Statistical physics of interacting agents on complex networks COST 1P04OCP10.001

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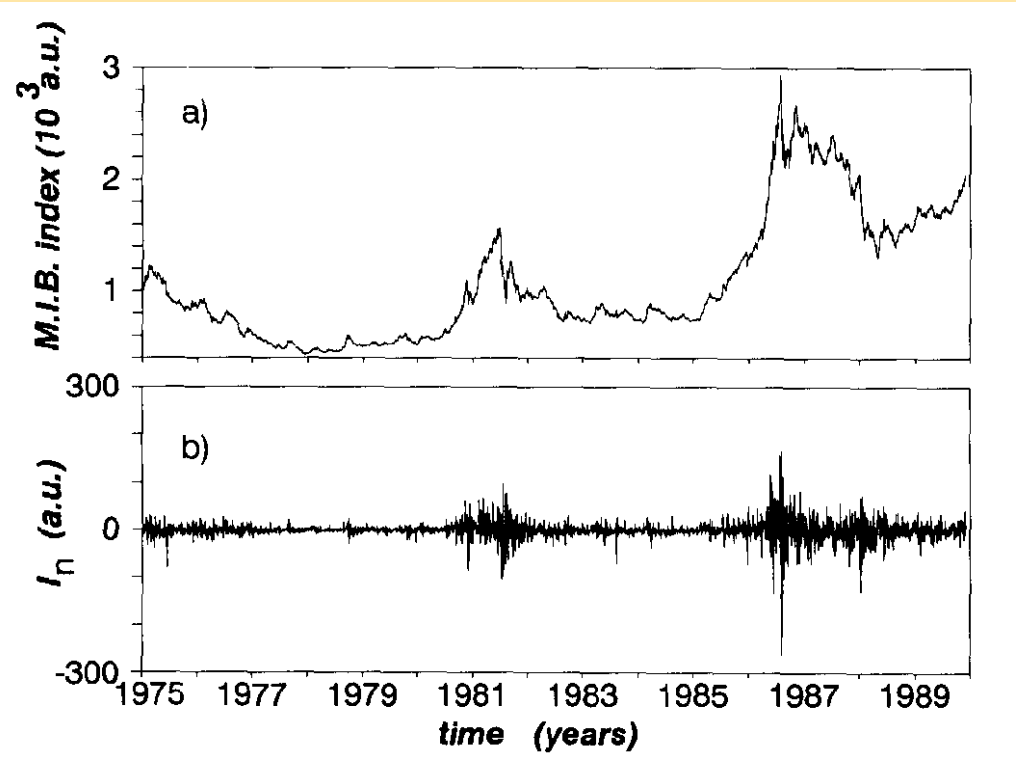
- Phenomenology of markets and networks
- Price fluctuations
- Imitation structures
- Social networks in electronic commerce
- Model of social divergence
- Thanks to MŠMT

Econophysical phenomenology

[R. N. Mantegna, Physica A 179, 232 (1991)]



Time evolution of Milano Stock market index and returns.

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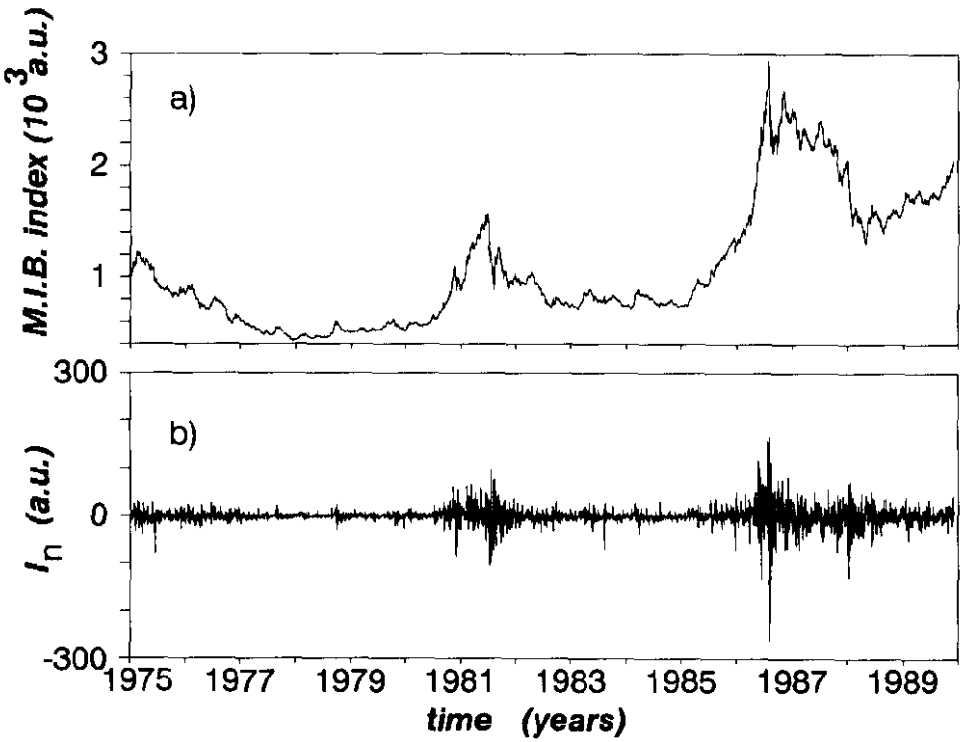

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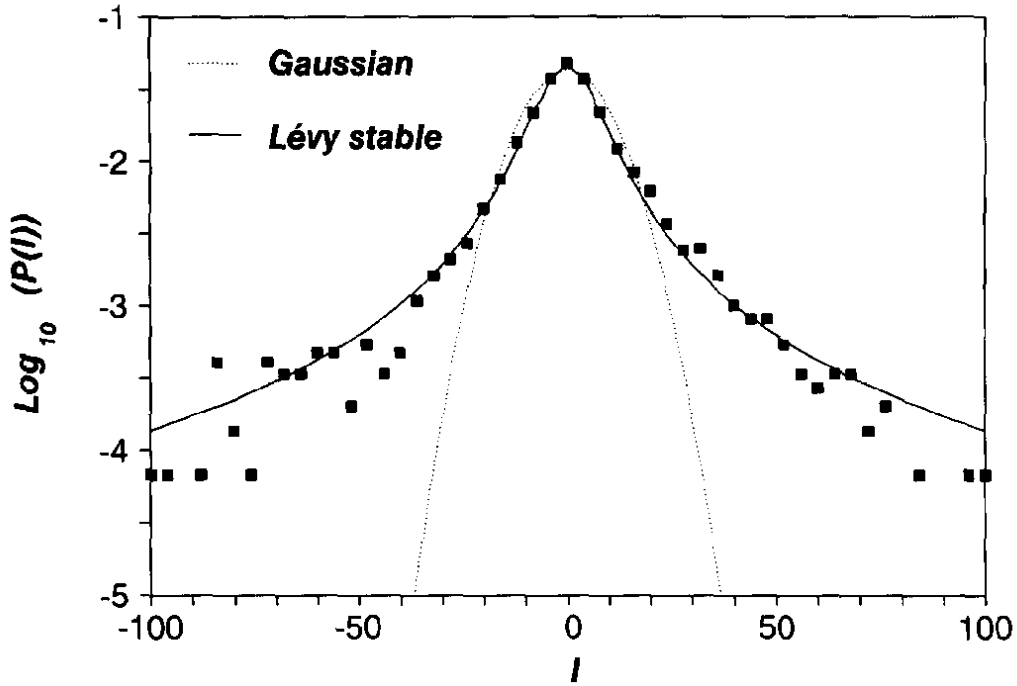


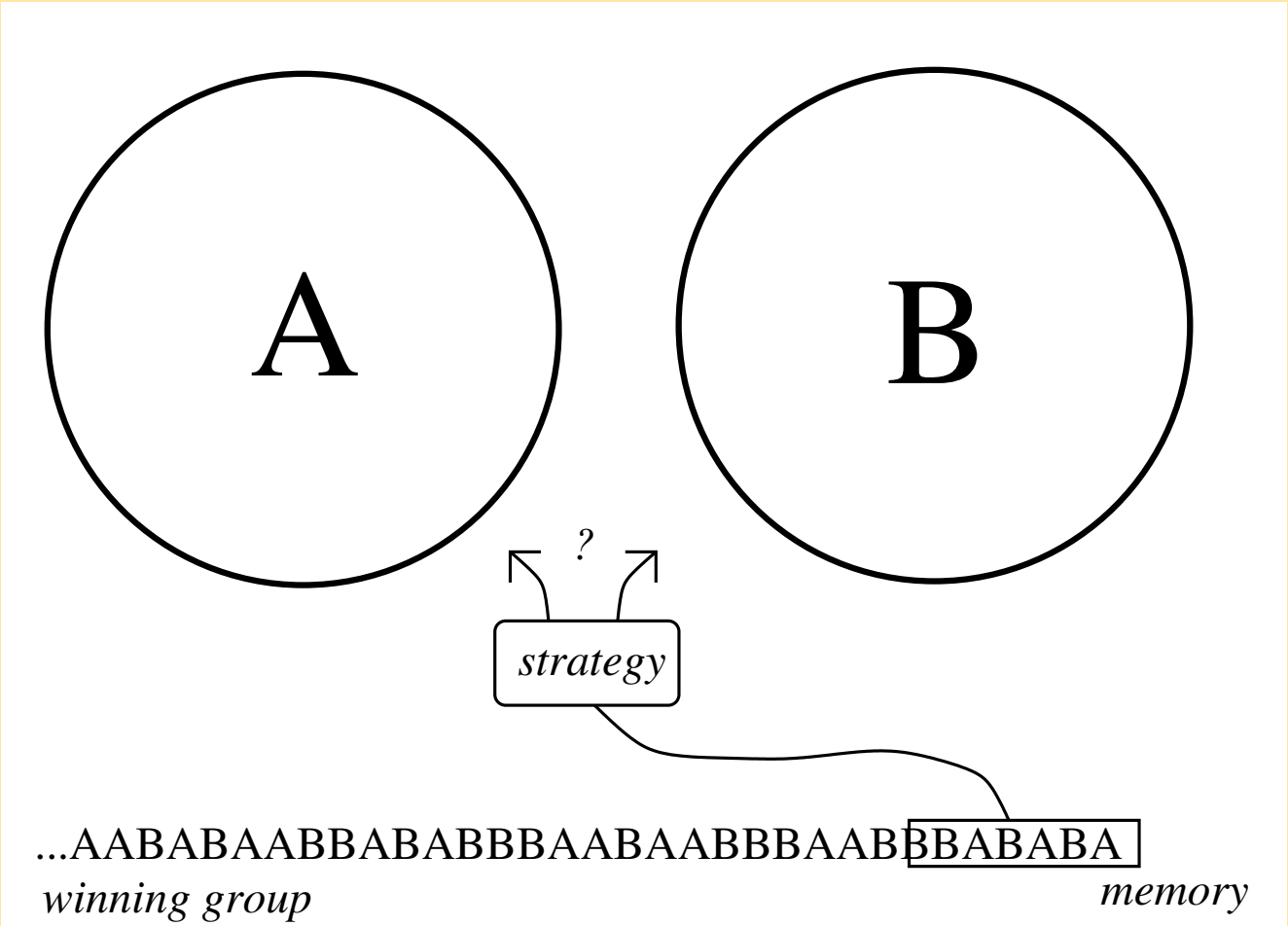
Histogram of time changes in Milano Stock market.

$P(I) \propto I^{-\alpha}$, $\alpha \simeq 2.4$.

Minority Game:

[W. B. Arthur Amer. Econ. Review 84,406 (1994).
D. Challet and Y.-C. Zhang, Physica A 246, 407 (1997).]

El Farol bar attendance problem: go to bar (B) or stay at home (A)?
Abstract formulation:



N players
S strategies,
memory length M.
Strategy with highest score is chosen.

Features: On-line adaptation. No optimal strategy possible.

Grand-canonical Minority Game []

N_p producers (always play) N_s speculators (play only if profitable)

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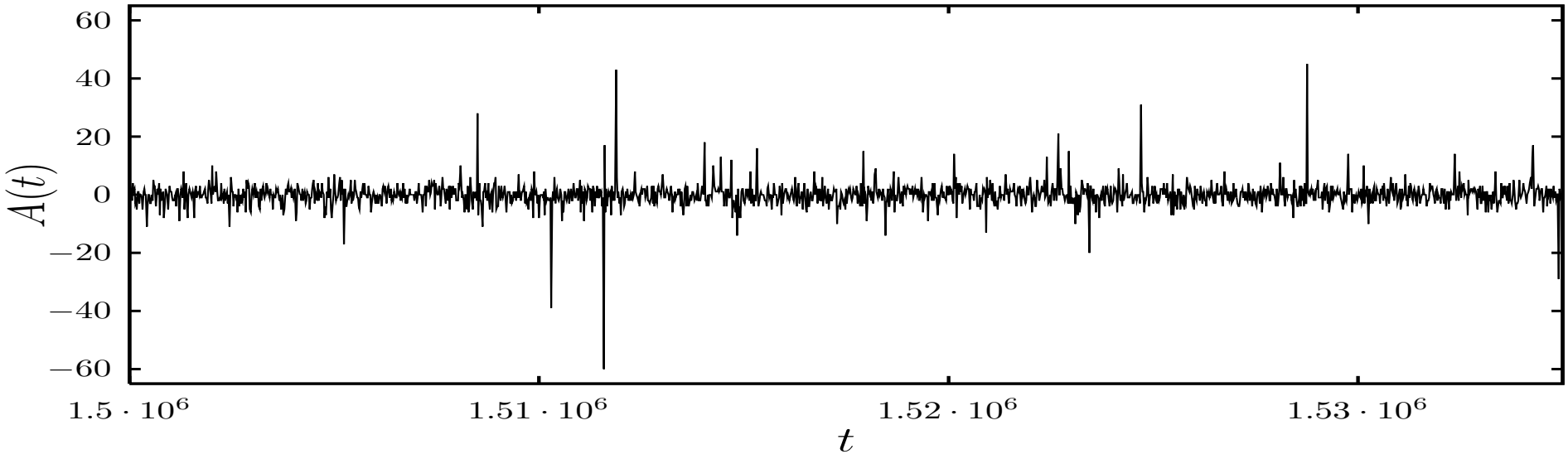

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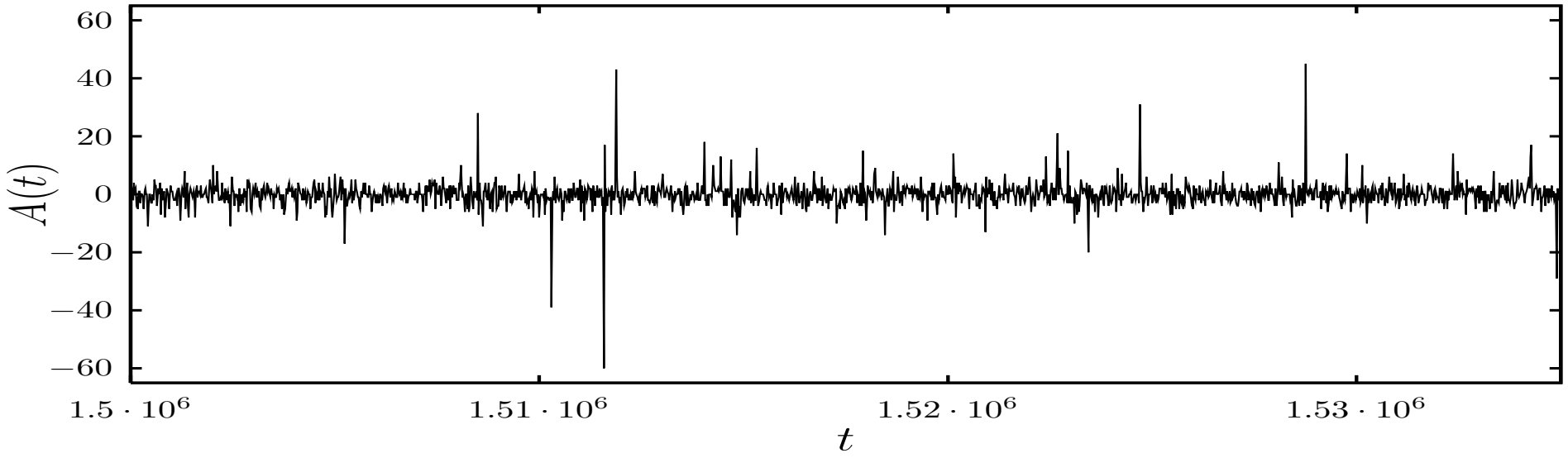

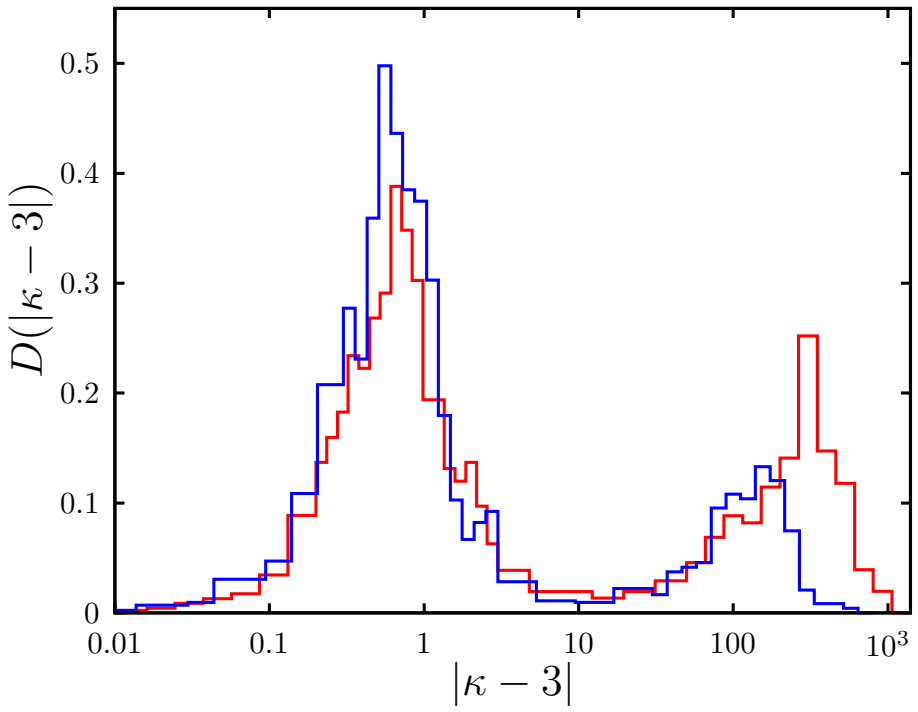

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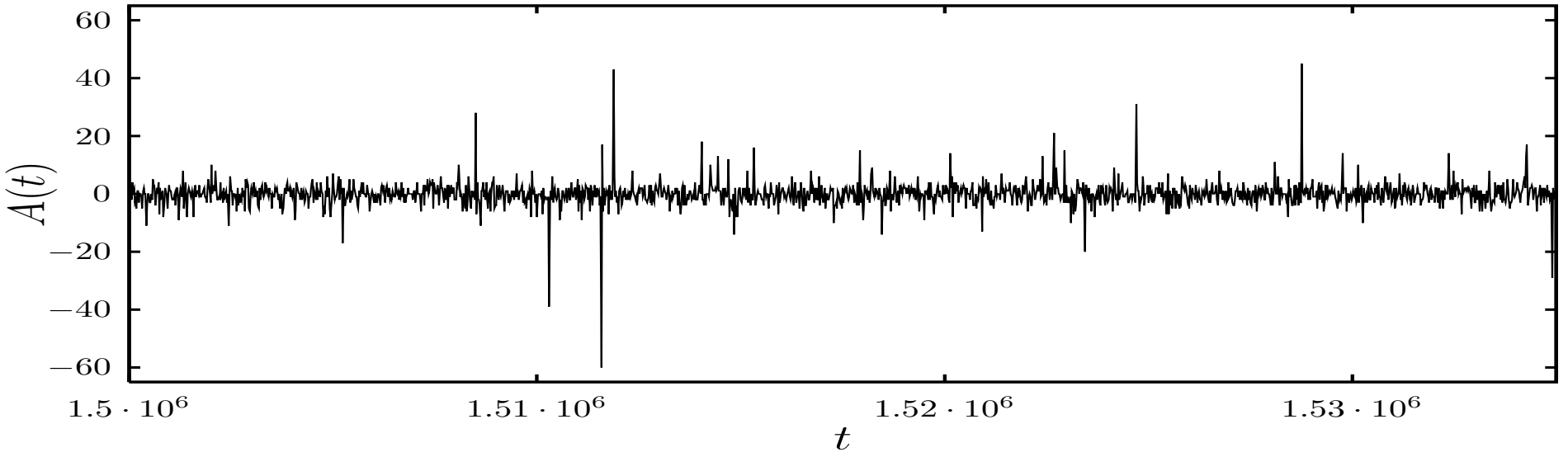
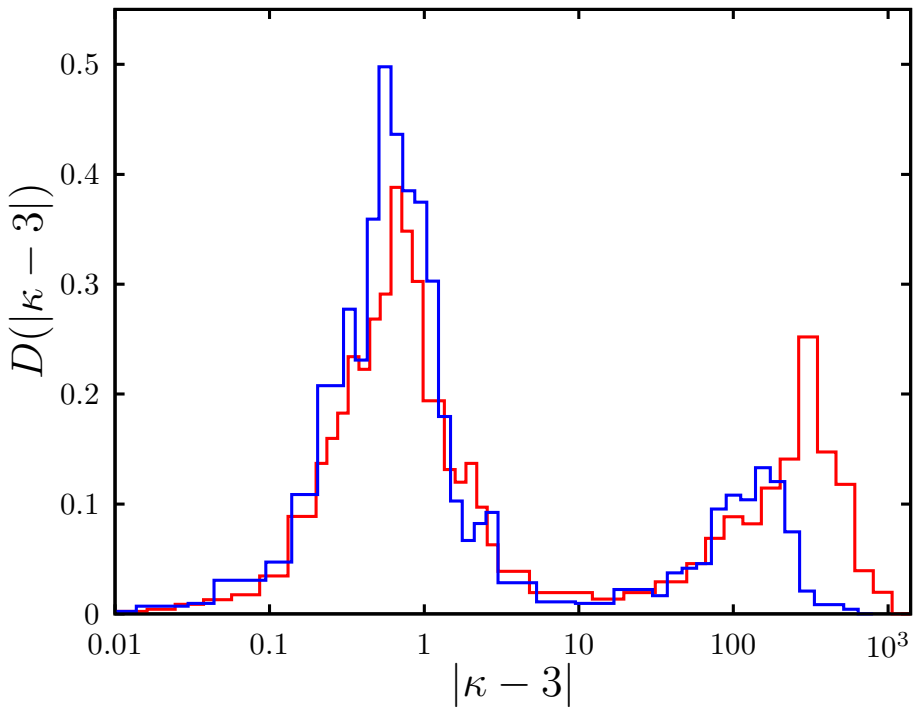
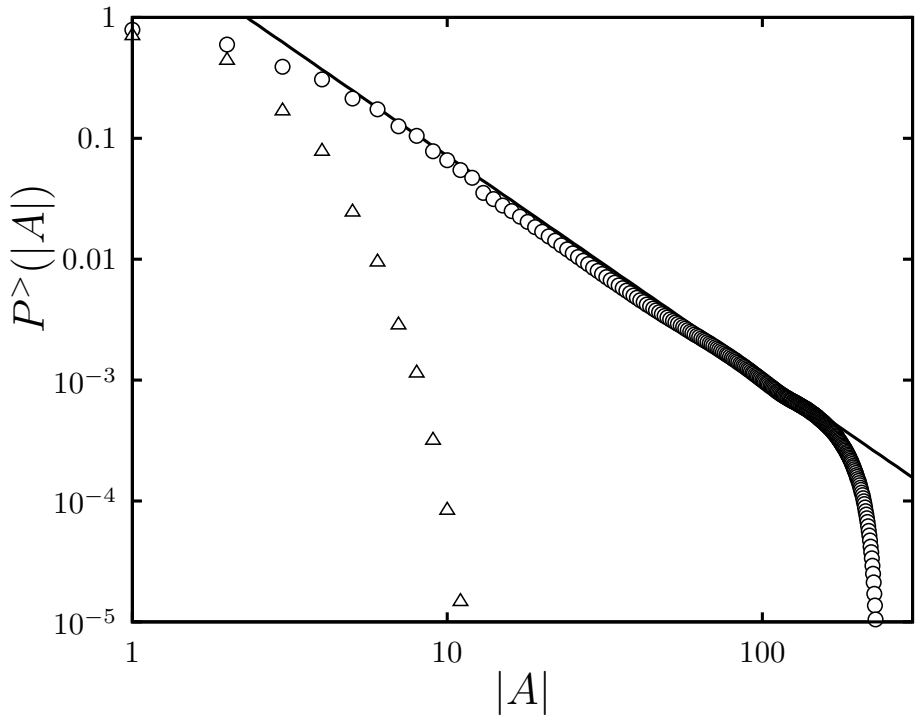


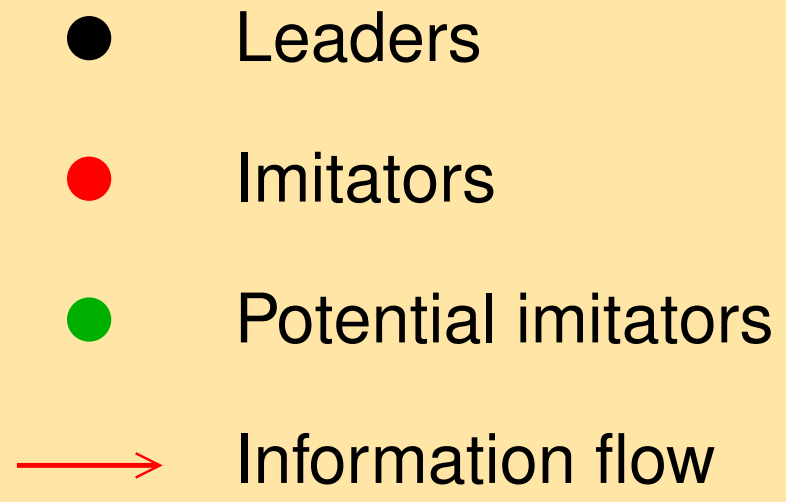




Social organization

Agents on social network imitate more successfull neighbors with probability p (and pay for it)

Agents are placed on linear chain. Imitation may occur along the link



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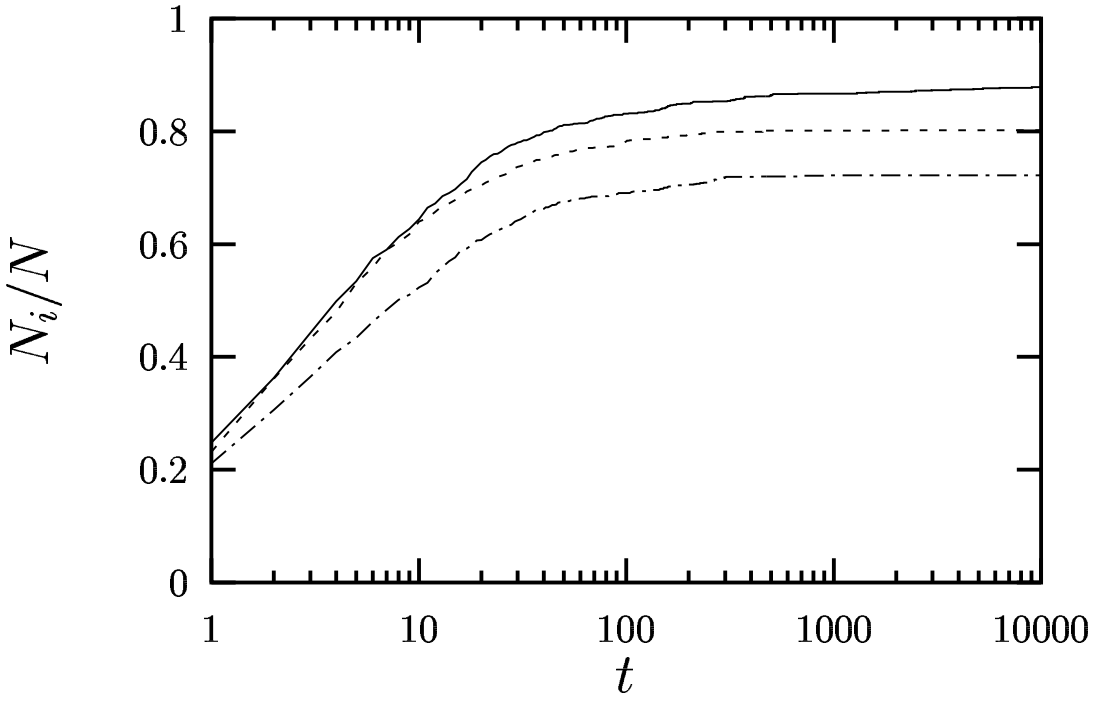
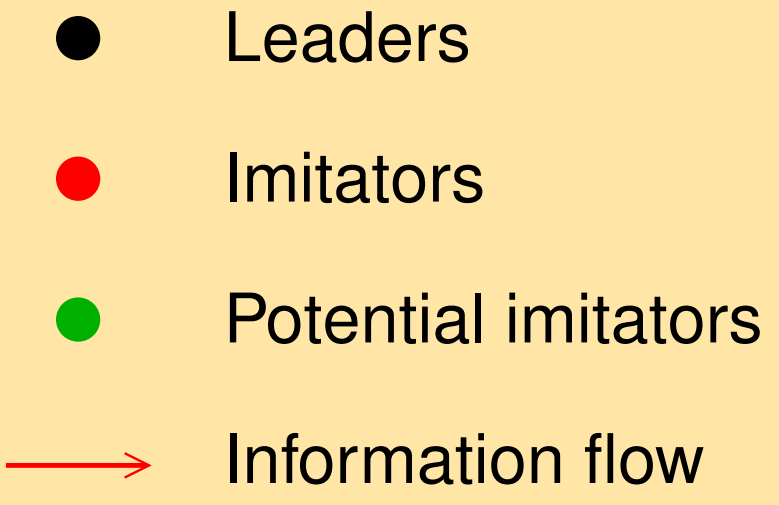
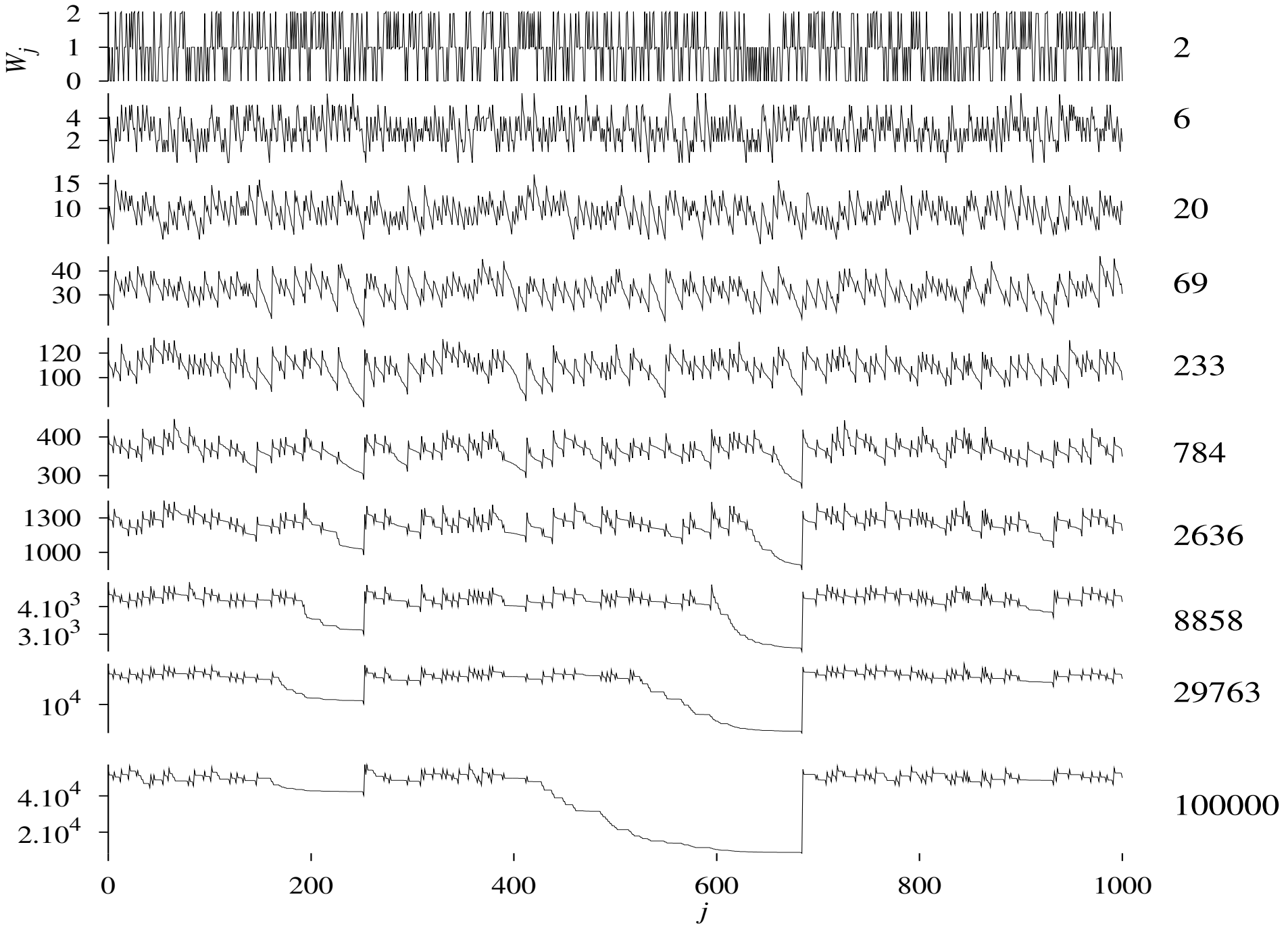


Time dependence of imitation. $p = 0.99$ (full line) 0.95 (dashed), 0.8 (dash-dotted)

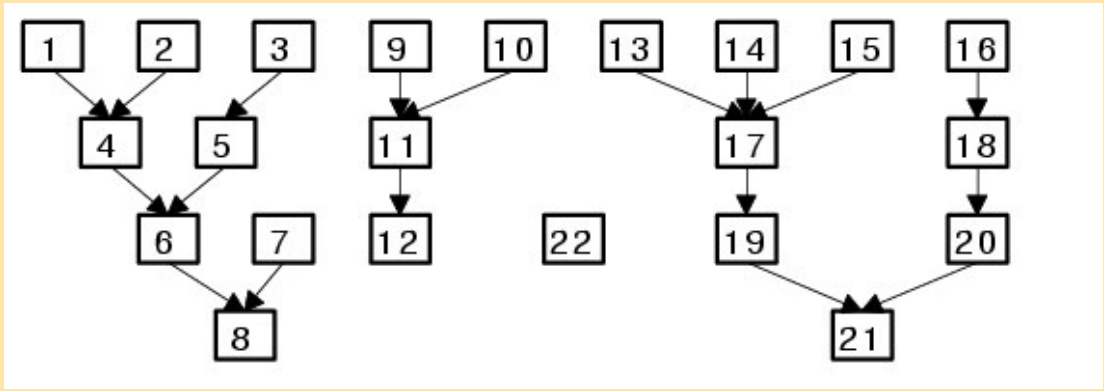




poverty islands created

Imitation on Barabási-Albert network [with H. Lavička]

No compact imitation clusters, rather tree-like structures:



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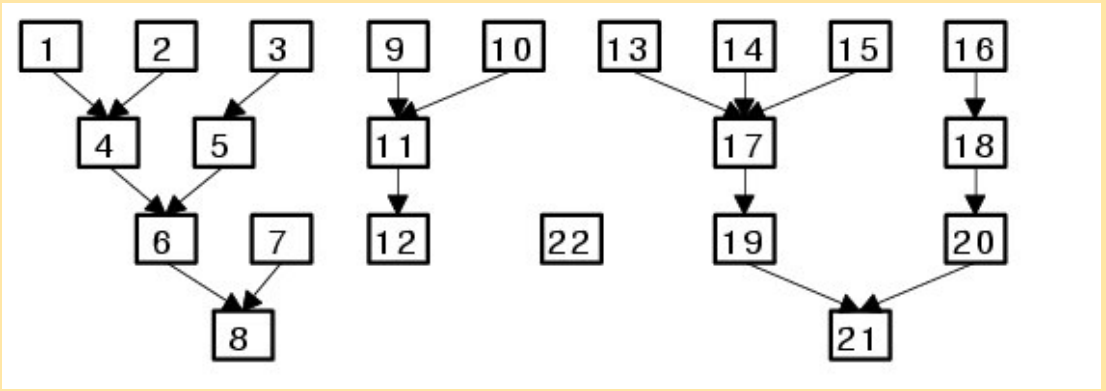
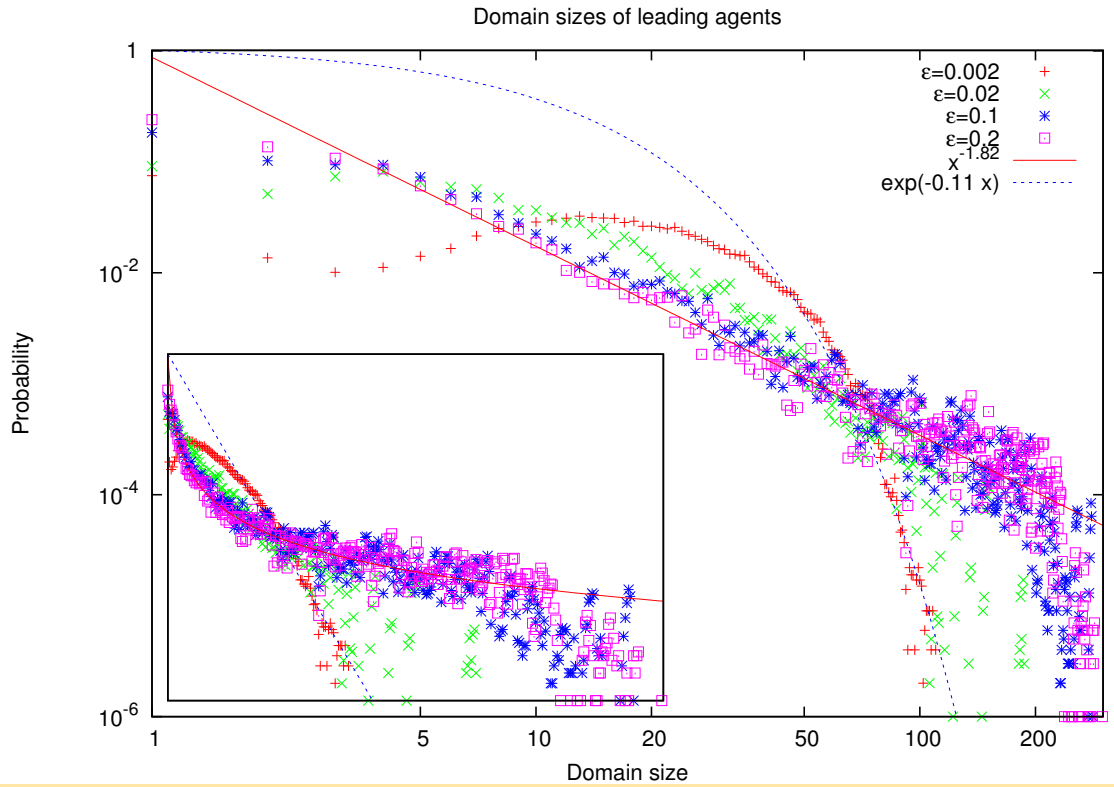
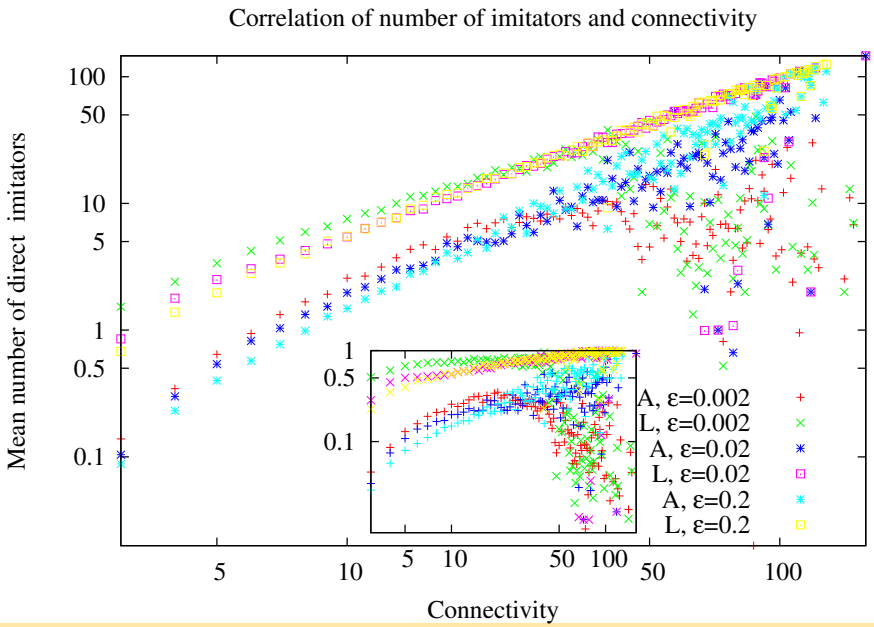


Domain-size statistics:

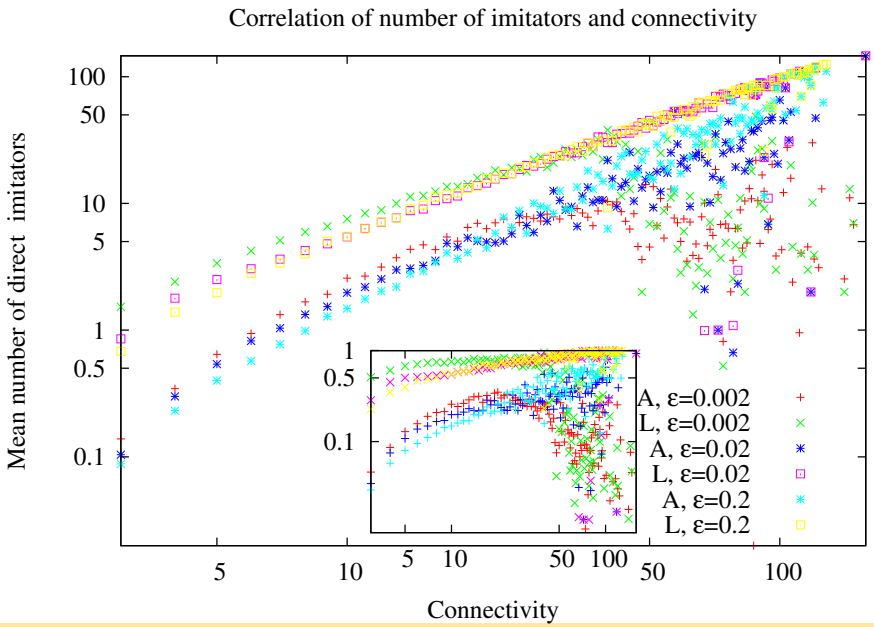


low fee: exponential
high fee: power-law

Forking:


Correlation of number of imitators and connectivity
Mean number of direct imitators
Connectivity
100
50
10
5
1
0.5
0.1
5
10
50
100
A, ε=0.002
L, ε=0.002
A, ε=0.02
L, ε=0.02
A, ε=0.2
L, ε=0.2

Forking: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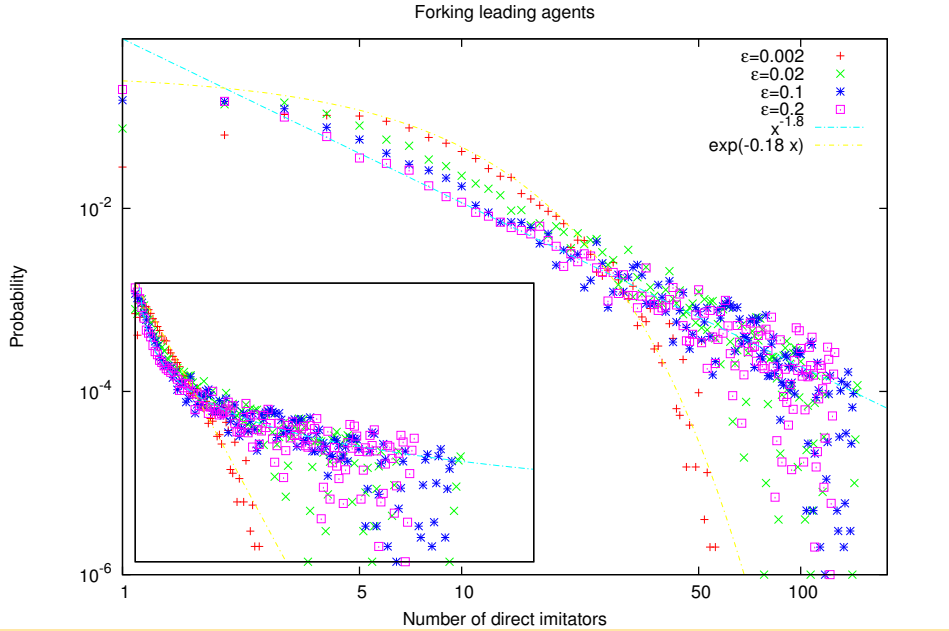
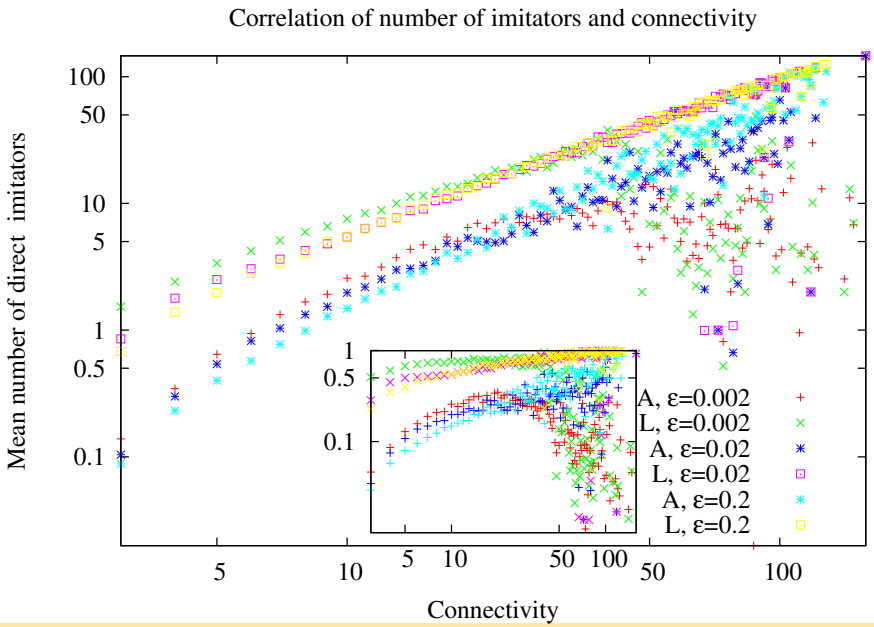
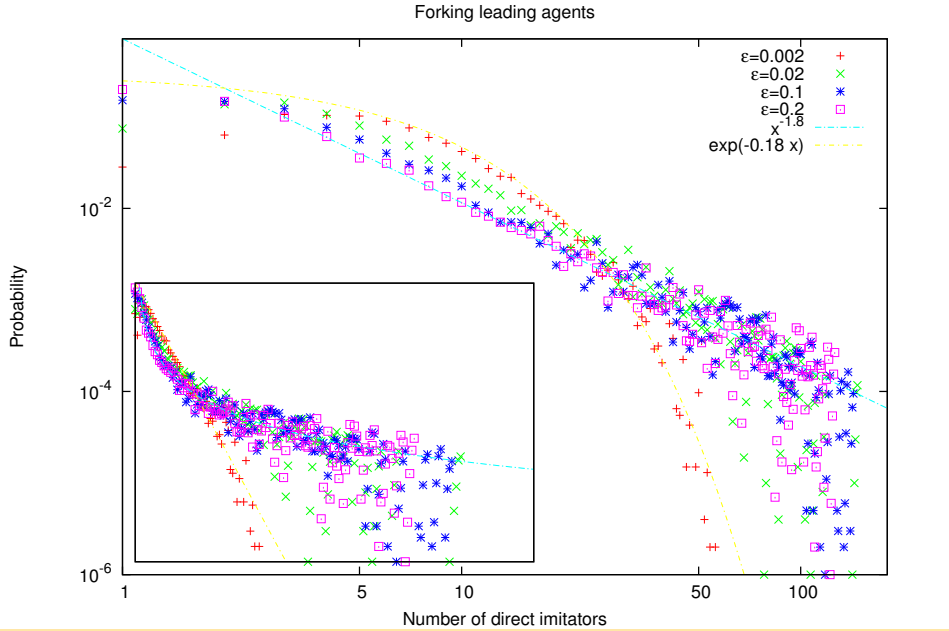
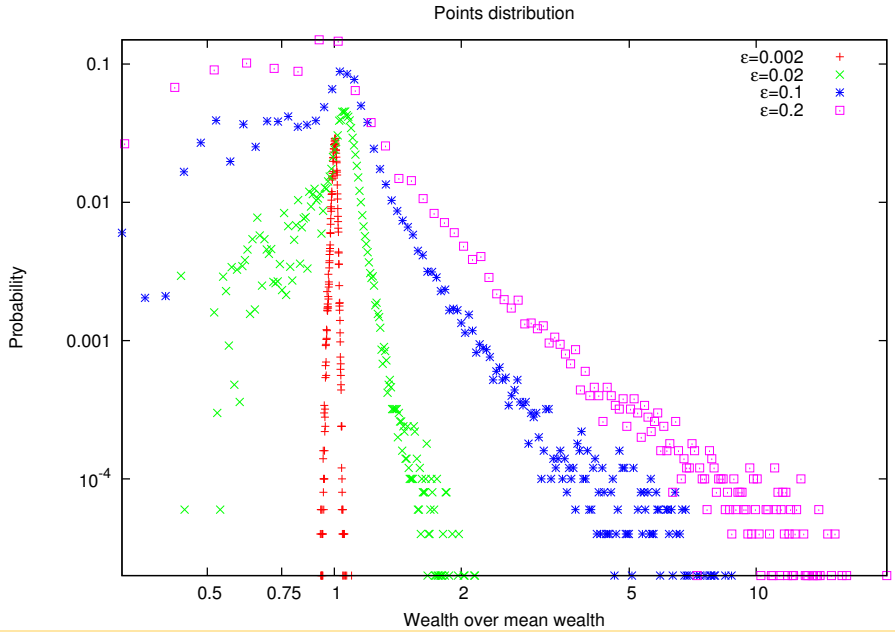

Forking leading agents
Probability
Number of direct imitators
ε=0.002
ε=0.02
ε=0.1
ε=0.2
x^-1.8
exp(-0.18 x)

Forking:





Wealth distribution:



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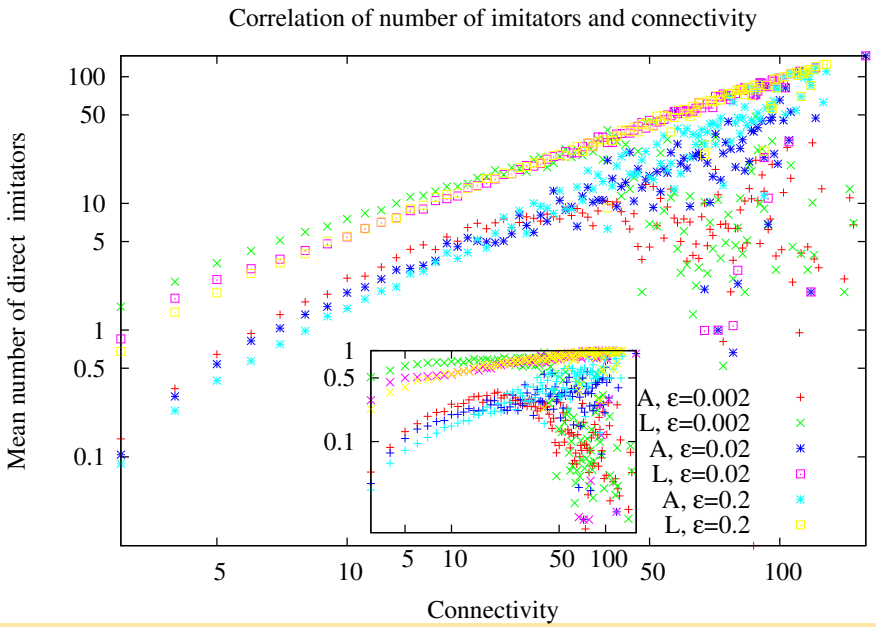

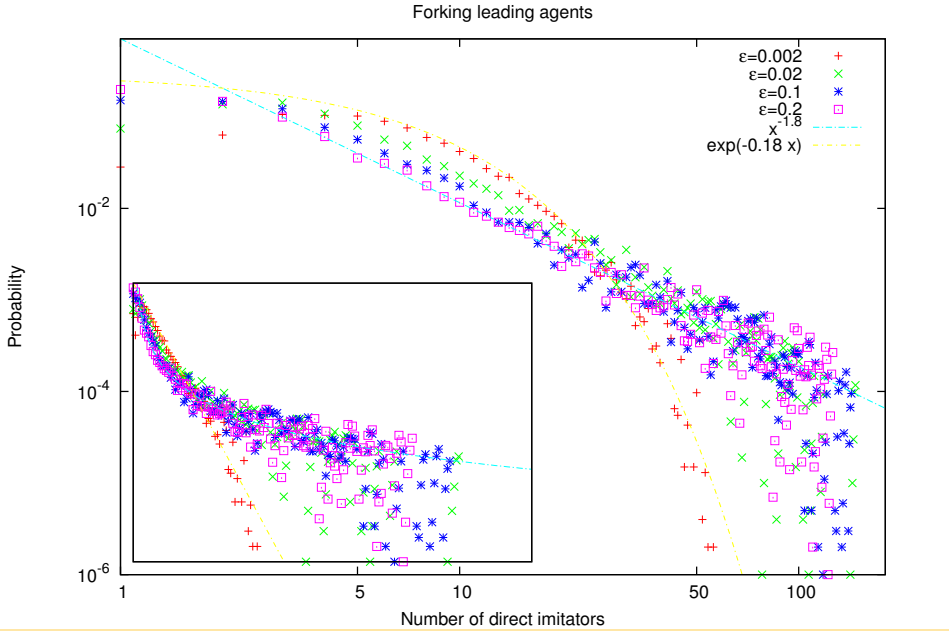

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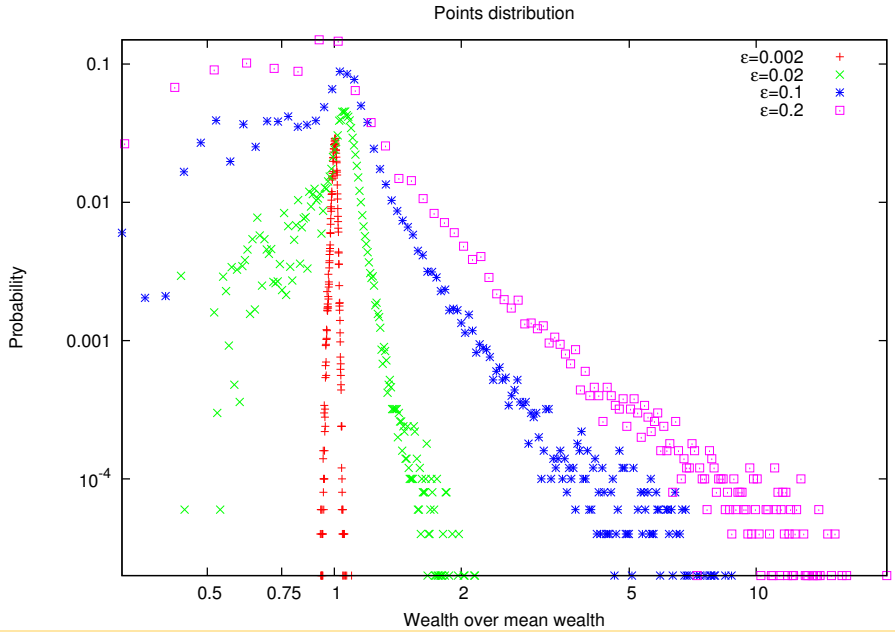

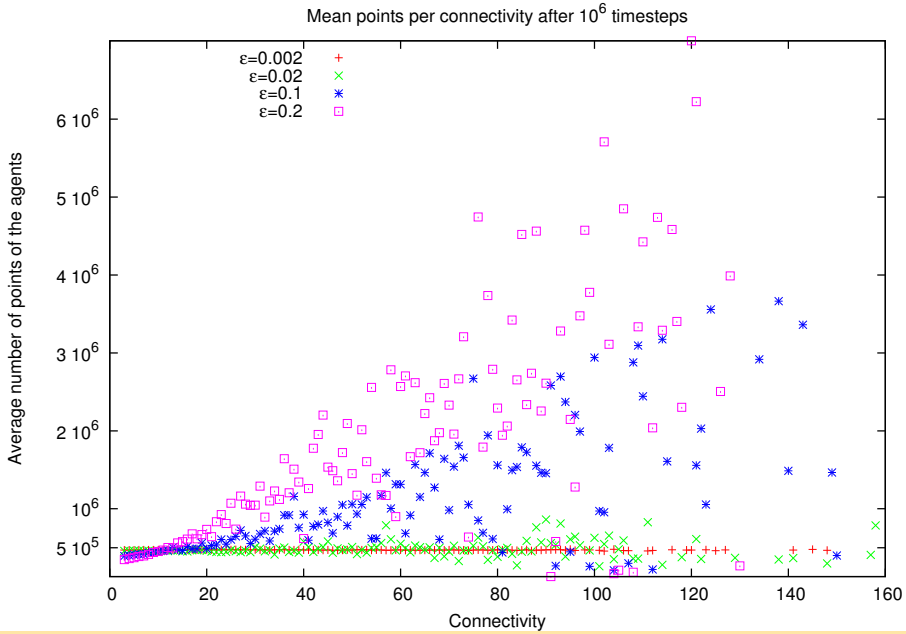

Modeling social divergence: Axelrod model []

Motivation: global versus fragmented culture (language etc.)

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Agents on lattice; F features each; Every feature value from $\{0, 1, \ldots, q-1\}$.

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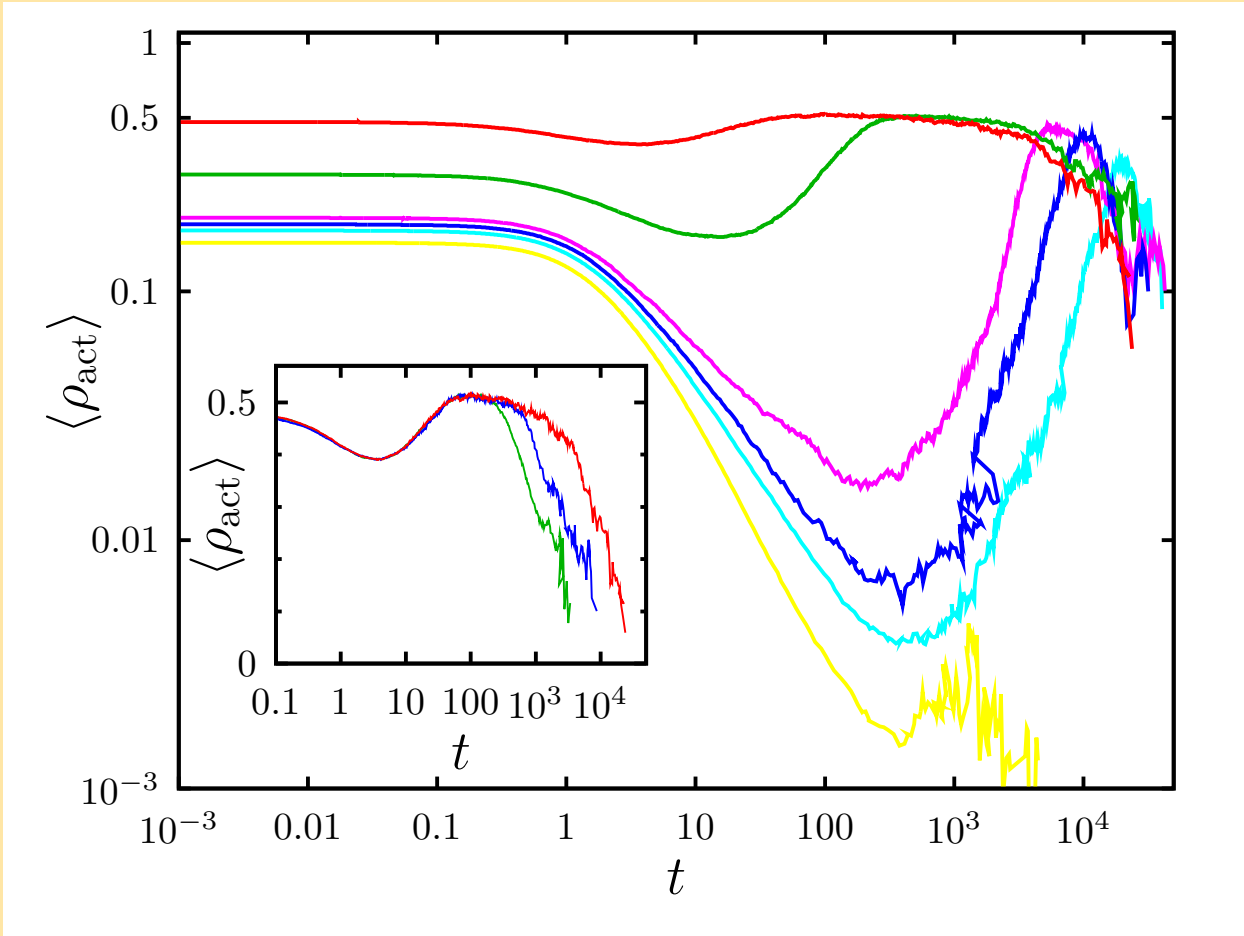

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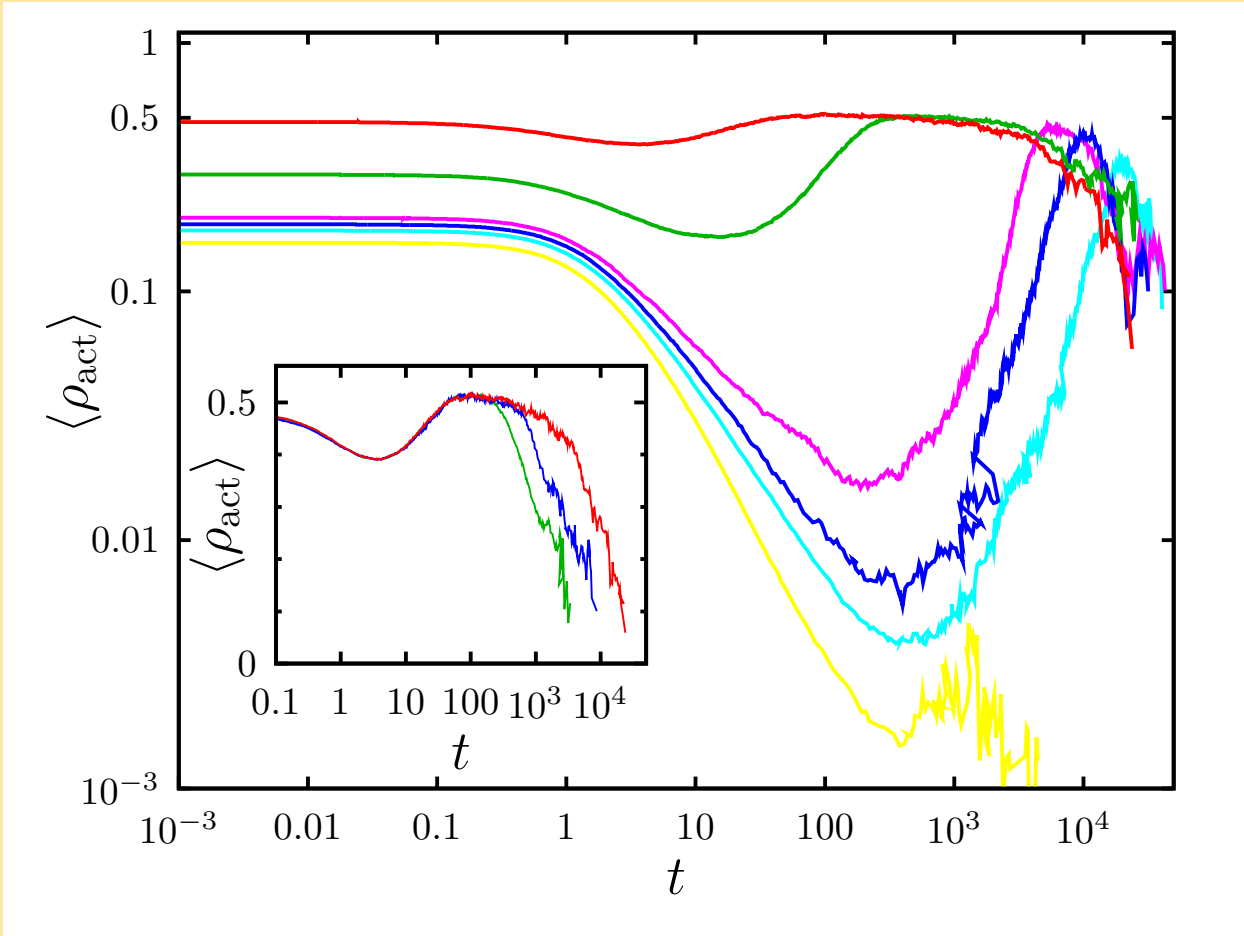
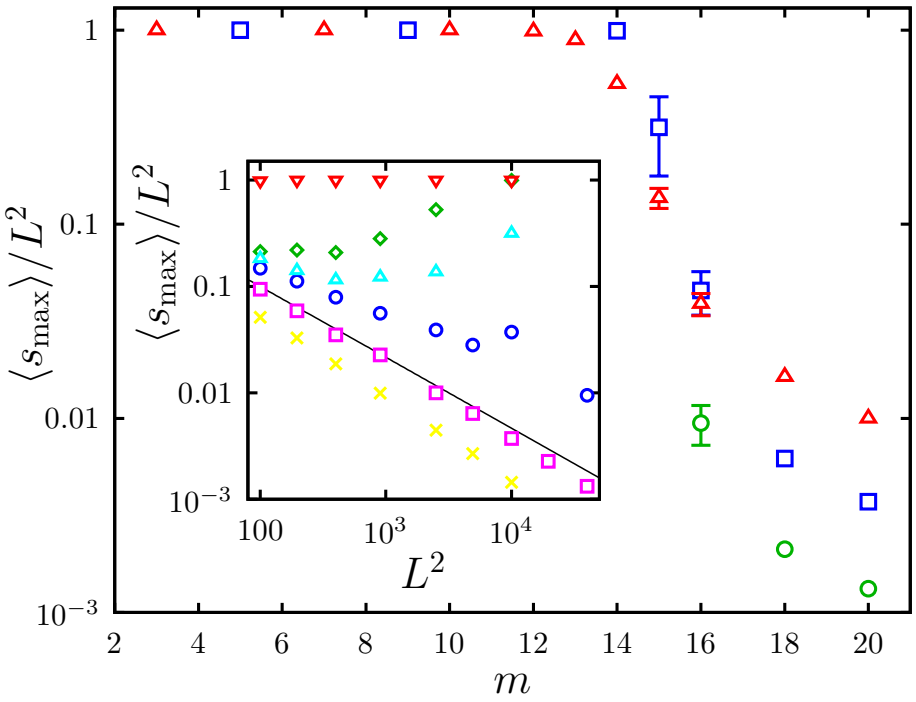
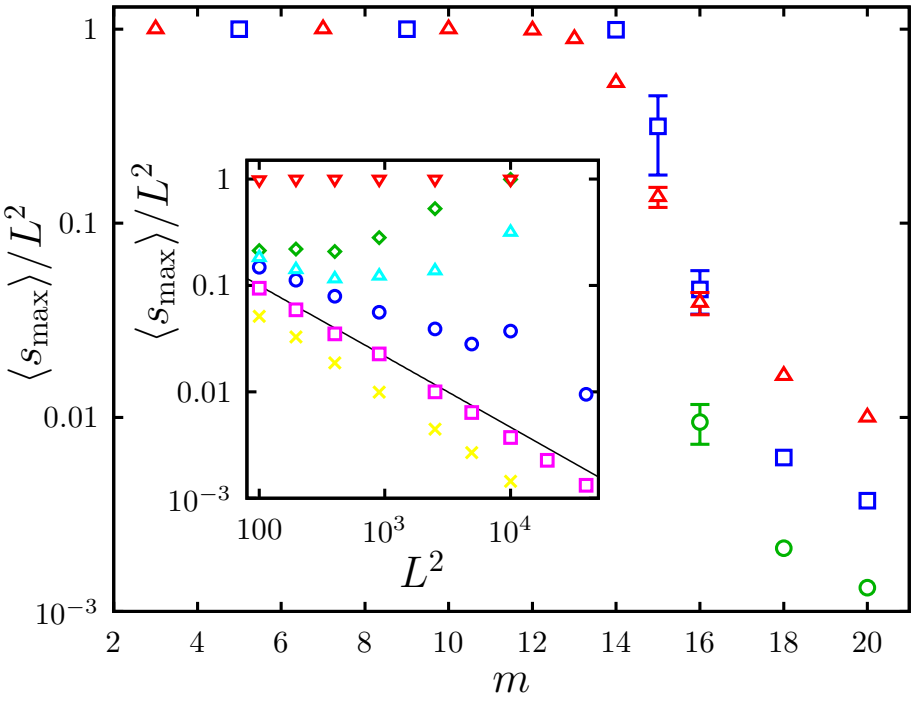
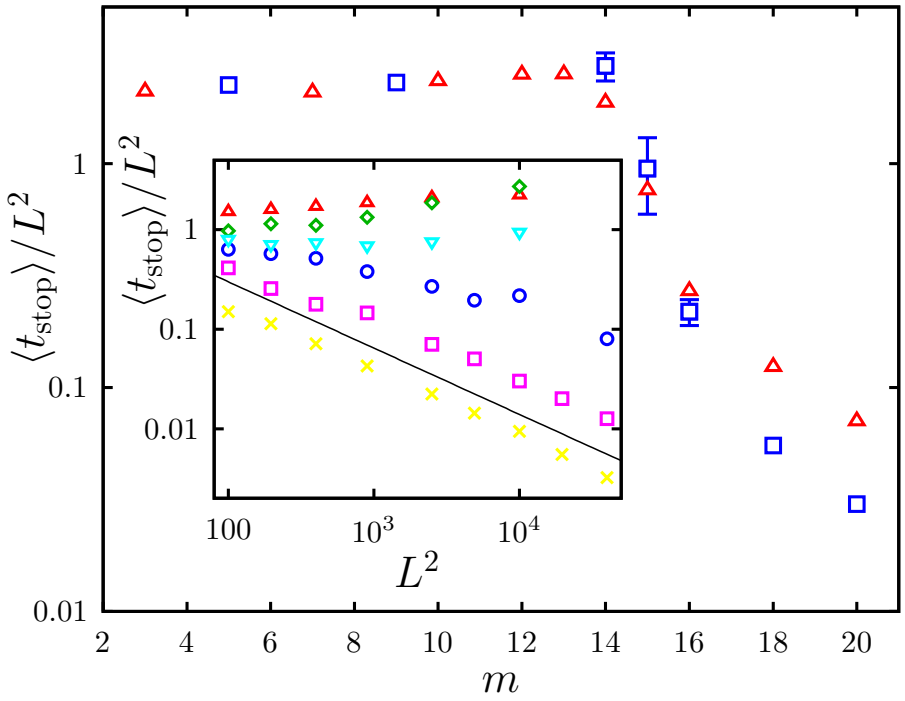


$F = 3$


1
0.1
0.01
10^{-3}
$\langle s_{\max} \rangle / L^2$
2
4
6
8
10
12
14
16
18
20
m
1
0.1
0.01
10^{-3}
$\langle s_{\max} \rangle / L^2$
100
10^3
10^4
L^2

$F = 3$





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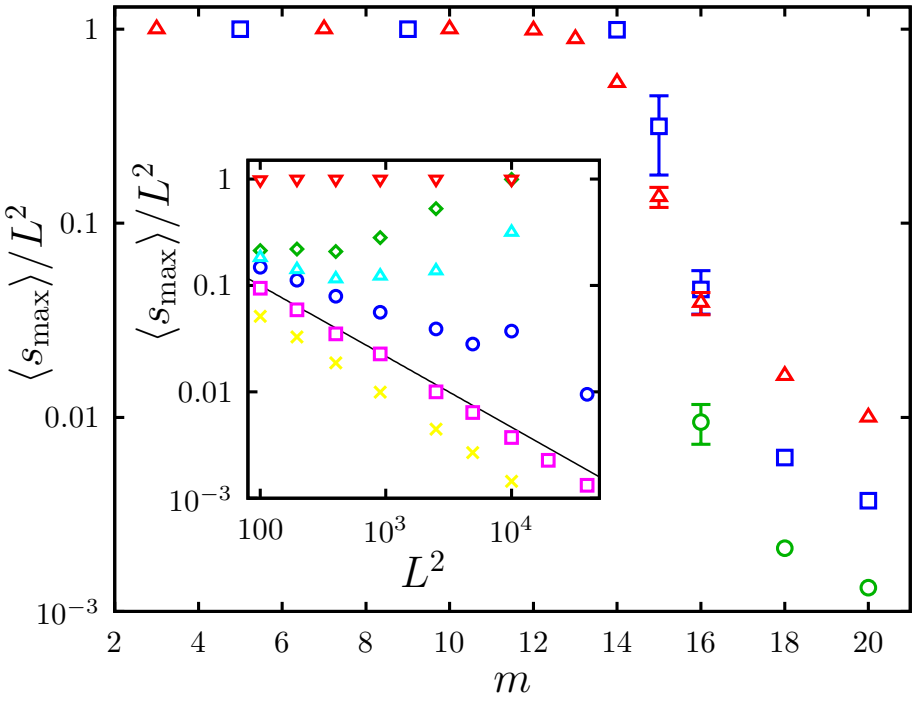
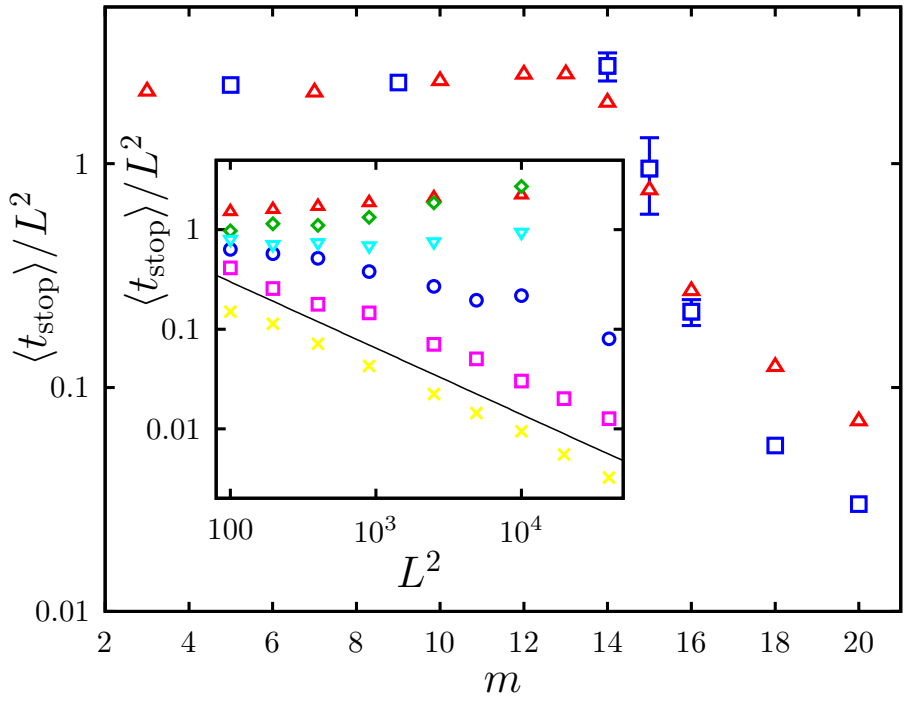
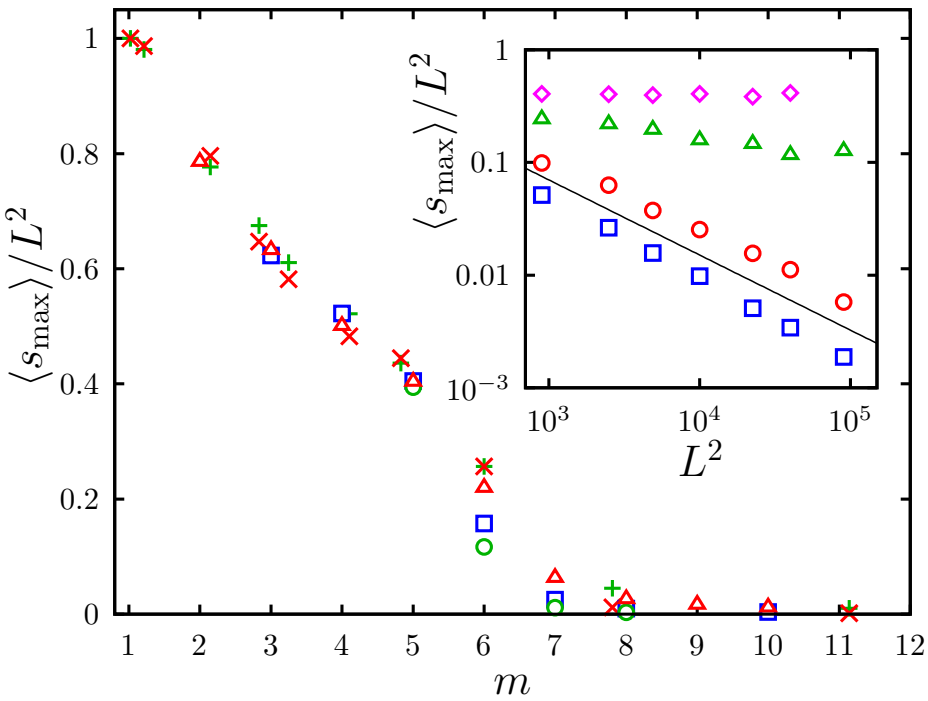
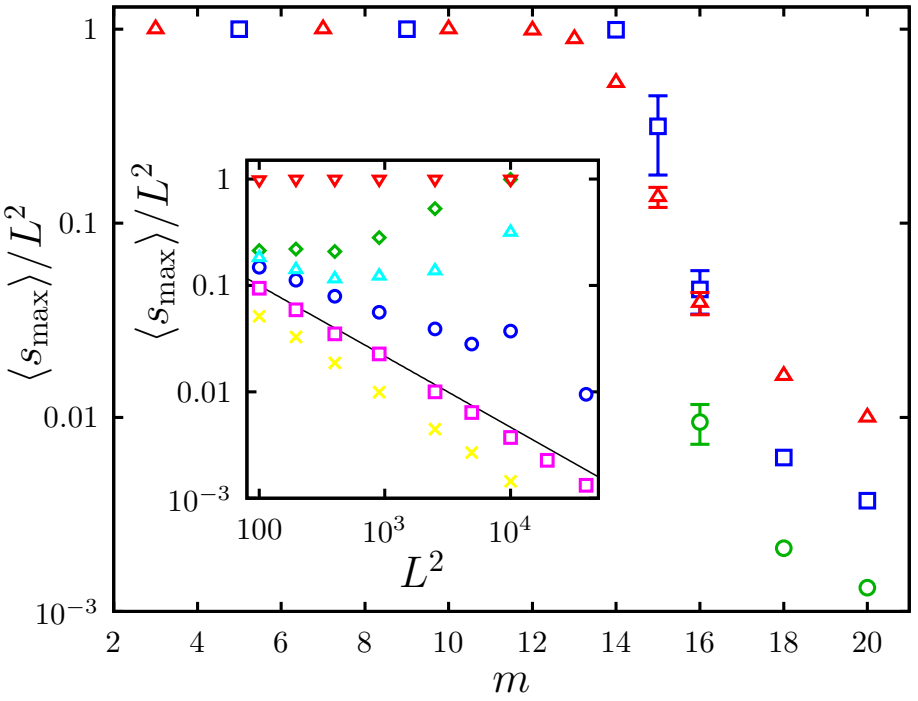

$\langle s_{\max} \rangle / L^2$
m
L^2

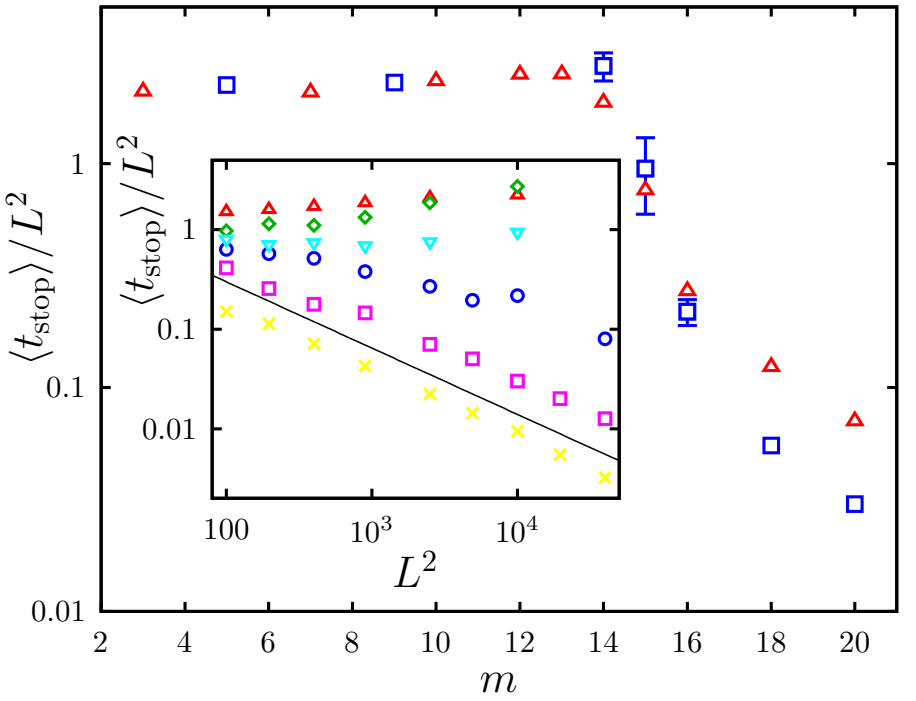

$\langle t_{\text{stop}} \rangle / L^2$
m
L^2

$F = 2$


$\langle s_{\max} \rangle / L^2$
m
L^2

$F = 3$


$\langle s_{\max} \rangle / L^2$
m
L^2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m
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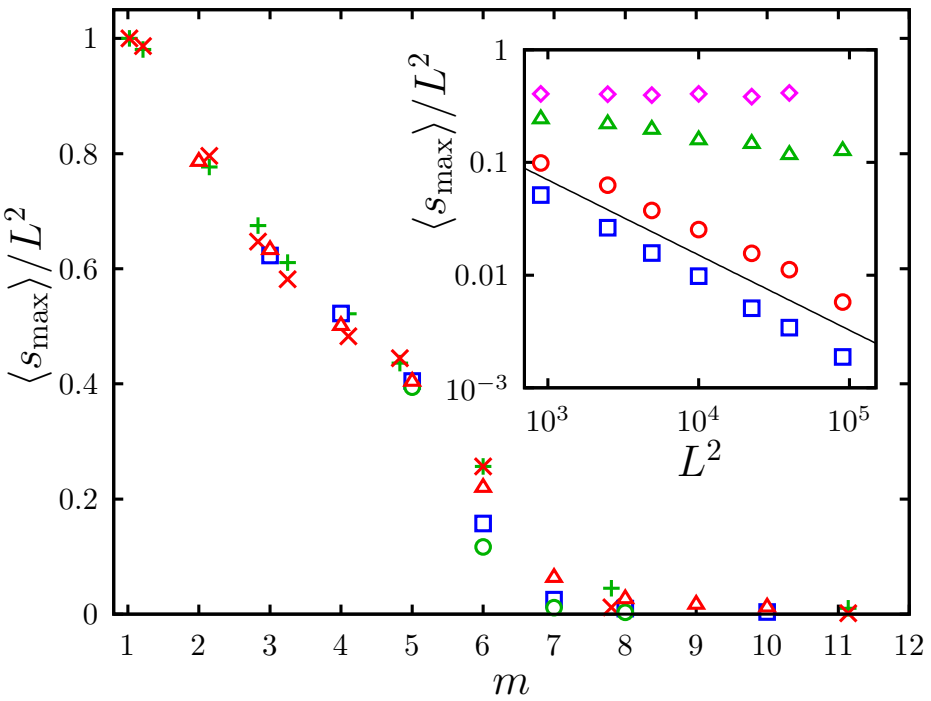
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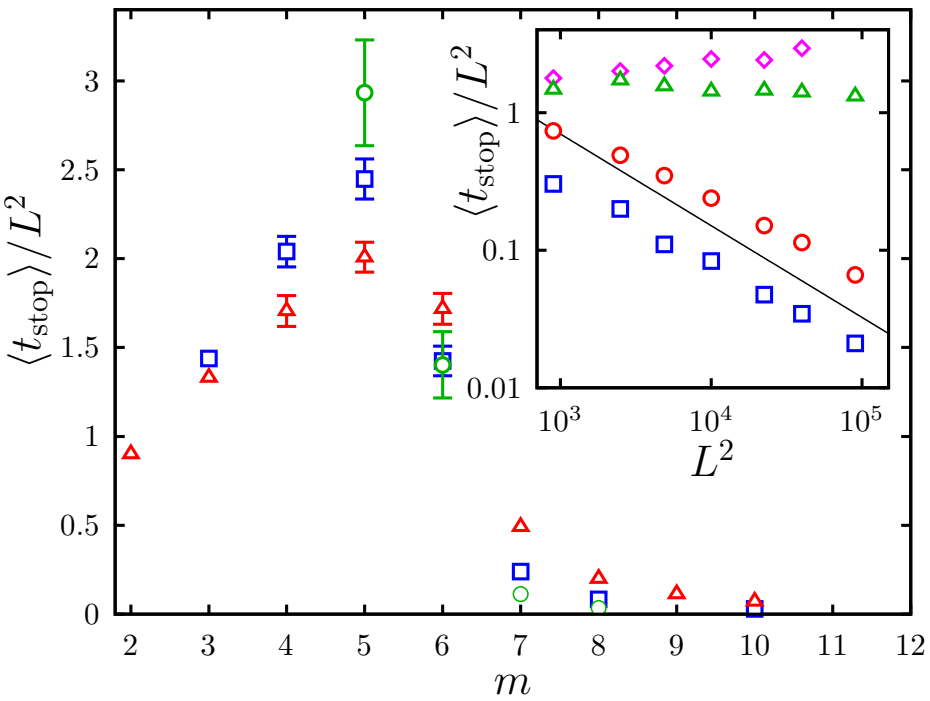
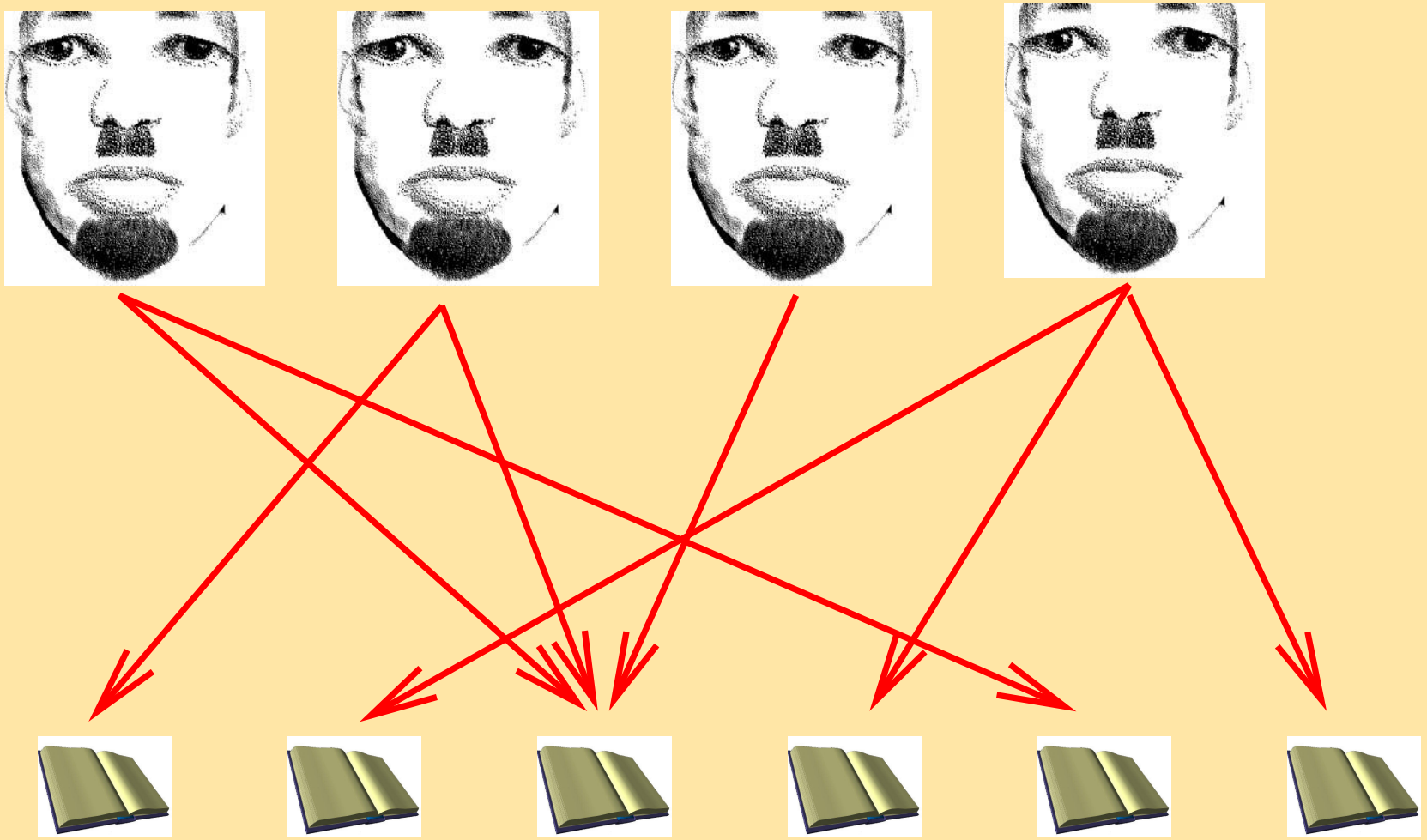
m
L^2

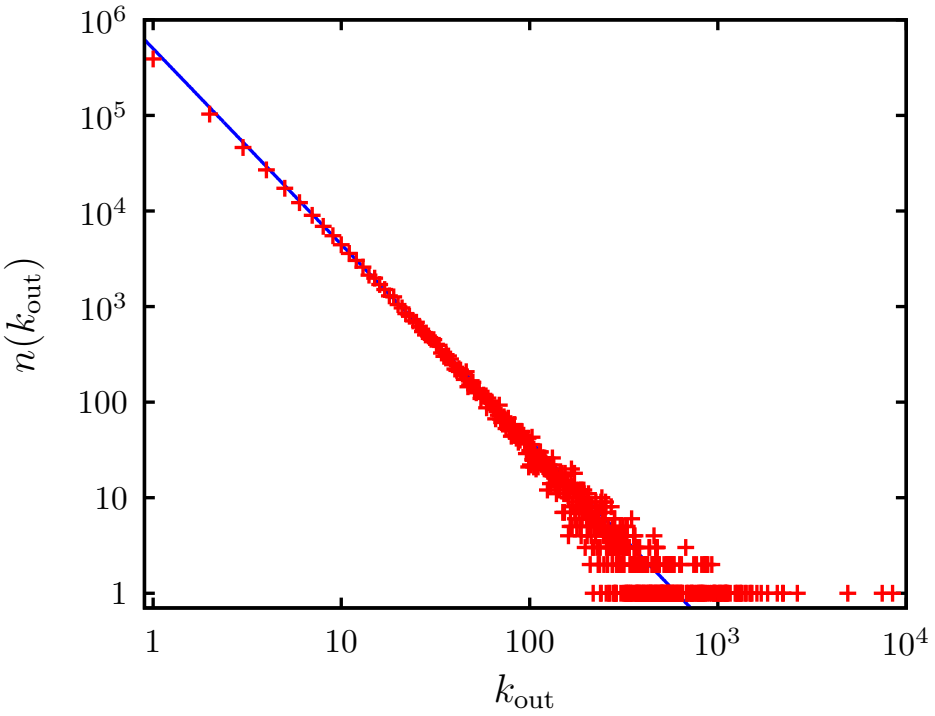

$\langle t_{\text{stop}} \rangle / L^2$
m
L^2

Rewiewers' network in e-commerce: www.amazon.com [with Y.-C. Zhang]



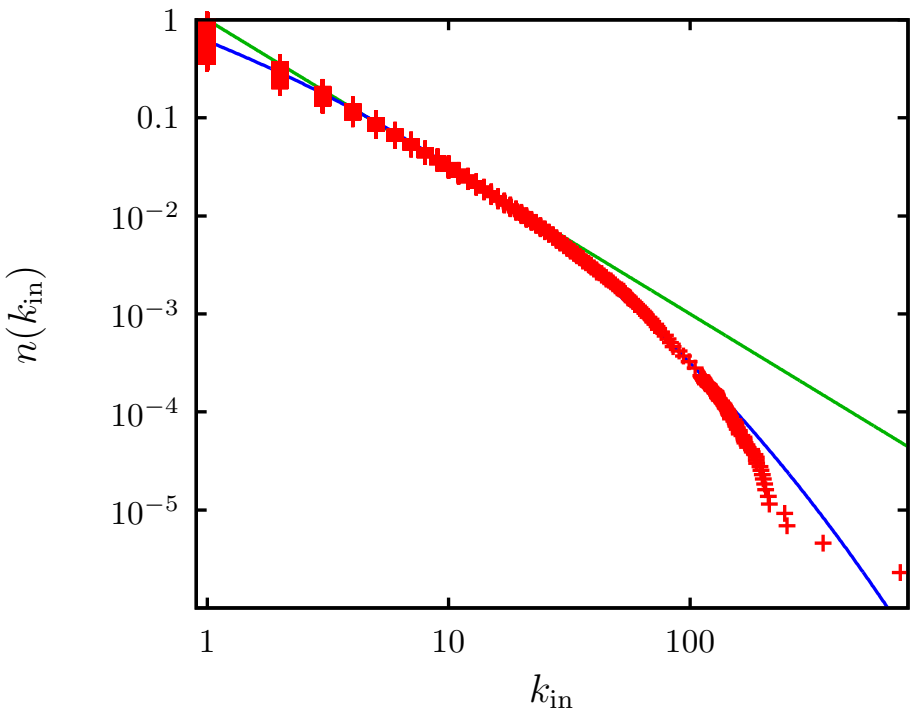
- Write a robot to download rewiews automatically
- Statistics of the network
- Spectral properties of the network

Statistics of the network



Distribution of books per reviewer.

Line: $k_{\rm out}^{-\gamma}$, $\gamma = 2.05$.

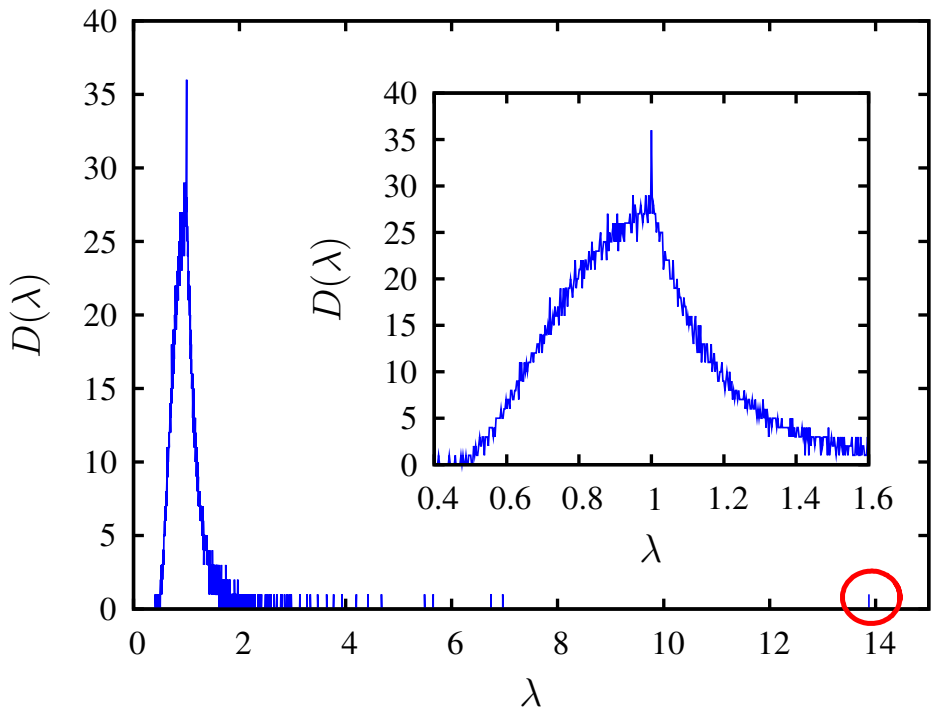


Distribution of rewiewers per book.

Green line: $k_{\rm in}^{-\gamma}$, $\gamma = 1.5$.

Blue line: $\exp(-5\,k_{\rm in}^{0.2})$

Spectral properties of the network (5000×5000 subnet)



$M_{br} = 1$ if r reviews book b. $A = M^T M$ describes indirect interaction between reviewers.

Histogram of eigenvalues of A

Inset: detail of the central part. Red circle: largest eigenvalue.

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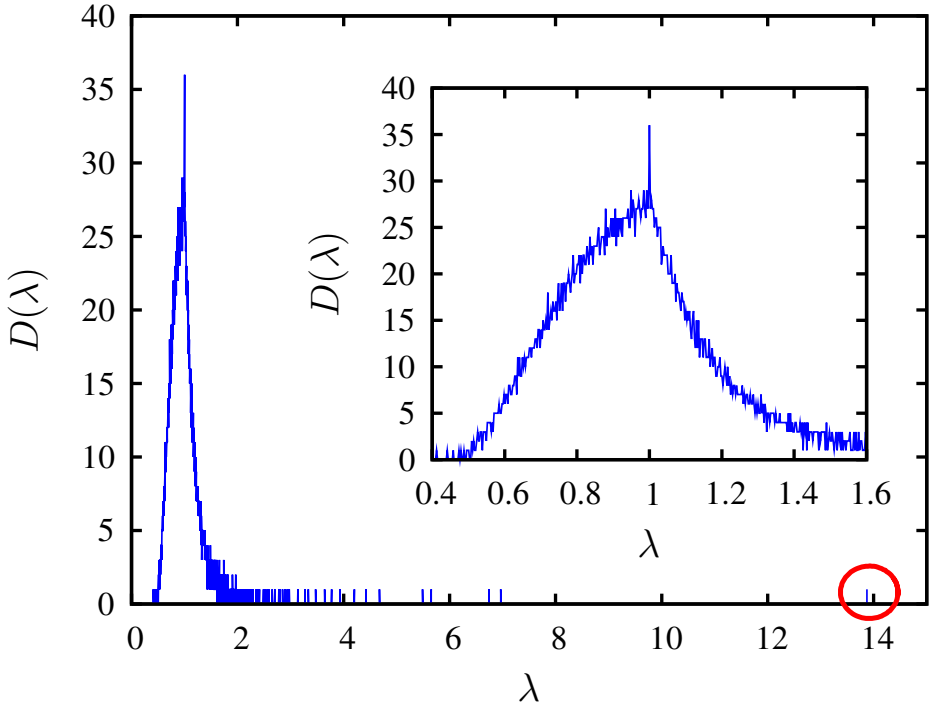

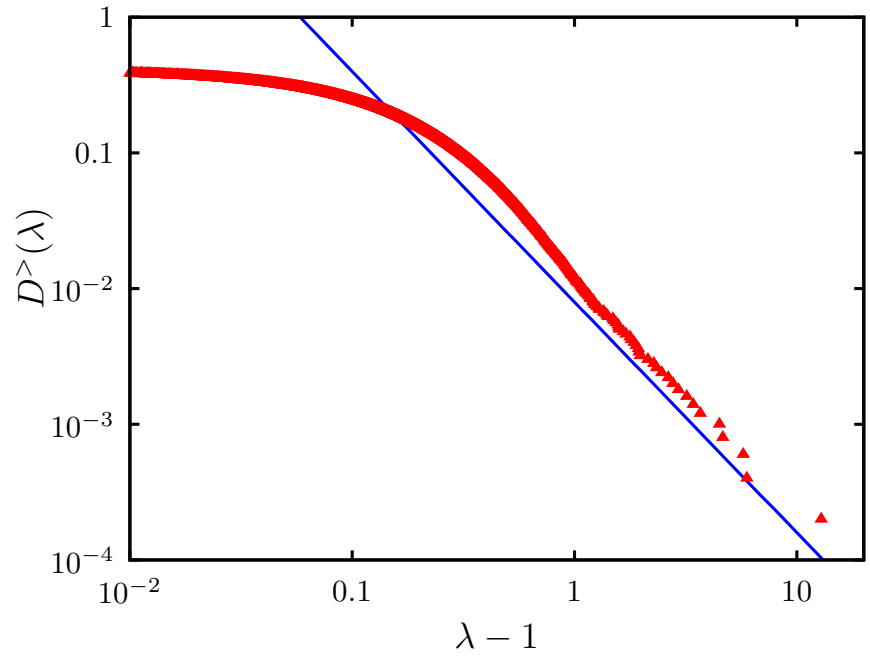

Right tail of the integrated density of states.

Line: $(\lambda - 1)^{-\gamma}$, $\gamma = 1.7$.

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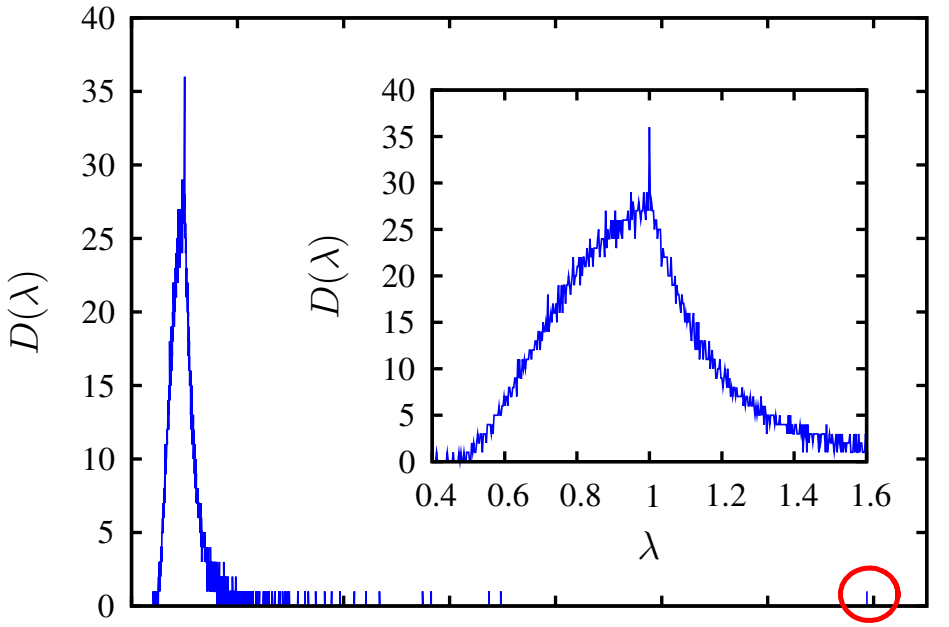

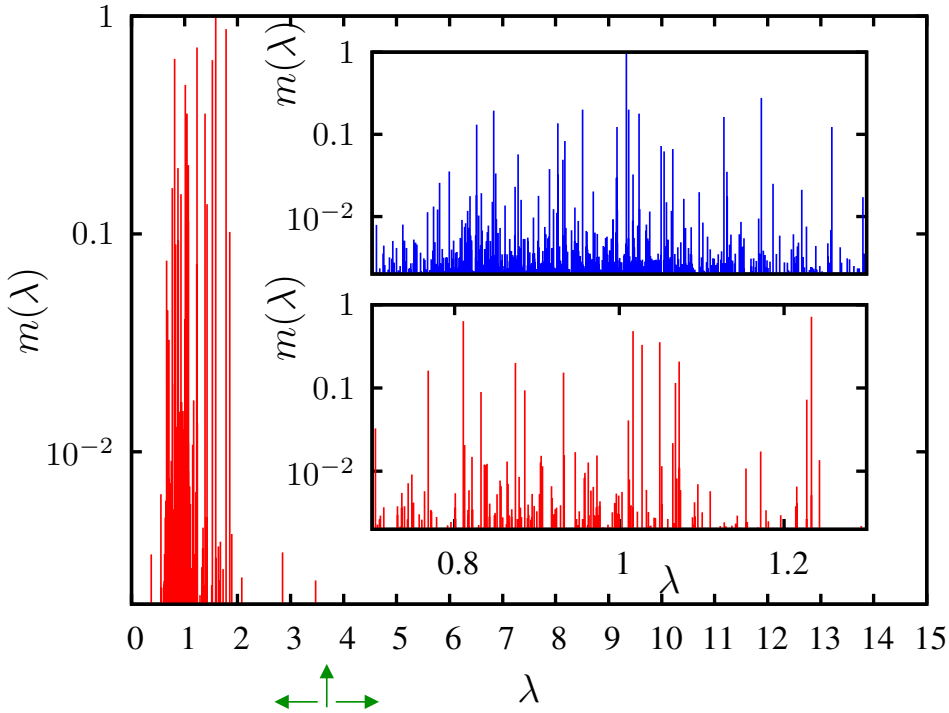

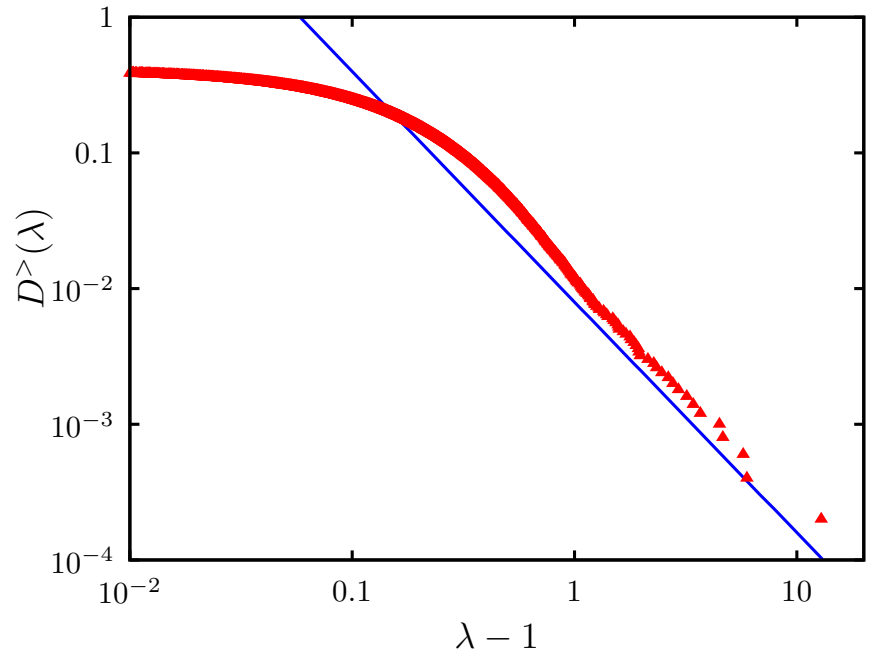

Right tail of the integrated density of states.

Line: $(\lambda - 1)^{-\gamma}$, $\gamma = 1.7$.

Inverse participation ratio $m(\lambda) = \sum_i v_{i\lambda}^4$.

Inset: detail for $N_r = 5000$ (red) and $N_r = 2000$ (blue). Compare to RM

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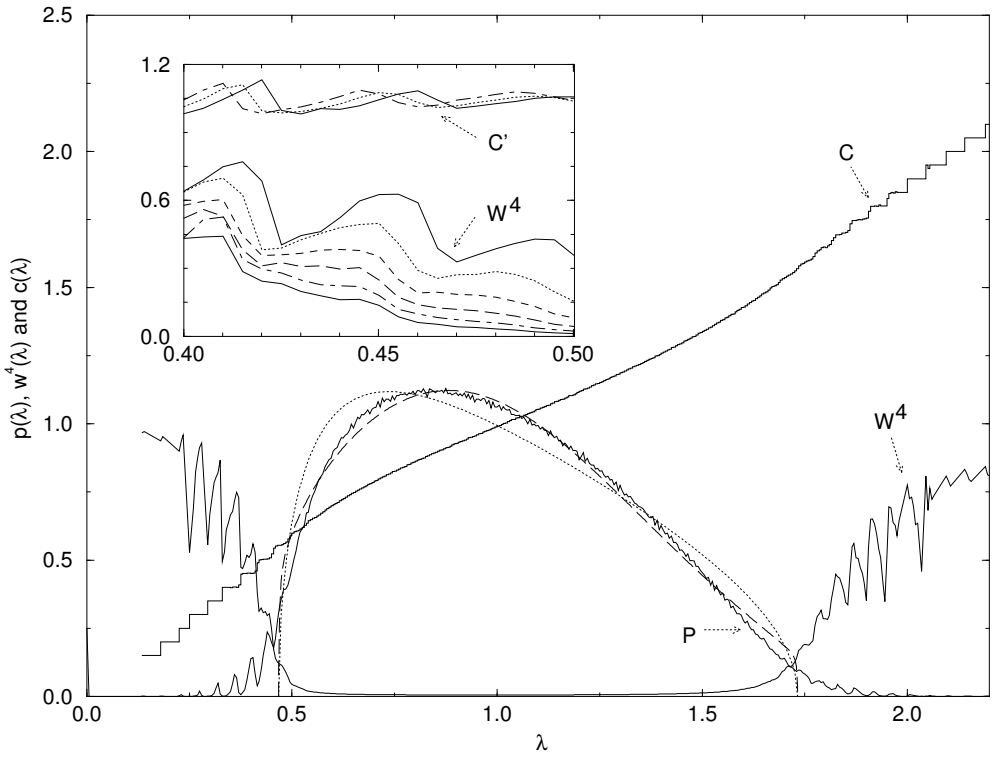
- Thanks to colllaborators: Y.-C. Zhang, H. Lavička

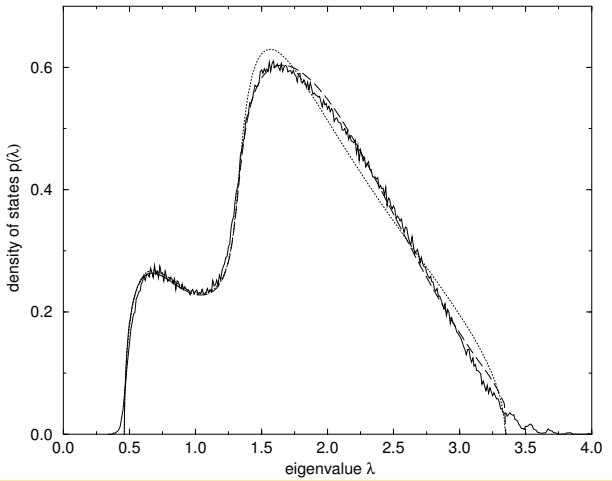
Spectra

[G. Biroli, R. Monasson, *J. Phys. A: Math. Gen.* **32**, L255 (1999); R. Monasson, *Eur. Phys. J. B* **12**, 555 (1999).]



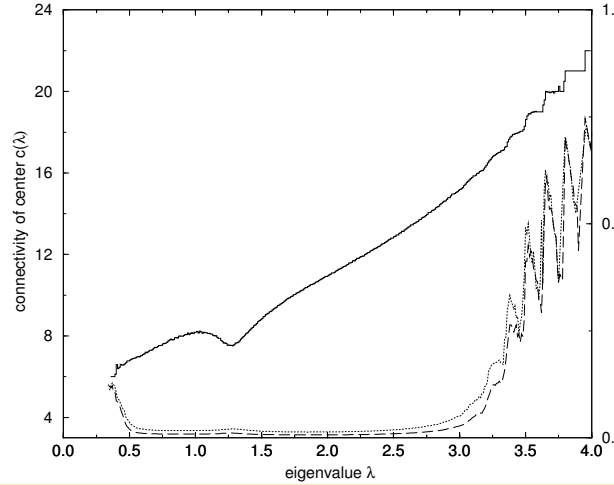
Random network.

Density of states $p(\lambda)$, inverse participation ratio $w^4(\lambda)$ and connectivity of the centers $c(\lambda)$ (divided by q) averaged over 2000 samples for $q = 20$, $N = 800$.



Small-world network $K = 3$, $q = 5$.

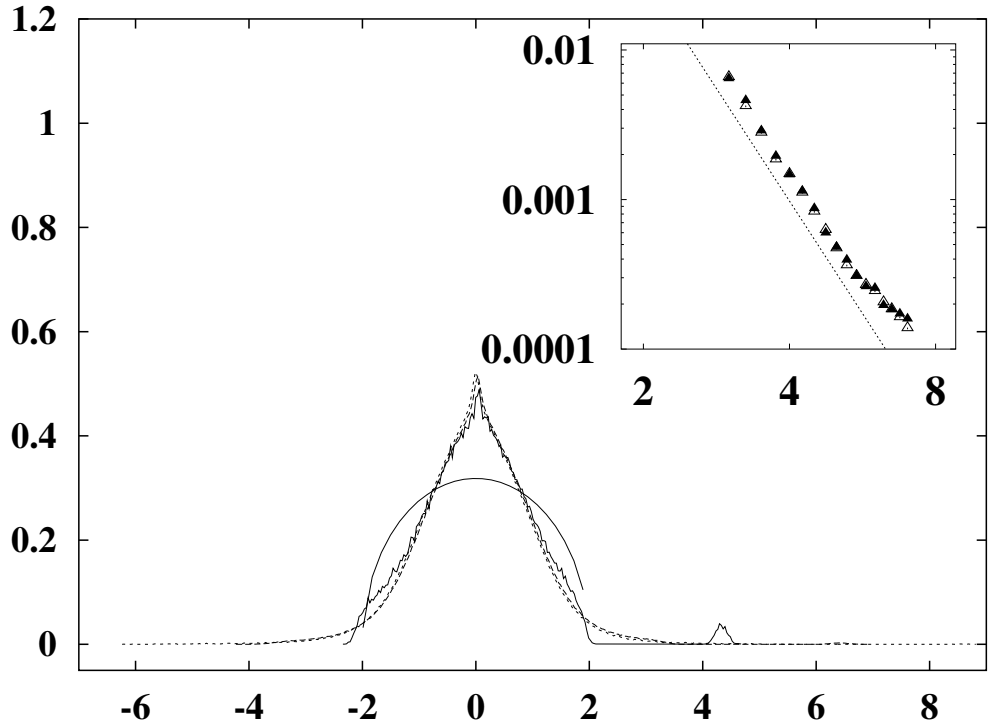
Density of states from numerics, EMA and SDA approximation.



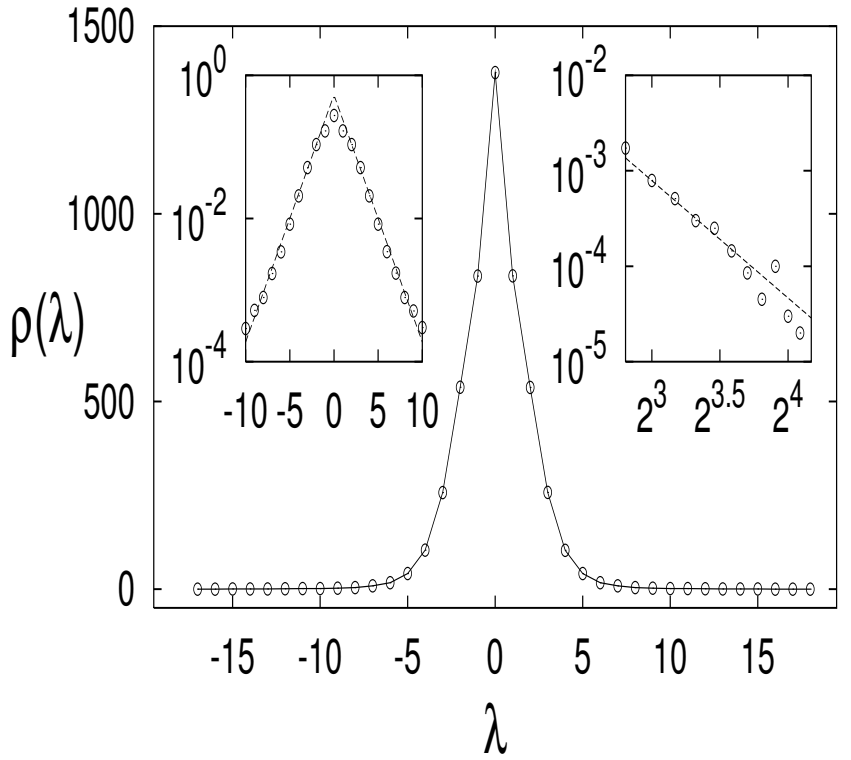
Inverse participation ratio for N = 256 and N = 512 averaged over 1000 samples.

Scale-free networks

[Illes J. Farkas, Imre Derenyi, Albert-Laszlo Barabasi, Tamas Vicsek, *Physical Review E* **64**, 026704 (2001); K.-I. Goh, B. Kahng, and D. Kim, *Phys. Rev. E* **64**, 051903 (2001); S. Bilke and C. Peterson, *Phys. Rev. E* **64**, 036106 (2001).]



Density of states. Power-law tail.



Density of states.

Compare to reviewer matrices