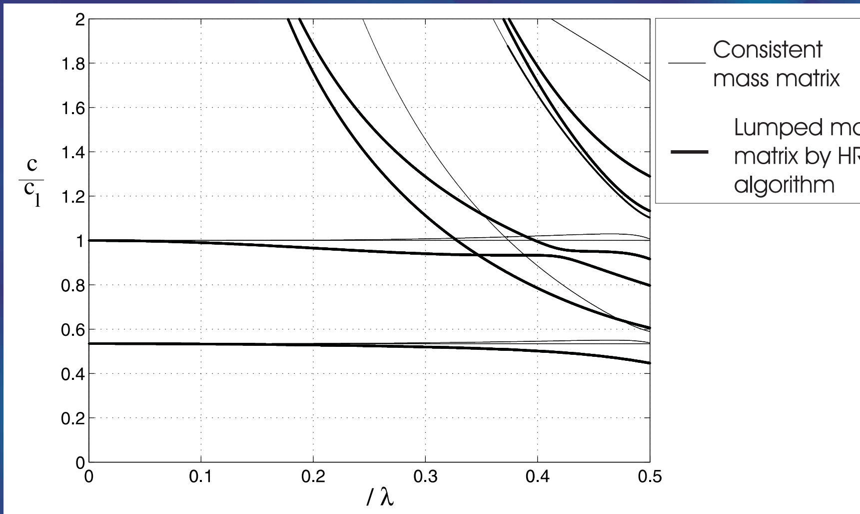
Comparing element types

Row sum and HRZ used.

linear



quadratic



Similar performance—advantage lost.