Left-handed metamaterials

Peter Markoš, Institute of Physcis, SAV, Bratislava

February 15, 2005

Abstract

Left handed metamaterials are man made composites which possess, in a given frequency interval, negative effective permittivity and permeability. I give a short review of electro-magnetic properties of such materials, and describe methods of numerical analysis of the transport of EM wave through left-handed structures.

Maxwell Equations

$$\text{div } \vec{D} = \rho \quad \text{rot } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\text{div } \vec{B} = 0 \quad \text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\rho = 0 \qquad \vec{J} = 0 \qquad \vec{D} = \varepsilon \vec{E} \qquad \vec{B} = \mu \vec{H}$$

ε . . . permittivity

μ . . . permeability

In general, ε a μ are *complex tensors*.

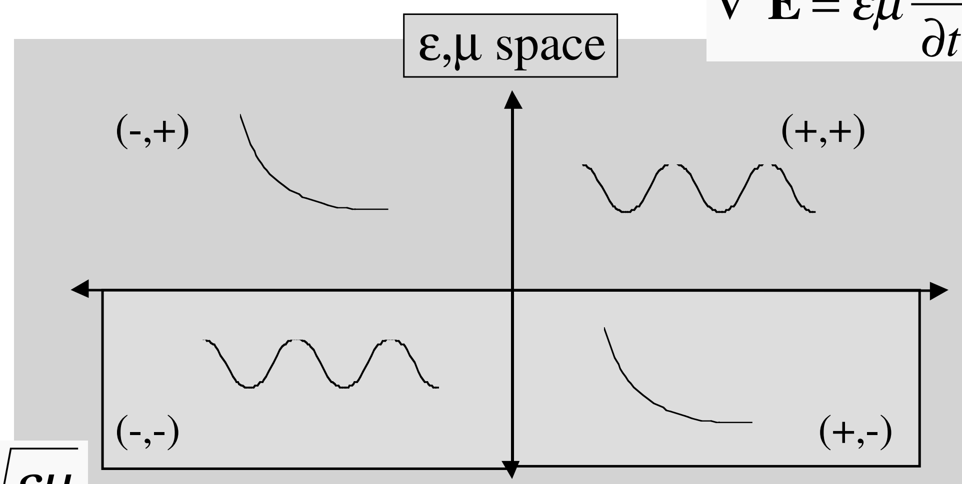
Index of refraction: $n = \sqrt{\varepsilon\mu}$

Impedance: $z = \sqrt{\frac{\mu}{\varepsilon}}$

Veselago

We are interested in how waves propagate through various media, so we consider solutions to the *wave equation*.

$$\nabla^2 \mathbf{E} = \varepsilon\mu \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad n = \sqrt{\varepsilon\mu}$$



$$k = \omega\sqrt{\varepsilon\mu}$$

Sov. Phys. Usp. 10, 509 (1968)

Plane monochromatic wave

$$k \times E = \frac{\omega}{c}\mu H \qquad k \times H = -\frac{\omega}{c}\varepsilon E \tag{1}$$

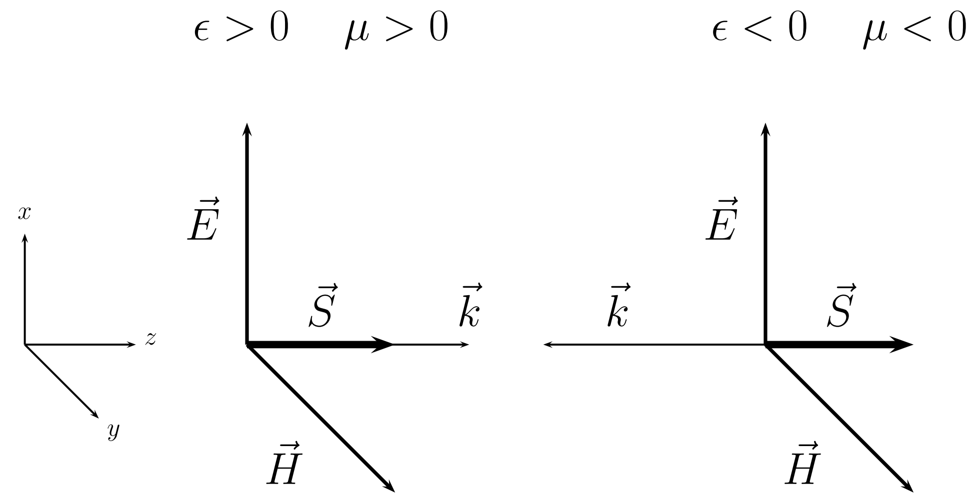
Wave vector:

$$k^2 = \frac{\omega^2}{c^2}\varepsilon\mu \tag{2}$$

There is no physical reason to require both ε and μ to be positive.

In fact, *negative* values $\varepsilon < 0$ and $\mu < 0$ allow wave propagation, too.

Left-handed rule



Left-handedness: vectors $\vec{E}$, $\vec{H}$ and $\vec{k}$ follow left hand rule.

Poynting vector: $\vec{S} = \vec{E} \times \vec{H}$. $\qquad \vec{k} \propto \varepsilon \vec{E} \times \vec{H}$

$\vec{k}$ and $\vec{S}$ are *anti-parallel*: $\vec{k}.\vec{S} < 0$.

Dispersion

Energy of the EM field: we need general relation

$$U = \frac{\partial(\varepsilon\omega)}{\partial\omega} E^2 + \frac{\partial(\mu\omega)}{\partial\omega} H^2 \tag{3}$$

Causality:

$$\frac{\partial(\varepsilon\omega)}{\partial\omega} > 0 \quad \text{and} \quad \frac{\partial(\mu\omega)}{\partial\omega} > 0$$

Therefore: left-handed material *must be* dispersive.

Because of Kramers-Kronig, EM losses are unavoidable in LHM.

Group velocity

Definition of the group velocity is the main source of misunderstandings in the analysis of the EM properties of LHM.

$$v_g = \frac{\partial \omega}{\partial k} \quad v_g = \frac{c}{\partial (n\omega)/\partial \omega}$$

One can show that $v_g > 0$ is positive. However, no analysis has been done for anisotropic systems with complex ε and μ.

We do not need v_g. What we need is the direction in which the energy flows.

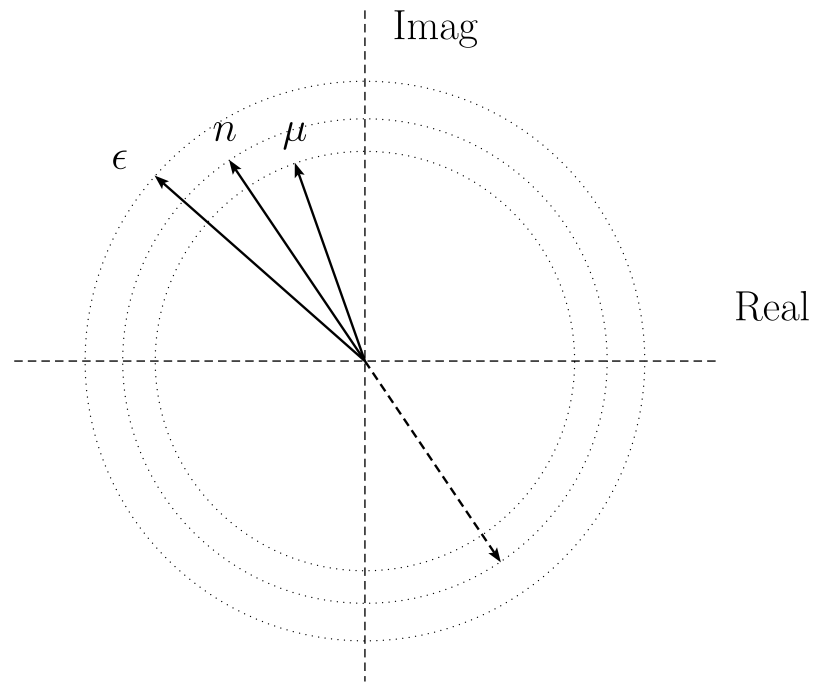
Index of refraction

We require:

$$n'' > 0 \quad \text{and} \quad z' > 0$$

$$n = n' + in'' = \sqrt{\varepsilon\mu}$$

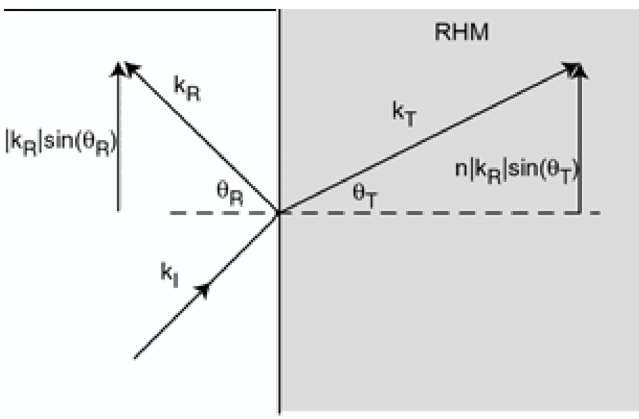
$\varepsilon = |\varepsilon|e^{i\phi_\varepsilon}$,
$\mu = |\mu|e^{i\phi_\mu}$,
$n = |n|e^{i\phi_n}$
$\phi_n = \frac{1}{2}(\phi_\varepsilon + \phi_\mu)$

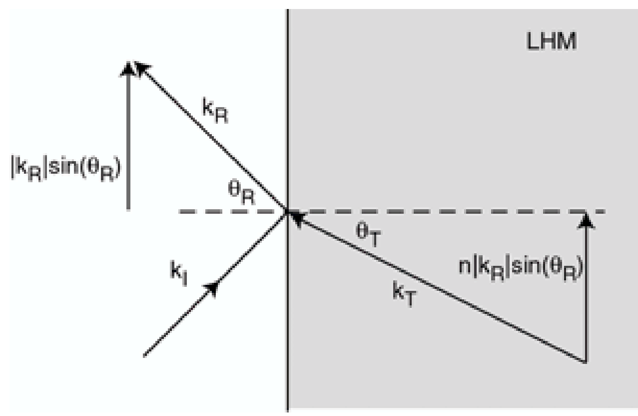


If $\varepsilon' < 0$ and $\mu' < 0$, then also $n' < 0$. Therefore: LHM possess *negative* refractive index $n' < 0$.

“Reversal” of Snell’s Law

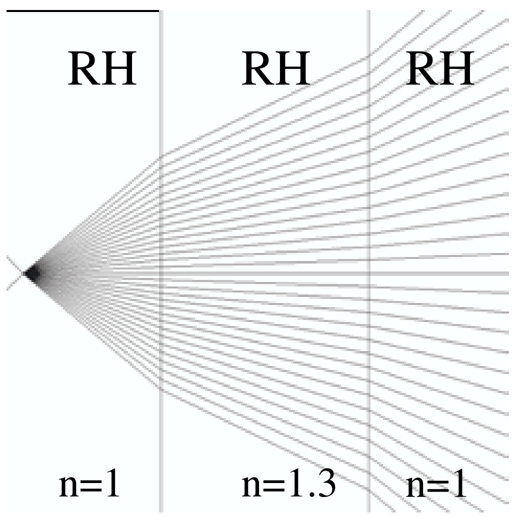
Right Handed Media | Left Handed Media

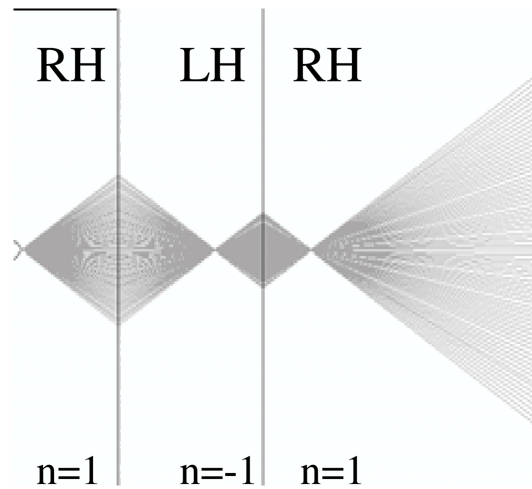




$$k_T = nk_I$$

Focusing in a Left-Handed Medium





Perfect lens?

Veselago showed that thin LH slab works as lens.

Pendry: *for* $n = -1$ *(*$\varepsilon = -1$ *and* $\mu = -1$*) such lens is perfect*
it has perfect resolution (it create *perfect image*).

Possibility to have lens with resolution smaller that the wave length is the main motivation of the studies of LHM

Origin of the perfect image

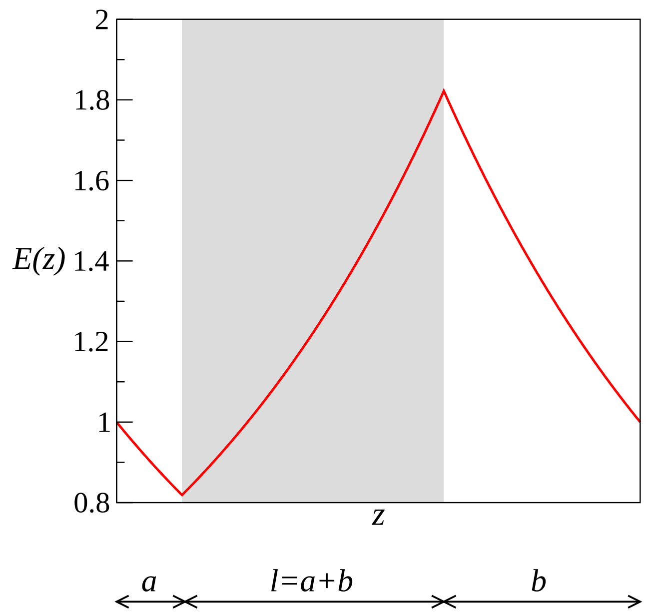
LHM slab amplifies evanescent modes, which decrease exponentially in the RH medium.

For $\varepsilon = -1$ and $\mu = -1$:

Transmission of the EM wave:

$$E = |E|e^{ikz} \qquad T = 1$$

$$E = |E|e^{-kz} \qquad T = e^{+k\ell}$$



Resolution of the LHM lens

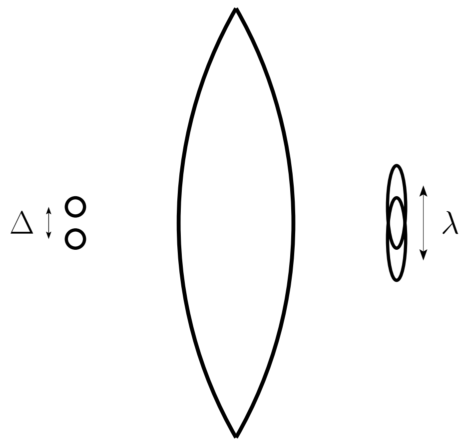
$$\omega^2 = k_x^2 + k_z^2 \tag{4}$$

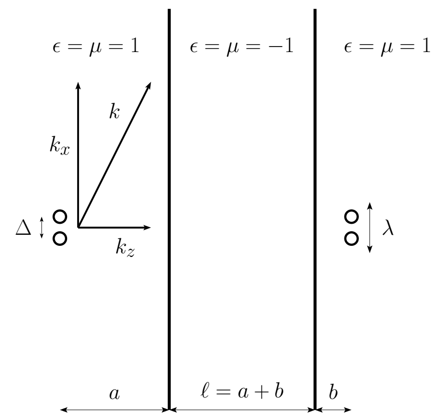
Classical lens: Only components with $k_x < \omega/c$ participate on the reconstruction. $s(x) = \sum_{k_x}^{\omega/c} s(k_x)e^{ixk_x}$ Resolution: $\Delta \sim \omega/c$

LH lens: Also evanescent waves with $e^{-k_z z}$, $k_z = \sqrt{k_x^2 - \omega^2}$ participate on the reconstruction of the image. $s(x) = \sum_{k_x}^{\infty} s(k_x)e^{ixk_x}$

This enables perfect reconstruction of the source since also Fourier components with $k_x \gg 2\pi/\lambda$ are transmitted through LHM and used in the reconstruction.

Resolution of the LHM lens

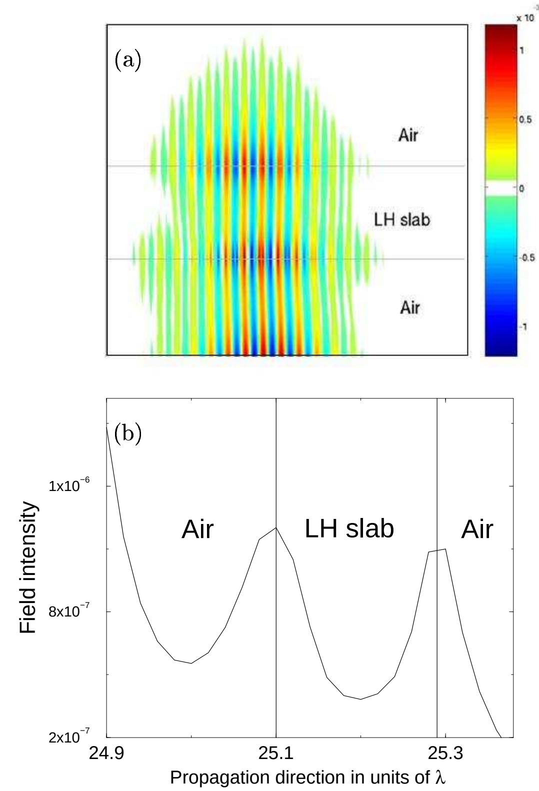




Ideal LH lens create perfect inage in the near-field.

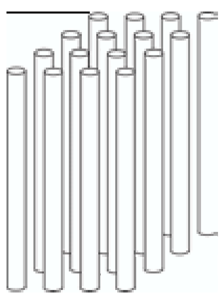
Real world: because of dispersion, we can not have $n = -1$ for all frequencies. There are also losses, anizotropy . . . . In any case, LH lens enables at least partial reconstruction of details smaller than the wave length.

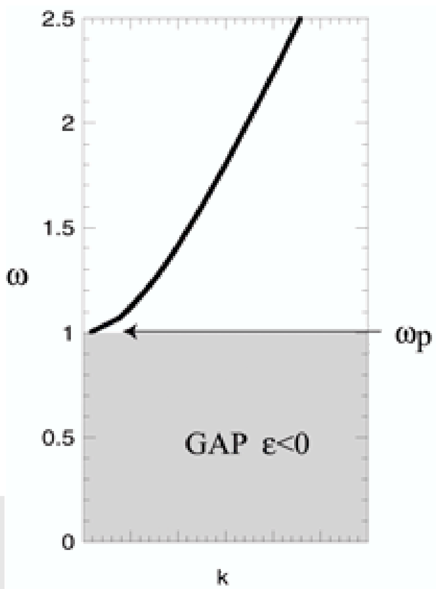
Surface plasmon

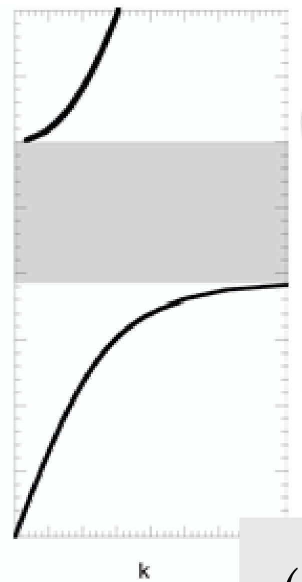


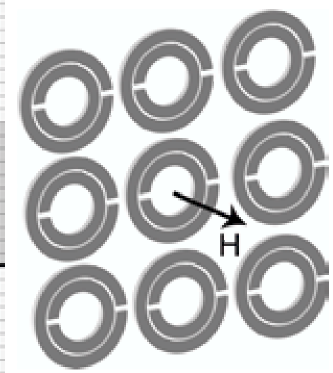
Physics: evanescent waves excites surface waves at the boundary LH - RH (Ruppin, Haldane).

Metamaterials Extend Properties









$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

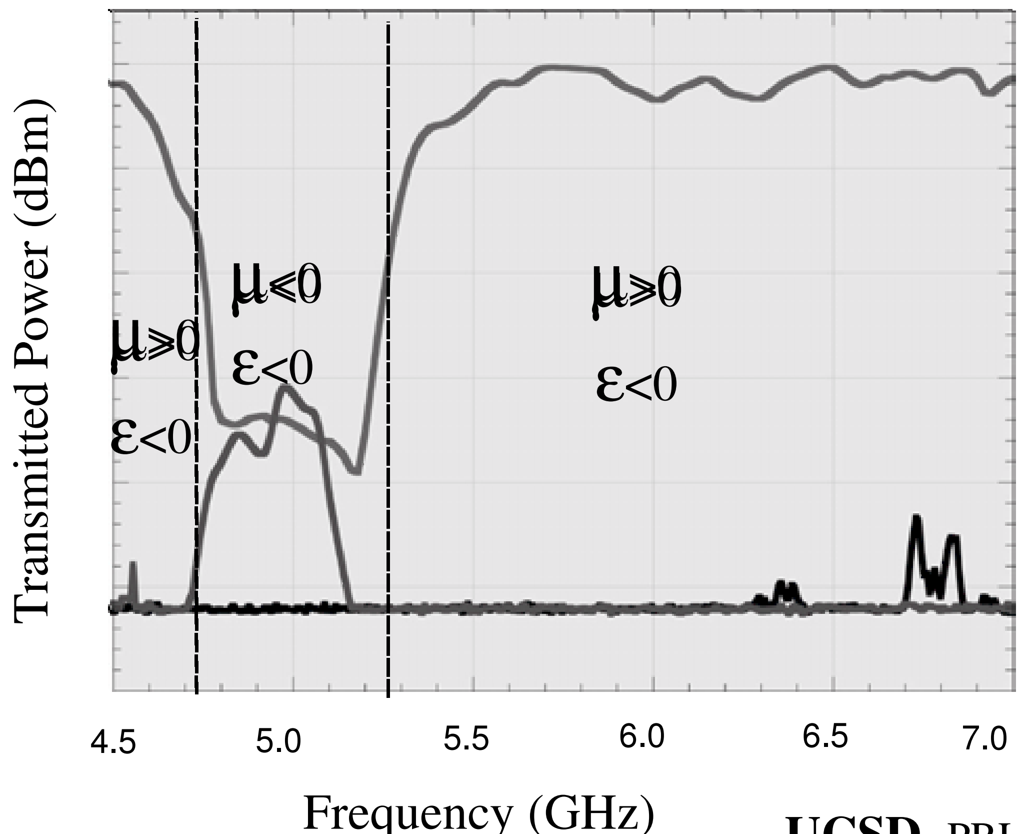
$$\mu(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2}$$

J. B. Pendry

First Left-Handed Test Structure

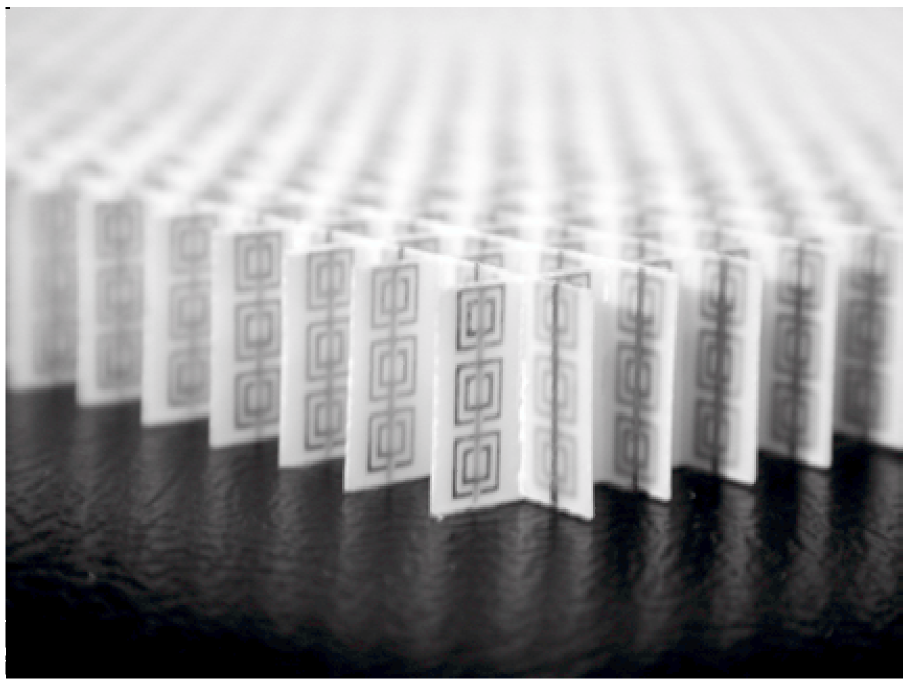
UCSD, PRL 84, 4184 (2000)

Transmission Measurements



UCSD, PRL 84, 4184 (2000)

A 2-D Isotropic Structure



UCSD, APL 78, 489 (2001)

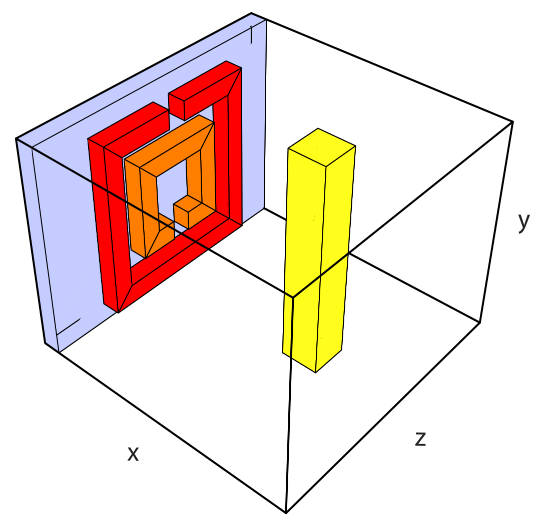
Structure of the unit cell

EM wave propagates in the z -direction

Periodic boundary conditions
are used in transverse directions

Polarization: p wave: E parallel to y
s wave: E parallel to x

For the p wave, the resonance frequency
interval exists, where with Re m_{eff} <0, Re e_{eff}<0
and Re n_p <0.

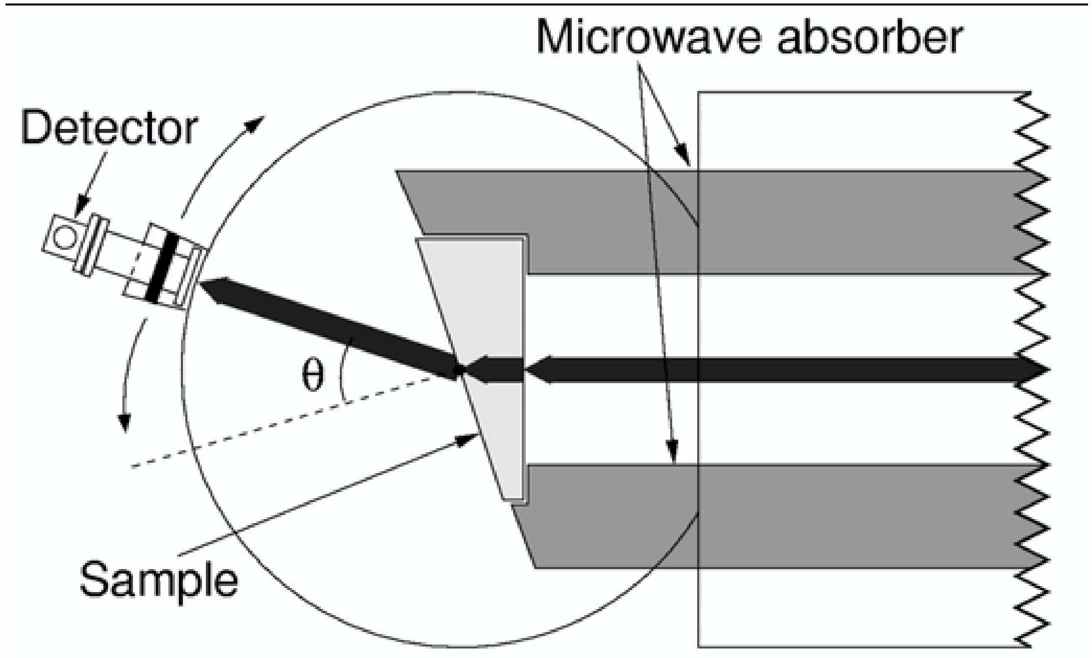


For the s wave, the refraction index $n_s = 1$.

Typical size of the unit cell: 3.3 x 3.67 x 3.67 mm

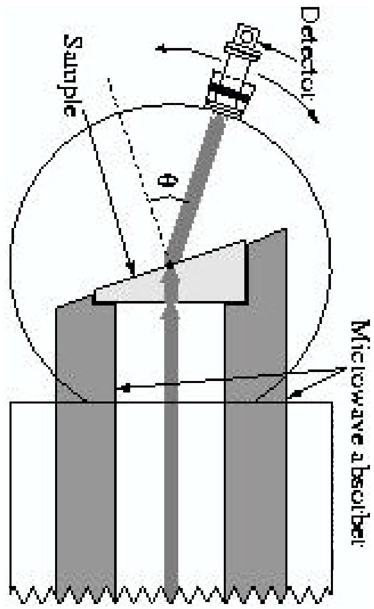
Typical permittivity of the metallic components: $e_{metal} = (-3+5.88\ i) \times 10^5$

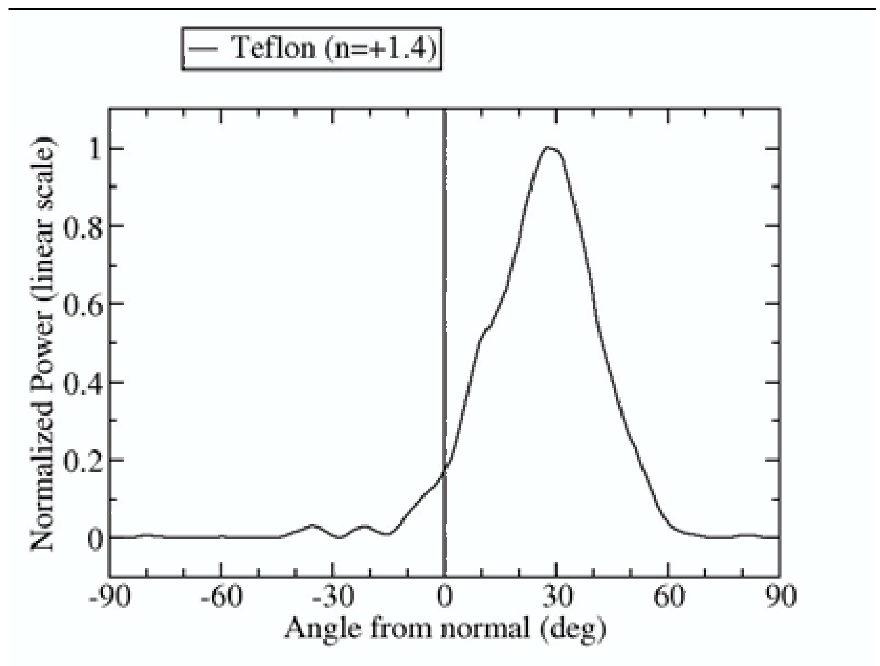
Measurement of Refractive Index



UCSD, Science 292, 77 2001

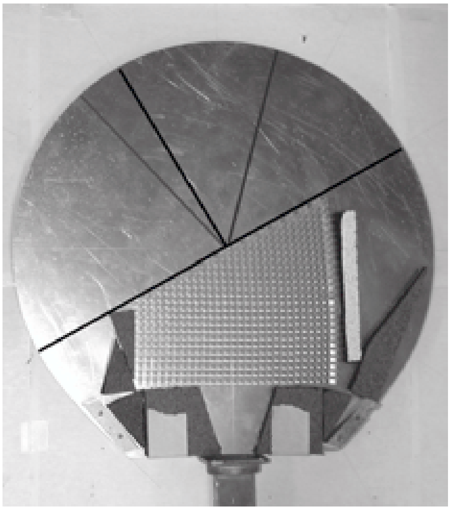
Measurement of Refractive Index

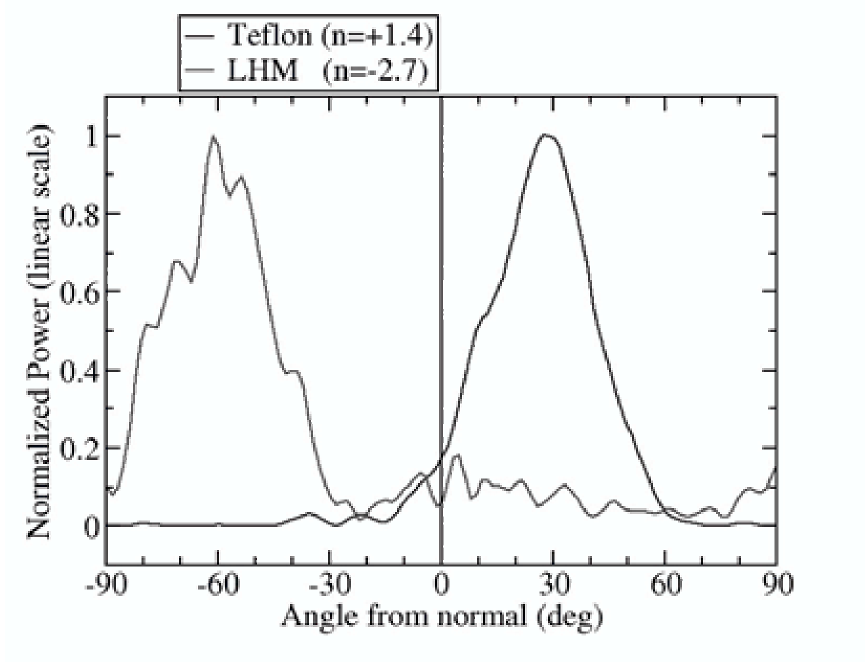




UCSD, Science 292, 77 2001

Measurement of Refractive Index





UCSD, Science 292, 77 2001

Objections: is left-handedness real?

- LH can not works because of huge energy losses (Garcia).
- $n < 0$ violates causality principle (Valanju)
- negative refraction shown in experiments is just a near field effect (Garcia)
- perfect lens do not exist (Walser, 't Hooft, . . .)
- One can not define effective parameters because of space dispersion (Efros)
- $\mu(\omega)$ has no physical meaning for large ω (Efros)

Effective parameters - numerical analysis

Consider *homogeneous* system of length L.

Find transmission t and reflection r from numerical simulations.

Express transmission and reflection as a function of refractive index n and impedance z:

$$t^{-1} = \left[\cos(nkL) - \frac{i}{2}\left(z + \frac{1}{z}\right)\sin(nkL)\right] \tag{5}$$

$$\frac{r}{t} = -\frac{i}{2}\left(z - \frac{1}{z}\right)\sin(nkL) \tag{6}$$

k is wave vector of EM wave *in vacuum*. $n = \sqrt{\varepsilon\mu}$ and $z = \sqrt{\mu/\varepsilon}$

Effective parameters

$$z = \pm\sqrt{\frac{(1+r)^2 - t^2}{(1-r)^2 - t^2}} \tag{7}$$

$$\cos(nkL) = X = \frac{1}{2t}\left(1 - r^2 + t^2\right) \tag{8}$$

we require $z' > 0$ and $n'' > 0$

$$e^{-n''kL}\left[\cos(n'kL) + i\ \sin(n'kL)\right] = Y = X \pm \sqrt{1 - X^2}. \tag{9}$$

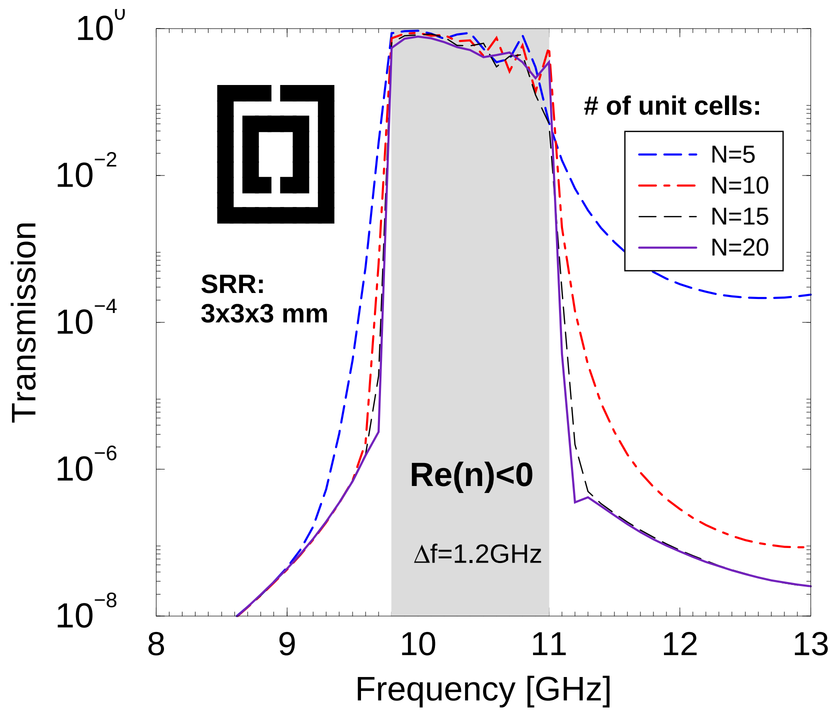
Assumptions:

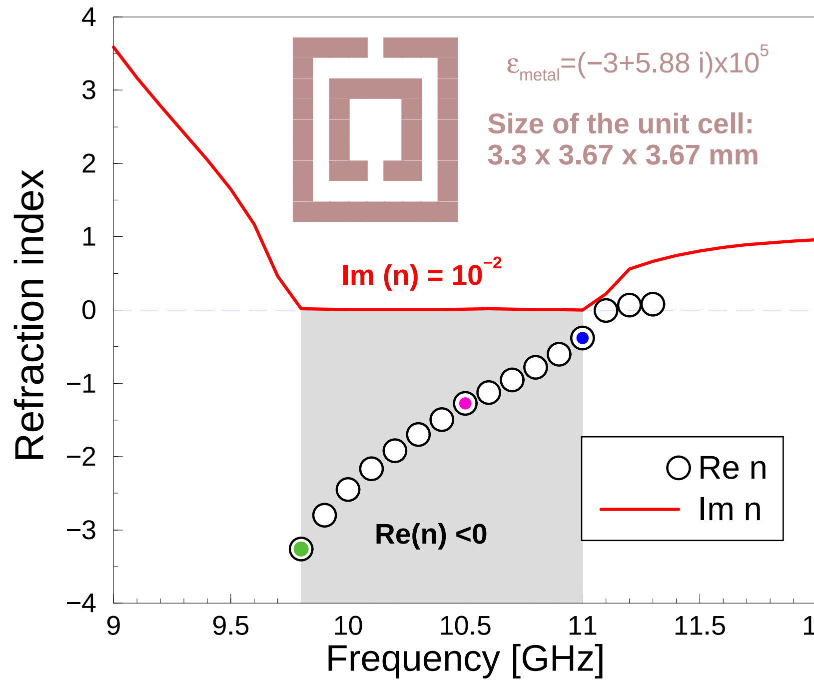
homogeneous material. Typical inhomogeneity must be $\ll$ wave length *inside* the sample.

This requirement is not always fulfilled.

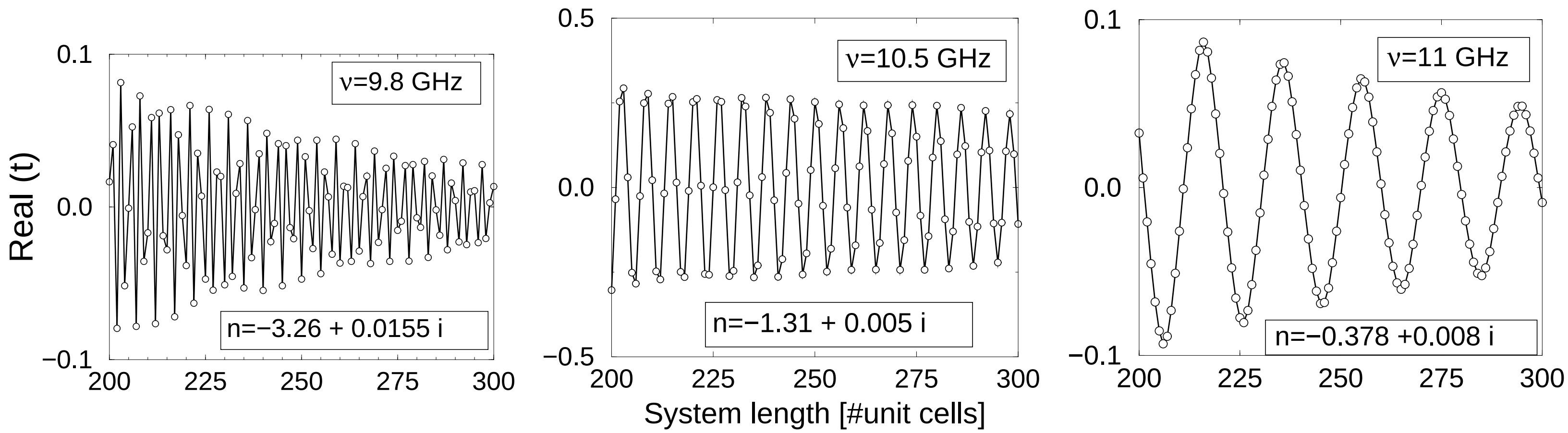
- typical size of the unit cell: 3-3.5 mm
- frequency of EM wave: 10 GHz
- wave length *in vacuum*: 3-4 cm
- refractive index inside the LH system: $0 < |n'| < 3$

Example: LHM


SRR:
3x3x3 mm
# of unit cells:
N=5
N=10
N=15
N=20
Re(n)<0
Δf=1.2GHz
Transmission
Frequency [GHz]


ε_metal=(−3+5.88 i)x10^5
Size of the unit cell:
3.3 x 3.67 x 3.67 mm
Im (n) = 10^−2
Re(n) <0
Re n
Im n
Refraction index
Frequency [GHz]

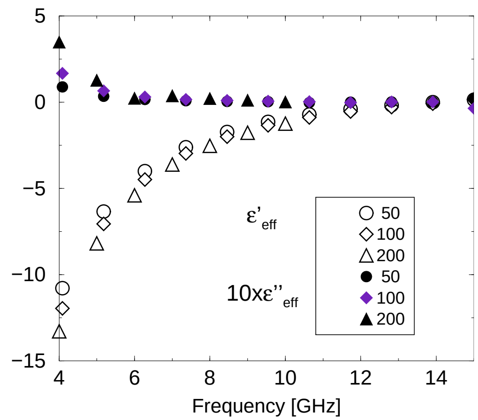
Example: LHM

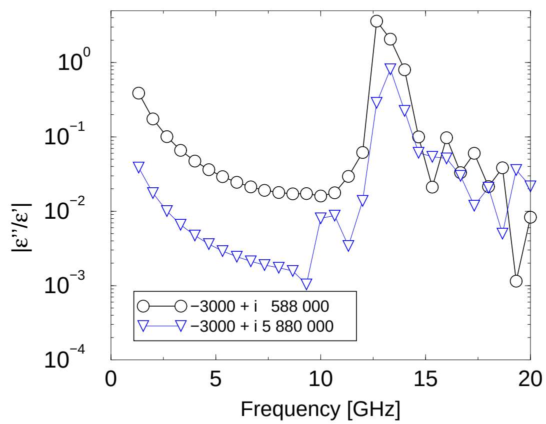


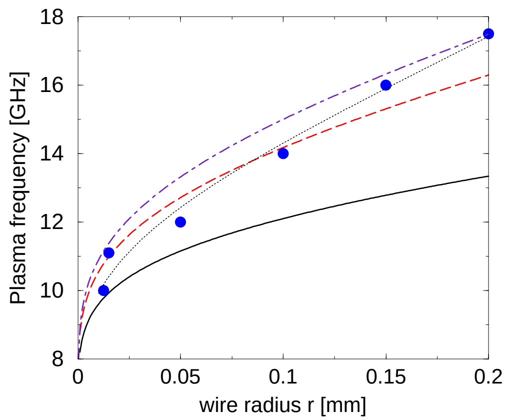
Note that wave inside the LH slab decays very slowly - losses are indeed small.

Periodic array of thin wires:

Effective permittivity:



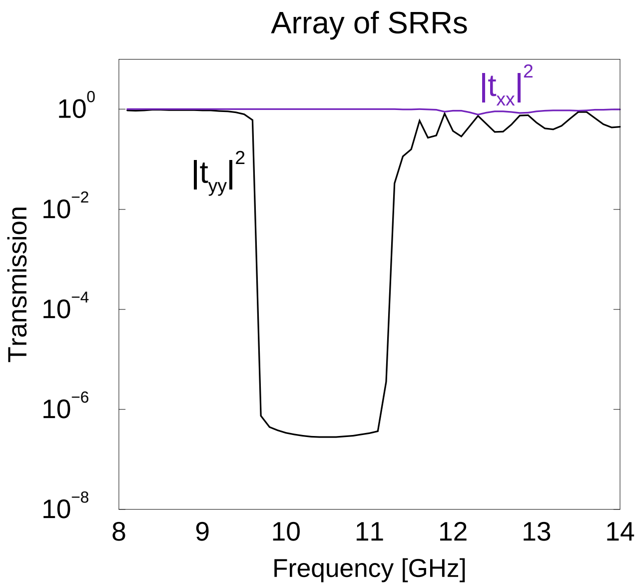


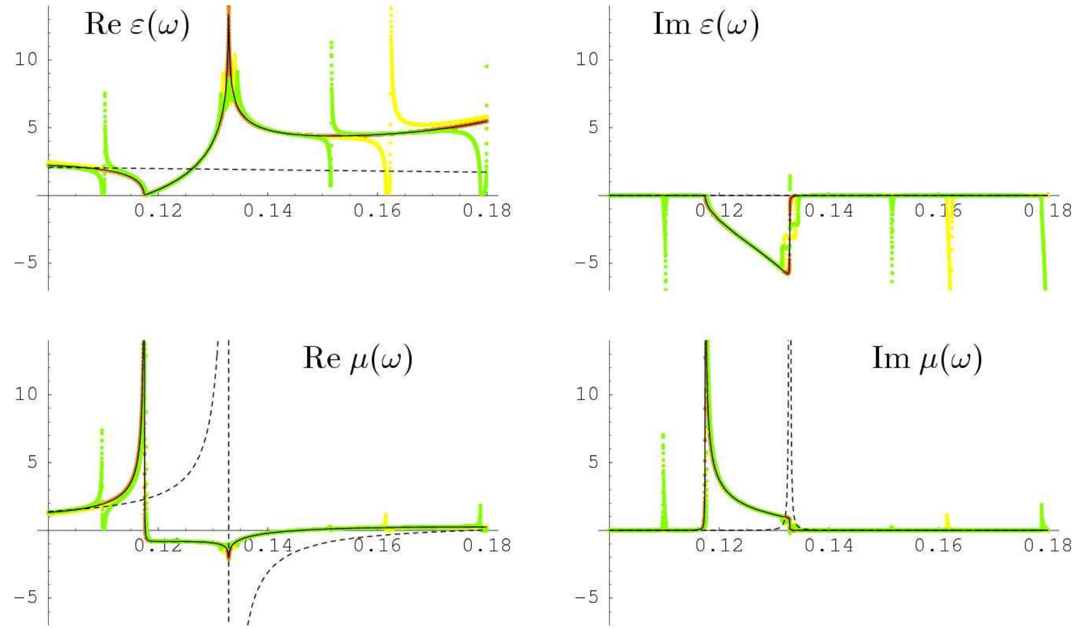


Drude formula: $\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}$

Numerical data agree with theory.

Array of SRR

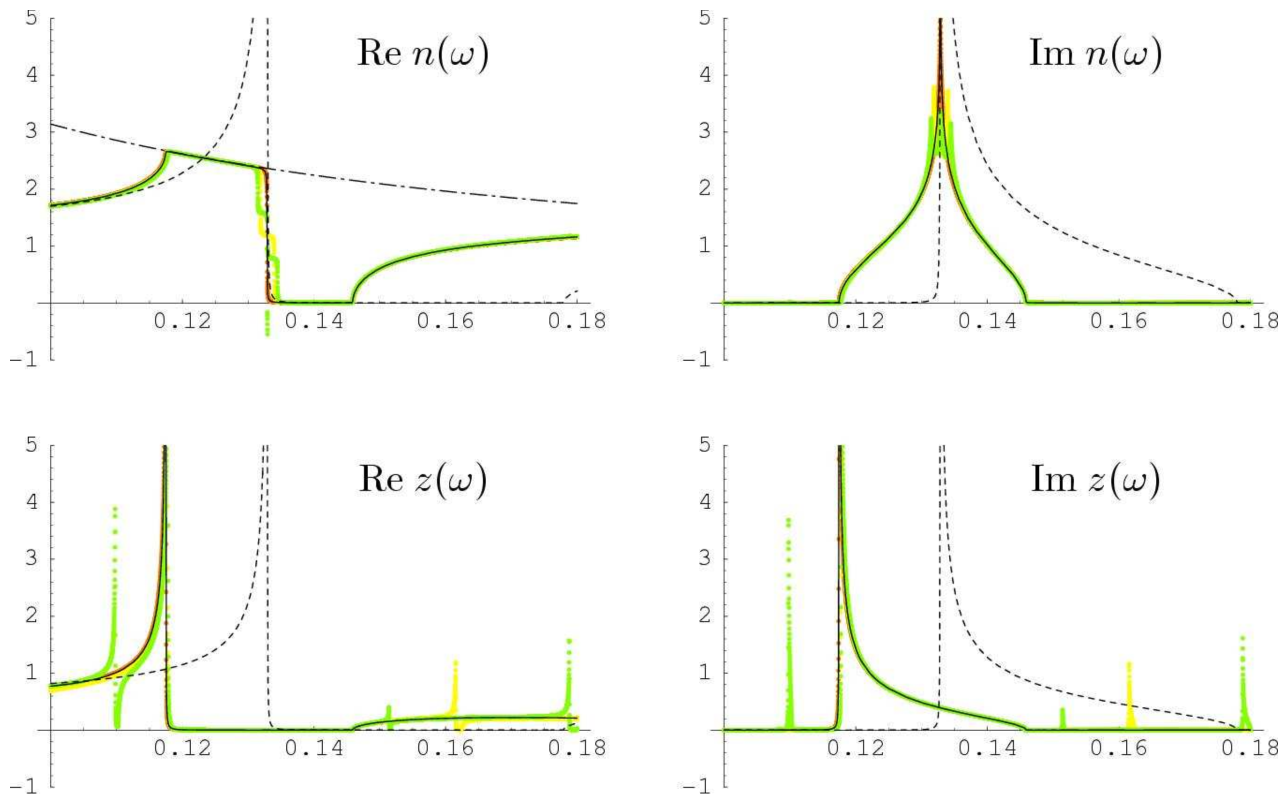




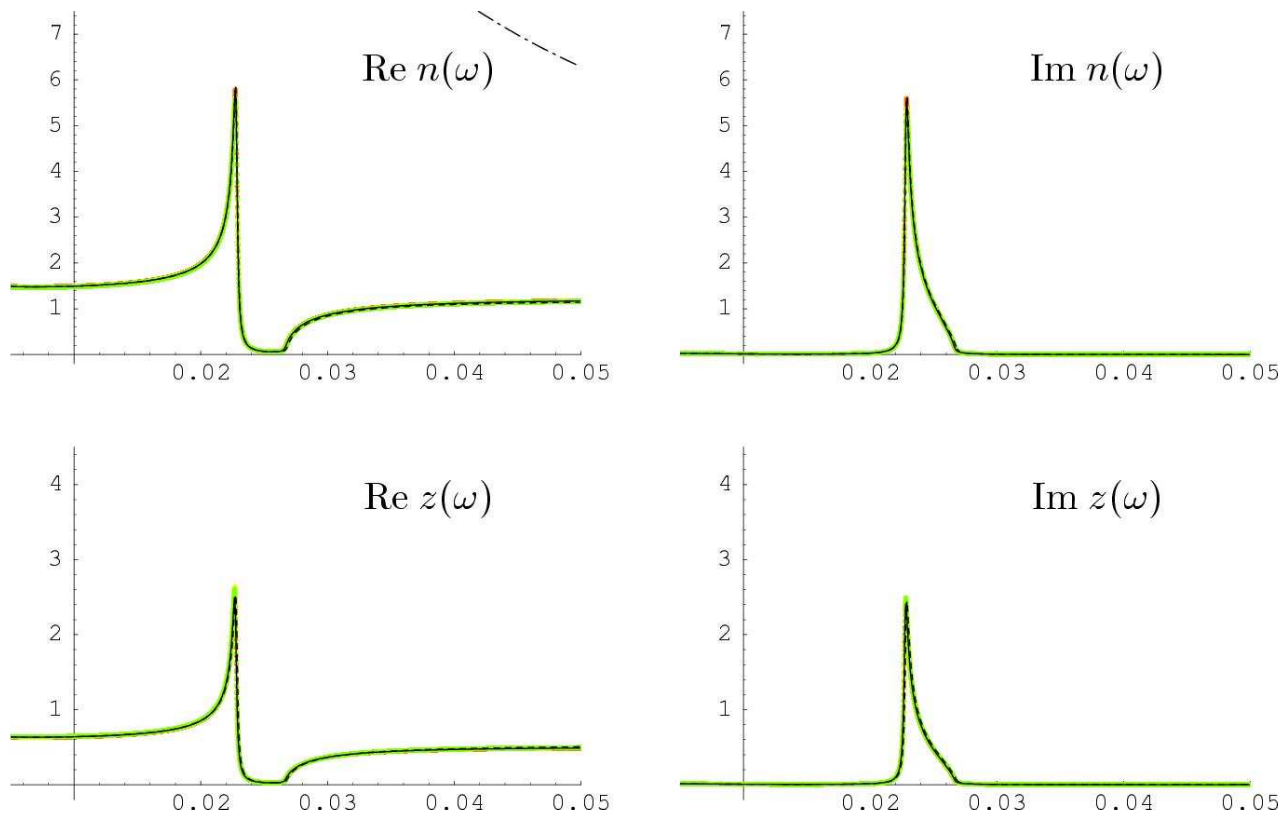
We expect resonance: $\mu(\omega) = 1 - \dfrac{\omega_p^2}{\omega^2 - \omega_0^2}$ and $\varepsilon = 1$.

We observed misshaped resonance in μ and antiresonance in $\epsilon(\omega)$.
Also, $\varepsilon'' < 0$.

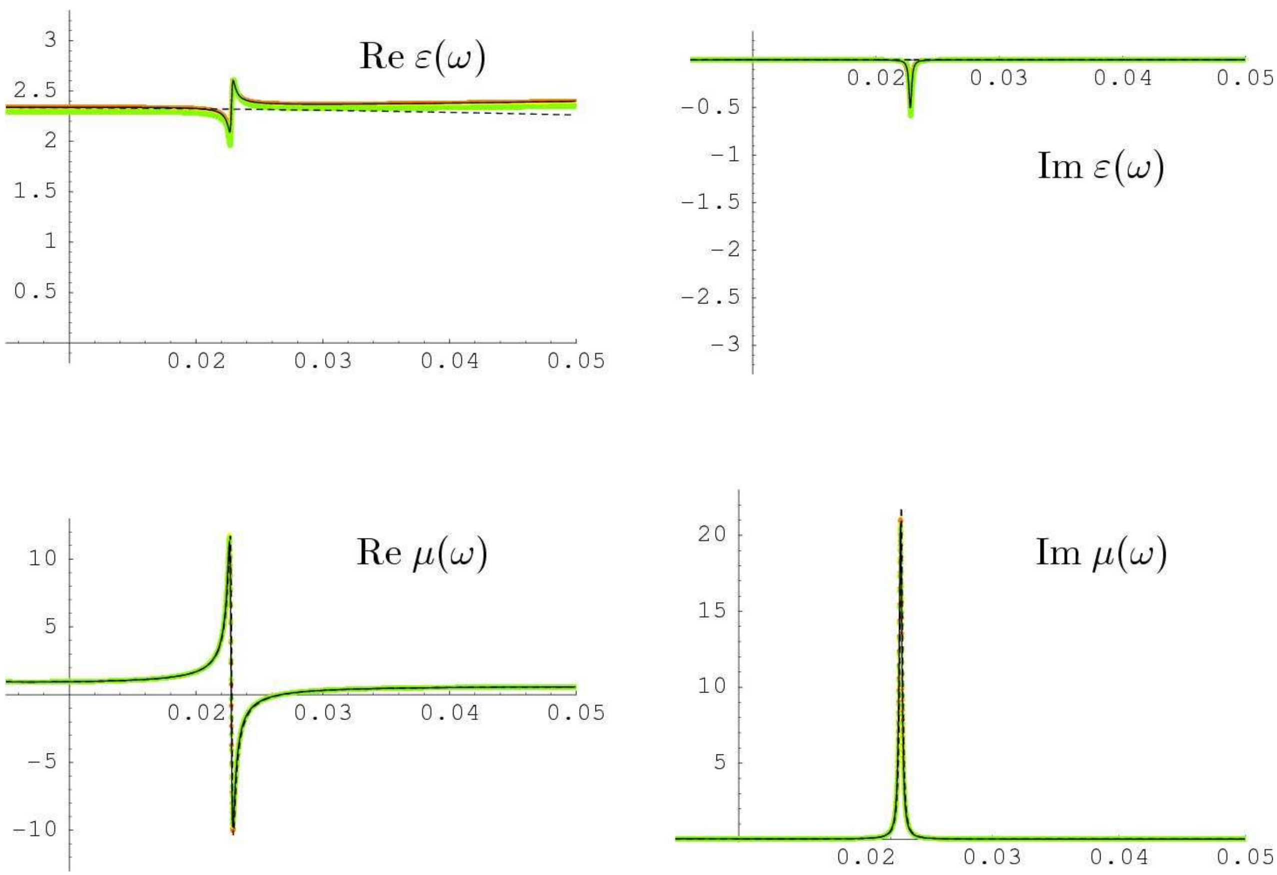
Reason: spatial periodicity of the metamaterial



Solution: shift of the resonance frequency below BZ



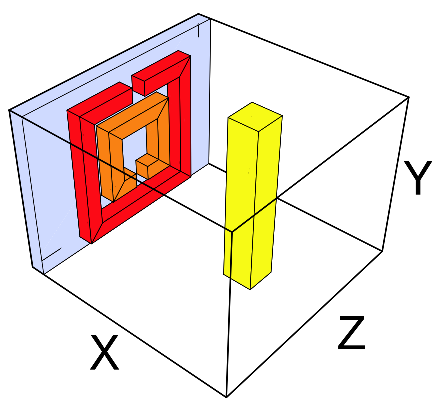
Almost perfect resonance in magnetic permeability

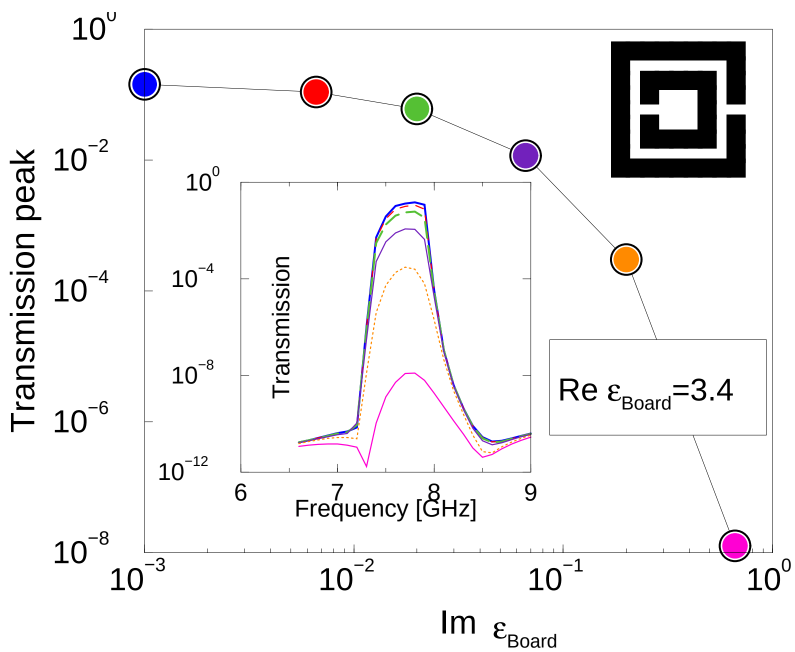


Other problems:

technology: it is difficult to create 2D and 3D samples

EM losses due to dielectric board:





Conclusions

- We believe that EM properties of LHM are not in contradiction with any physical law.
- Theory is perfect for homogeneous materials which, however, does not exists.
- System is anisotropic; shown structures are LH only in one direction and only for one polarization. We want homogeneous and isotropic samples

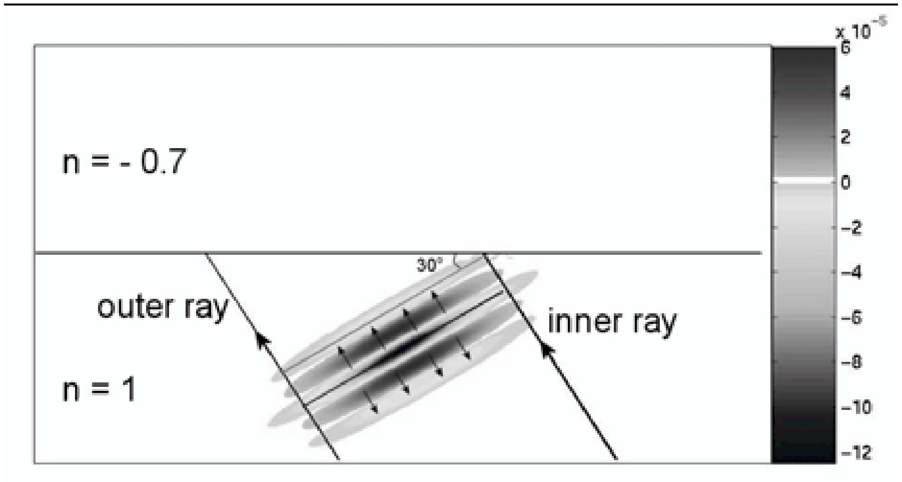
Appendix: Photonic crystals

Negative refraction of EM waves has been observed also at the boundary vacuum - PC

I believe that physics of this phenomena is different from LHM: note that the space period of PC is always comparable with the wavelength. Therefore we have no effective ε and μ.

Advantage: PC consists from dielectric rods. There are no EM losses. 2D PC are therefore much easier to prepare than LHM.

Photonic Crystals with negative refraction.

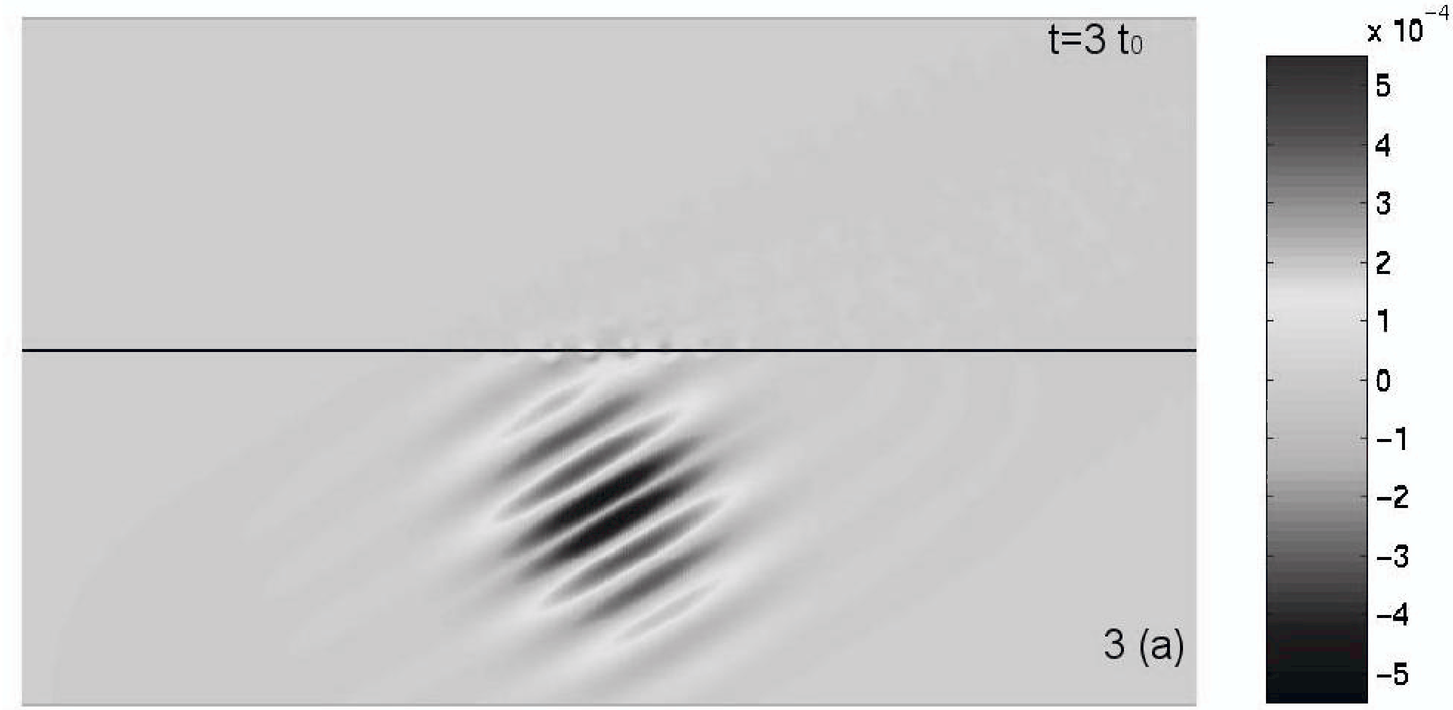


Photonic Crystal

vacuum

FDTD simulations were used to study the time evolution of an EM wave as it hits the interface vacuum/photonic crystal.
Photonic crystal consists of an hexagonal lattice of dielectric rods with ε=12.96. The radius of rods is r=0.35a. a is the lattice constant.

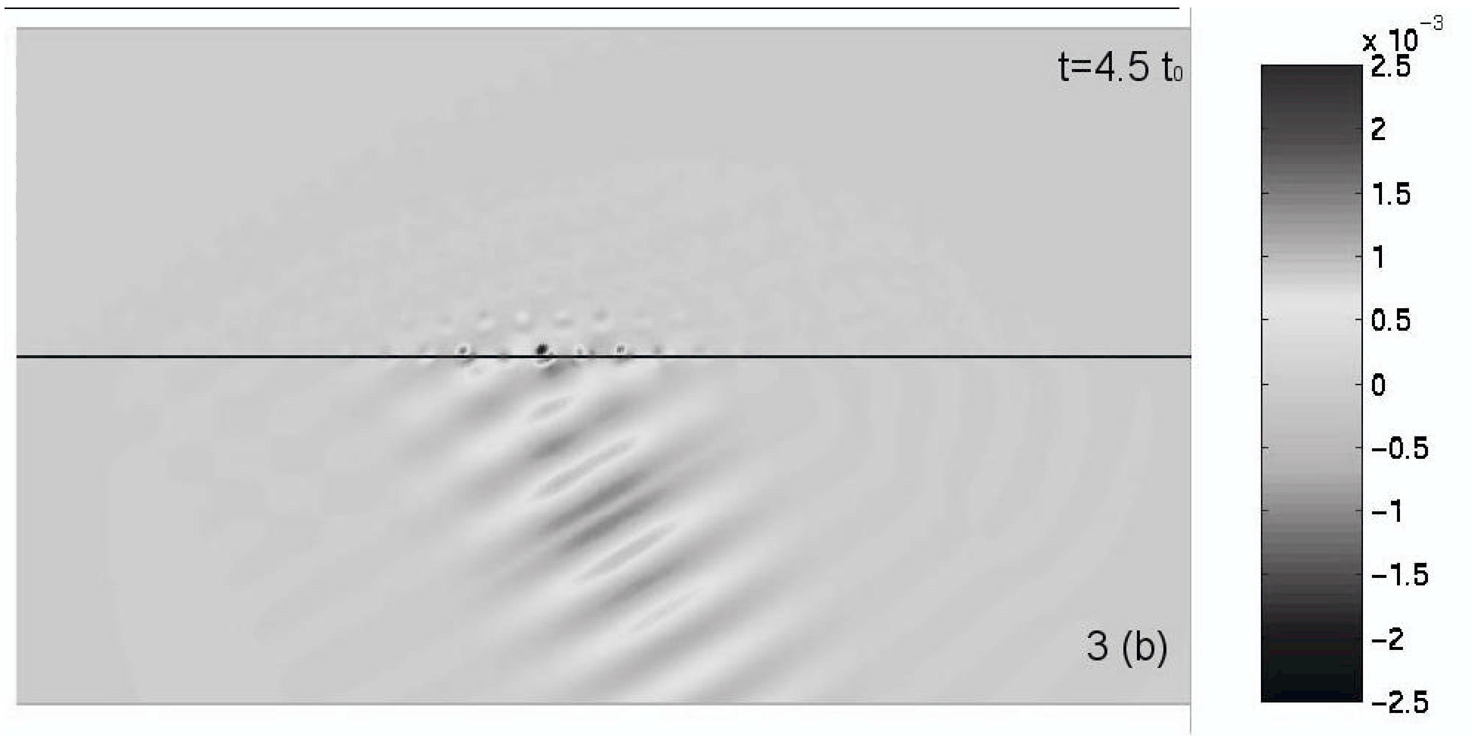
Photonic Crystals with negative refraction.



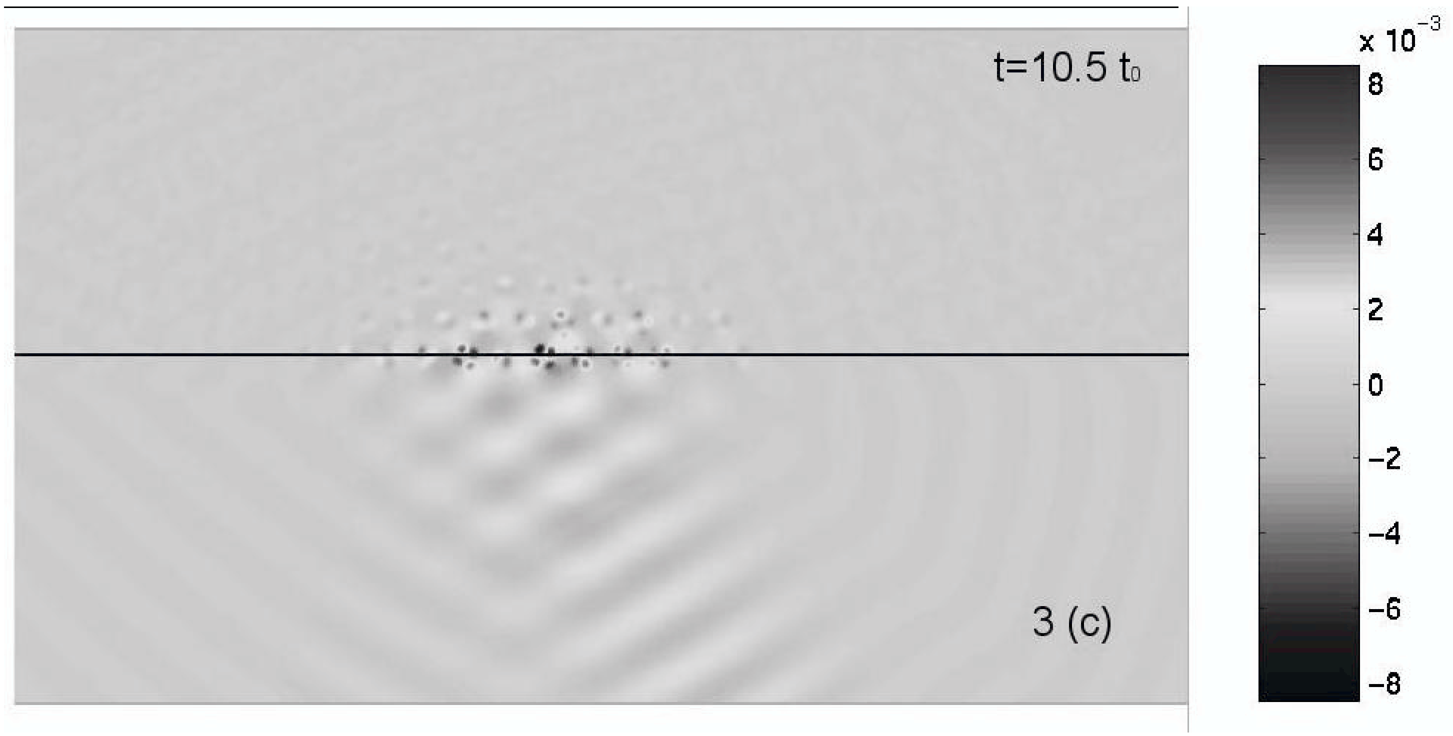
$t_0 = 1.5T$

$T = \lambda / c$

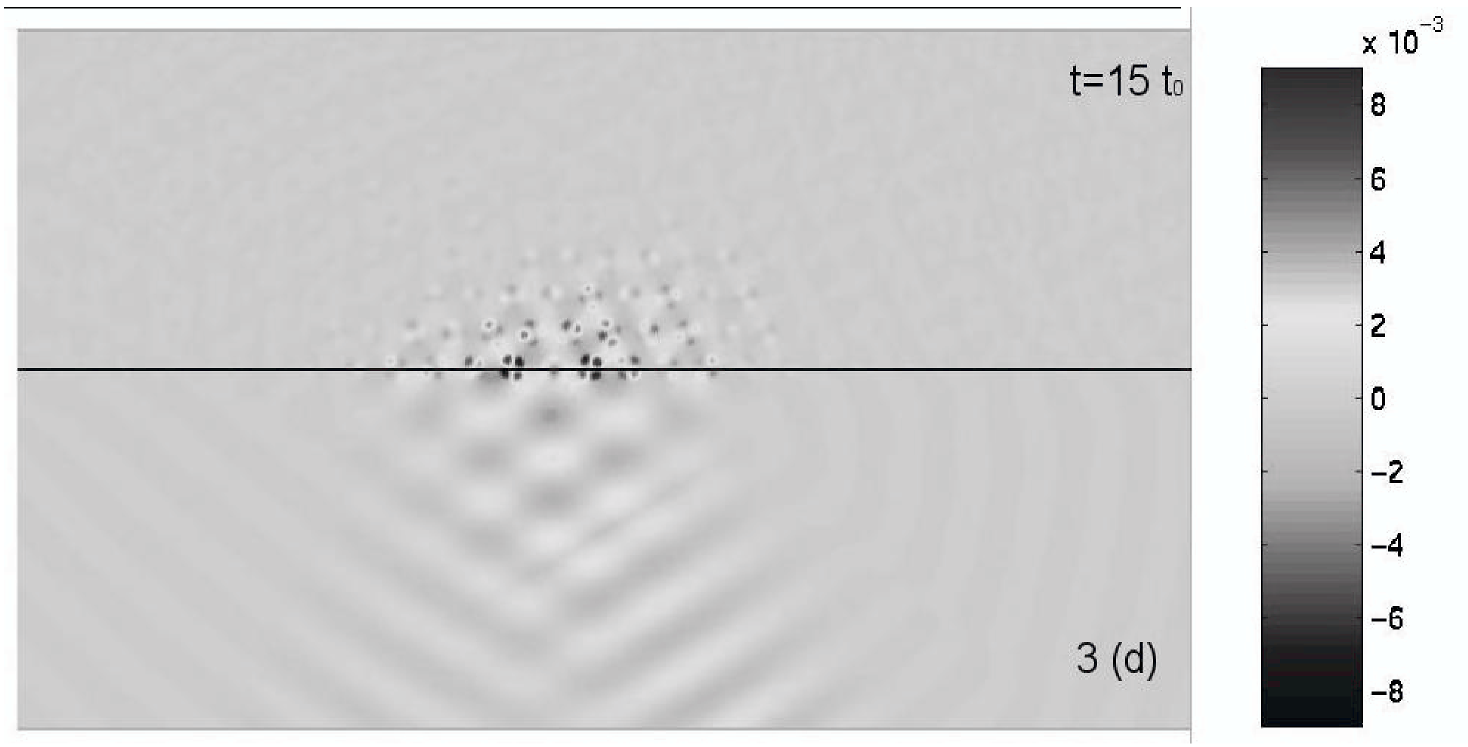
Photonic Crystals with negative refraction.



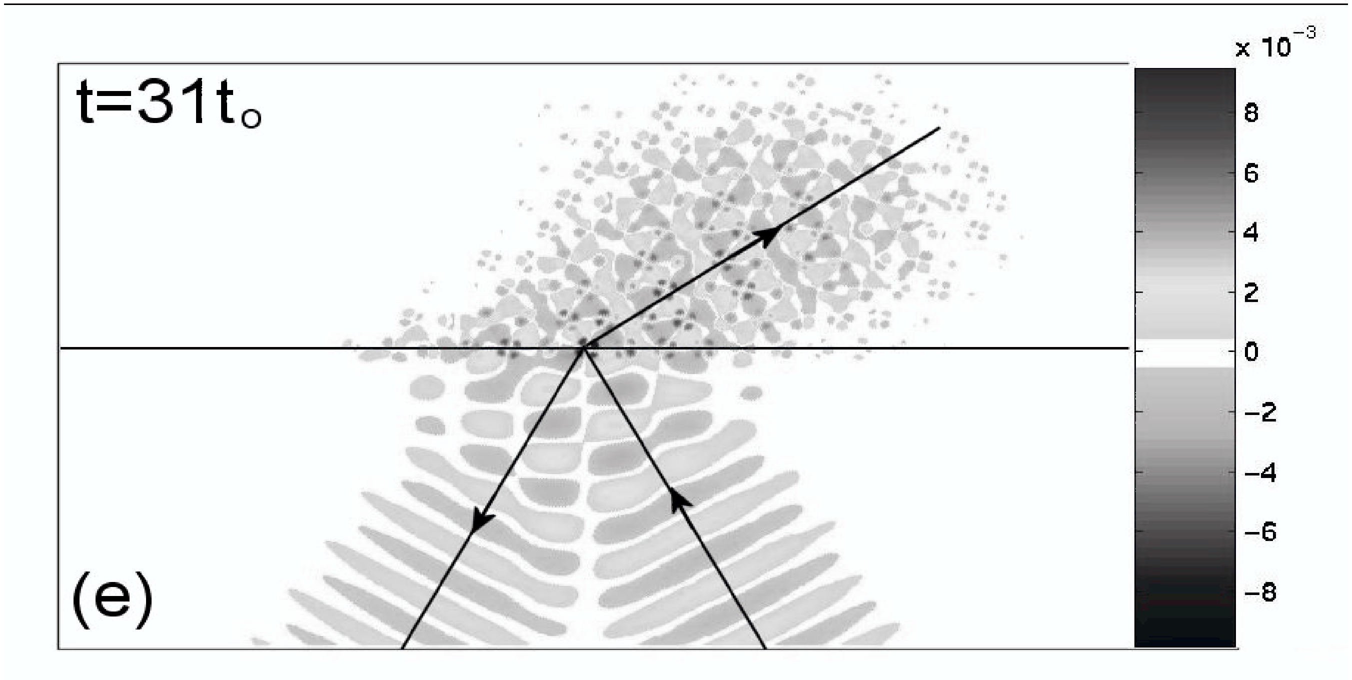
Photonic Crystals with negative refraction.



Photonic Crystals with negative refraction.

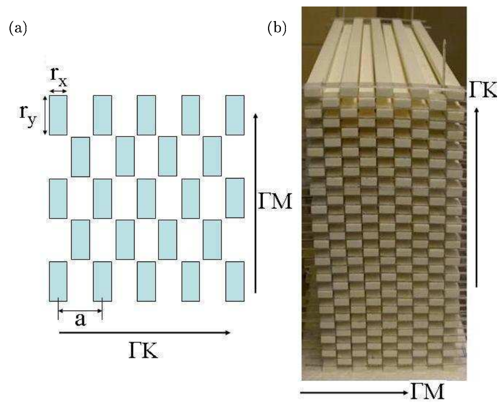


Photonic Crystals: negative refraction



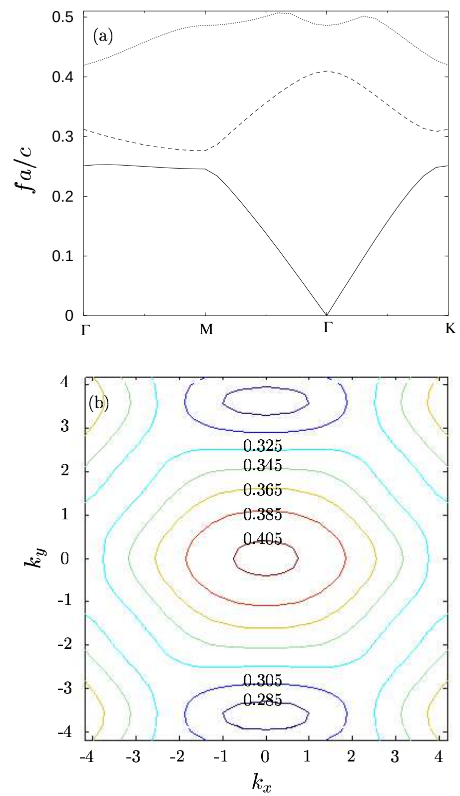
The EM wave is trapped temporarily at the interface and after a long time, the wave front moves eventually in the negative direction.
Negative refraction was observed for wavelength of the EM wave
$\lambda = 1.64 - 1.75\, a$ (a is the lattice constant of PC)

Theory

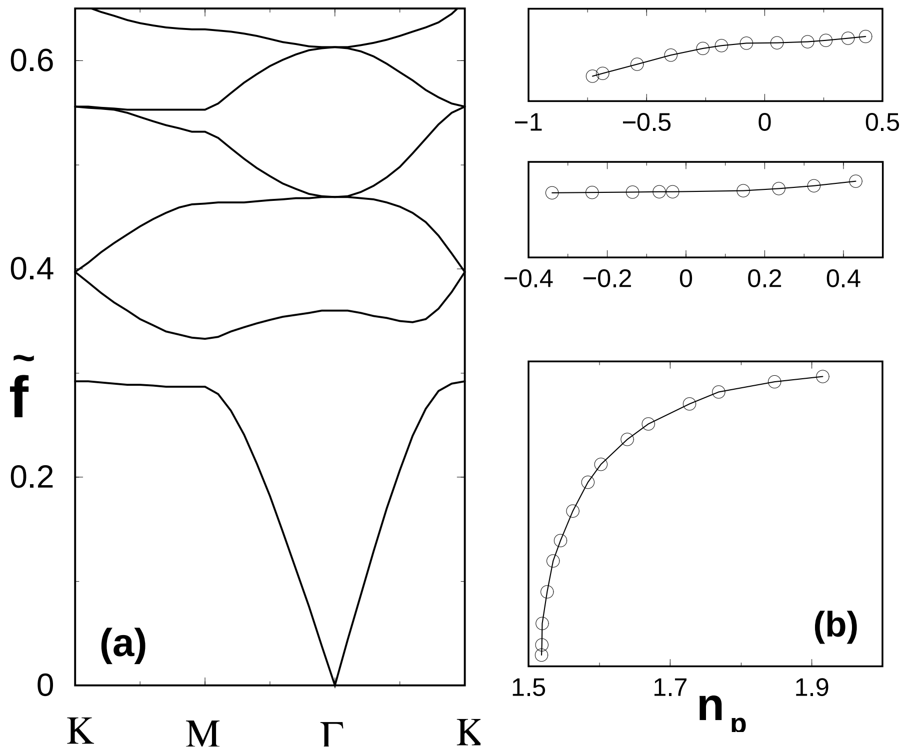


$$n = \frac{\partial E}{\partial k}$$

Plot $E = E(\vec{k})$.
Close to $E = E_{\max}$ expect $n < 0$.
Find lattice for which $n \approx -1$
in all directions.



"Refractive index"



Acknowledgement

I used figures of D. R. Smith, C. M. Soukoulis, Th. Koschny, S. Foteinopoulou, [Phys. Rev. B 66, 235107 (2003)], and R. Moussa [Phys. Rev. B 71, 085106 (2005)]