Bunching instability in surface growth

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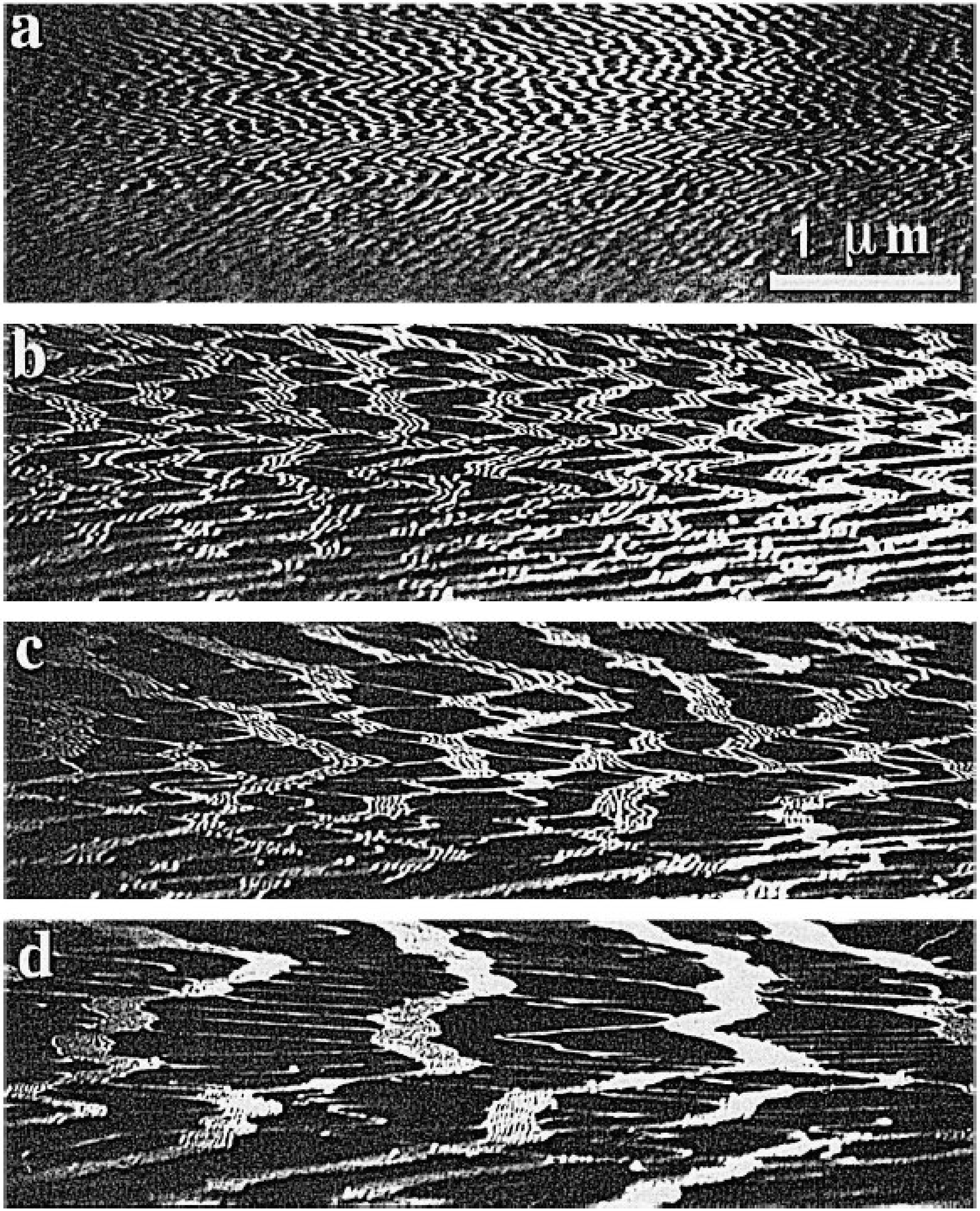
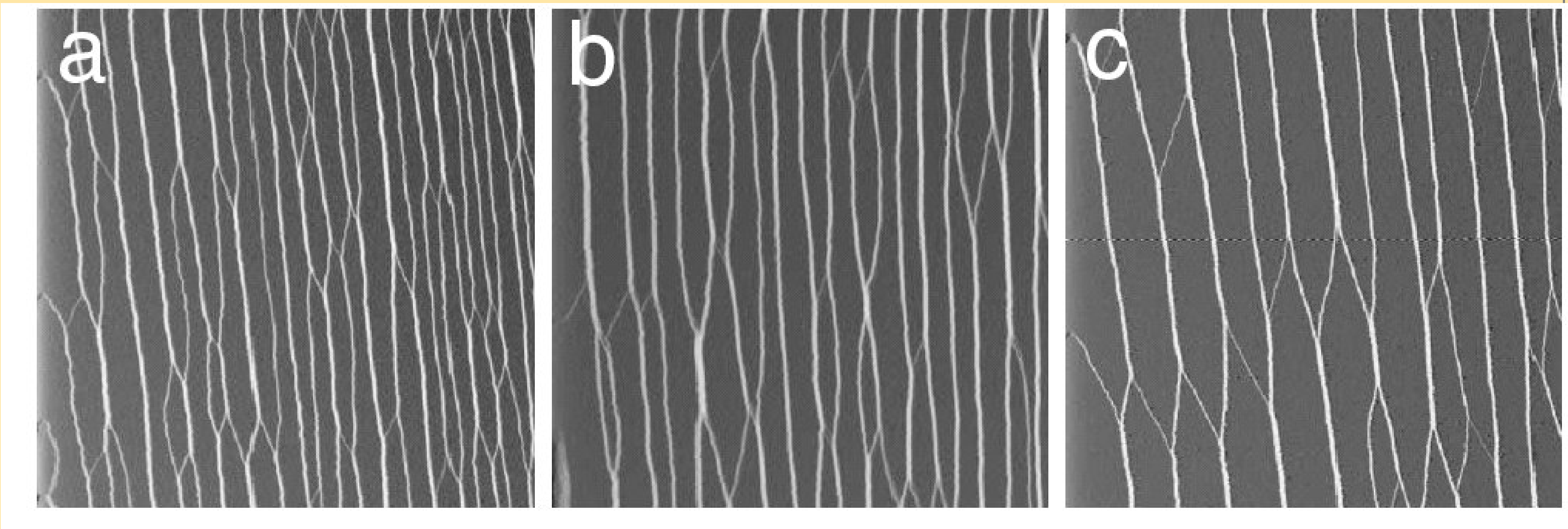
- Experiments
- One-dimensional model
- Evolution of bunches
- Stationary bunch profile
- Analytical

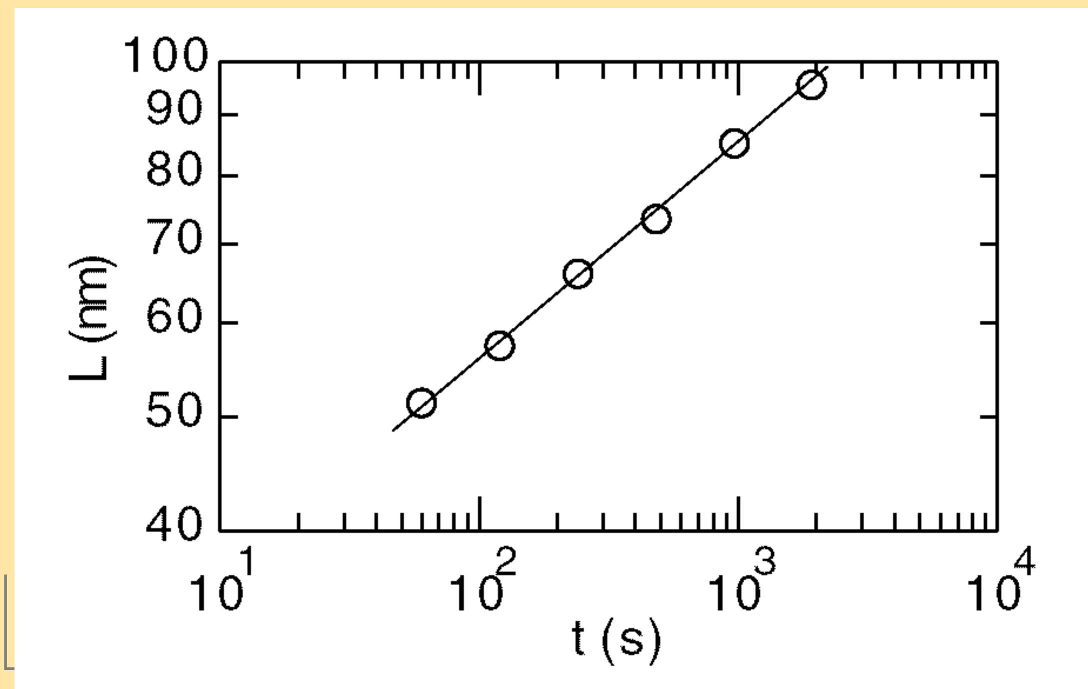


[A. V. Latyshev et al. Appl. Surf. Sci 130-132, 139 (1998).]

REM images of Si(001) surface, DC for 1 min (b) 3 min (c) 14 min (d).

[K. Sudoh and H. Iwasaki, Phys. Rev. Lett. 87, 216103 (2001).]

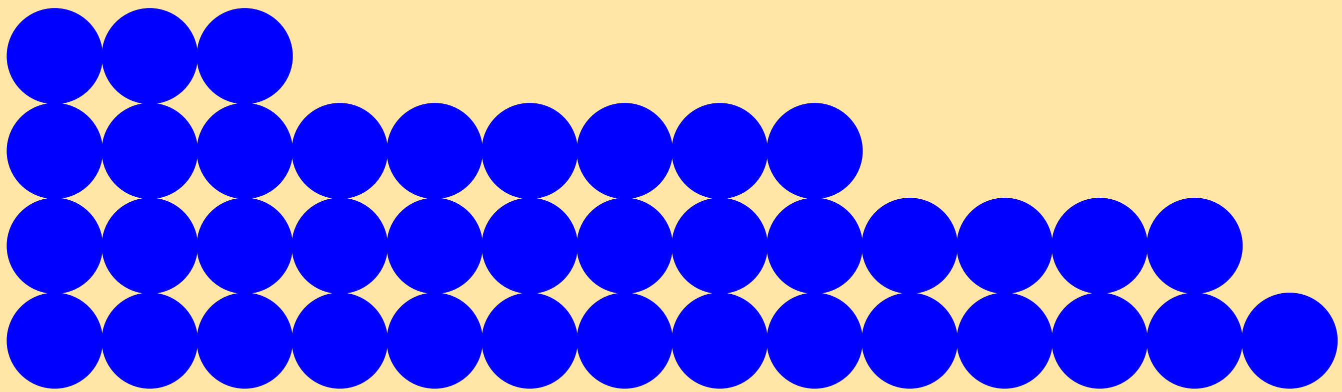




1300 nm 3 1300 nm STM images of Si(113) surfaces taken at room temperature after annealing at 600 degrees for (a) 1 min, (b) 8 min, and (c) 32 min. width.

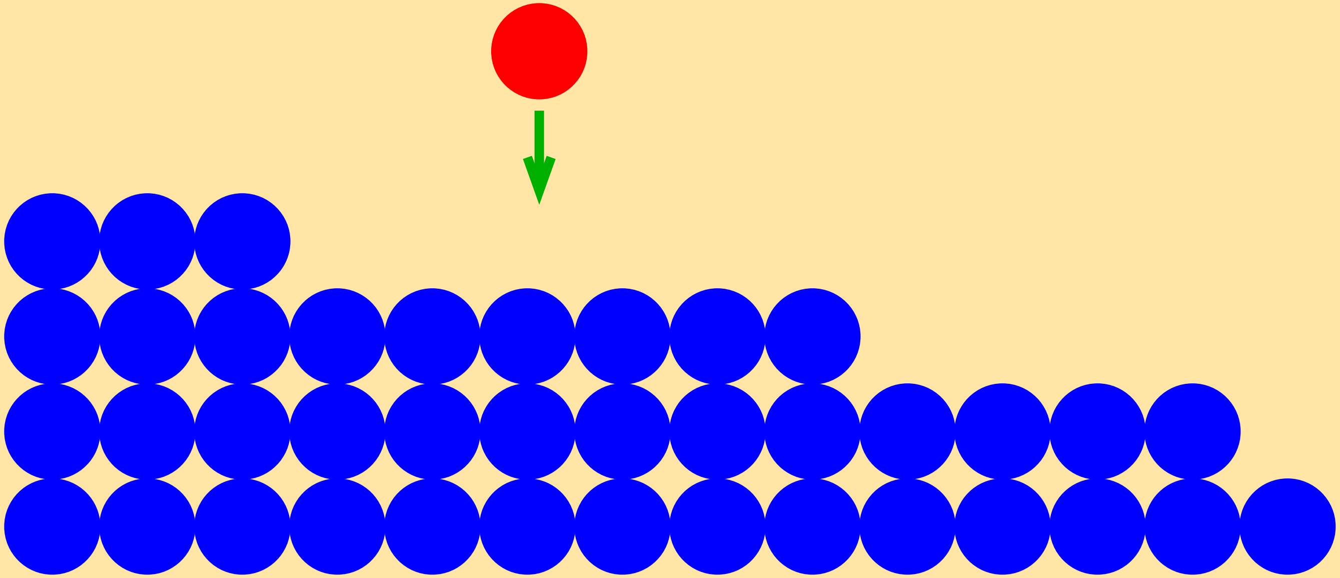
Time dependence of the average terrace.

One-dimensional model



There are S steps located at positions $x_1, x_2, ..., x_S$ on a chain of length L. Periodic boundary conditions.

One-dimensional model



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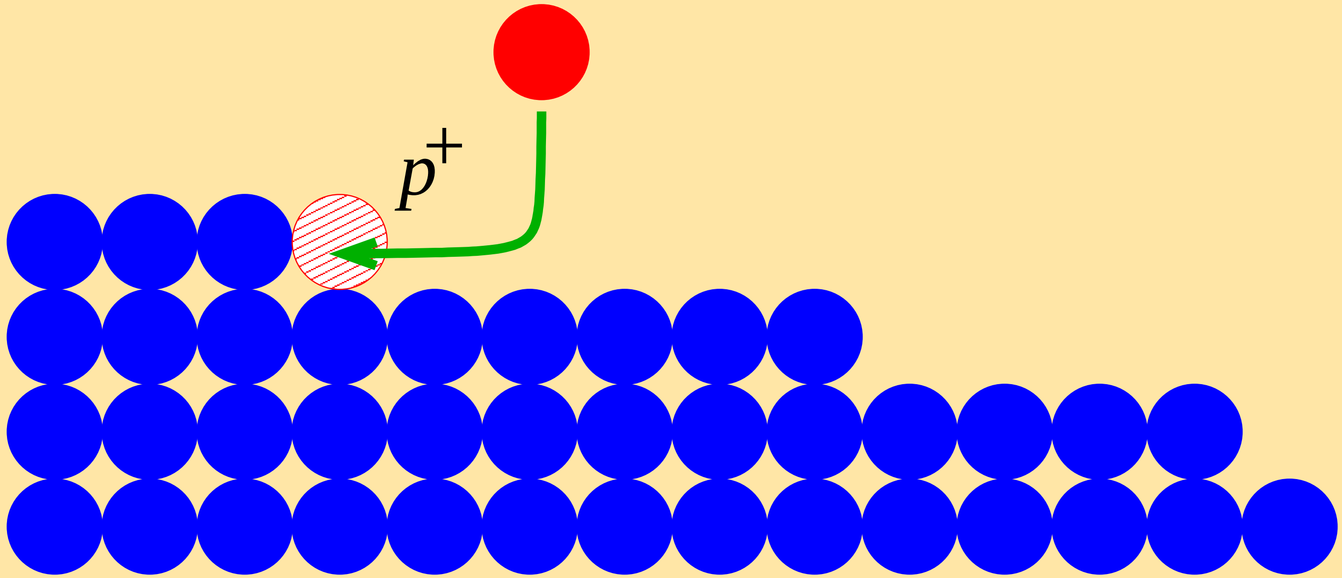


There are S steps located at positions $x_1, x_2, ..., x_S$ on a chain of length L. Periodic boundary conditions.

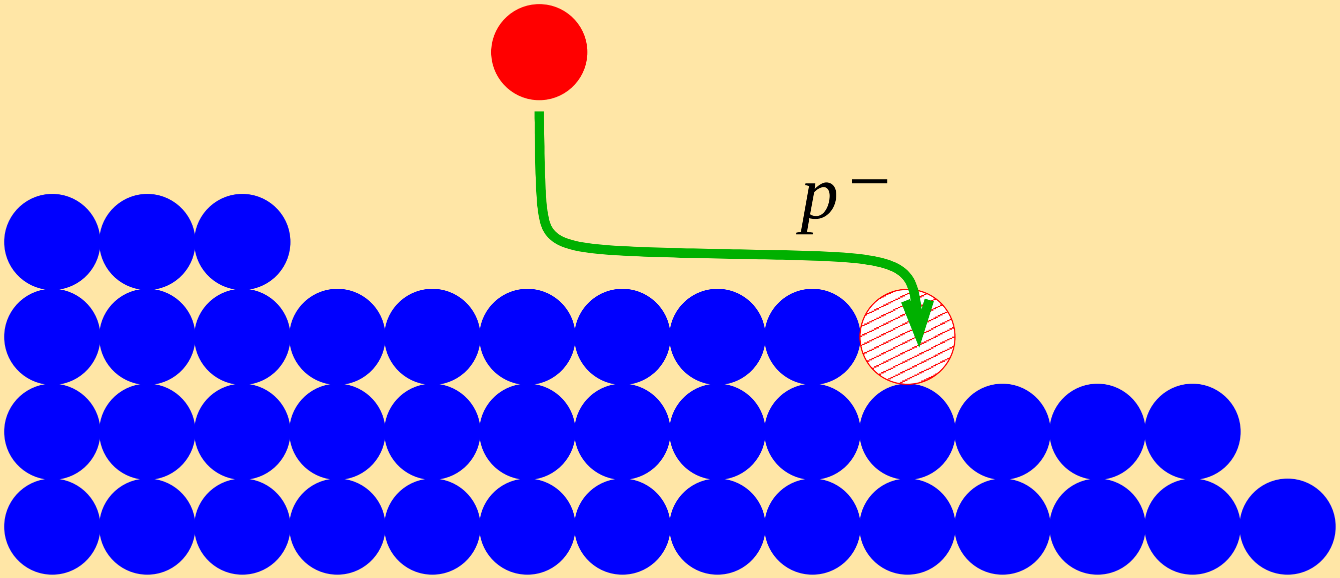
For a terrace of length $l = l_i \equiv x_i - x_{i-1}$ the probabilities of attachment on left is p^+ on right p^-

$$p^{\pm} = \frac{1}{2}\,\frac{1 \pm b + d\,l}{1 + d\,l}$$

$b \in (-1, 1)$......"ballance" ES barrier

$d \in [0, \infty)$...."inverse diffusion constant"

One-dimensional model



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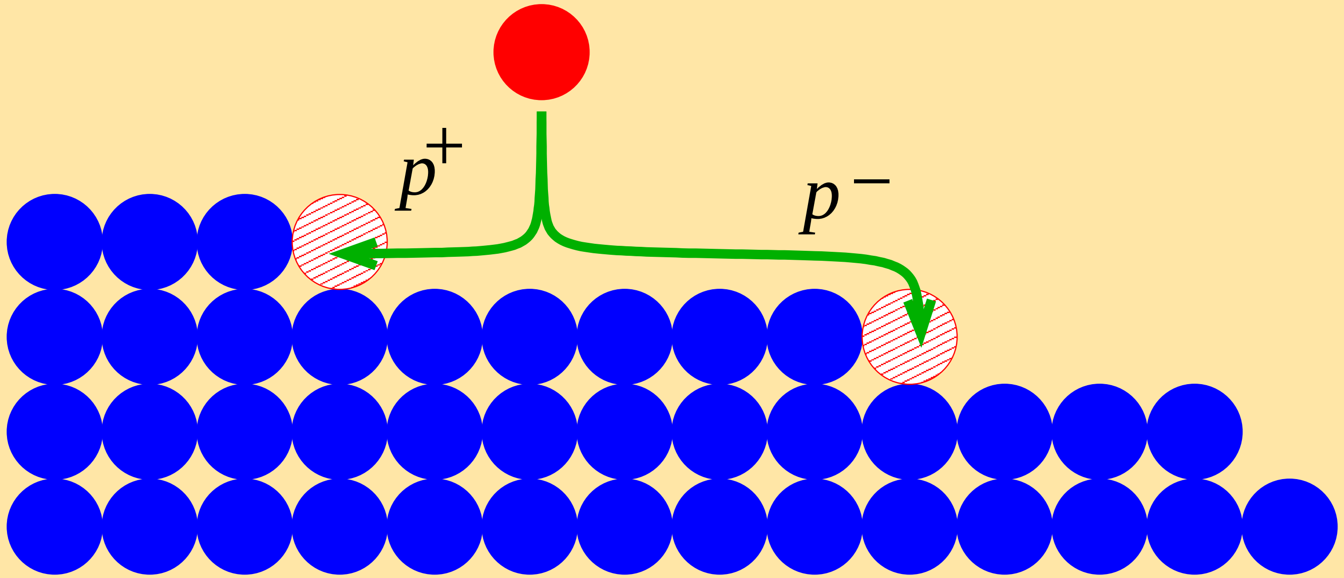

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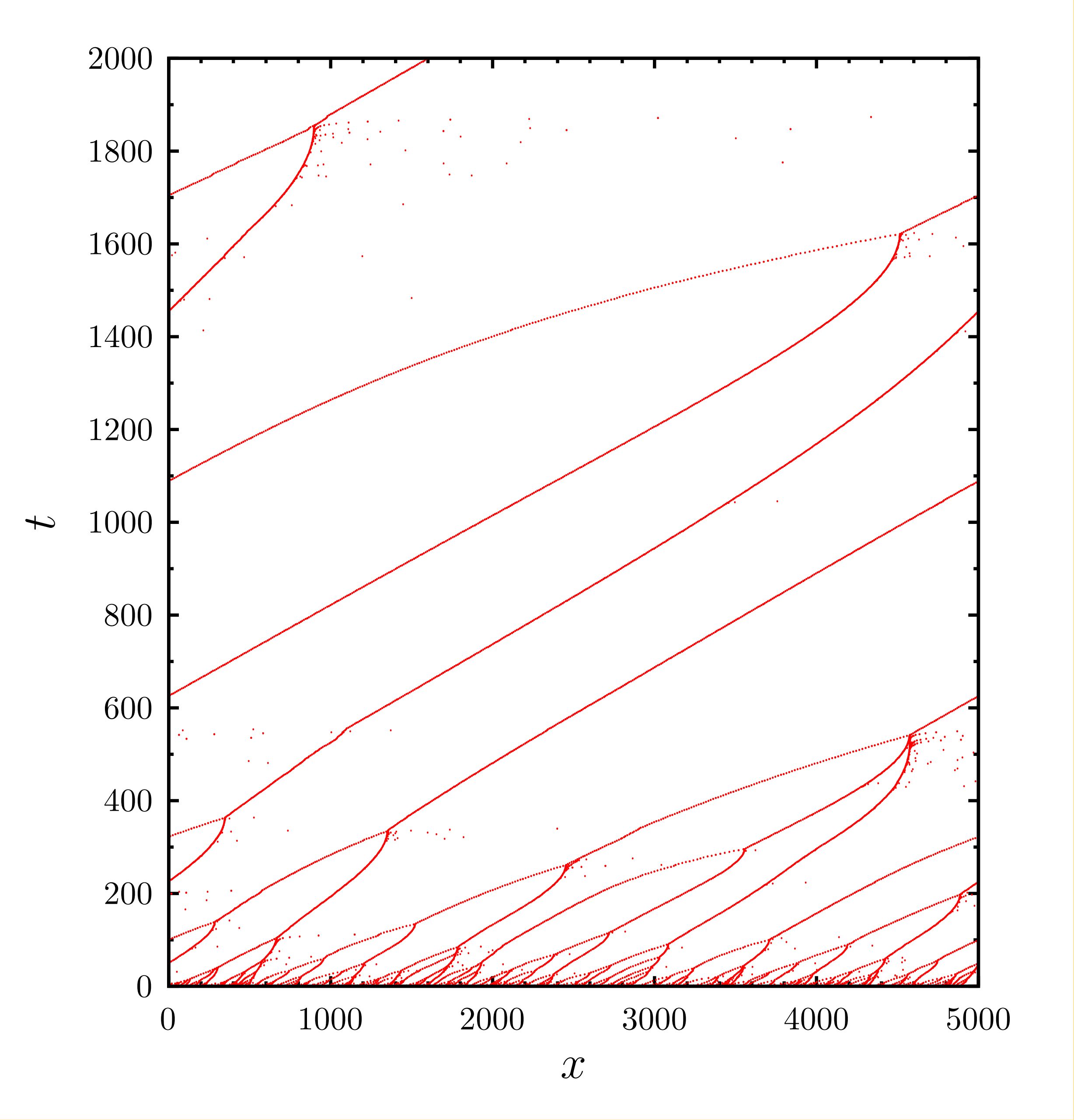
Launch simulation

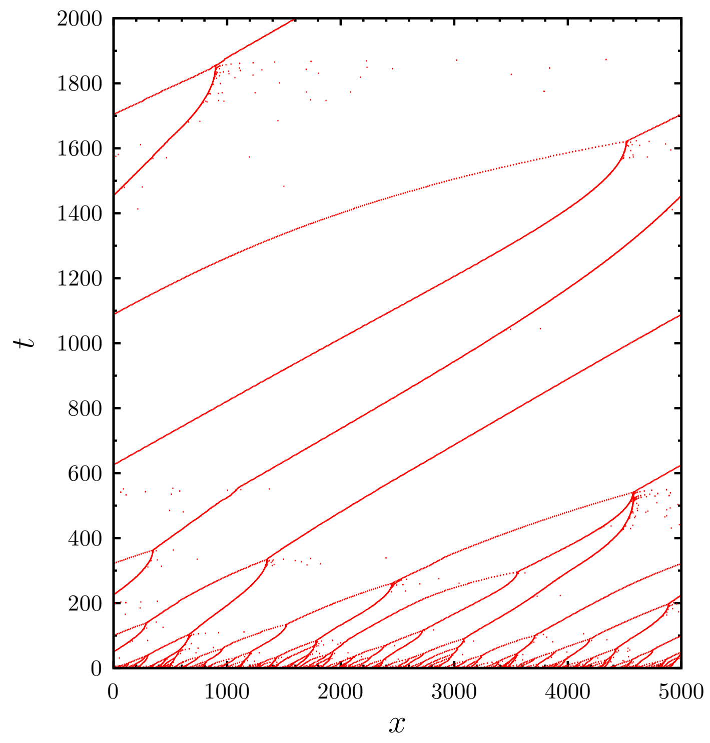
Space-time diagram



$L = 5 \cdot 10^3$, $\bar{l} = 5$,

$b = -1$, $d = 0$

Space-time diagram



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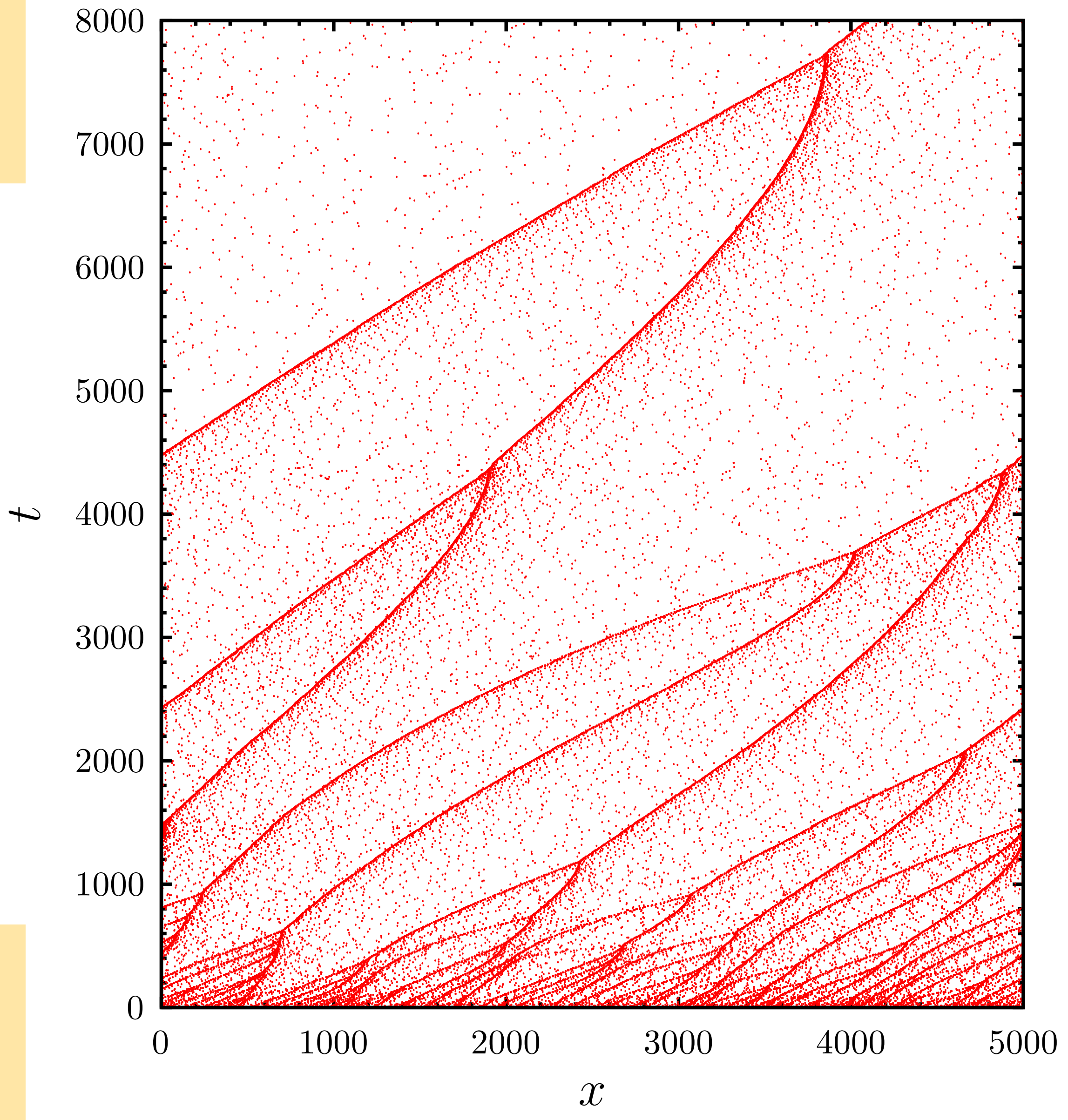
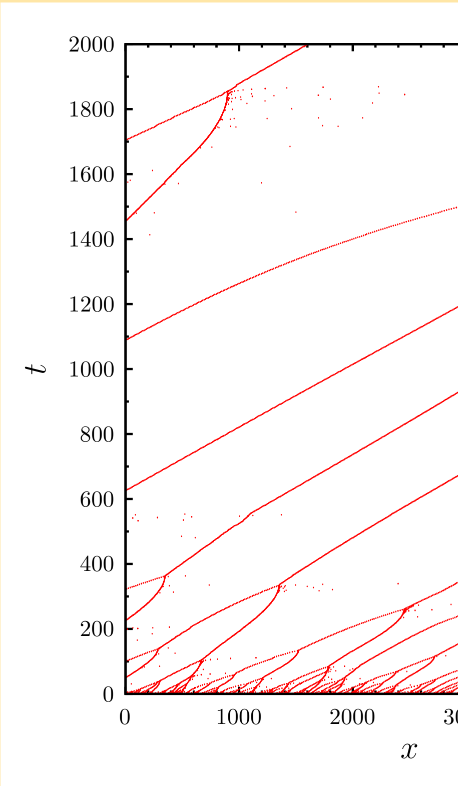
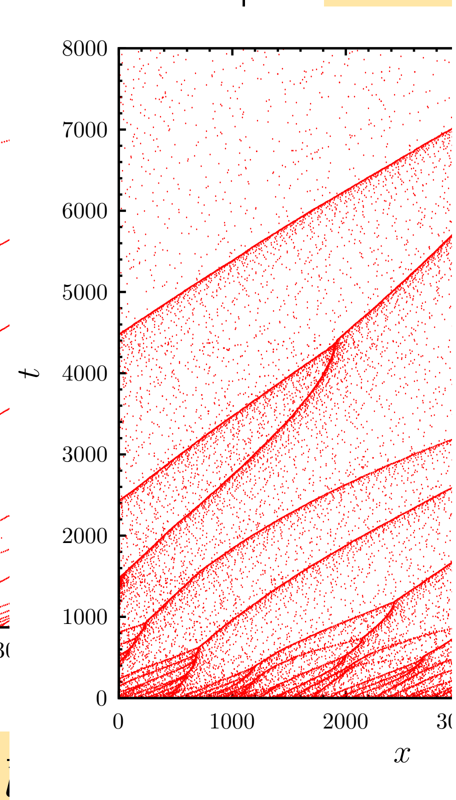


$L = 5 \cdot 10^3$, $\bar{l} = 5$,

$b = -0.6$, $d = 0$

Space-time diagram





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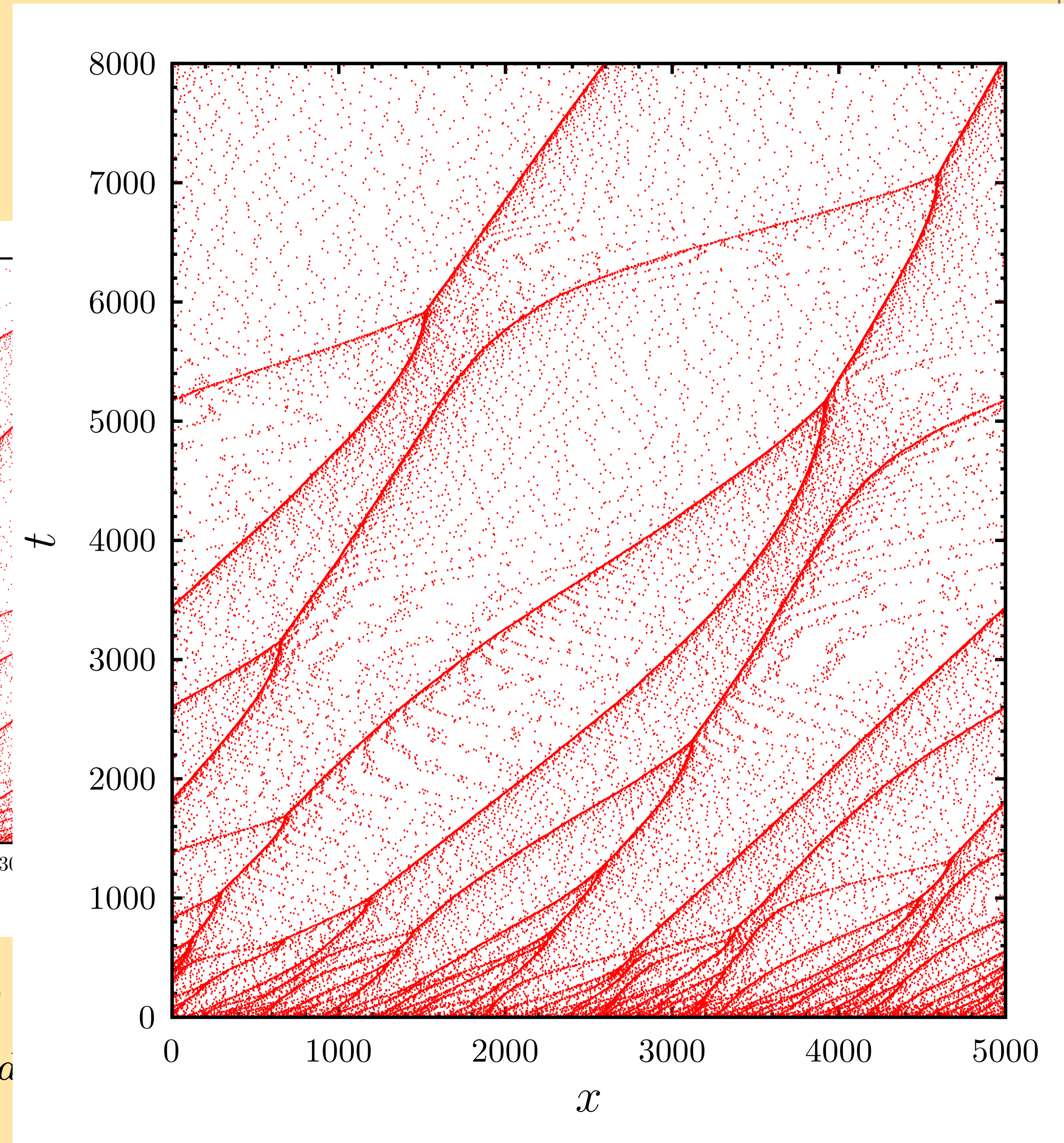
$b = -0.6$, d ...



$L = 5 \cdot 10^3$, $\bar{l} = 5$,

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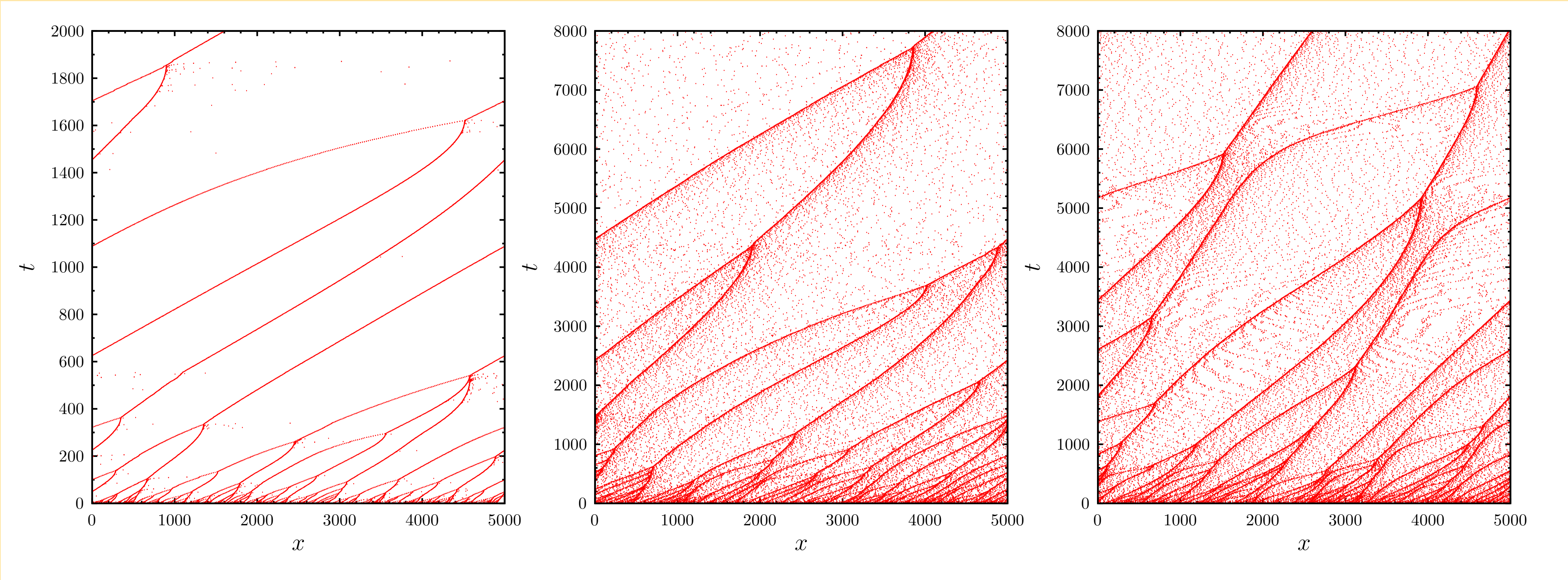

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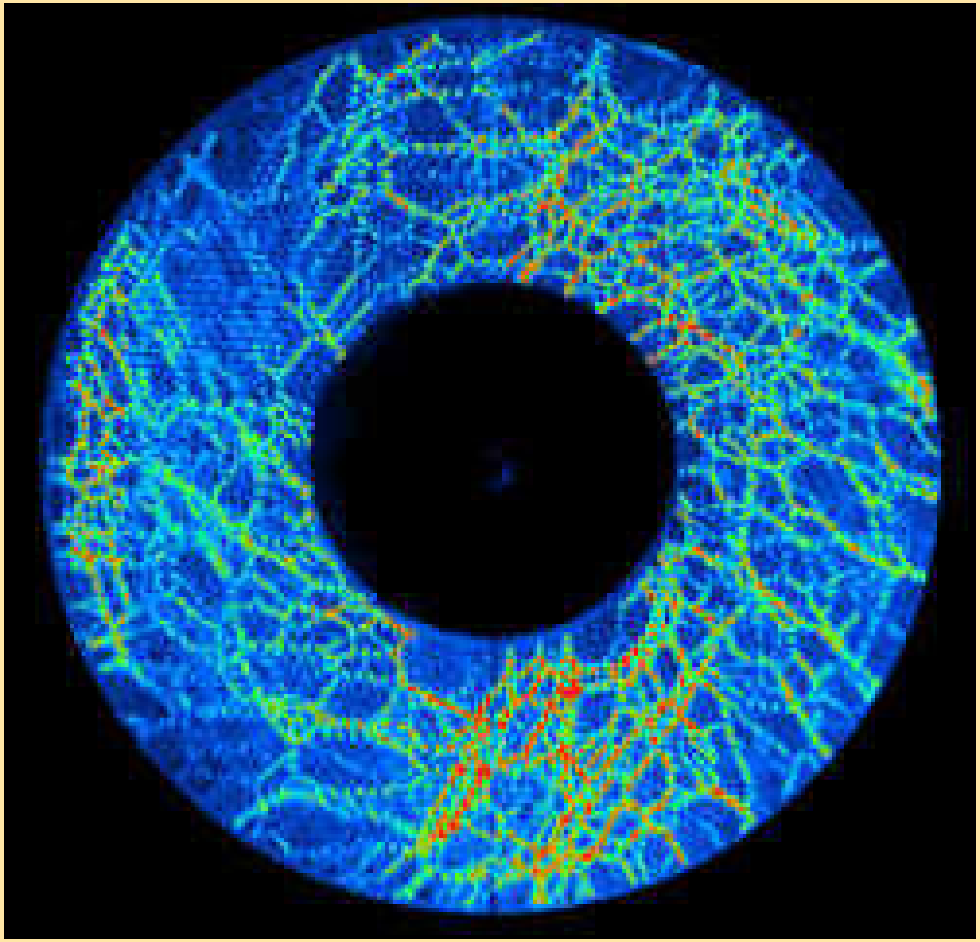
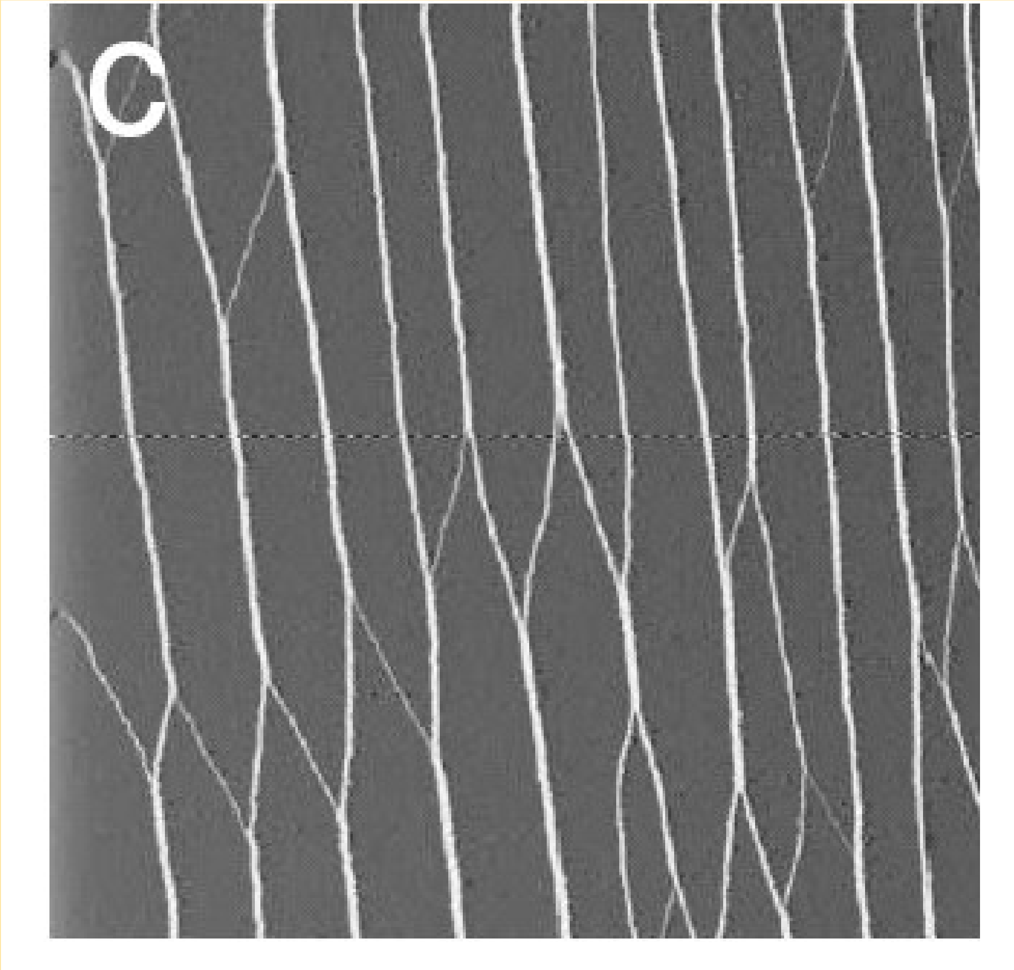
Measured quantities

General problem: very strong, complex topological correlations



Force chains in sheared sand.



Bunches.

Measured quantities

- Density-density correlation function.

$C_n(x,t) = \sum_{x'} \langle n(x',t) n(x'+x,t) \rangle$, with $n(x,t) = \sum_{i=1}^{S} \delta(x_i(t) - x)$.

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- Distribution of distances between bunches

$P_{\mathrm{domain}}(x,t) = \langle \sum_i^{S_k} \delta(x_{k,i} - x_{k,i-1} - x) \rangle$

Bunch of size k: at least $k > 1$ steps at the same position. Bunch positions $x_{k,i}, i = 1, 2, ..., S_k$.

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- Fluctuations in terrace widths

illustration

$\Delta = \frac{1}{\bar{l}S} \sum_{i=1}^{S} (x_i - x_{i-1})^2$ with $\bar{l} = \frac{1}{S} \sum_{i=1}^{S} x_i - x_{i-1} = \frac{L}{S}$

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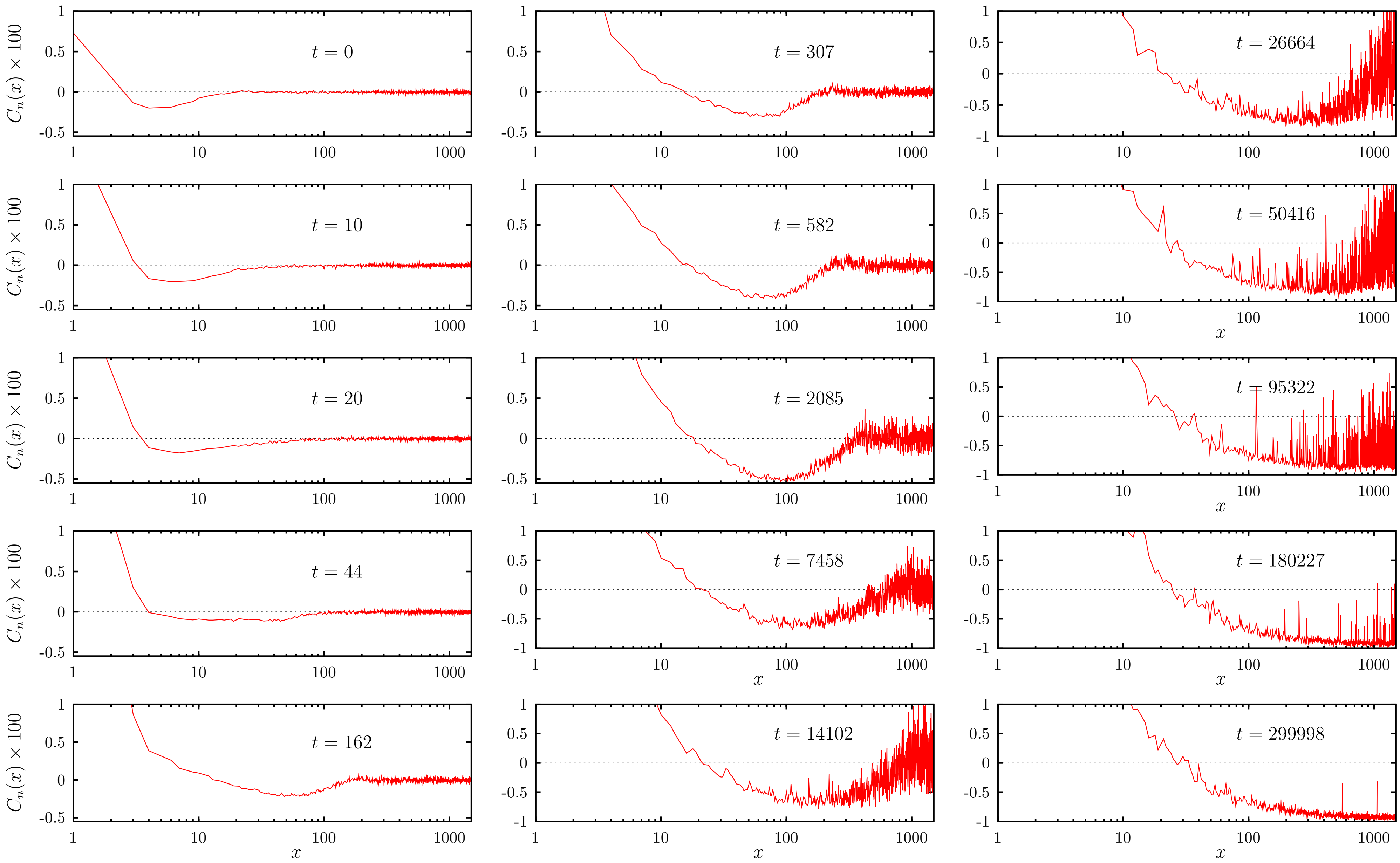
- Stationary profile

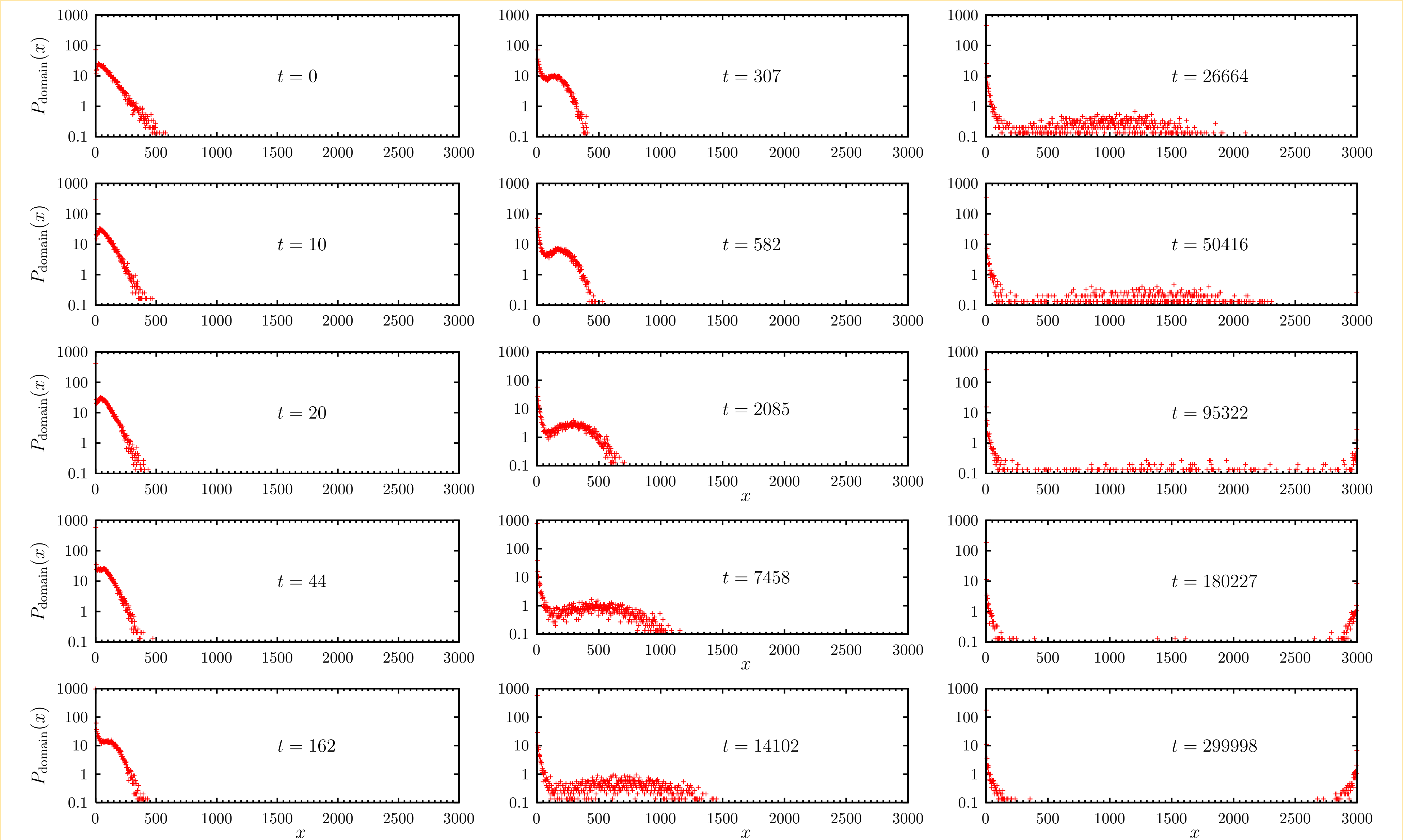
$h(x) = \sum_{i=1}^{S} \theta(x_{k,i} - x)$

Density-density correlation function



$L = 3000$, $b = -0.3$, $\bar{l} = 10$, $d = 0.01$, average over 500 runs.

Distribution of distances between bunches (“domain sizes”)



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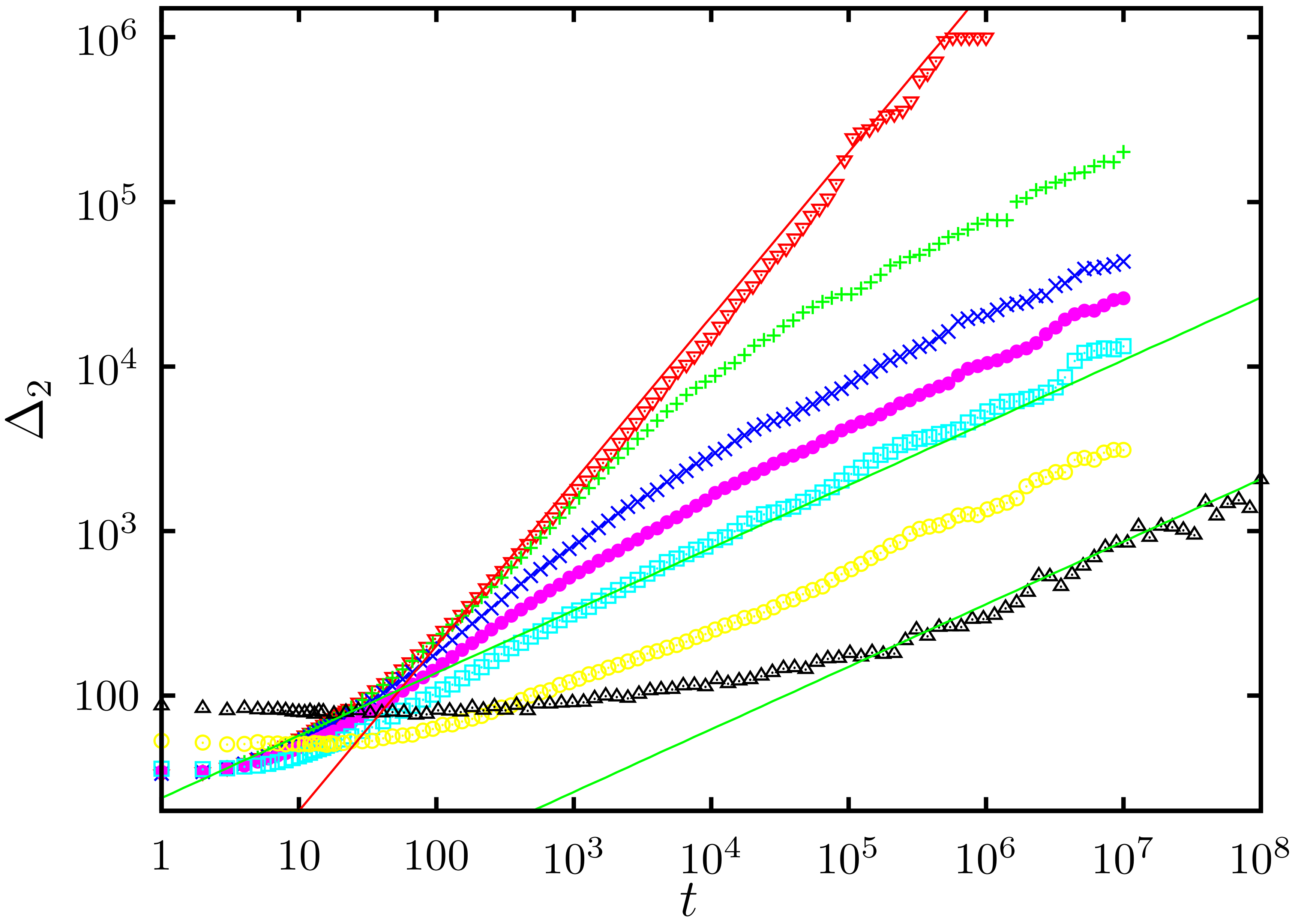
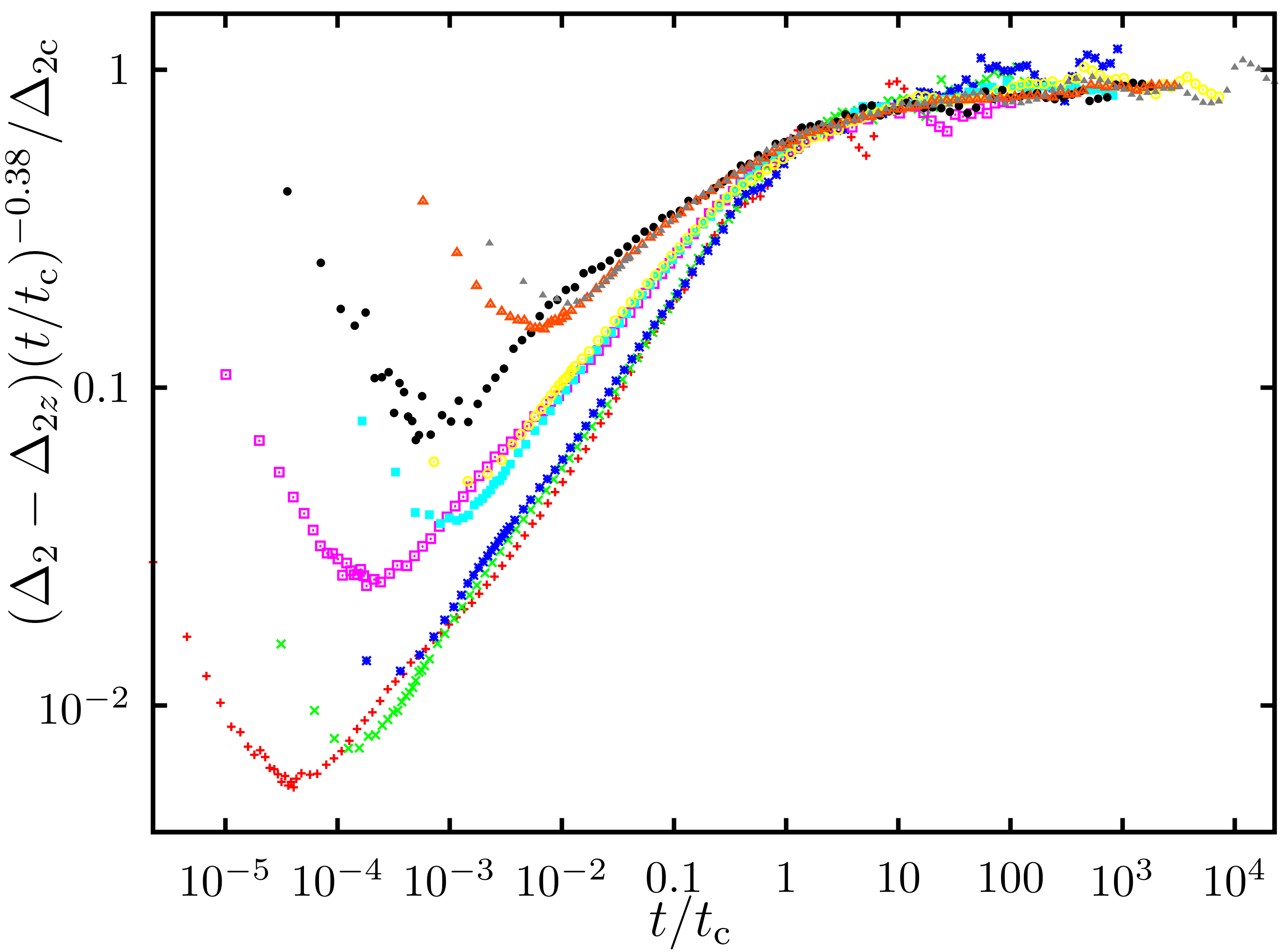


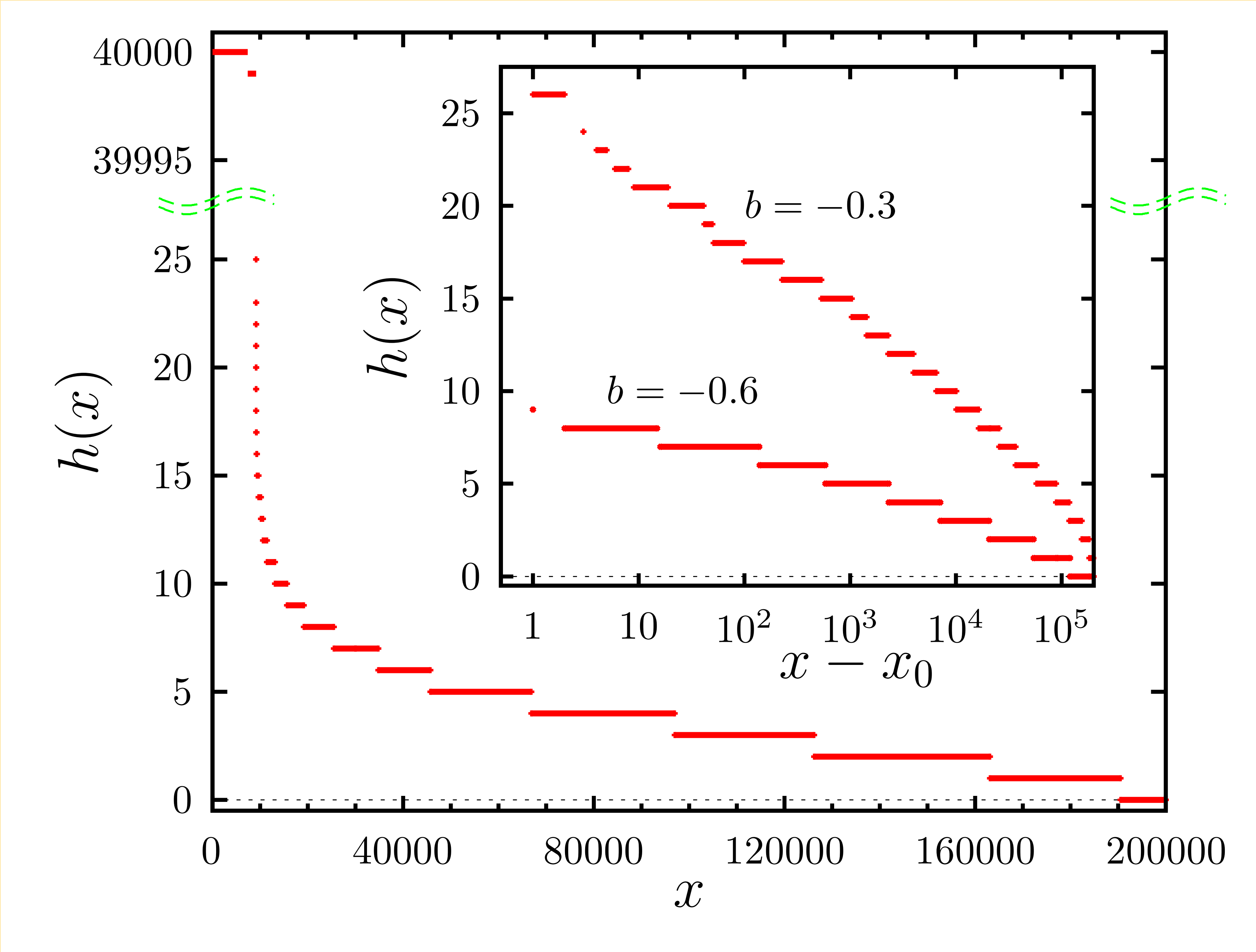
$L = 10^6, 2 \cdot 10^5, \ldots\ \bar{l} = 5,\ b = -0.9,\ d = 0,\ 0.0001,\ 0.001,\ 0.003,\ 0.01,\ 0.1,\ 1$



Rescaled: $b = -0.9, -0.6, -0.3$.

bunch distance $\sim \Delta_2 \sim t^{0.38}$

Bunch profile... ...Logarithmic singularity

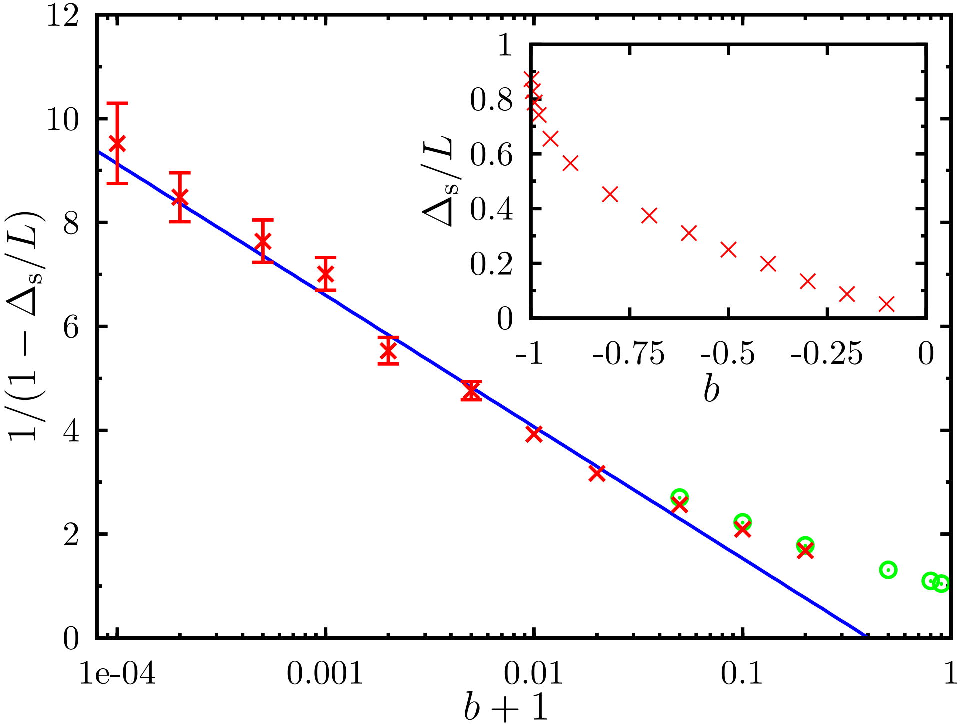


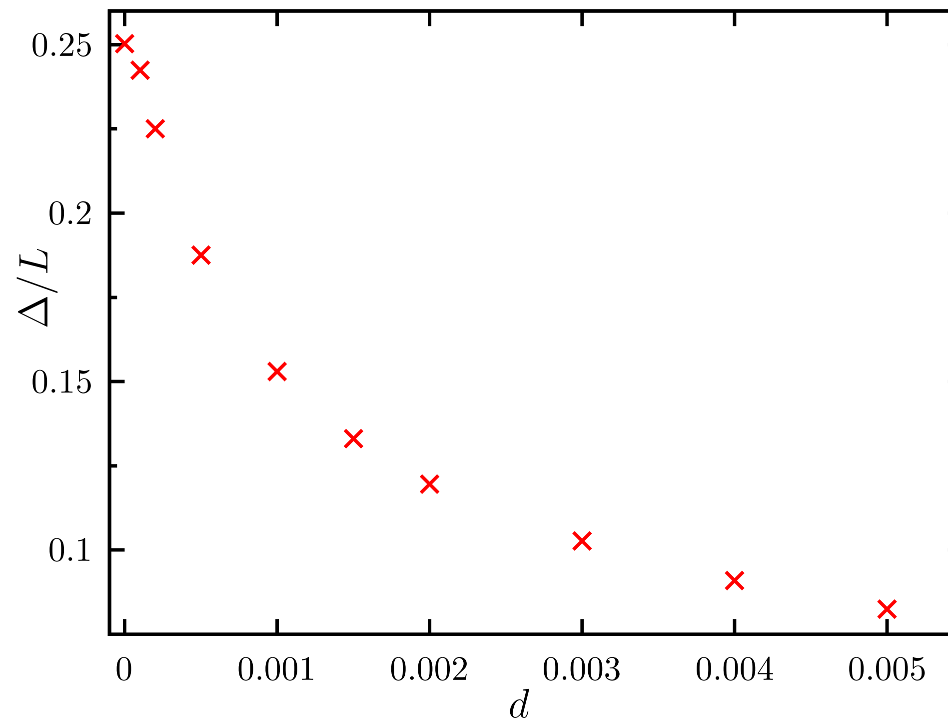
$\bar{l} = 5$

$d = 0$

to analytical

Stationary regime





$L = 10^5$ ($\times$), 10^4 ($\odot$), $\bar{l} = 5$, (inset: $L = 10^4$ $\bar{l} = 10$),

$d = 0$; line: $\sim -\ln(b+1)$

$L = 10^4$, $\bar{l} = 10$,

$b = -0.5$

$$\Delta_s/L = 1 - \frac{B}{A - \ln(b+1)}, \quad b \to -1$$

Or: $b + 1 \sim \exp(-\frac{B}{1-\Delta_s/L})$

Analytic treatment

Probability of advancing (length of the sample is L):

$$Prob\{x \to x+1\} = \frac{1}{2L}\left[l_+ + l_- + b\left(\frac{1}{1+d\,l_+} - \frac{1}{1+d\,l_-}\right)\right]$$

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Fokker-Planck equation for motion of the step

$$\frac{\partial P(x,\tau)}{\partial \tau} = -\frac{\partial}{\partial x}\left\{\left[1 - b\,\frac{P'(x,\tau)}{2(P(x,\tau)+d)^2}\right] \Big/ \left[1 - \left(\frac{P'(x,\tau)}{2P^2(x,\tau)}\right)^2\right]\right\}$$

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Look for solution in the form $P(x,\tau) = \Phi(x - v\tau)$. This leads to equation

$$v\Phi(x) + c = \frac{1 - b\,\frac{\Phi'(x)}{2(\Phi(x)+d)^2}}{1 - \left(\frac{\Phi'(x)}{2\Phi^2(x)}\right)^2}$$

Hint for solution

For d=0

$$x = 2\int_0^{\frac{1}{2\Phi(x)}} \frac{cy+v}{\sqrt{(b^2+4c(c-1))y^2+4v(2c-1)y+4v^2}-by}\mathrm{d}y$$

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$$\Phi(x) \simeq \underbrace{\frac{\sqrt{b^2+4c(c-1)}+b}{4(c-1)}}_{K}\,\frac{1}{x}$$

and profile

$$\boxed{h(x) \simeq K(\ln L - \ln x)}$$

to simulations

Conclusions

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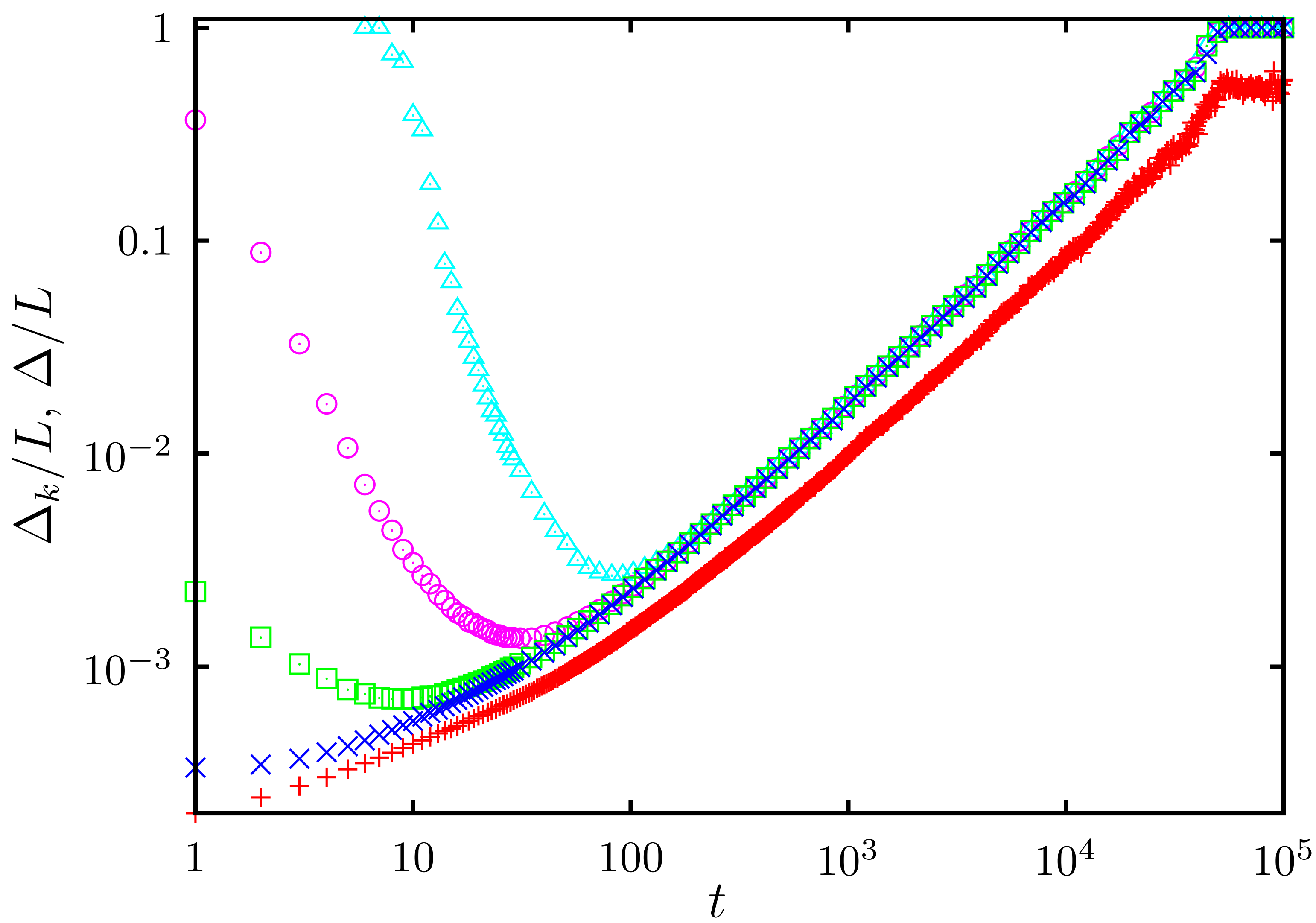
- Distances between bunches grow as $\sim t^{\alpha}$, $\alpha \simeq 0.38$ for $d > 0$, $\alpha = 1$ for $d = 0$
- Stationary bunch profile: logarithmic singularity

Outlook

- Crossover between $d = 0$ and $d > 0$
- Numerical solution of equation for stationary profile

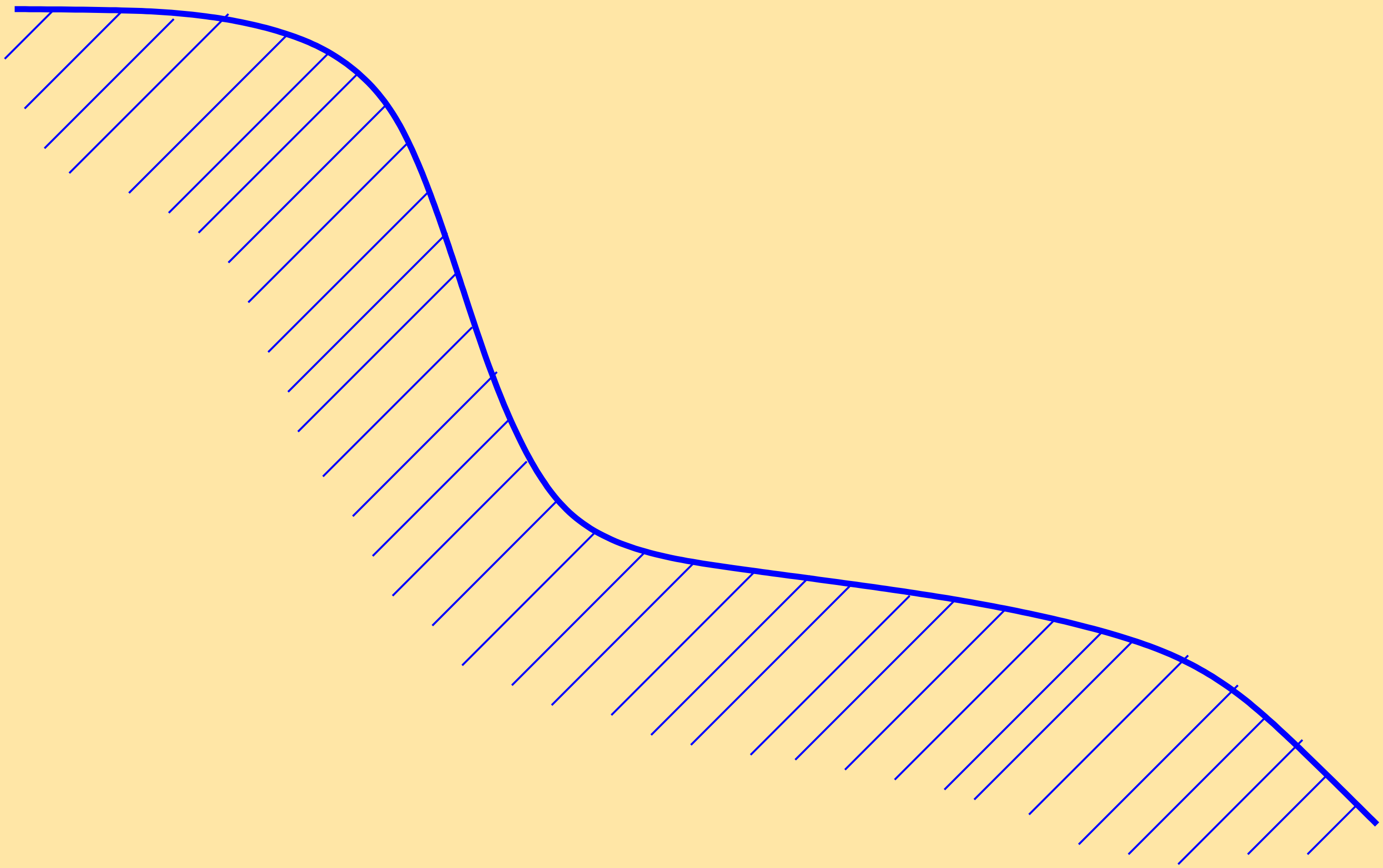


back

Comparison of various definitions of bunch.

Δ ($+$), Δ_k for $k = 2$ ($\times$), 4 ($\boxdot$), 6 ($\odot$), 8 ($\triangle$). $L = 10^5$, $\bar{l} = 5$, $b = -0.9$, $d = 0$.

Illustration of quantities Δ and Δ_k



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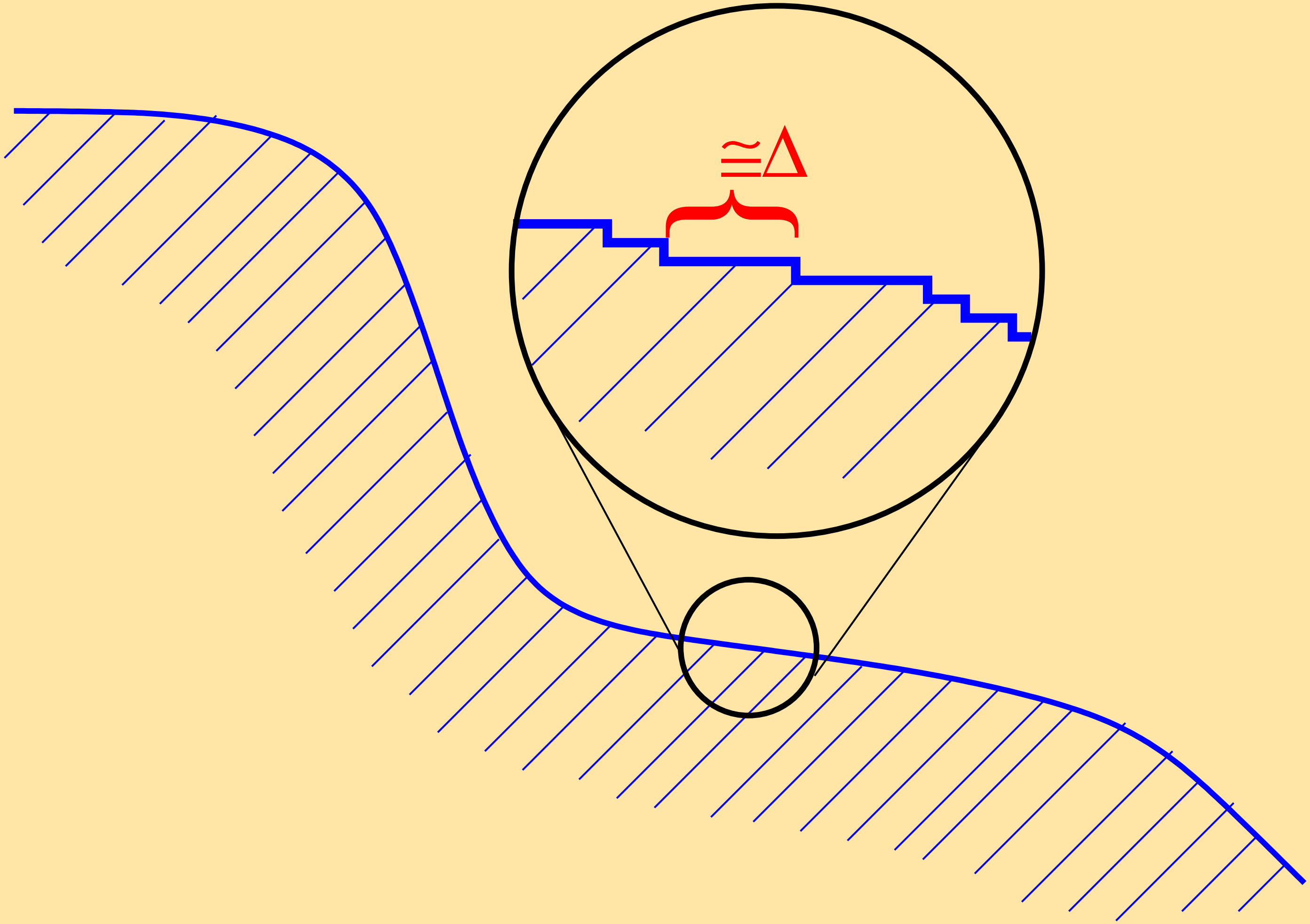

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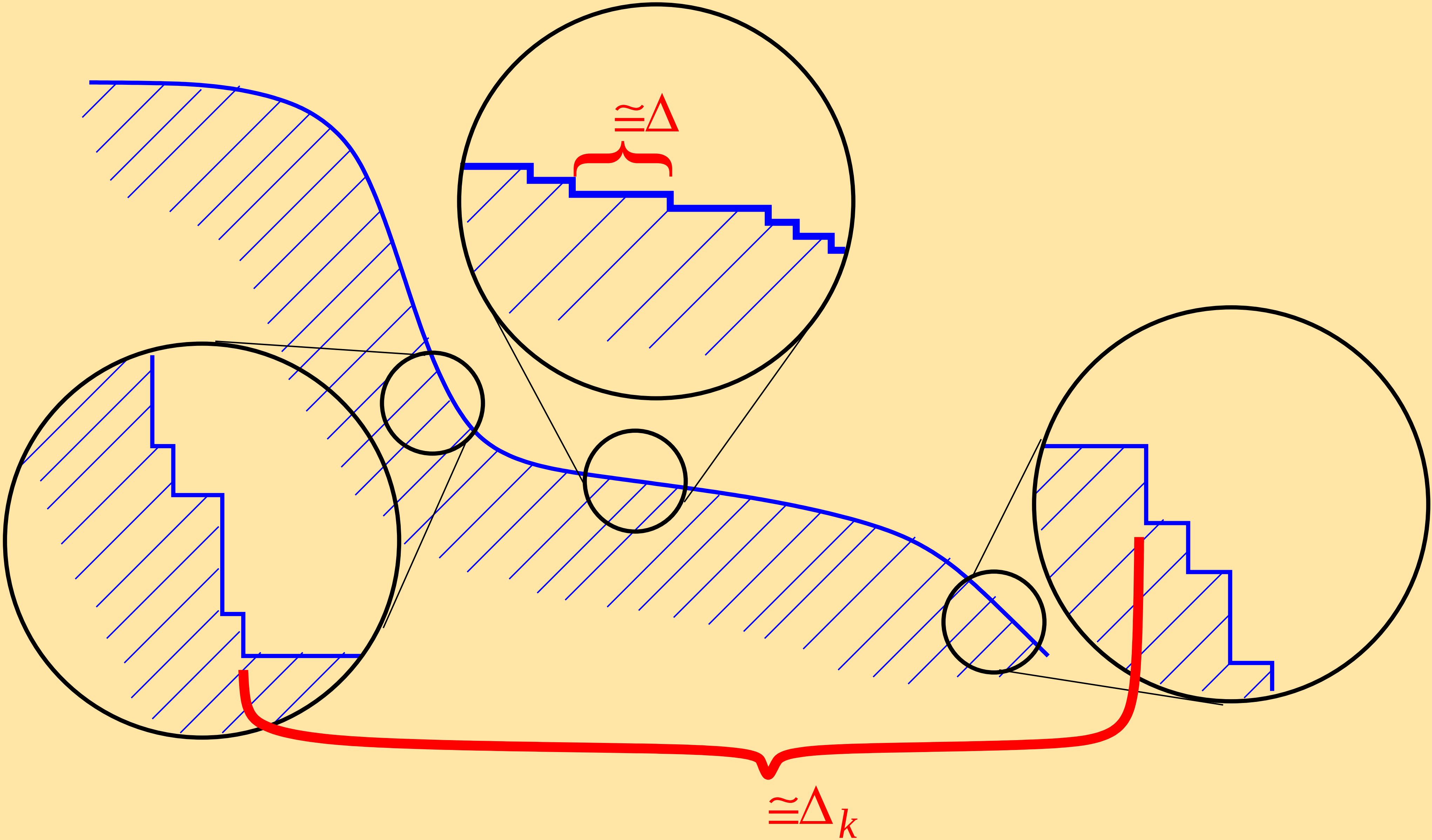

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