Econophysics: statistical physics of interacting agents

František Slanina

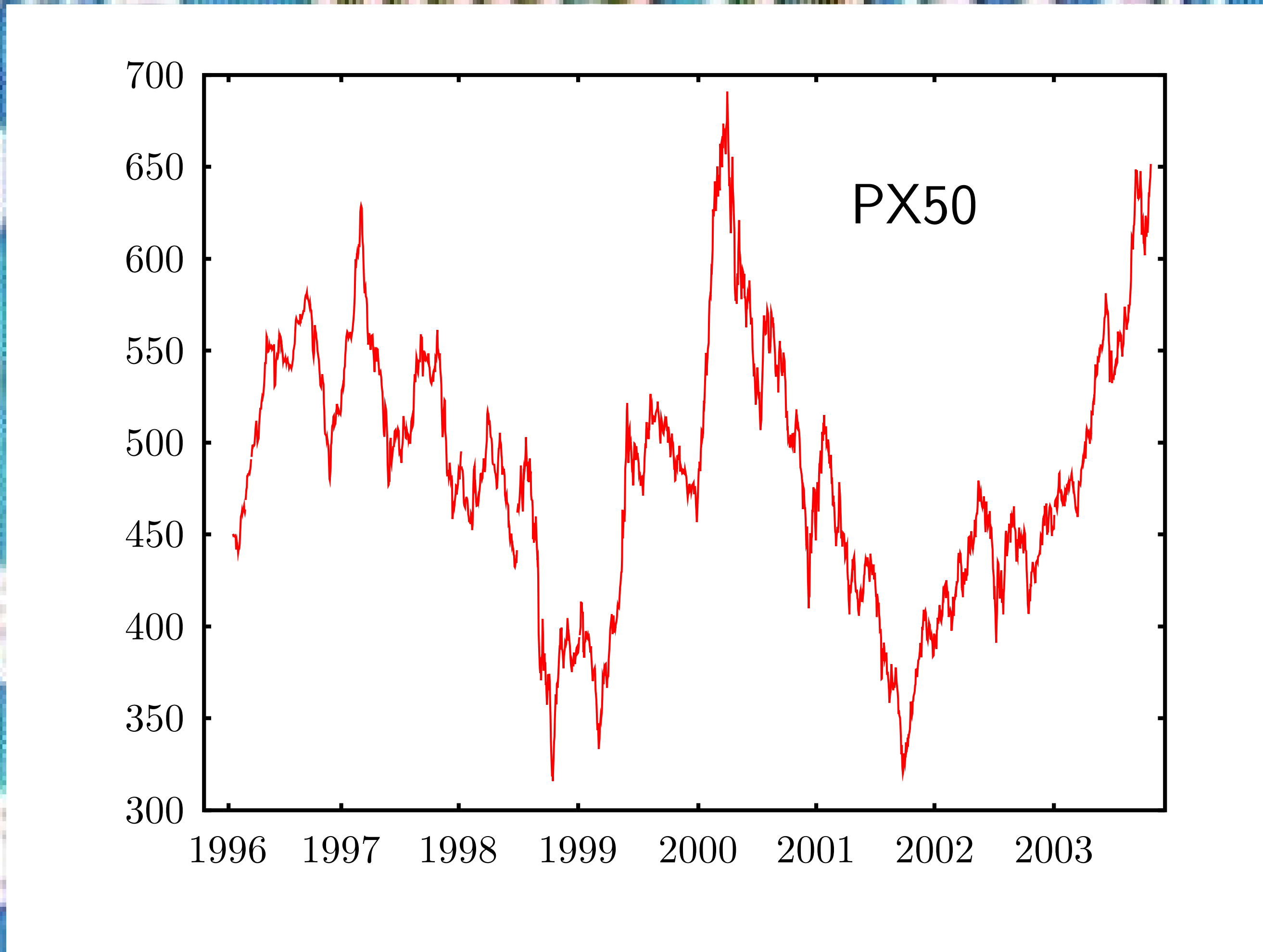
Institute of Physics, Academy of Sciences of the Czech Republic, Prague

`slanina@fzu.cz`

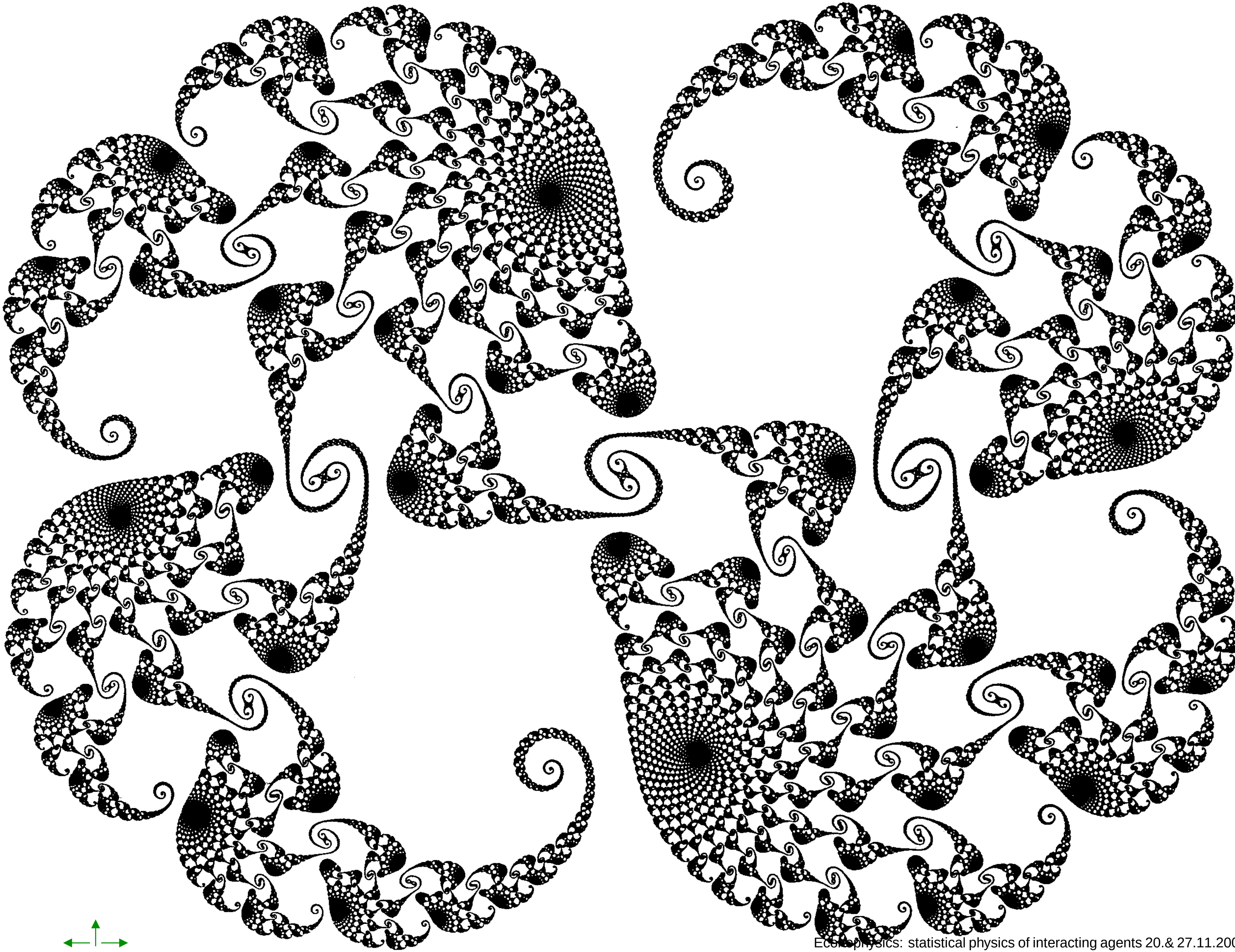
- Scaling, criticality and power laws
- Price fluctuations and book of orders
- Wealth distribution
- Imitation in Minority Game
- Sznajd model
- Thanks to GAČR 202/01/1091

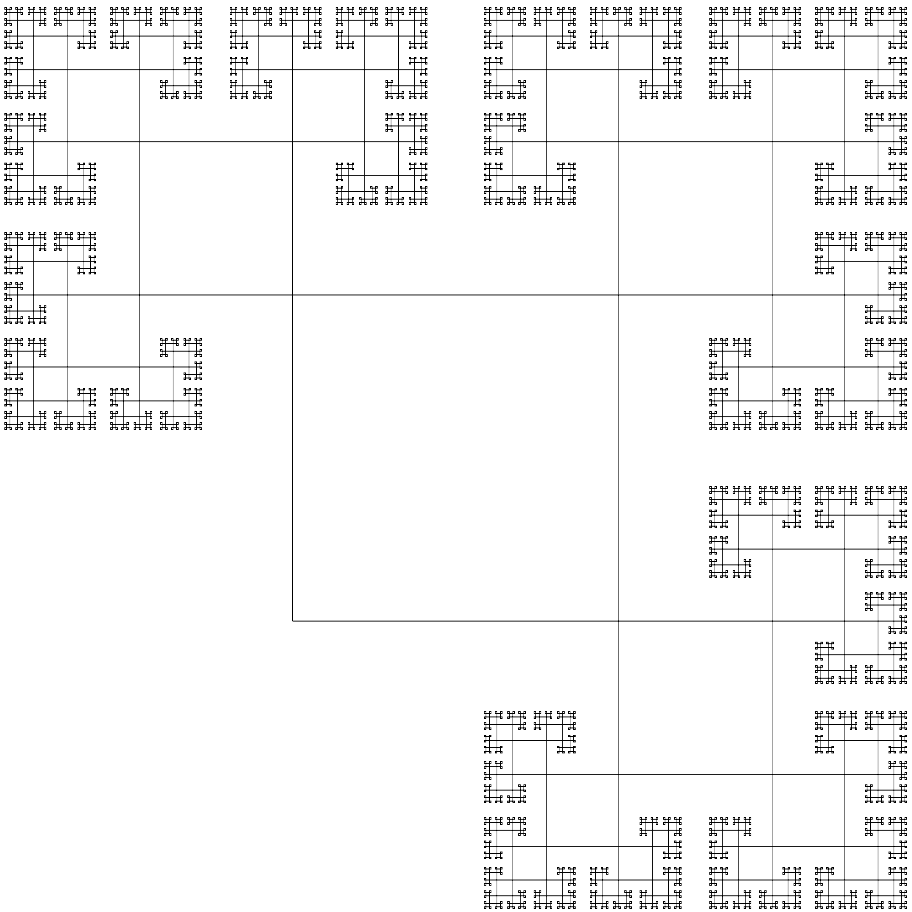
Stock market crashes

Stock market crashes

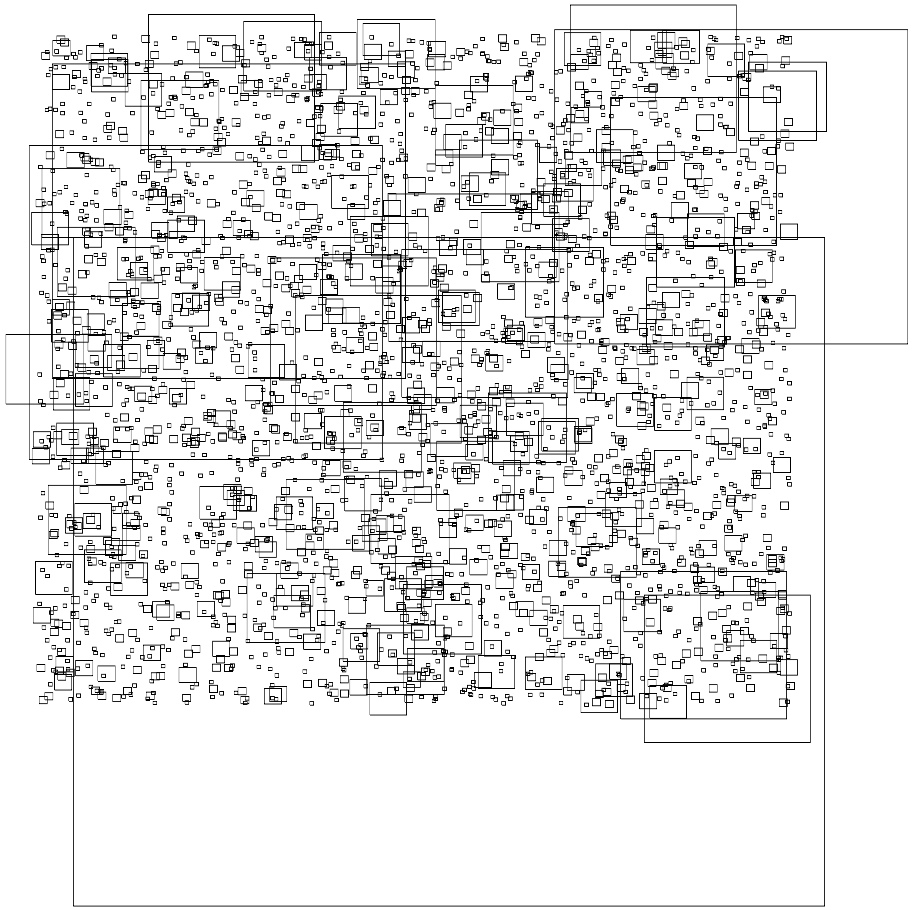


Prague Stock market index PX50.



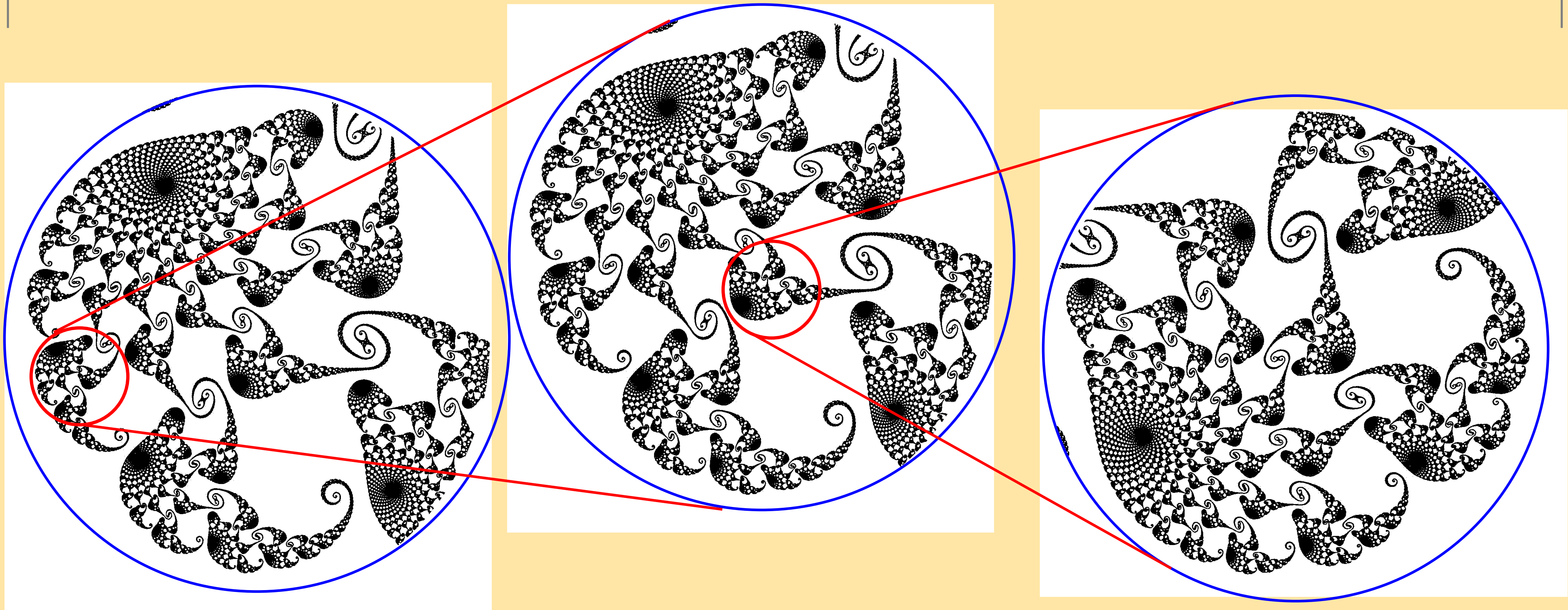


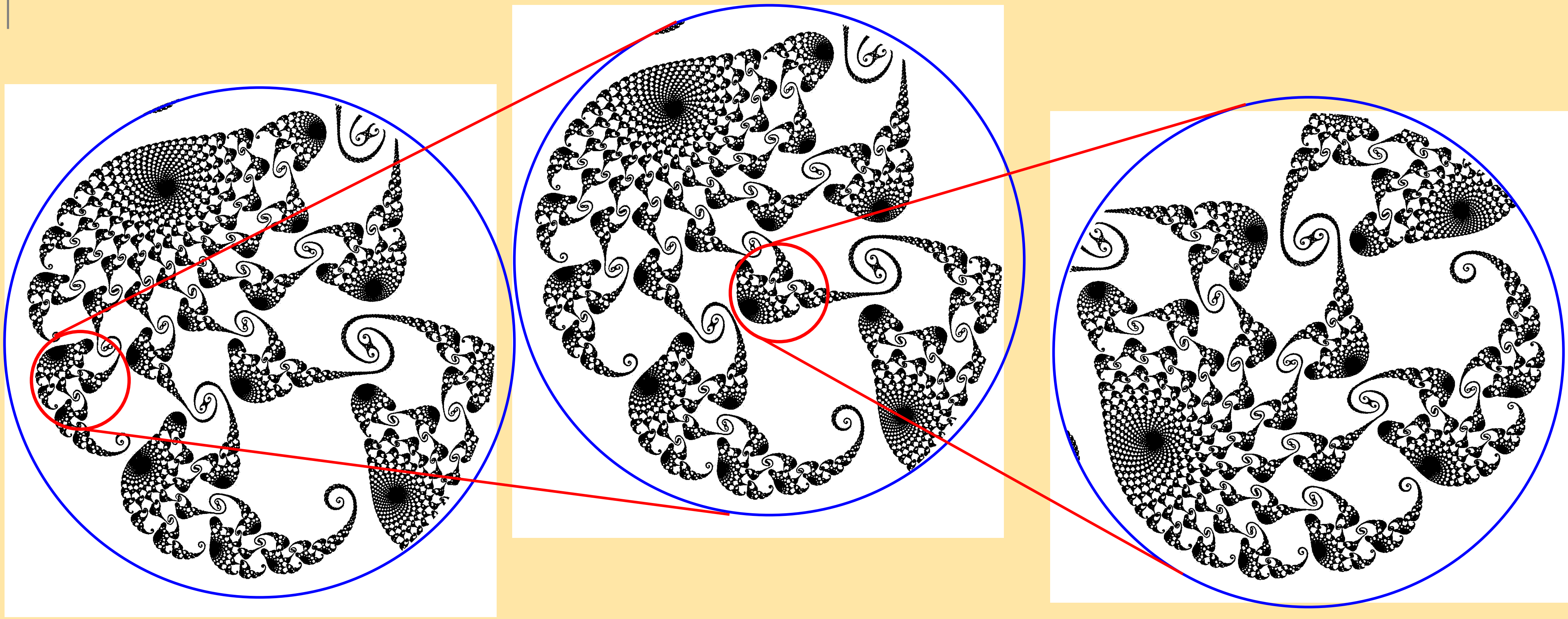
Regular fractal



Random fractal

$$P(s) \sim s^{-\tau}, \tau = -\frac{\ln 3}{\ln 0.47} = 1.455072...$$





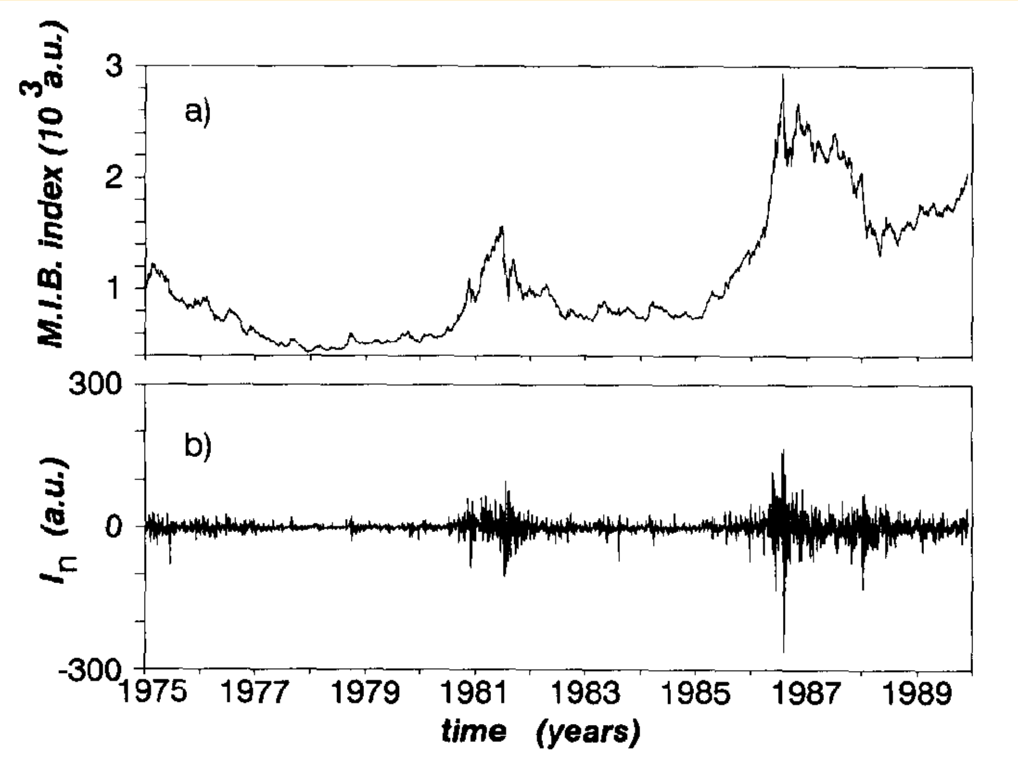
Change of scale:

$$s \longrightarrow bs \Rightarrow P(s) \propto s^{-\tau} \longrightarrow b^{-\tau} s^{-\tau} \propto s^{-\tau}$$

...power-law unchanged!

Econophysical phenomenology

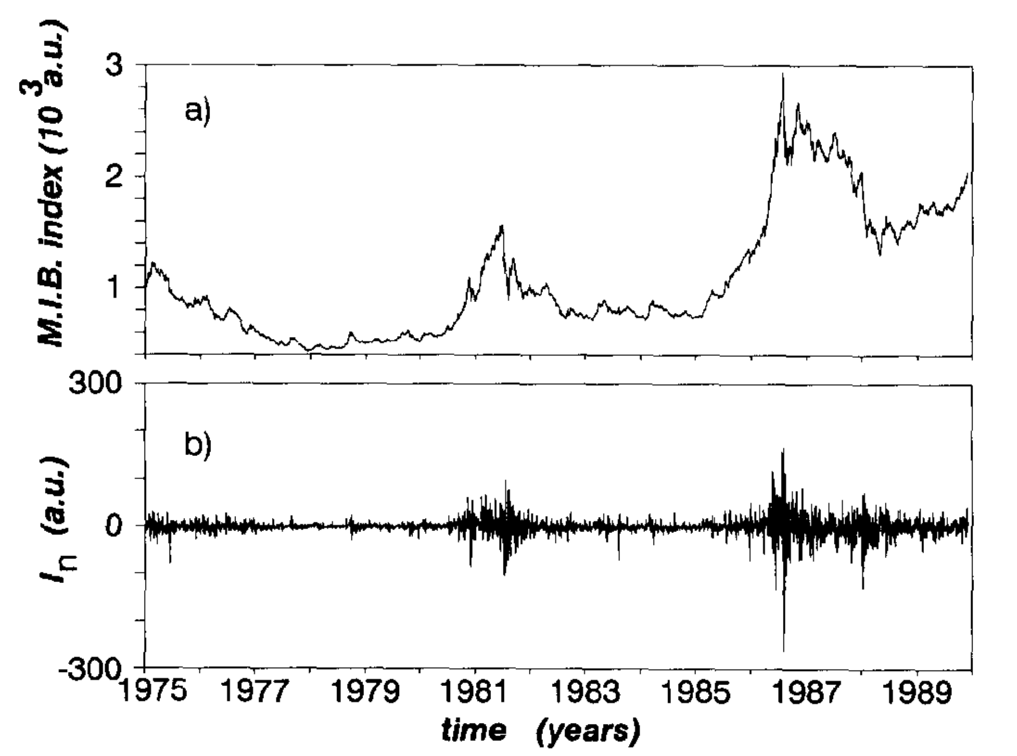
[R. N. Mantegna, Physica A 179, 232 (1991)]



Time evolution of Milano Stock market index and returns.

Econophysical phenomenology

[R. N. Mantegna, Physica A 179, 232 (1991)]



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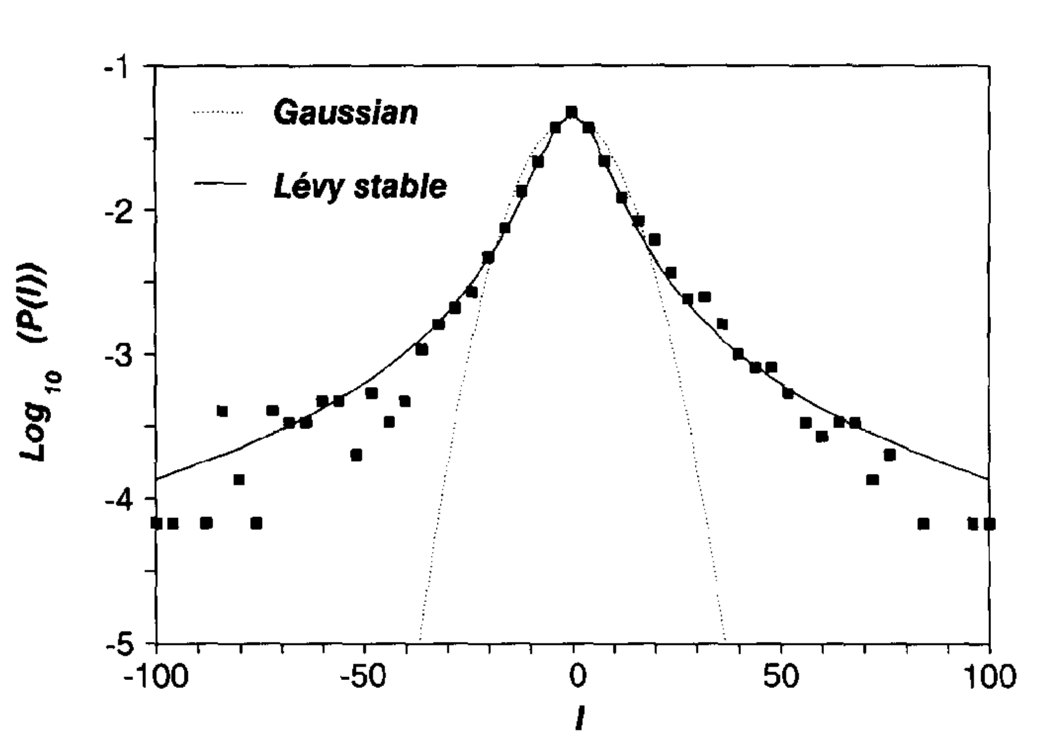
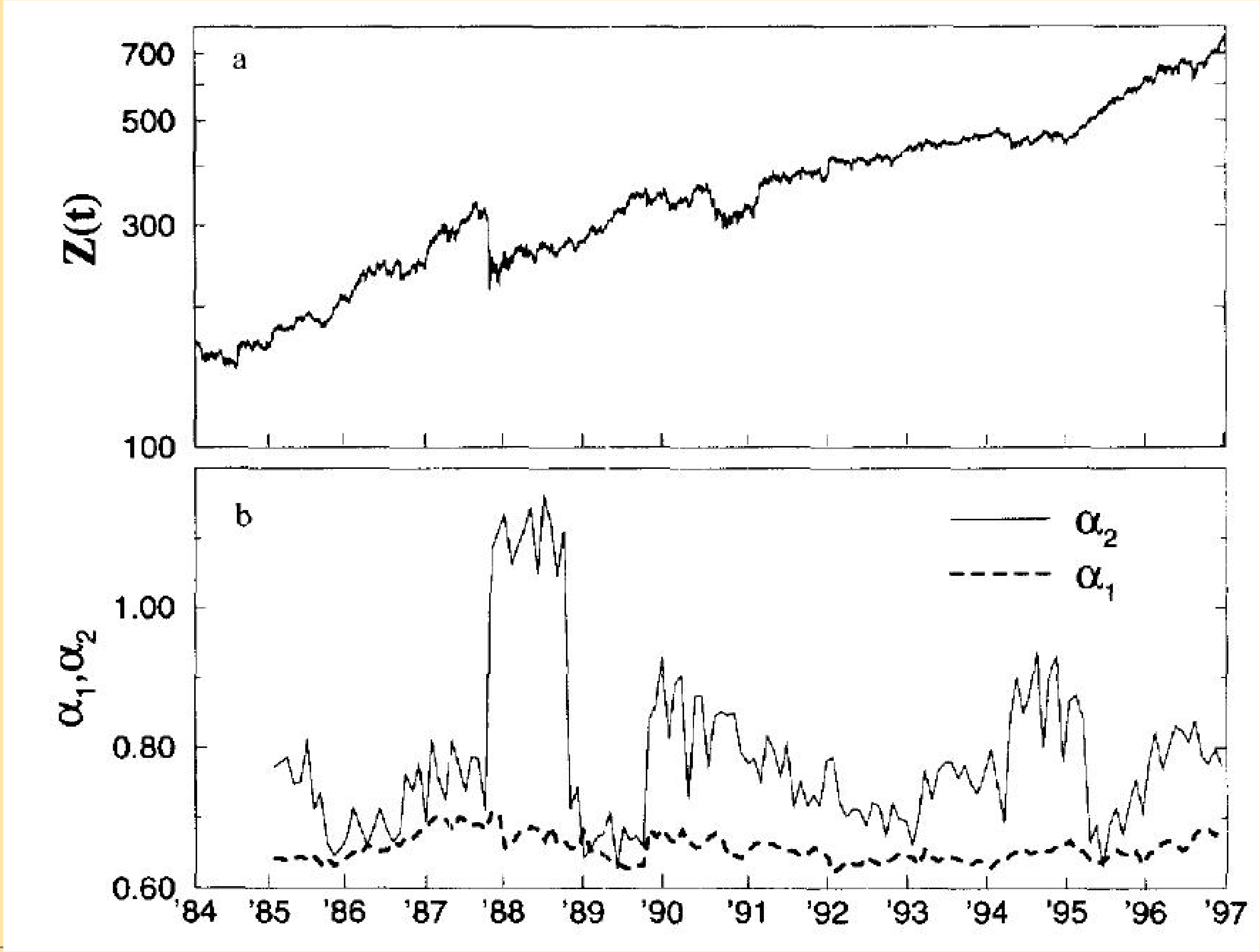


Histogram of time changes in Milano Stock market.

$P(I) \propto I^{-\alpha}$, $\alpha \simeq 2.4$.

Price fluctuations : Lévy walks

[R. N. Mantegna and H. E. Stanley, Nature 376, 46 (1995); Y. Liu, P. Cizeau, M. Meyer, C.-K. Peng and H. E. Stanley, Physica A 245, 437 (1997).]

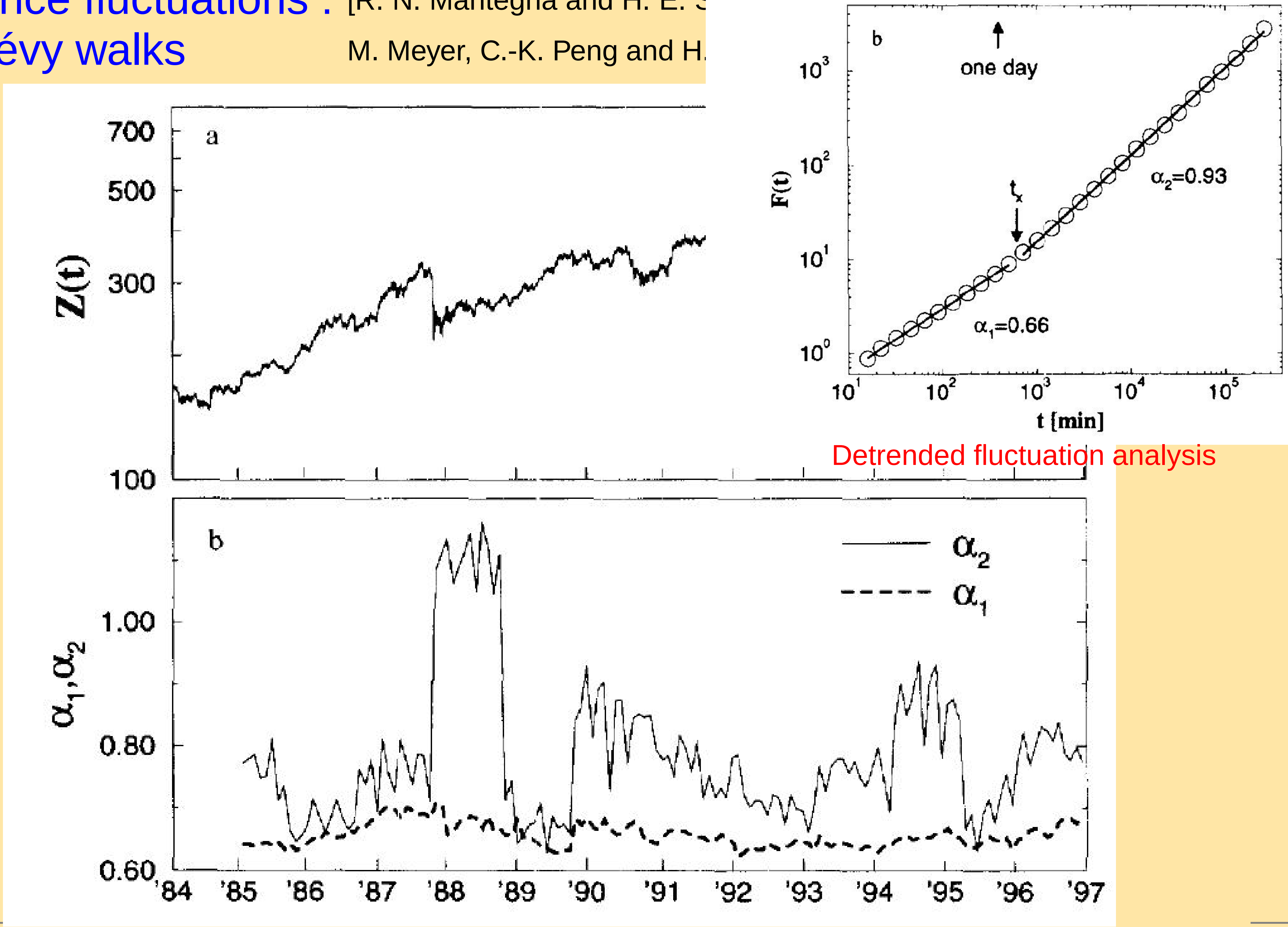


S&P 500 index

Price fluctuations : Lévy walks

[R. N. Mantegna and H. E. S

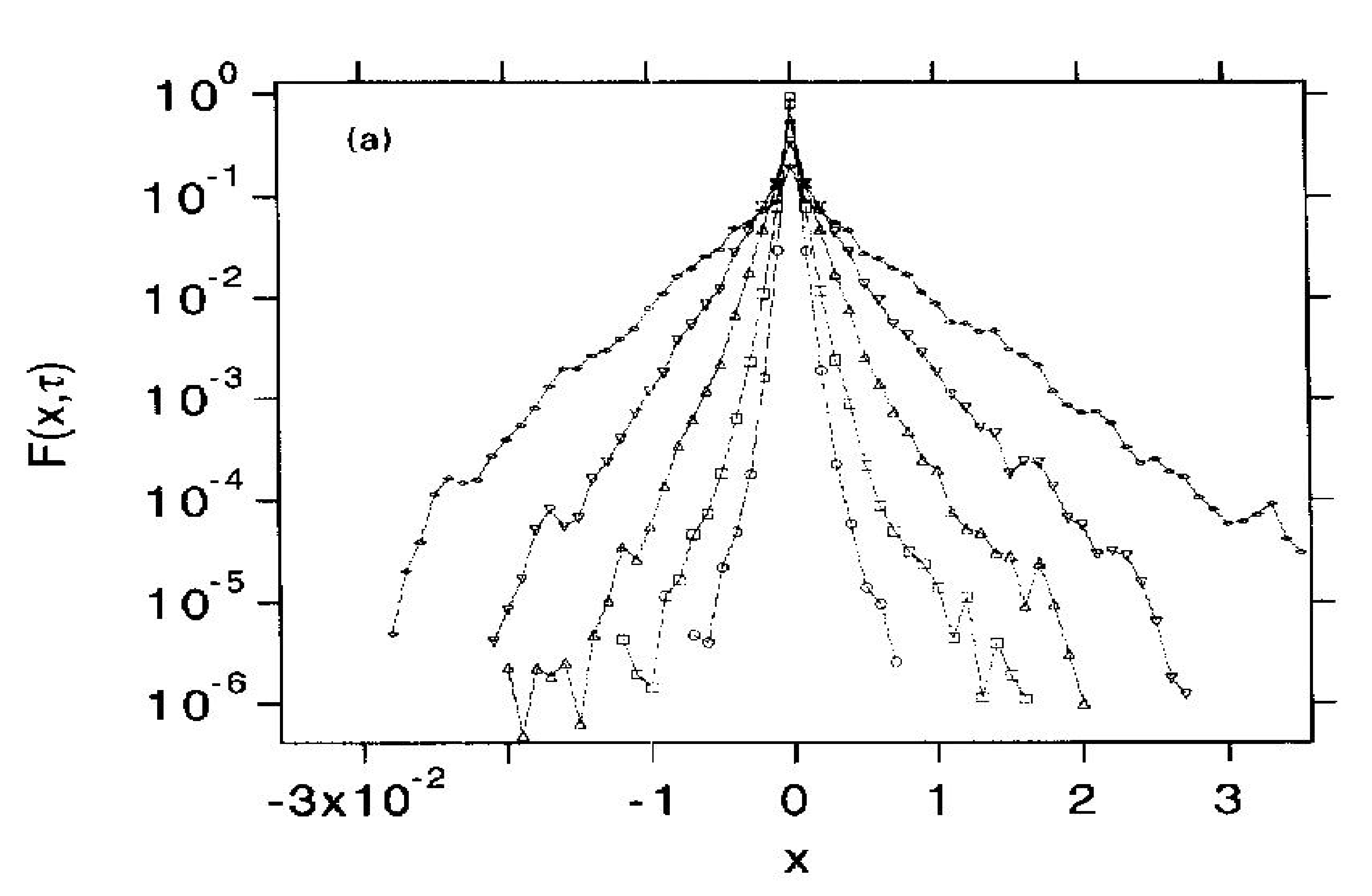
M. Meyer, C.-K. Peng and H.



Detrended fluctuation analysis

S&P 500 index

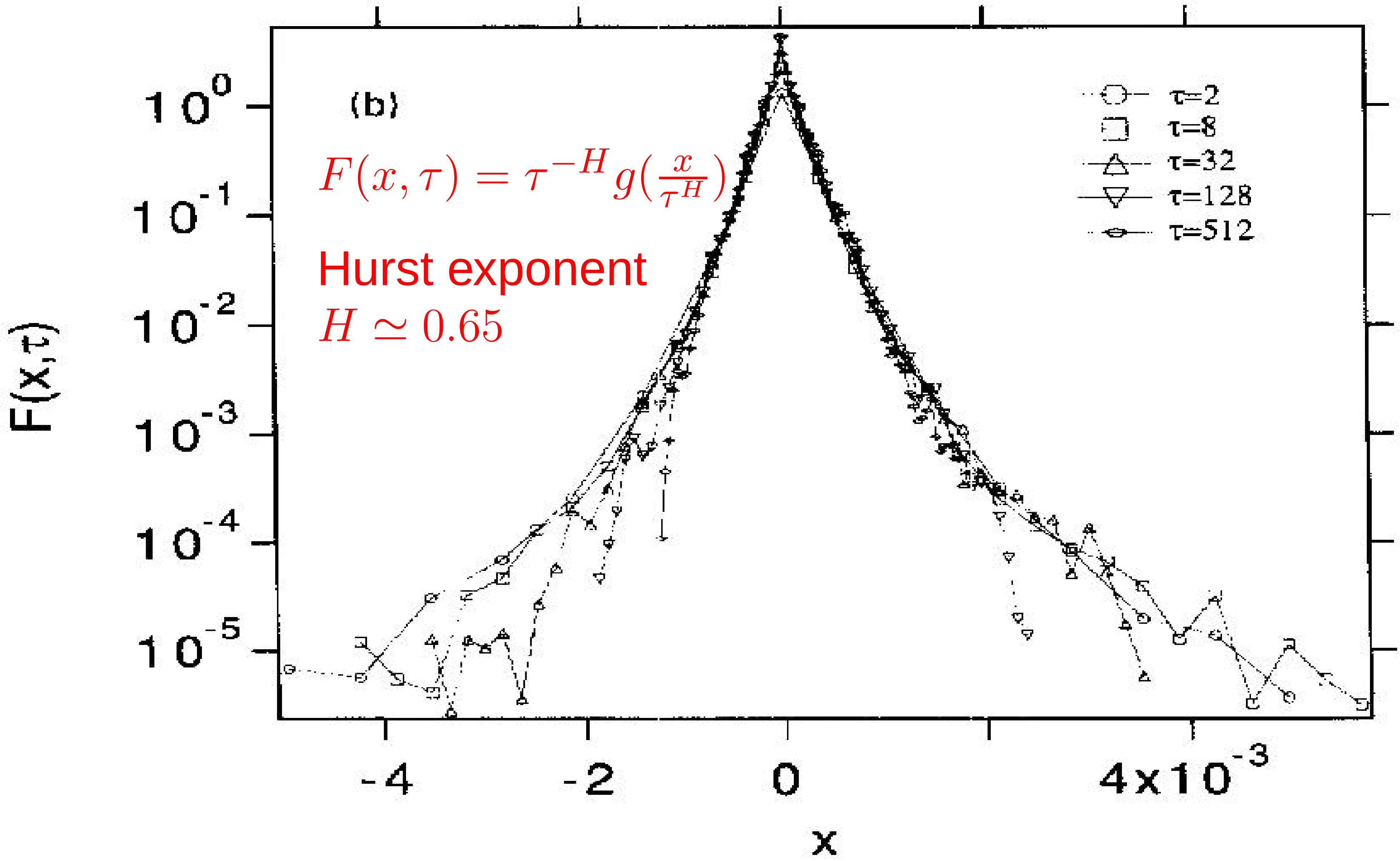
Scaling [S. Galluccio, G. Caldarelli, M. Marsili and Y. -C. Zhang, Physica A 245, 423 (1997).]



DEM ⟶ USD

Scaling

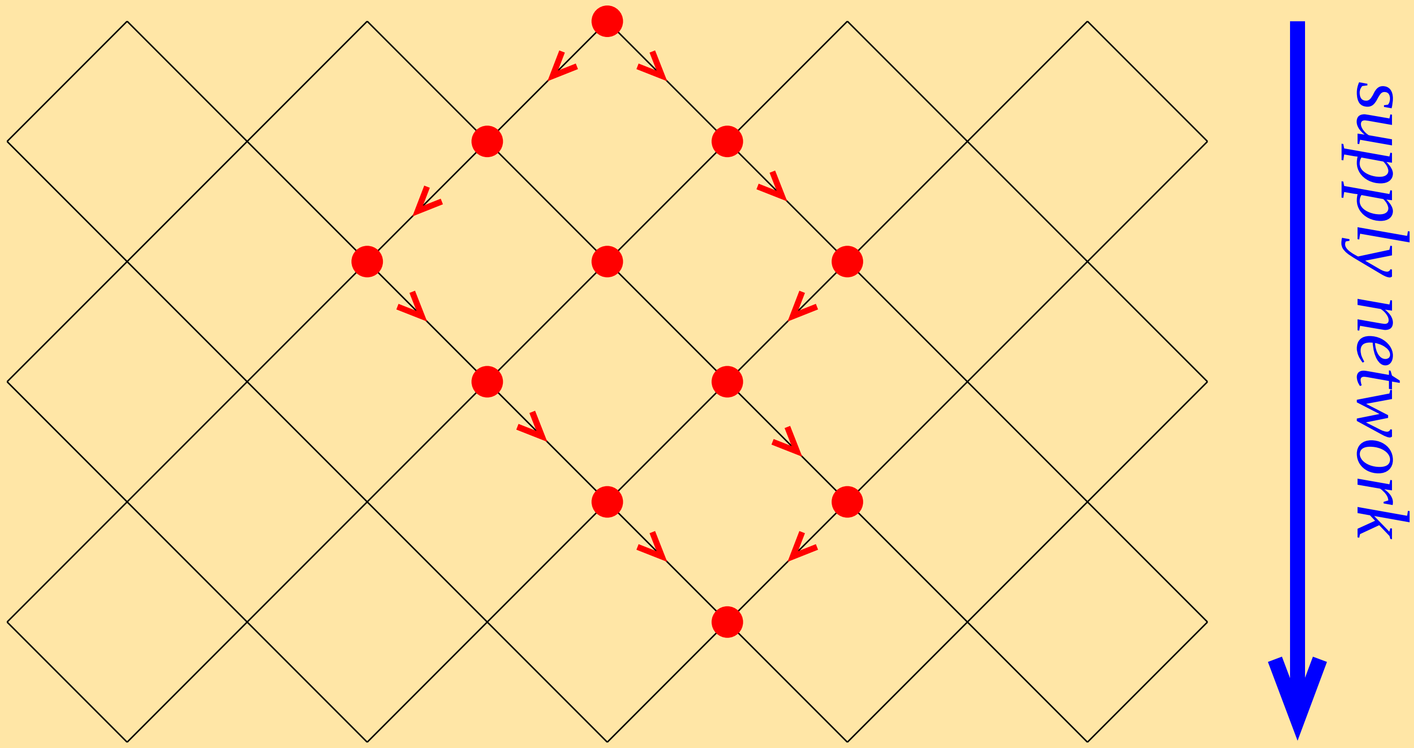
[S. Galluccio, G. Caldarelli, M. Marsili and Y. -C. Zhang, Physica A 245, 423 (1997).]



DEM ⟶ USD

Self-organized critical models

"Directed sandpile", P. Bak et al.



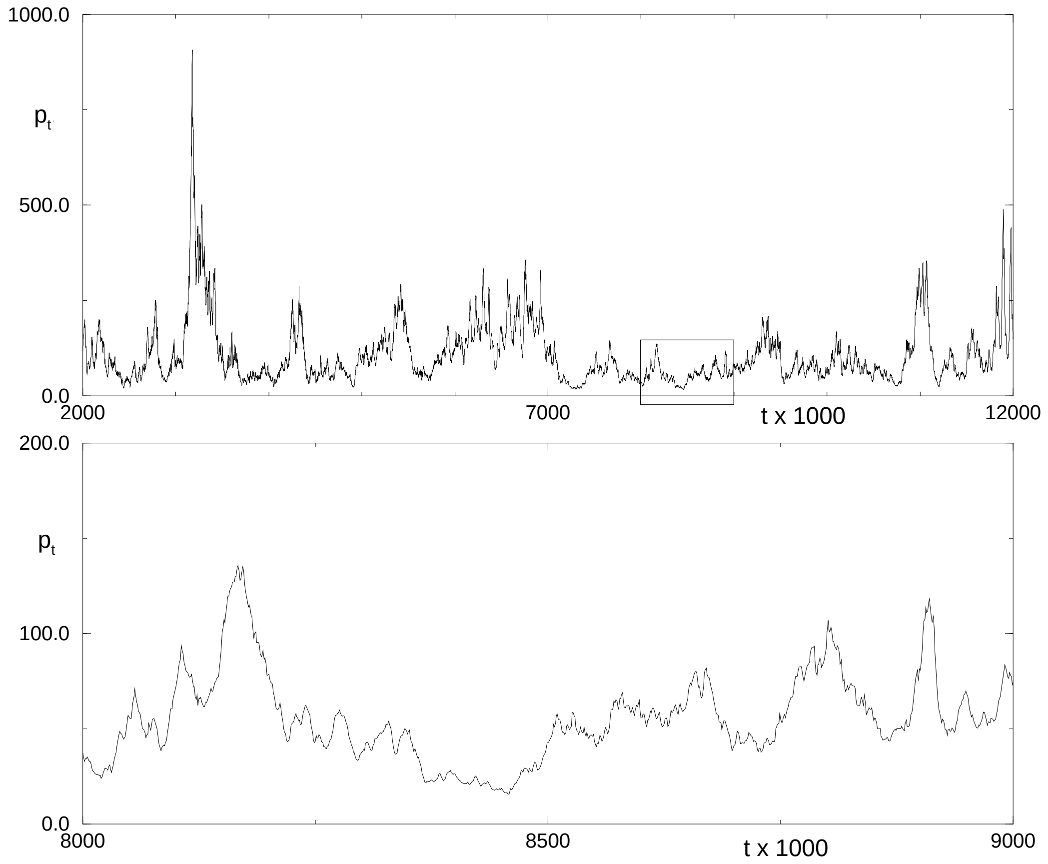
Exactly soluble: avalanche = pair of annihilating random walkers

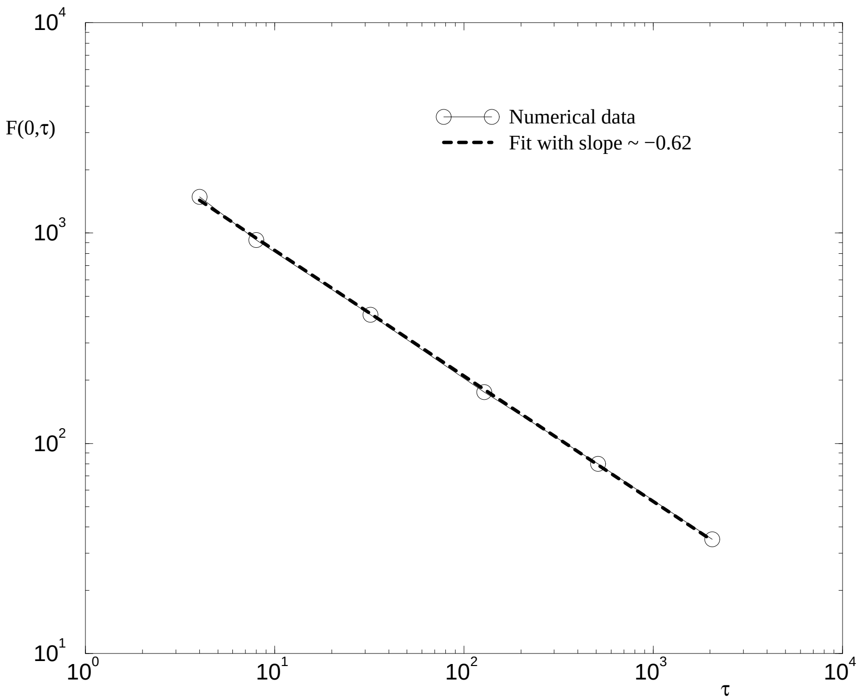
Distribution of durations: $P(\Delta t) \sim (\Delta t)^{-3/2}$.

Cont-Bouchaud herding model: mean-field percolation

Distribution of cluster sizes: $P(s) \sim s^{-5/2}$

CMZ model [Caldarelli G., Marsili M., Zhang Y.C., Europhys. Lett. 40, 479 (1997).]

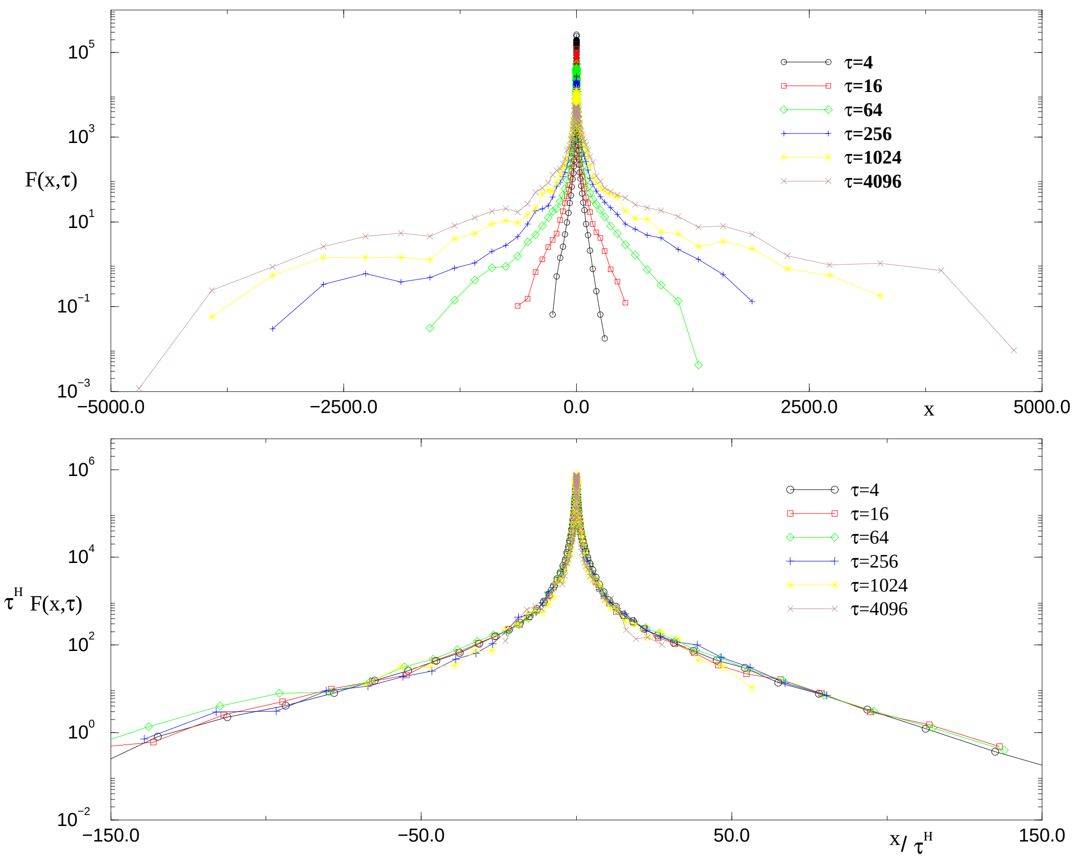


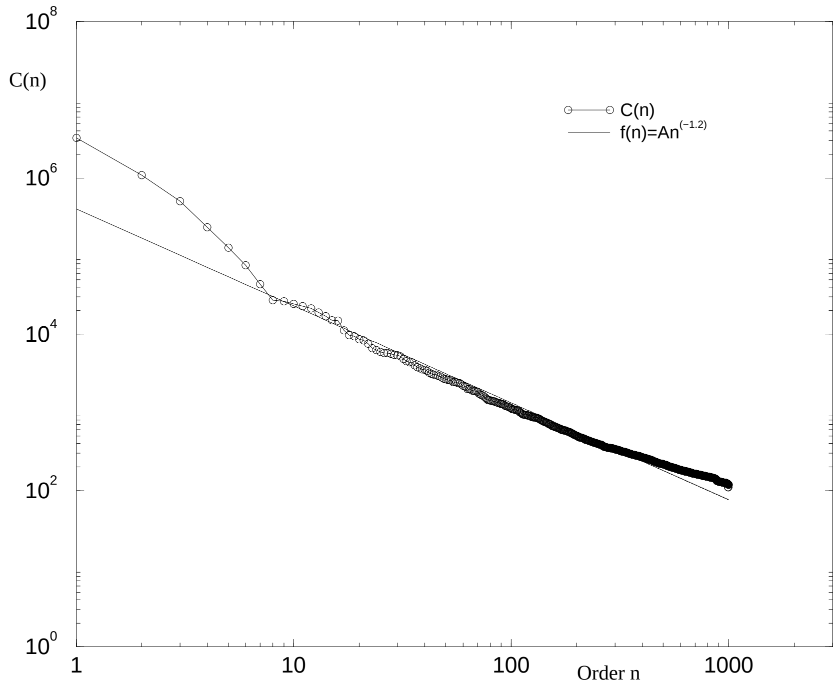


Price history, 1000 agents.

Probability of return to the original price, $H = 0.62$.

CMZ model [Caldarelli G., Marsili M., Zhang Y.C., Europhys. Lett. 40, 479 (1997).]

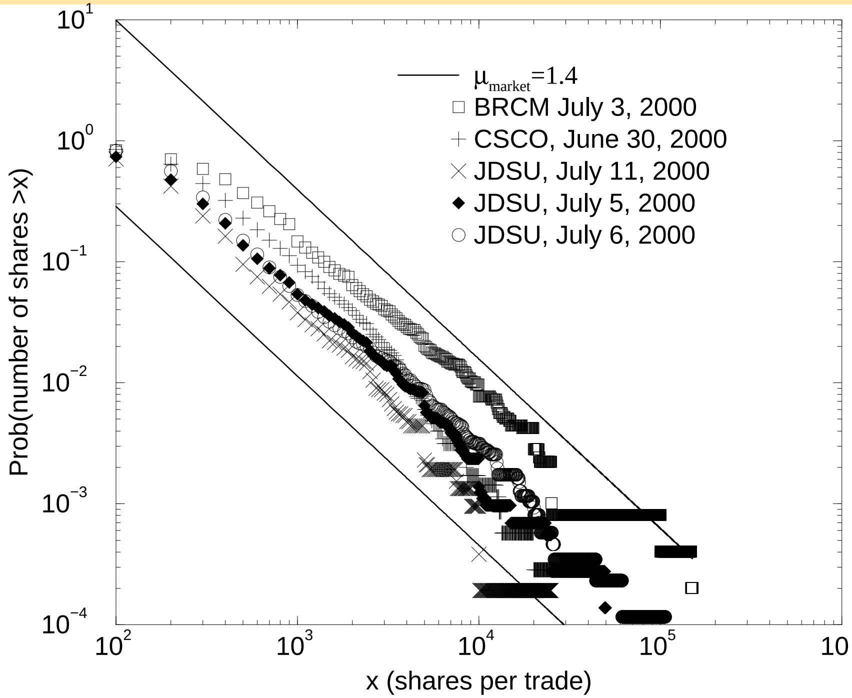


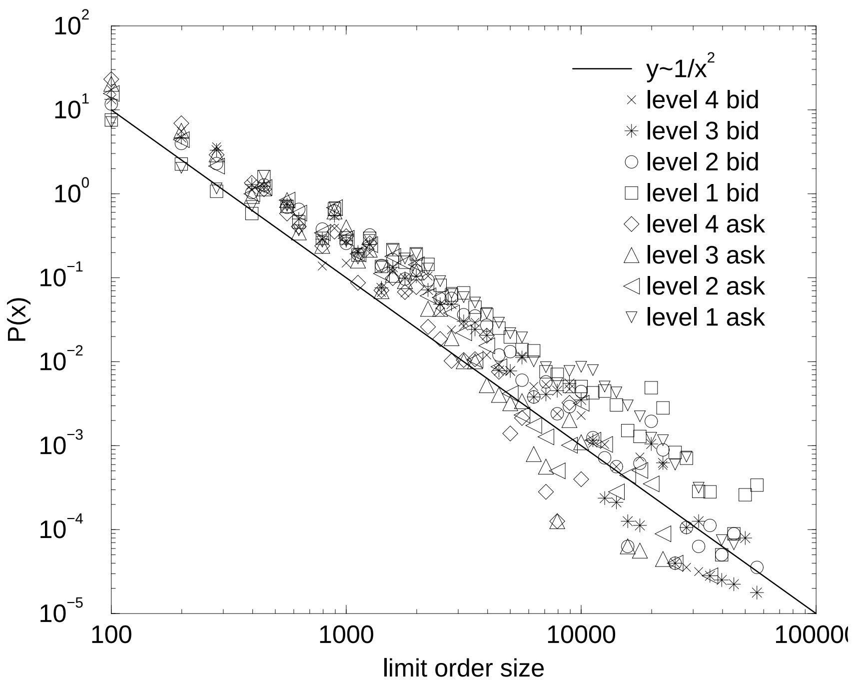


Price changes and scaling.

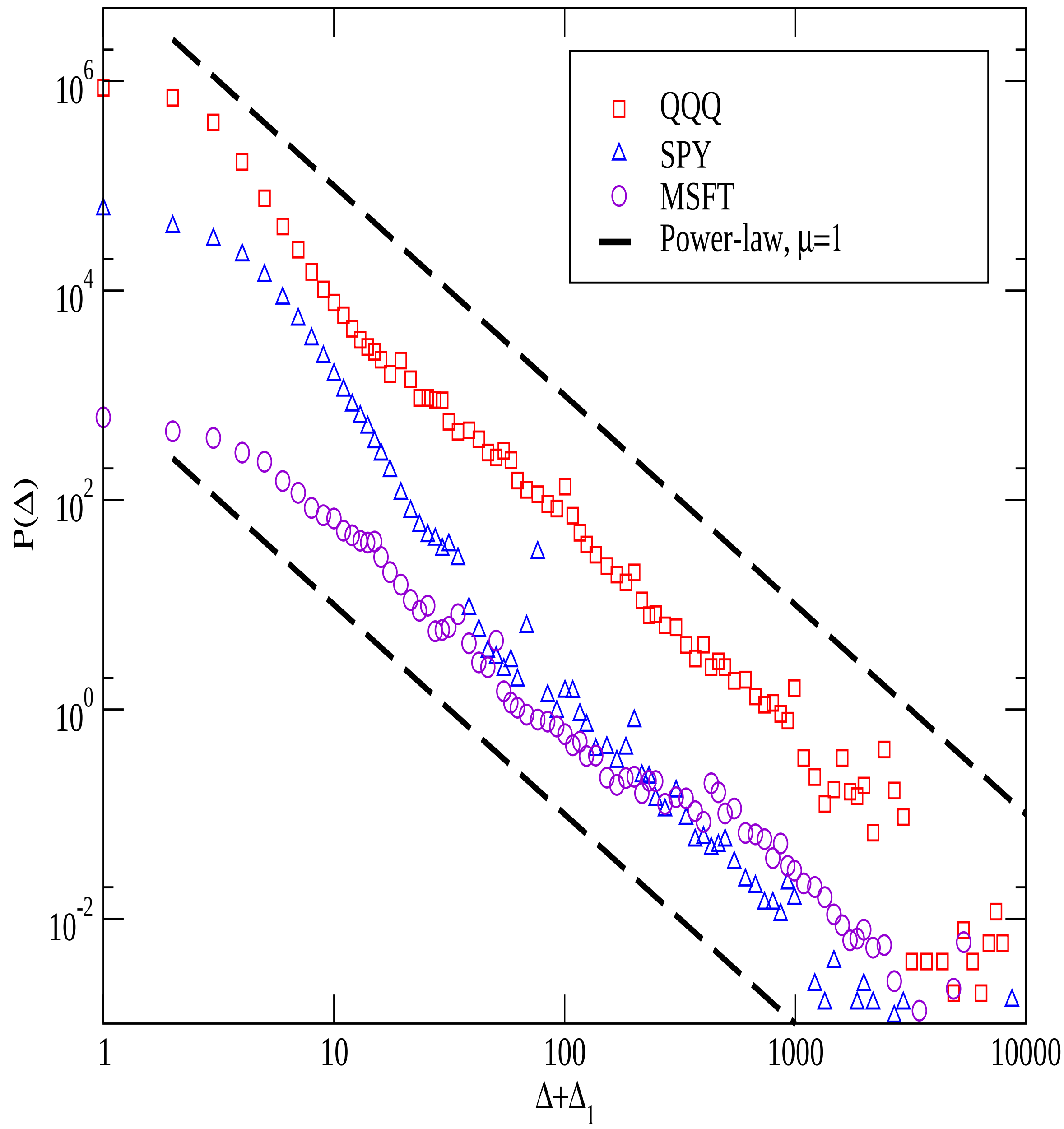
Wealth distribution (Zipf plot), exponent 1.2.

Order book statistics



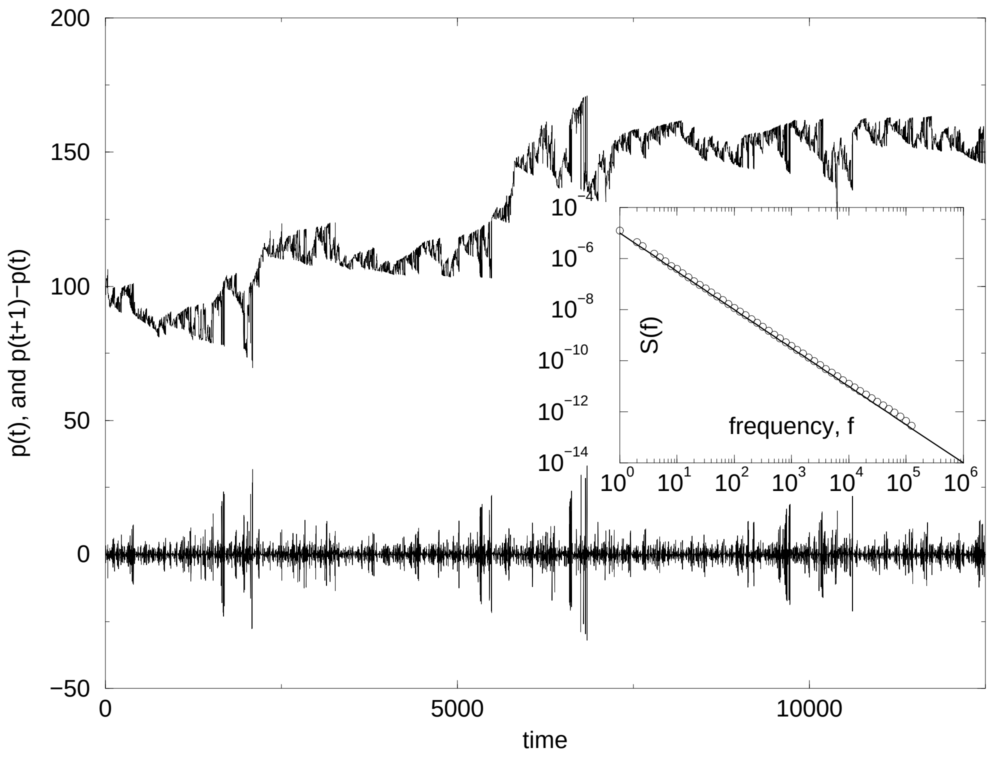


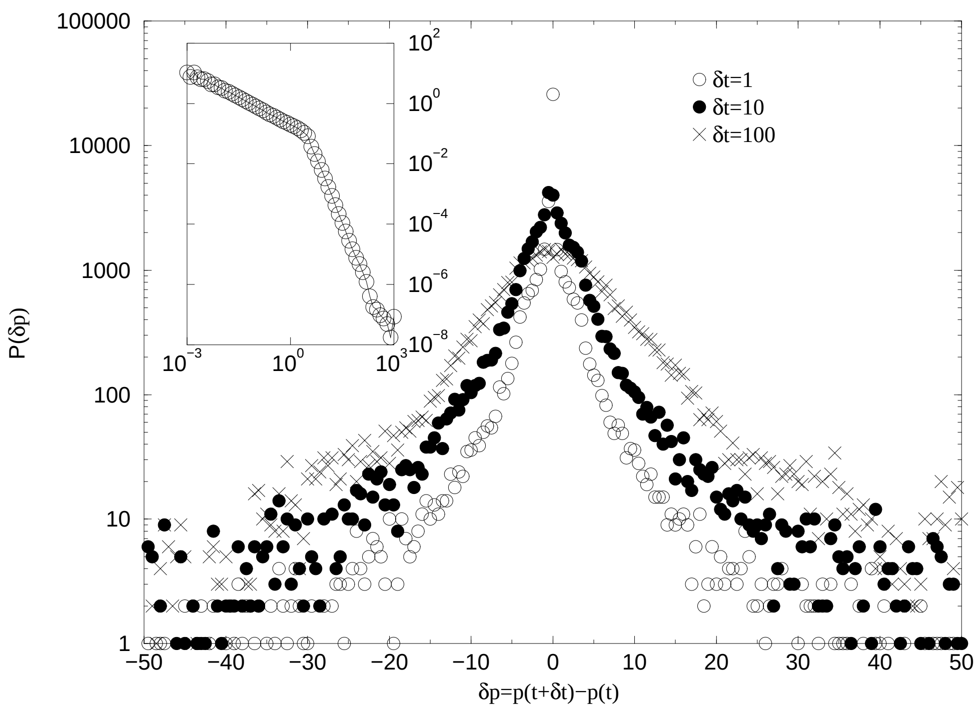
Distribution of volumes



Distribution of distances from current price

Stock market: Maslov model [S. Maslov, Physica A 278, 571 (2000)]



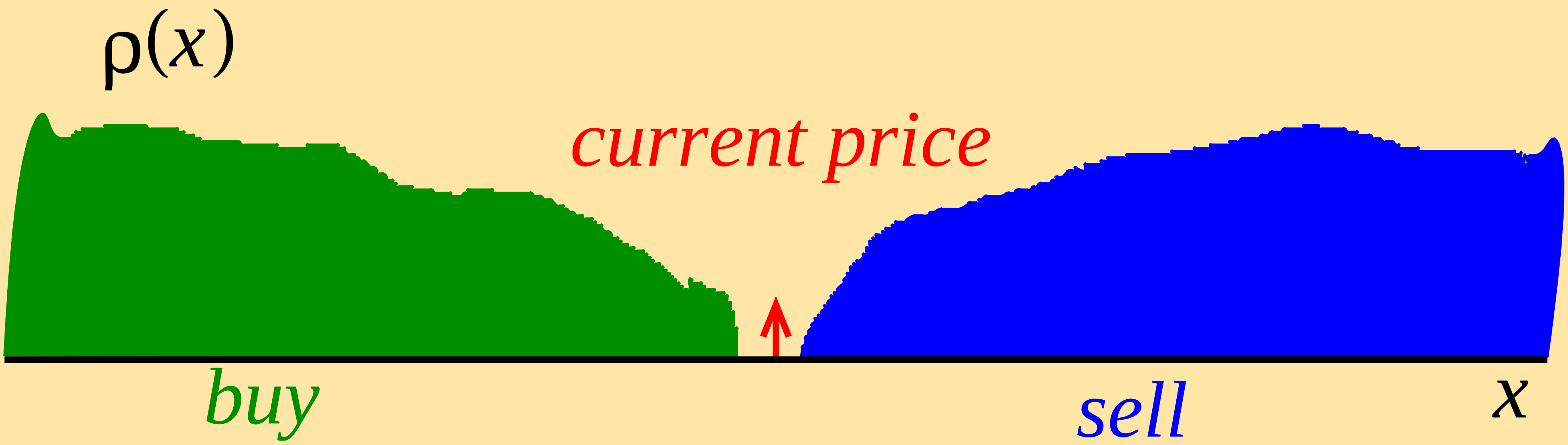


Price history, power spectrum ($H = 1/4$).

Price changes. Exponents 0.6 and 3 in the inset.

Maslov model: mean-filed solution

[F.S., Phys. Rev. E 64, 056136 (2001).]



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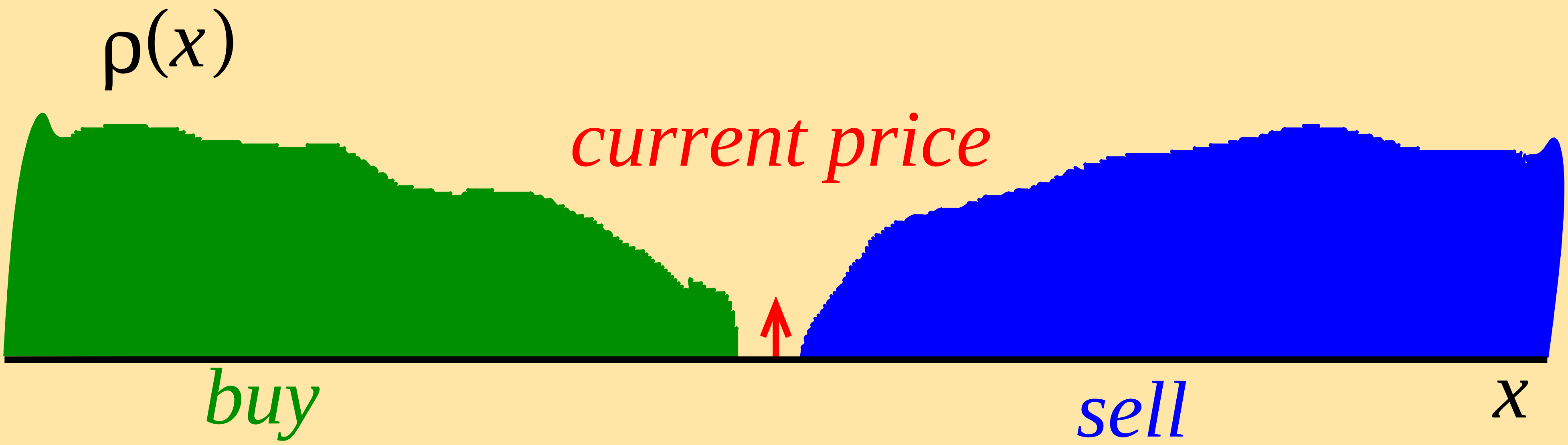

Mean-field approx.:
homogeneous upper ρ_+, lower ρ_-; order volume s.
potential price changes: $x_+ = s/\rho_+$, $x_- = s/\rho_-$

vector $X = \begin{pmatrix} x_+ \\ x_- \end{pmatrix}$ performs

a random matrix multiplicative process.

$$\boxed{X \to X' = TX}$$

$$T = \begin{cases} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} & \text{buy order} \\ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} & \text{sell order} \\ \frac{1}{2}(3 - 1/p) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \text{limit order} \end{cases}$$

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Probability distribution of price changes (after m limit and one market order):

$$P(x) = \sum_{m=0}^{\infty} (1-p)p^m \int \mathrm{d}x' \, P(x') \, \delta\left(x - \frac{3}{2} \, x' \, \left(\frac{3 - \frac{1}{p}}{2} \right)^m \right)$$

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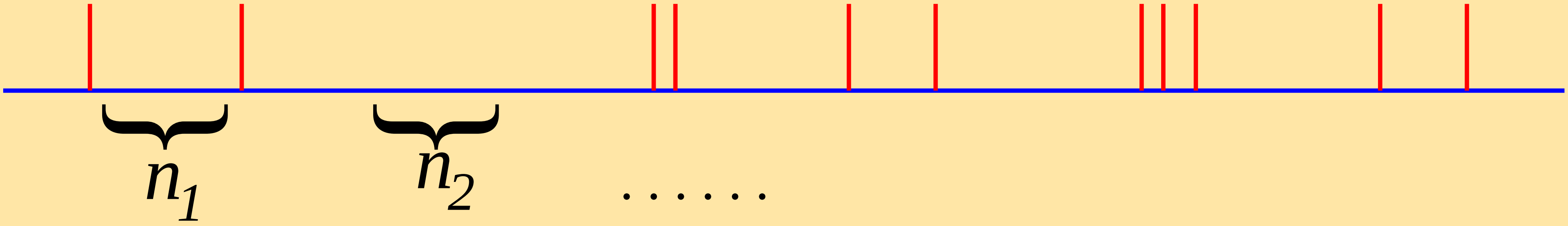
assuming power-law tail $P(x) \sim x^{-1-\alpha}$: $\displaystyle \sum_{m=0}^{\infty} (1-p)p^m \left(\frac{3}{2} \left(\frac{3 - \frac{1}{p}}{2} \right)^m \right)^\alpha = 1 \, .$

non-trivial solution: $\alpha = 1$

Interacting gaps model [Lev Muchnik, F.S., and Sorin Solomon, Physica A 330, 232 (2003).]



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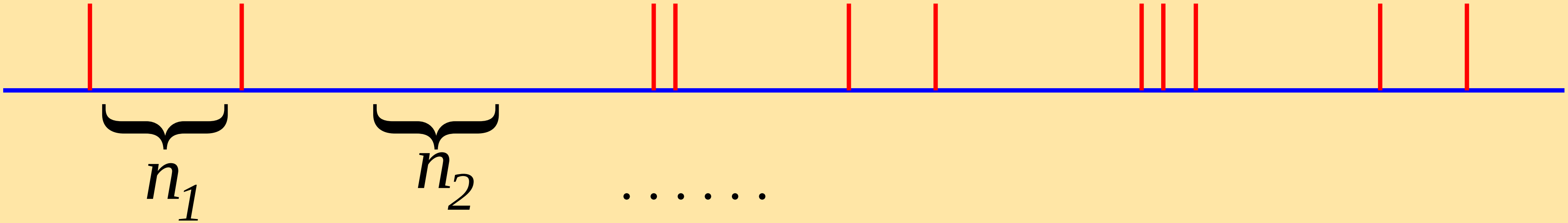

“collision” of a pair of intervals (n_i, n_j):

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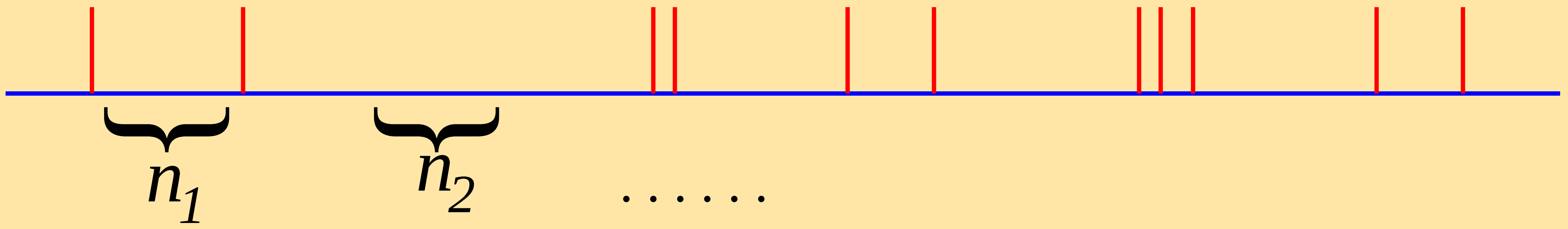

“collision” of a pair of intervals (n_i, n_j):

1. a collapse with probability p:

$$n_i(t+1) = n_i(t) + n_j(t) - 1$$

$$n_j(t+1) = 1$$

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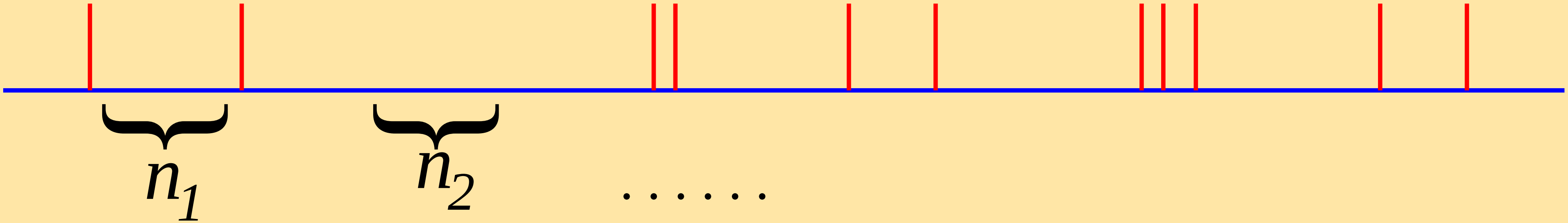

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2. a reaction with probability $1 - p$

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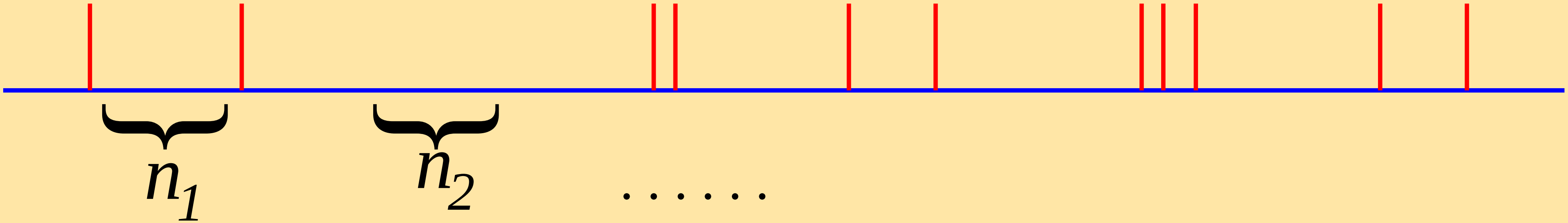

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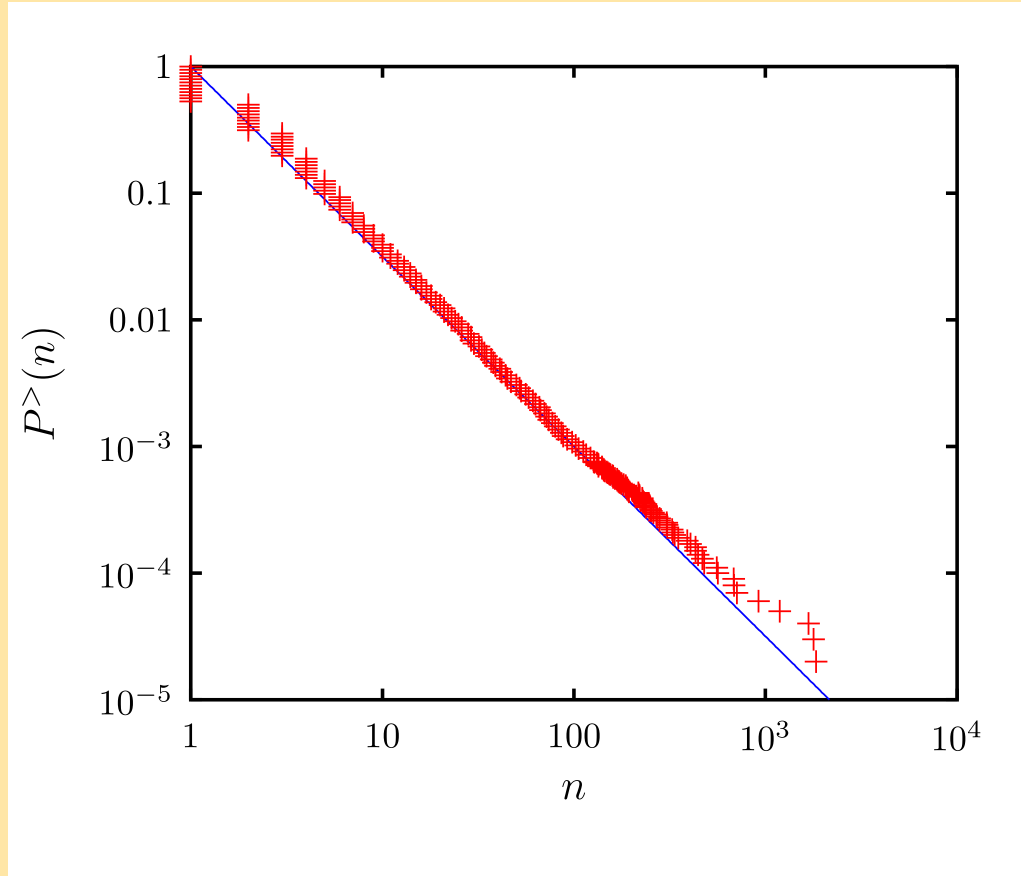
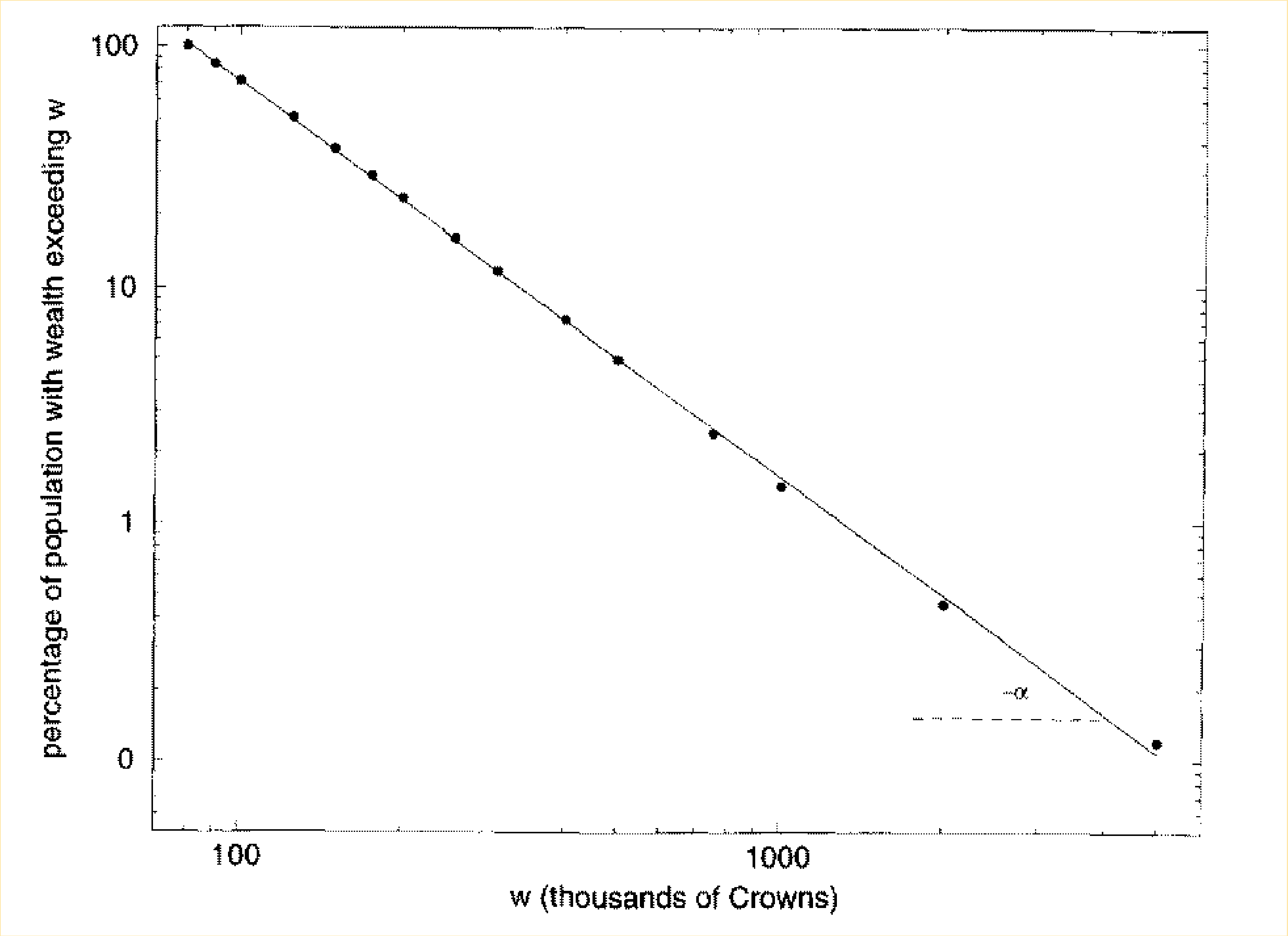


$P^{>}(n) \sim n^{-\alpha}$, $\alpha = 3/2$

Wealth distribution

Empirical wealth distribution [M. Levy, S. Solomon Physica A 242, 90 (1997)]



Pareto distribution $P(w) \sim w^{-\alpha-1}$, $\alpha \simeq 1.4$

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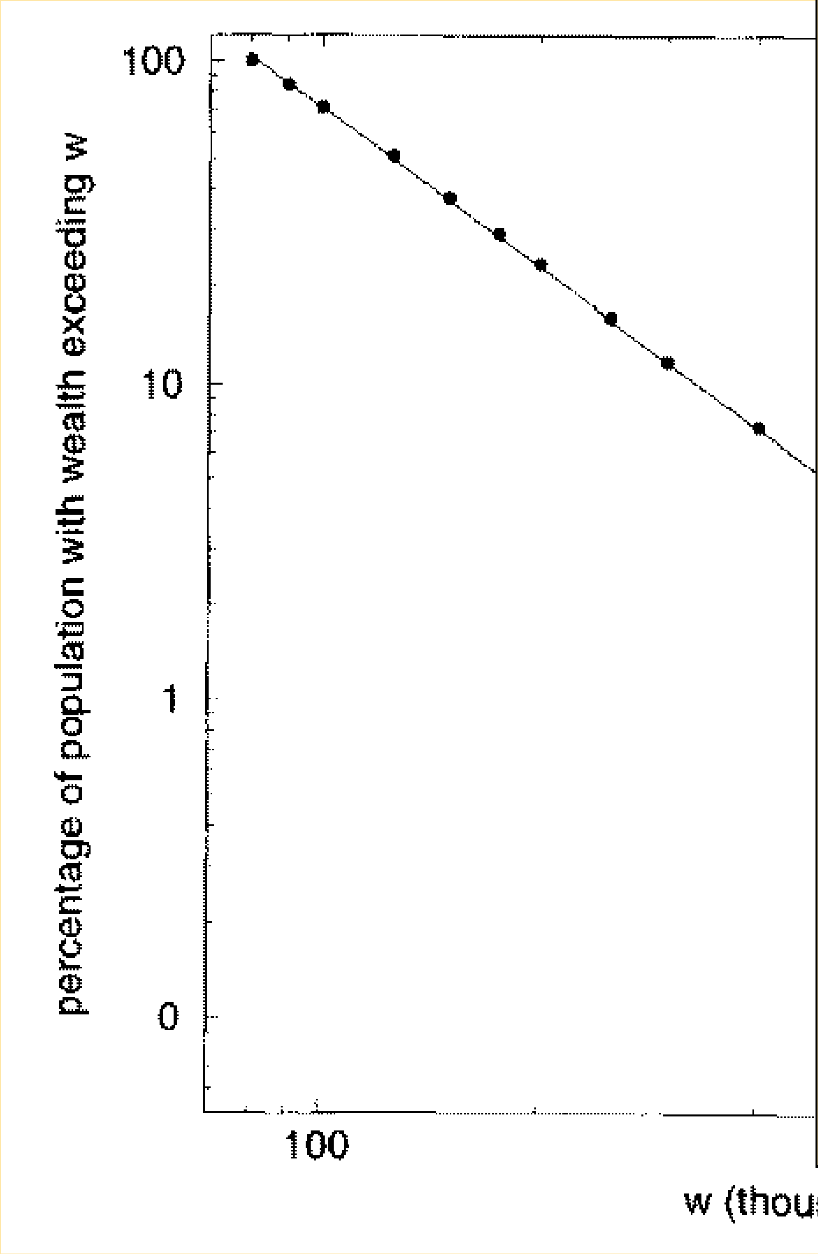
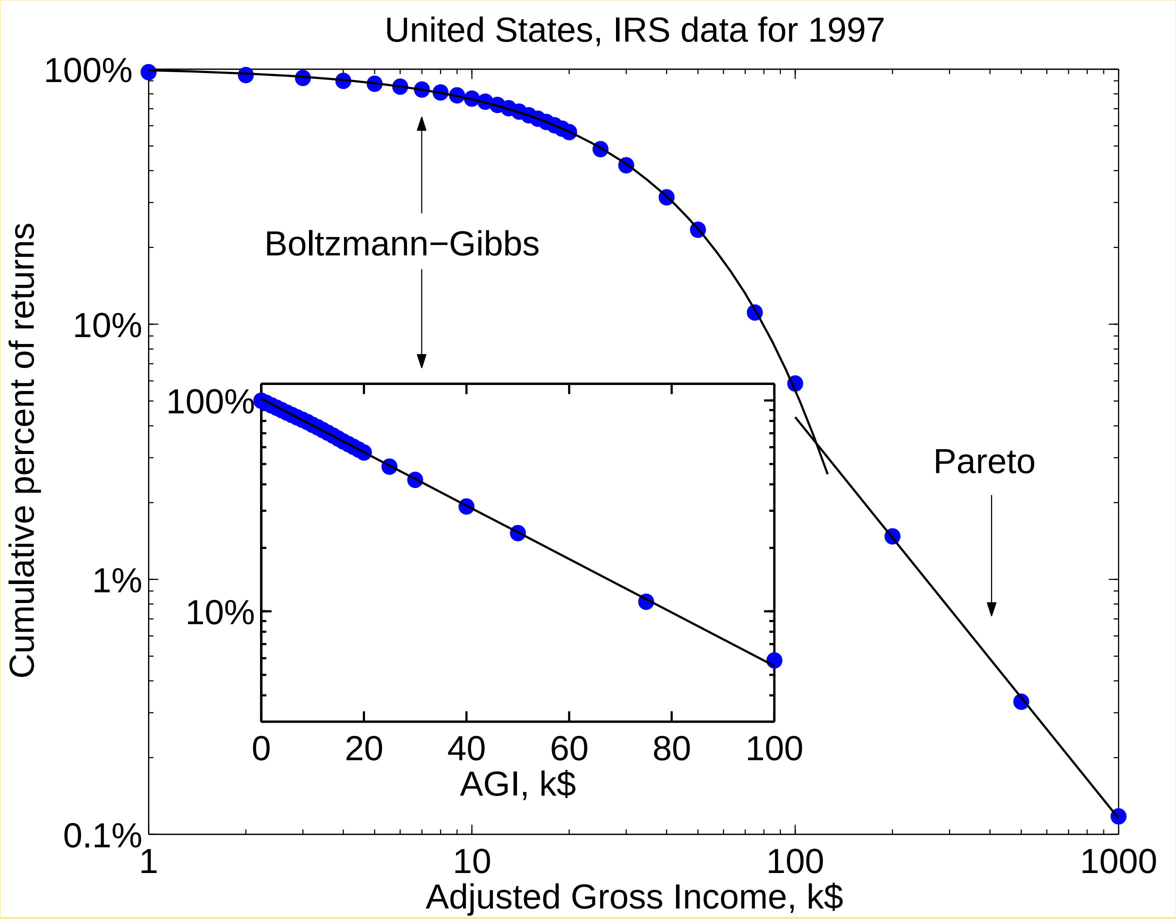


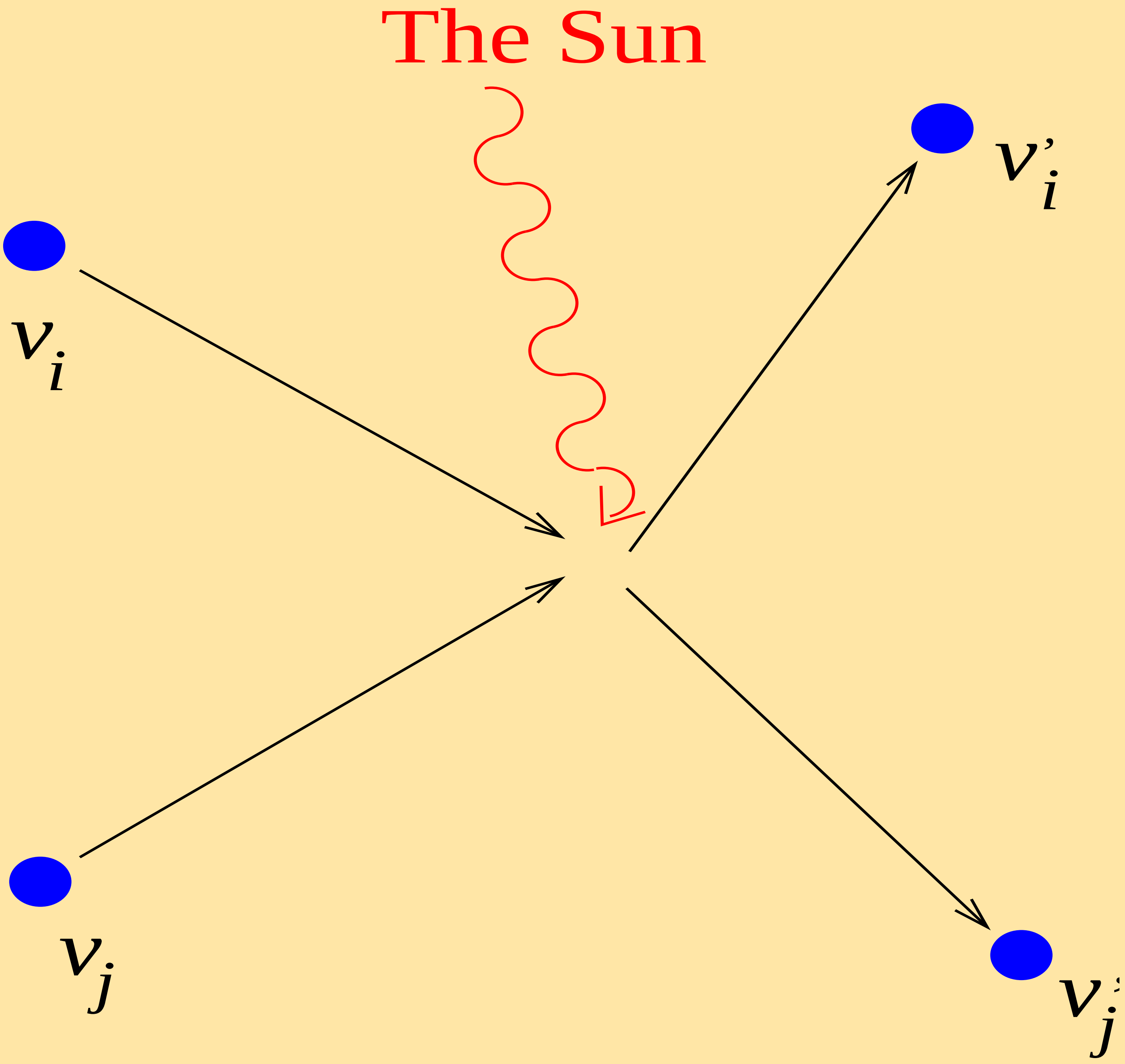


Exponential distribution at low incomes

Pareto at high incomes (V.M. Yakovenko et al.)

Pareto distribution $P(w) \sim w^{-\alpha-1}$, $\alpha \simeq 1.4$

Wealth distribution: inelastic gas [F.S. at WEHIA2002; submitted to Phys. Rev. E]



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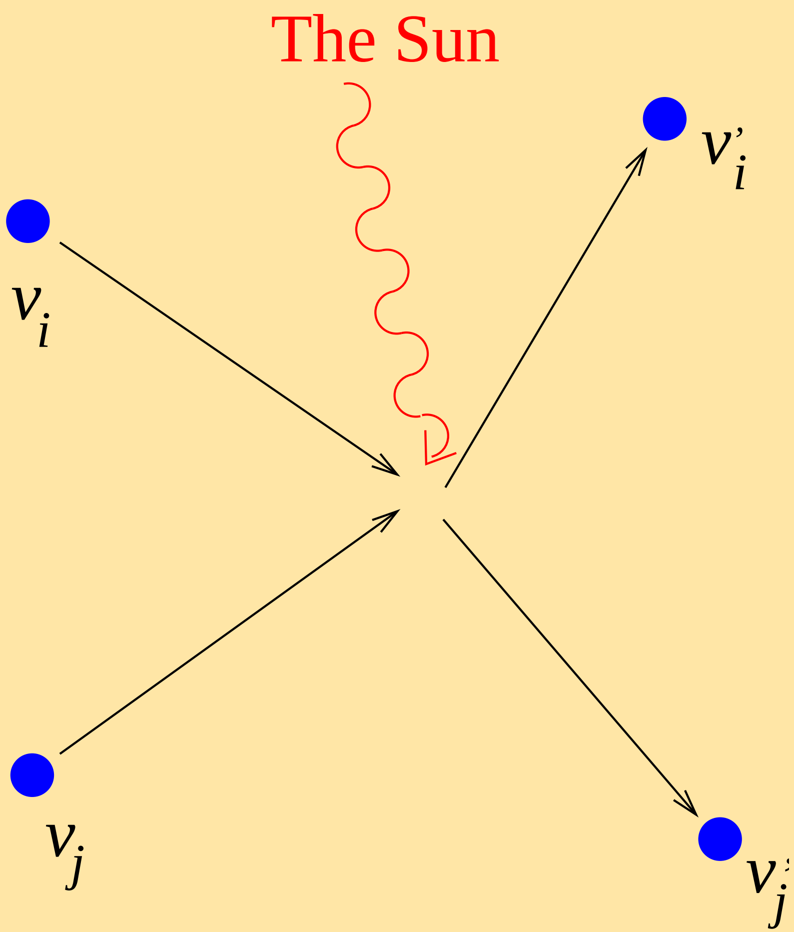

$$\begin{pmatrix} v_i' \\ v_j' \end{pmatrix} = \begin{pmatrix} 1+\varepsilon-\beta & \beta \\ \beta & 1+\varepsilon-\beta \end{pmatrix} \begin{pmatrix} v_i \\ v_j \end{pmatrix}$$

Granular gas analogy

Agents=grains; wealth v_i

Economic interaction= inelastic scattering

β measures intensity of exchange

ε measures average profit

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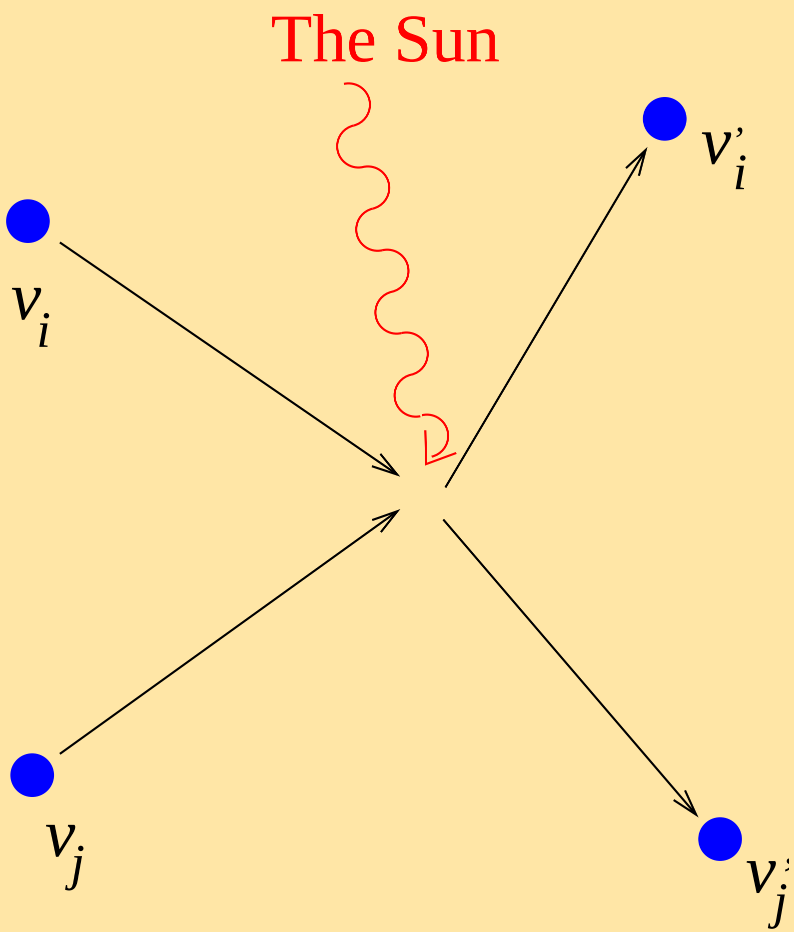

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Granular gas analogy

Agents=grains; wealth v_i
Economic interaction= inelastic scattering
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Probability density governed by a Boltzmann-like equation

$$\frac{\partial P(v)}{\partial t} + P(v) =$$
$$= \int P(v_1)P(v_2)\,\delta((1-\beta+\varepsilon)v_1 + \beta v_2 - v)\mathrm{d}v_1\mathrm{d}v_2$$

Average wealth grows exponentially: $\bar{v}(t) = \bar{v}(0)\,\mathrm{e}^{\varepsilon t}$

$$\Downarrow$$

Scaling solution: $P(v,t) = \frac{1}{\bar{v}(t)}\,\Phi(\frac{v}{\bar{v}(t)})$

Average wealth grows exponentially: $\bar{v}(t) = \bar{v}(0)\,\mathrm{e}^{\varepsilon t}$

Scaling solution: $P(v,t) = \frac{1}{\bar{v}(t)}\,\Phi(\frac{v}{\bar{v}(t)})$

Laplace transform: $\hat{\Phi}(x) = \int_0^\infty \Phi(w)\,\mathrm{e}^{-xw}\,\mathrm{d}w$

$$\varepsilon x \hat{\Phi}'(x) + \hat{\Phi}(x) = \hat{\Phi}((1-\beta+\varepsilon)x)\ \hat{\Phi}(\beta x)$$

...non-local differential equation

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...non-local differential equation

Power-law tail $\Rightarrow$ Looking for solution in the form

$\hat{\Phi}(x) = 1 - x + A\, x^{\alpha} + \ldots$ where $\alpha \in (1,2)$

$\Rightarrow \Phi(w) \sim w^{-\alpha-1}$ for $w \to \infty$.

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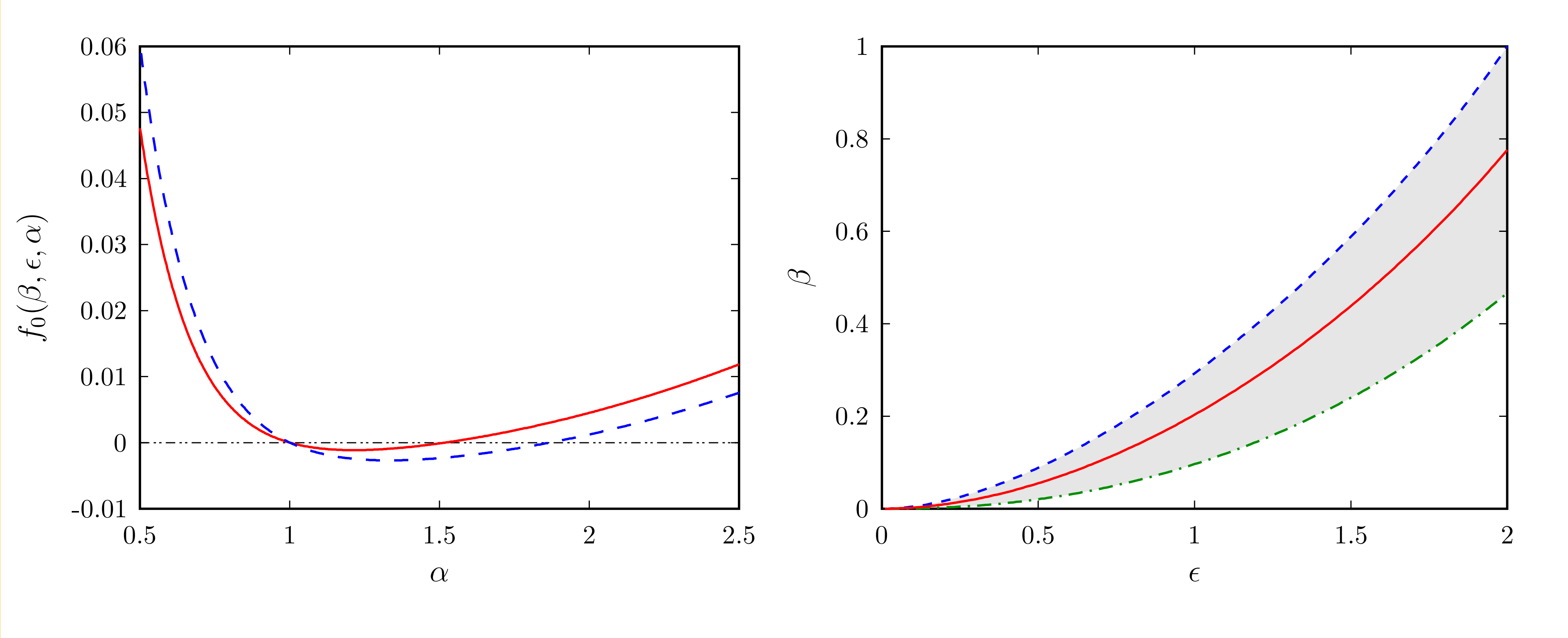
$f_0(\beta,\epsilon,\alpha) \equiv$

$$\boxed{(1+\varepsilon-\beta)^{\alpha} + \beta^{\alpha} - 1 - \varepsilon\alpha = 0}$$

$$(1+\varepsilon-\beta)^{\alpha}+\beta^{\alpha}-1-\varepsilon\alpha=0$$



Solution of the equation for $\varepsilon = 0.1$ and $\beta = 0.0025$ (red) and $\beta = 0.004$ (blue).

Solution in the range $\alpha \in (1,2)$ exists within the shaded region. Green line corresponds to $\alpha = 1$, blue line corresponds to $\alpha = 2$ and red line to the solution $\alpha = \frac{3}{2}$.

Limit of continuous trading: $\beta \to 0$, $\epsilon \to 0$, α fixed:

$$\beta = \frac{\alpha-1}{2}\,\varepsilon^2 - \frac{(\alpha-1)(2\alpha-1)}{6}\,\varepsilon^3 + \frac{1}{\alpha}\left(\frac{\alpha-1}{2}\right)^{\alpha}\,\varepsilon^{2\alpha} + \ldots$$

For $\alpha = \frac{3}{2}$:

$$\epsilon = \frac{1}{8}\;\frac{-3\sqrt{\beta}+17\,\beta-29\,\beta^{3/2}+15\,\beta^2+4\,\beta^{5/2}-4\,\beta^3+\sqrt{3}\sqrt{\left(3-2\sqrt{\beta}\right)\beta\left(2\sqrt{\beta}+1\right)^3\left(\sqrt{\beta}-1\right)^6}}{\sqrt{\beta}-3\,\beta+3\,\beta^{3/2}-\beta^2}$$

$$\beta = \frac{1}{4}\epsilon^2 - \frac{1}{12}\epsilon^3 + \frac{1}{16}\epsilon^4 - \frac{7}{144}\epsilon^5 + \frac{113}{2592}\epsilon^6 + \ldots$$

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$$\Rightarrow -\frac{1}{2}x\hat{\Phi}''(x) + \frac{\alpha-1}{2}\left(\hat{\Phi}'(x) + \hat{\Phi}(x)\right) = 0$$

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Solution with correct asymptotics $\hat{\Phi}(x) \to 0$ for $x \to +\infty$ is expressed through modified Bessel function

$$\hat{\Phi}(x) = C'\,x^{\alpha/2}\,K_\alpha(2\sqrt{\alpha-1}\sqrt{x})$$

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$$\hat{\Phi}(x) = C'\,x^{\alpha/2}\,K_\alpha(2\sqrt{\alpha-1}\sqrt{x})$$

Inverting Laplace transform:

$$\boxed{\Phi(w) = C\,w^{-\alpha-1}\exp(-\frac{\alpha-1}{w})}$$

Limit of continuous trading: $\beta \to 0$, $\epsilon \to 0$, α fixed:

$$\beta = \frac{\alpha-1}{2}\,\varepsilon^2 - \frac{(\alpha-1)(2\alpha-1)}{6}\,\varepsilon^3 + \frac{1}{\alpha}\left(\frac{\alpha-1}{2}\right)^{\alpha}\,\varepsilon^{2\alpha} + \ldots$$

$$\Rightarrow -\frac{1}{2}x\hat{\Phi}''(x) + \frac{\alpha-1}{2}\left(\hat{\Phi}'(x) + \hat{\Phi}(x)\right) = 0$$

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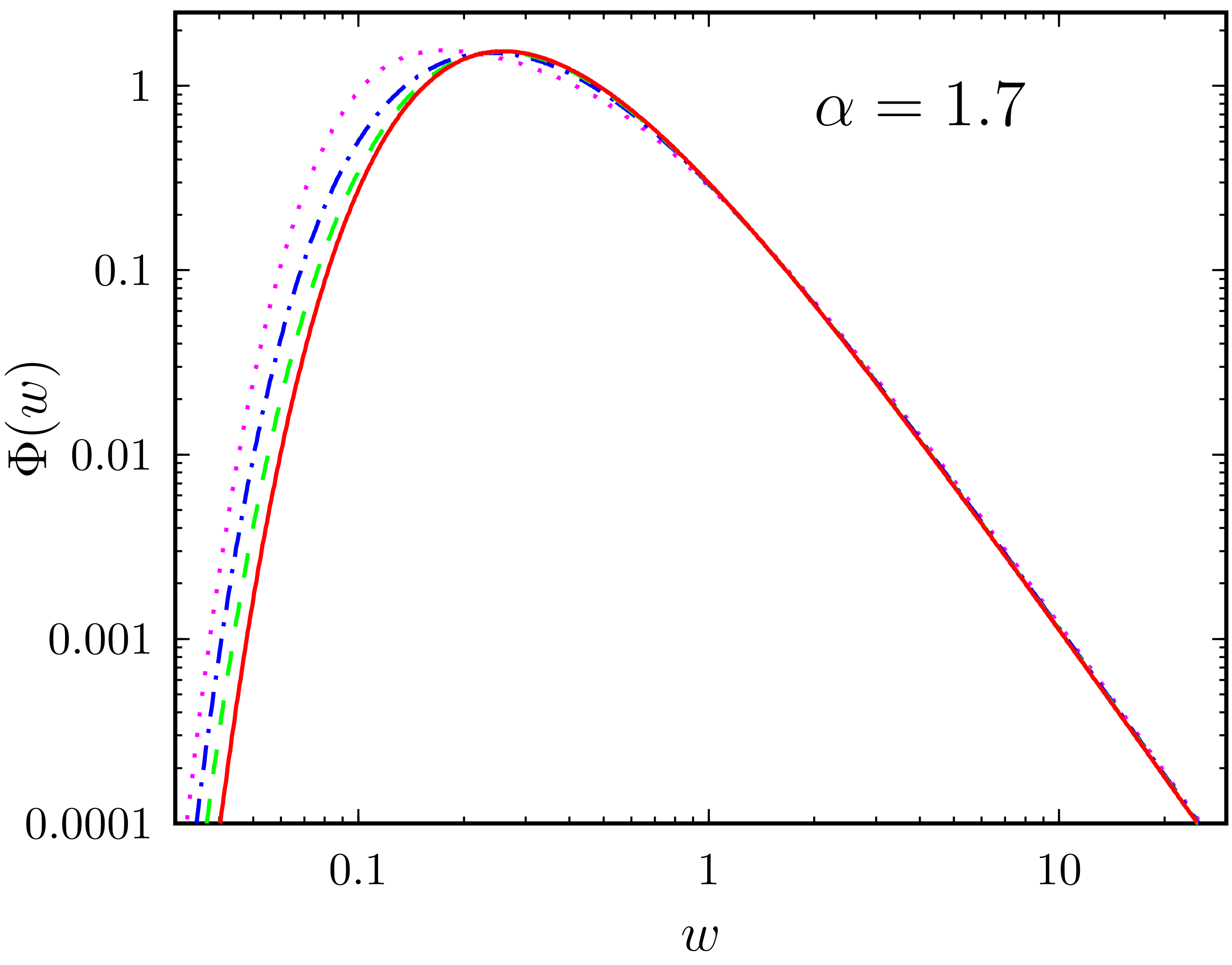
$$\boxed{\Phi(w) = C\,w^{-\alpha-1}\exp(-\frac{\alpha-1}{w})}$$

$$C = \frac{(\alpha-1)^\alpha}{\Gamma(\alpha)}$$

Systematic corrections: Expansion in ε

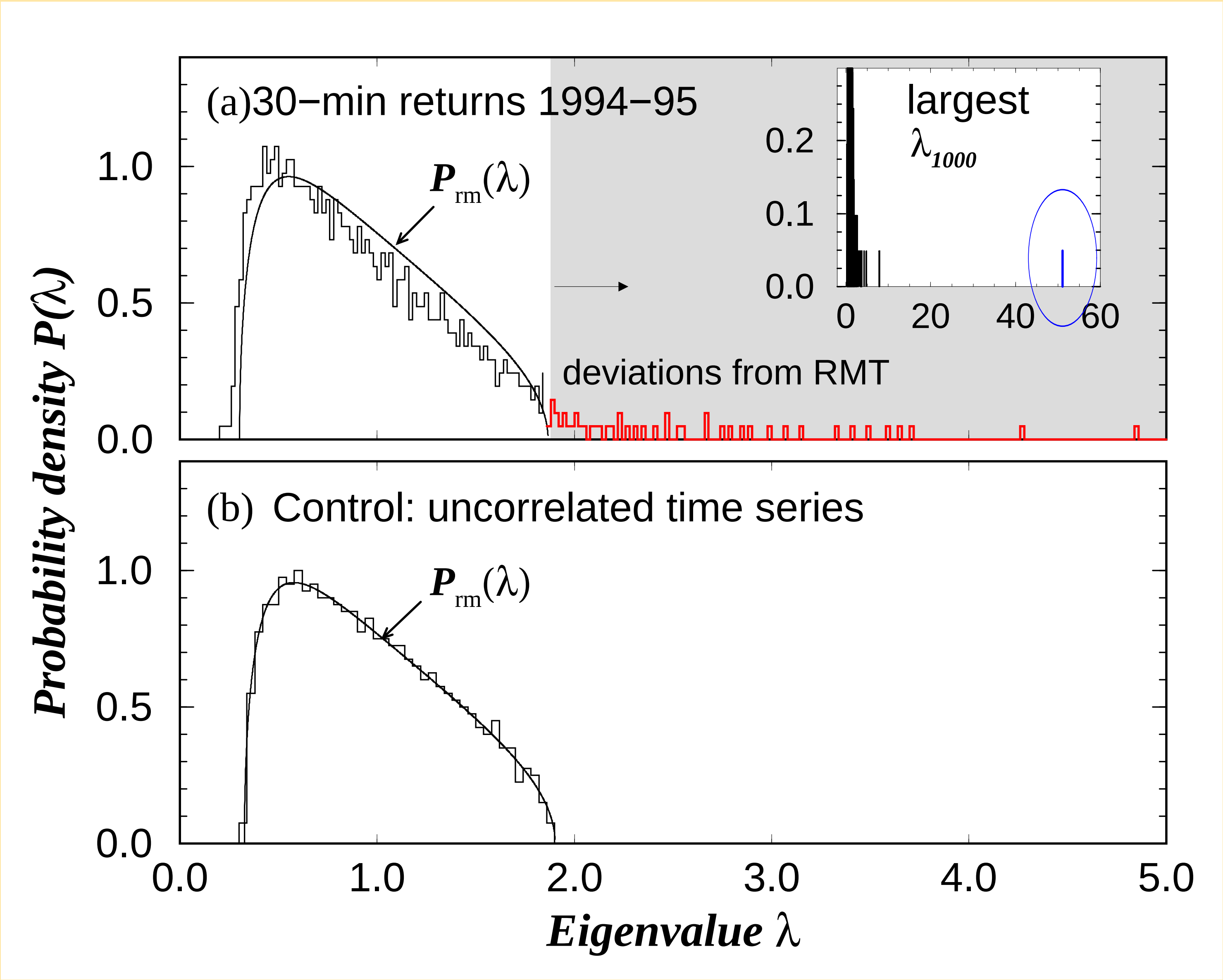


Wealth distribution for $\varepsilon \to 0$ (full line), $\varepsilon = 0.03$ (dashed line), $\varepsilon = 0.1$ (dash-dotted line), and $\varepsilon = 0.3$ (dotted line).

Imitation and herding

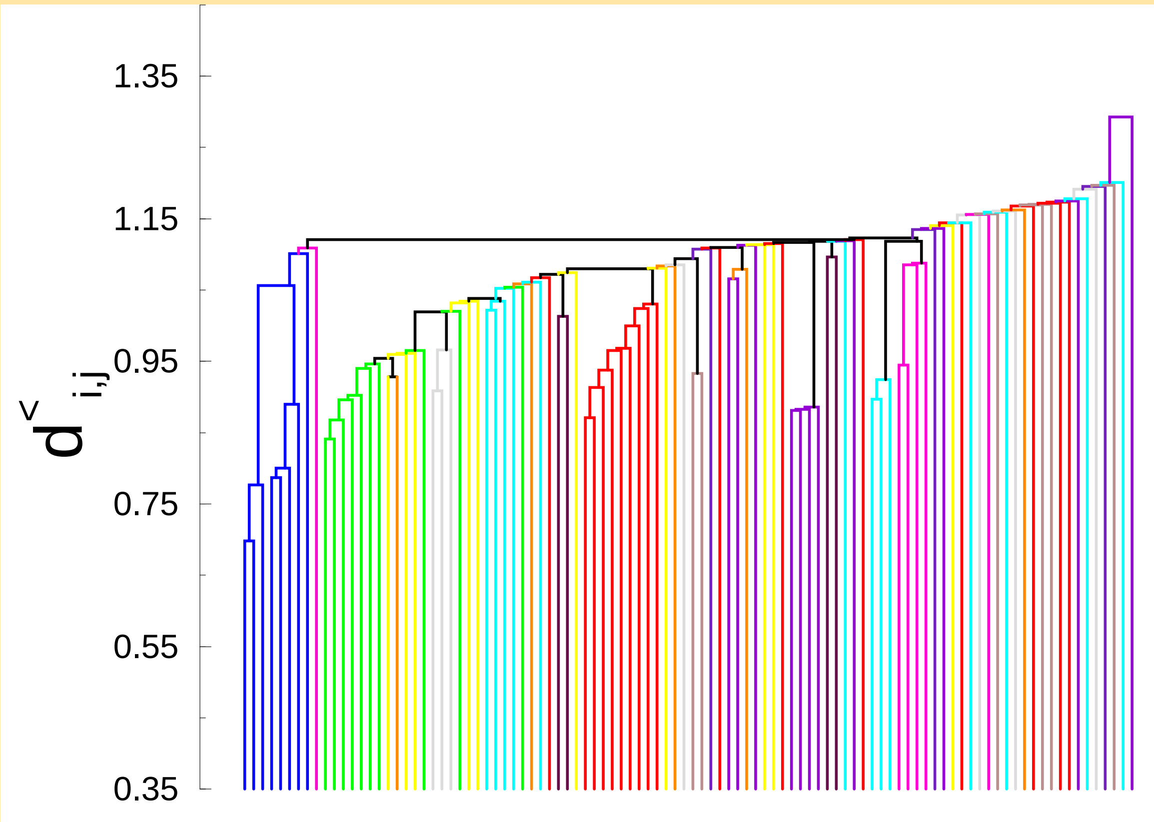
Random matrices

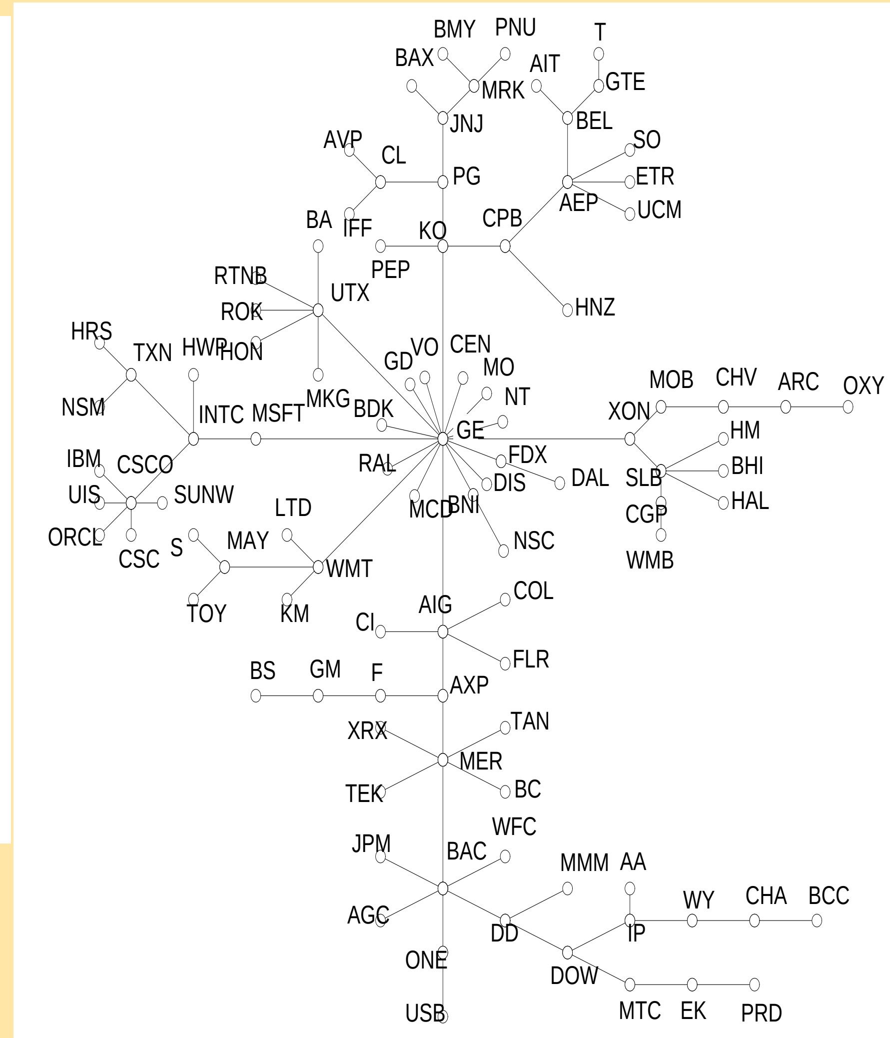
[L. Laloux, P. Cizeau, J.-P. Bouchaud, and M. Potters, Phys. Rev. Lett., 83, 1467 (1999); V. Plerou, P. Gopikrishnan, B. Rosenow, L. A. N. Amaral, and H. E. Stanley, Phys. Rev. Lett. 83, 1471 (1999).]



Industrial sectors can be attributed to each eigenvalue falling off the RMT spectrum.

Stock correlations [G. Bonanno, F. Lillo, R. N. Mantegna, Quantitative Finance 1, 96 (2001).]

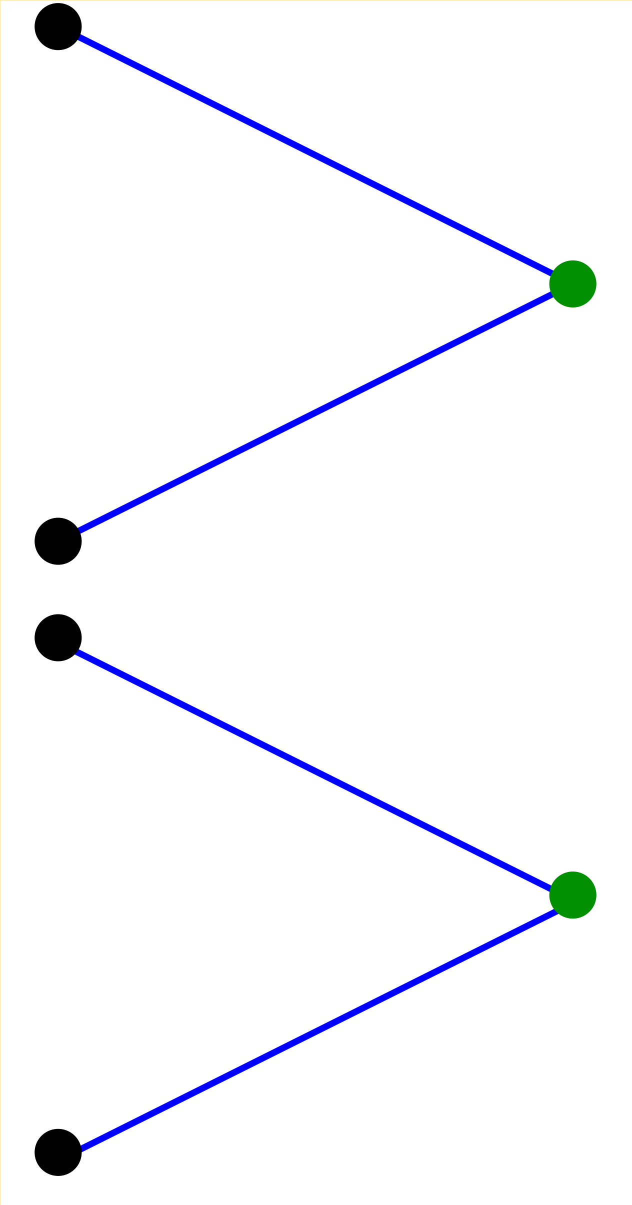




Ultrametric structure. Colors distinguish sectors, e.g. energy (blue), finance (green) etc. Time horizon 6h 30min.

Minimum spanning tree.

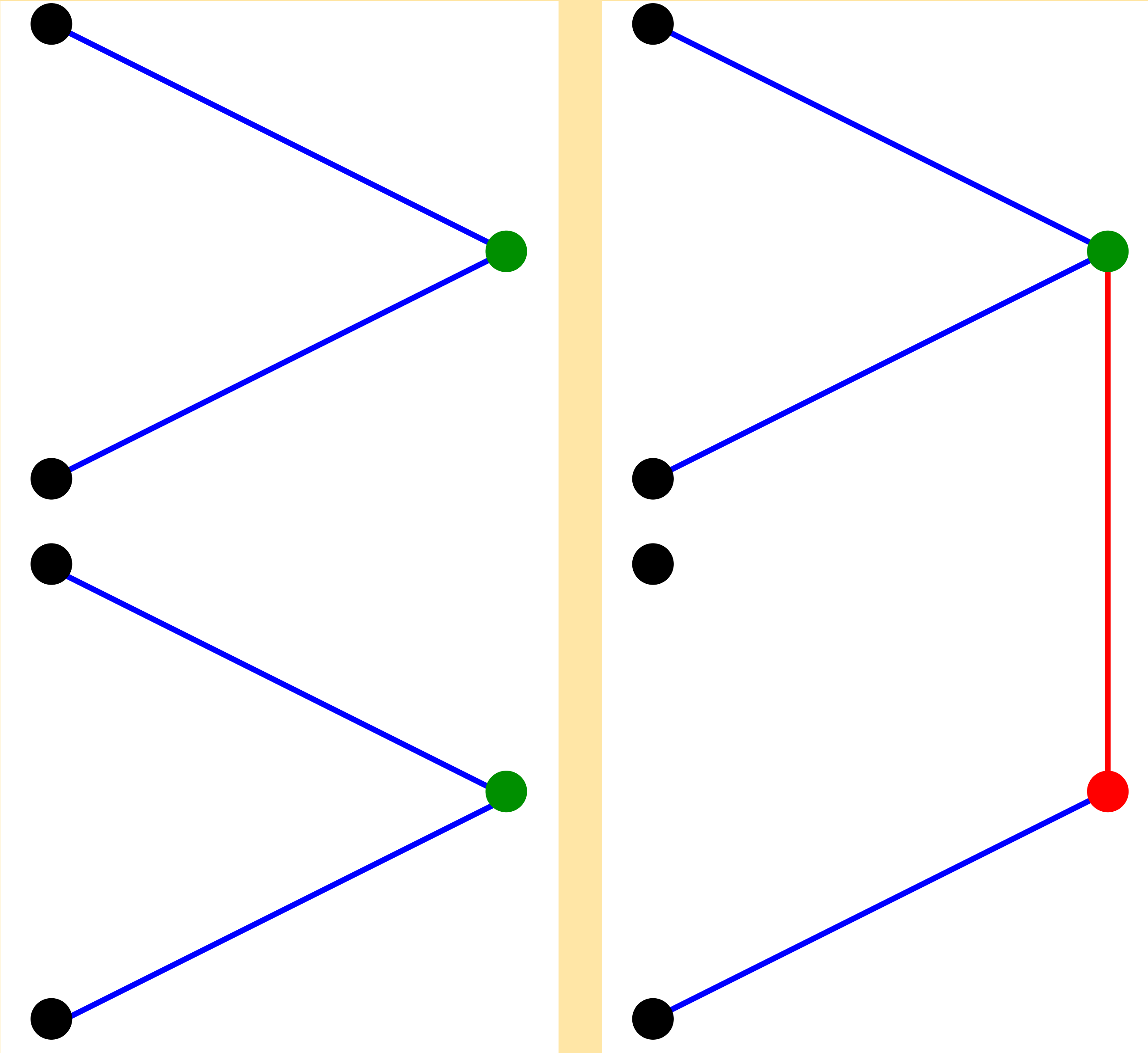
Imitation dilemma



No imitation:

fair game.

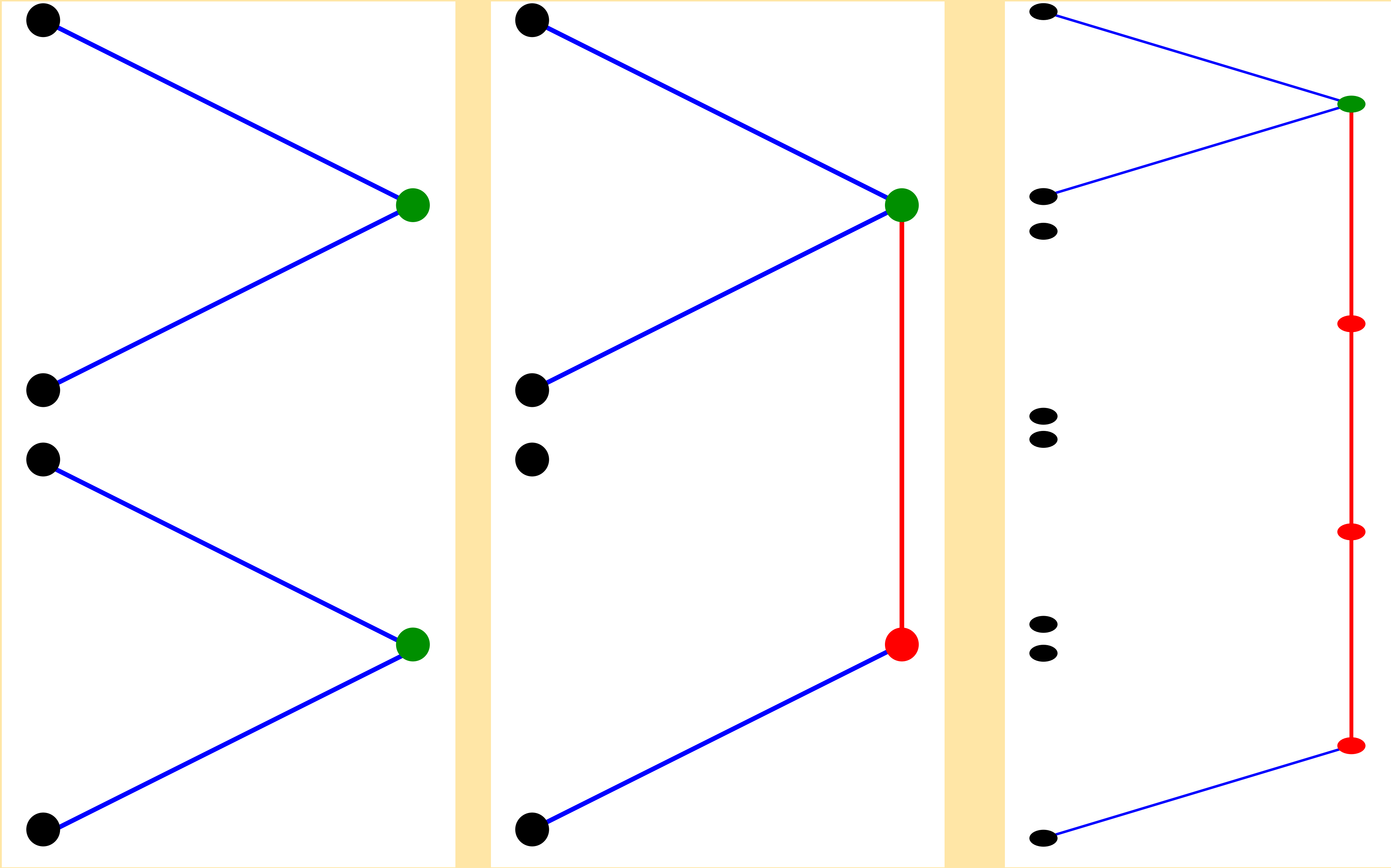
Imitation dilemma



No imitation: fair game.

Imitation provides comparative advantage.

Imitation dilemma



No imitation: fair game. | Imitation provides comparative advantage. | Too much imitation is dangerous.

Minority Game: [W. B. Arthur Amer. Econ. Review 84,406 (1994).
D. Challet and Y.-C. Zhang, Physica A 246, 407 (1997).]

El Farol bar attendance problem: go to bar (B) or stay at home (A)?

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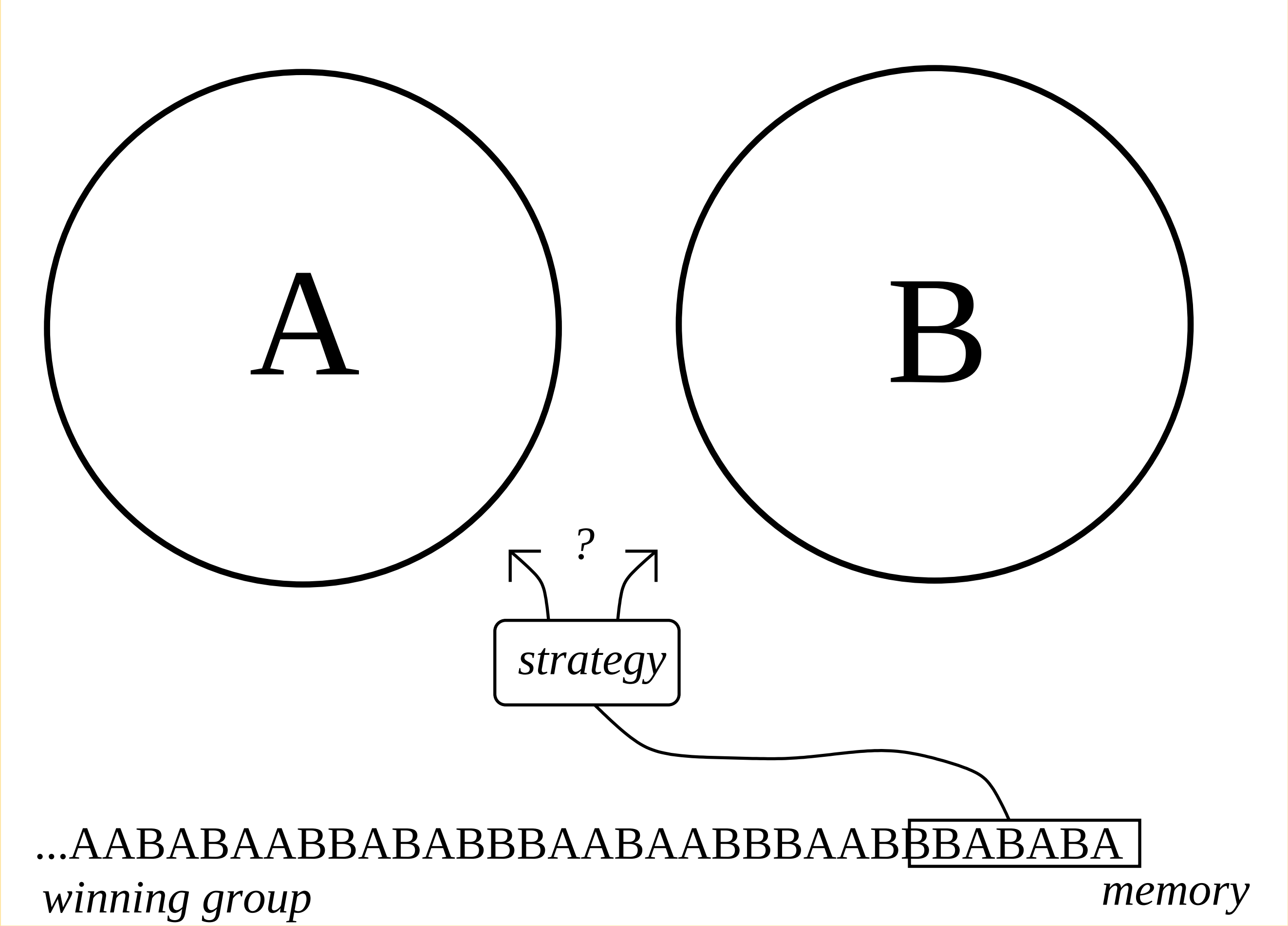
Abstract formulation:



N players

S strategies,

memory length M.

Strategy with highest score is chosen.

Minority Game:

[W. B. Arthur Amer. Econ. Review 84,406 (1994).
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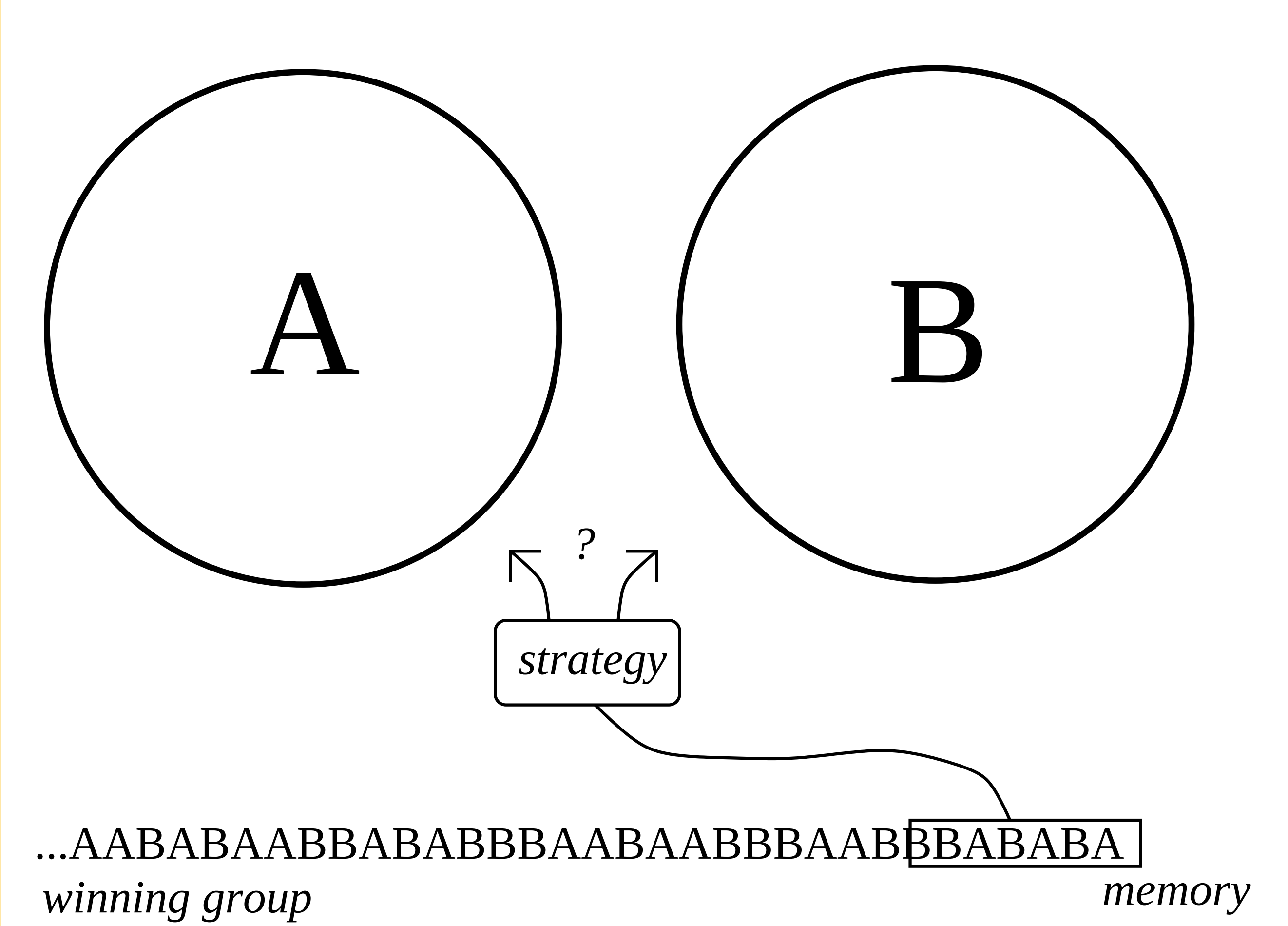

N players

S strategies,

memory length M.

Strategy with highest score is chosen.

Features: On-line adaptation. No optimal strategy possible.

Formalization:

Attendance: $A(t) = \sum_i a_i(t)$

Strategies' scores: $U_{j,s}(t+1) = U_{j,s}(t) - a_{j,s}^{\mu(t)}\, \mathrm{sign} A(t)$

Action: $a_j(t) = a_{j,s_M(t)}^{\mu(t)}$, where $U_{j,s_M(t)}(t) = \max_s U_{j,s}(t)$

Measure of ineffectivity: $\sigma^2 = \frac{1}{T} \sum_{t'=t-T}^{t} A^2(t)$

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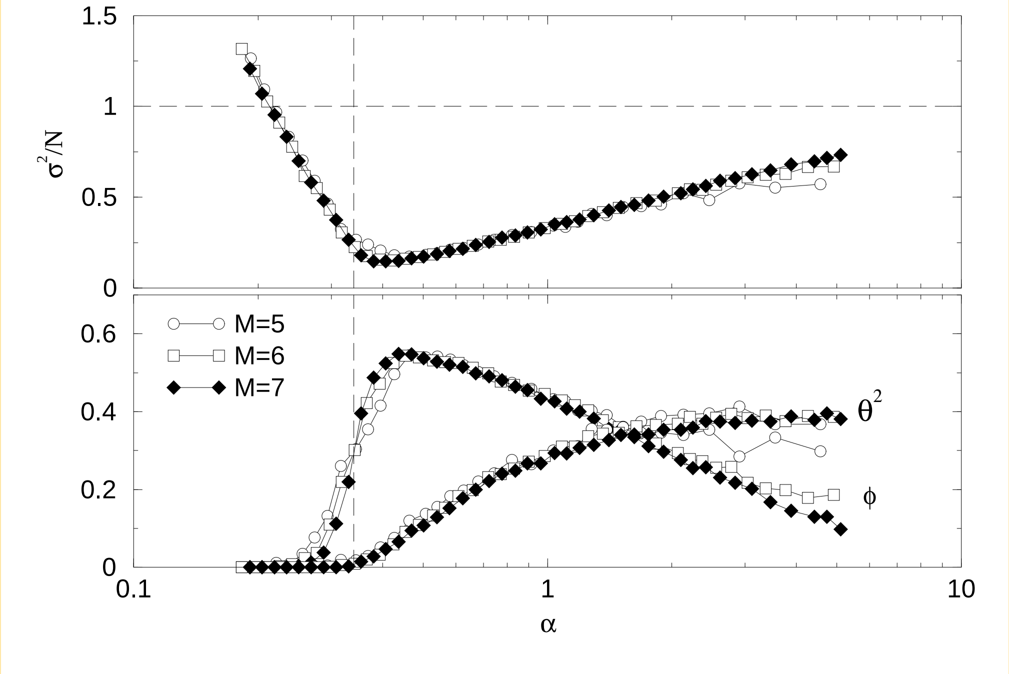
Simulations:



Top: σ^2/N versus $\alpha = 2^M/N$

with $M = 5, 6$ and 7.

Optimal value of the parameter $\alpha = 2^M/N$ exists ($\alpha_c \simeq 0.34$)

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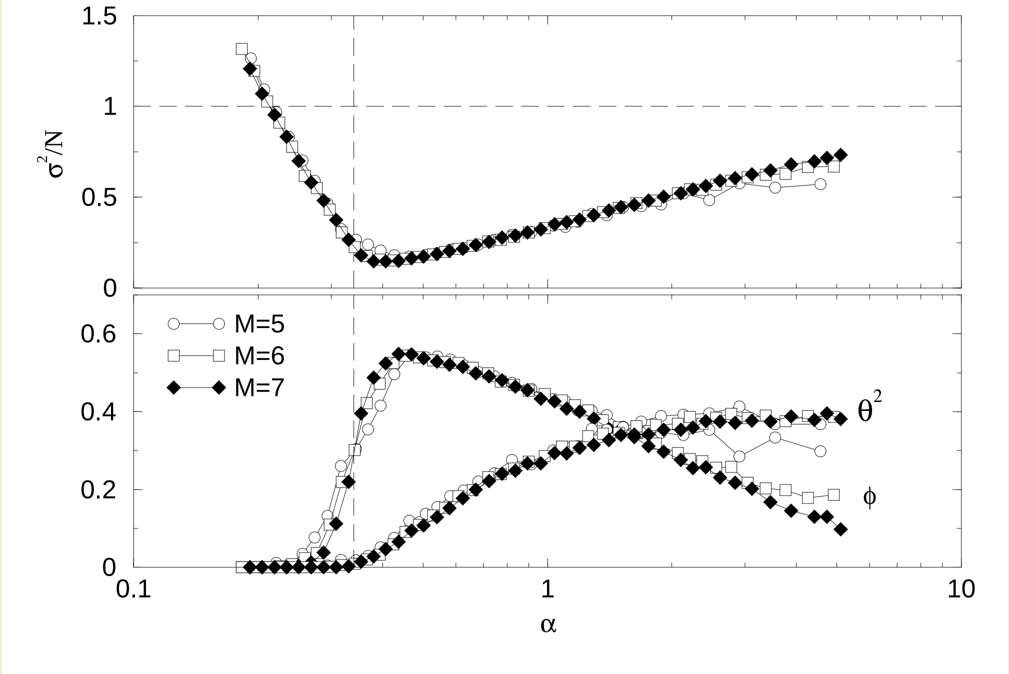

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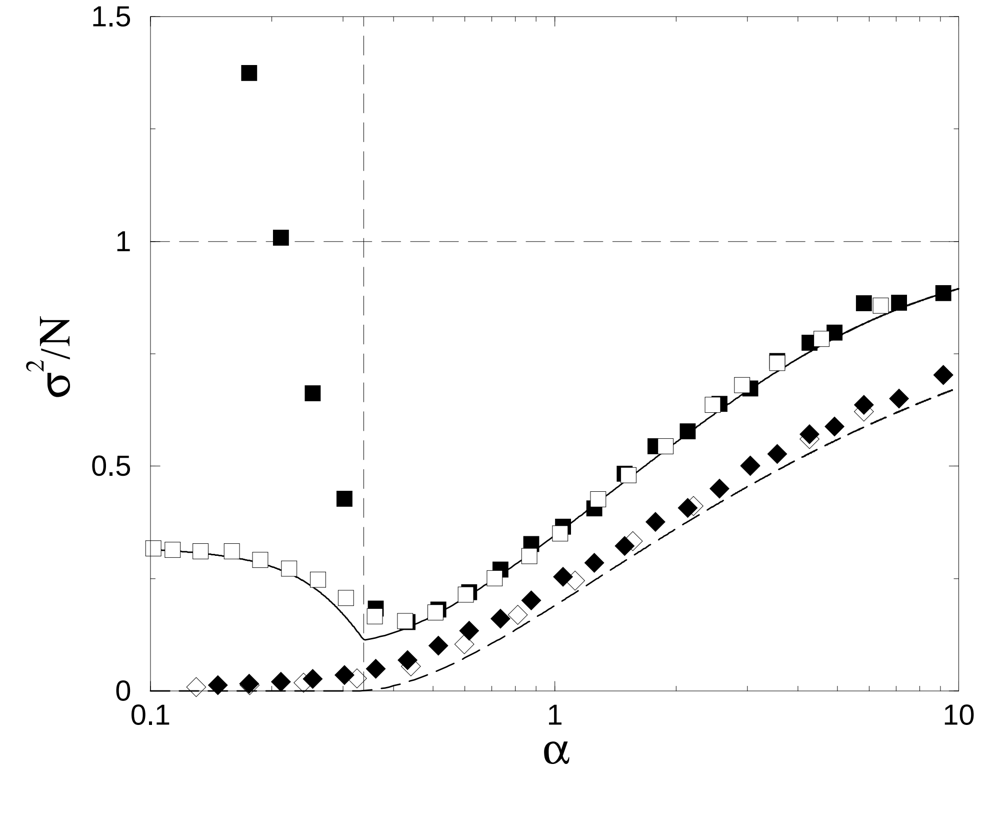
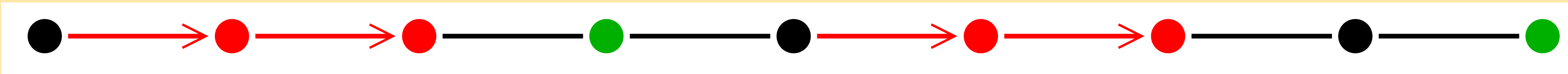
Analytical solution:

Replica method

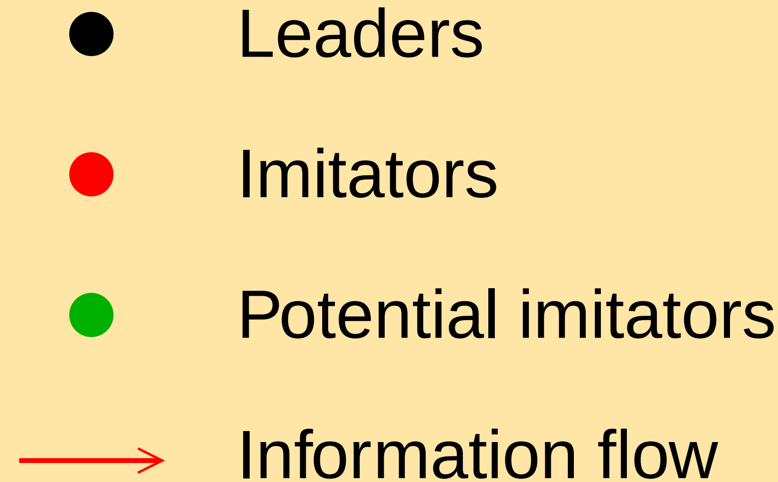


Social organization [F. S., Physica A 286, 367 (2000); Physica A 299, 334 (2001).]

Agents on social network imitate more successfull neighbors with probability p (and pay for it)



Agents are placed on linear chain. Imitation may occur along the link



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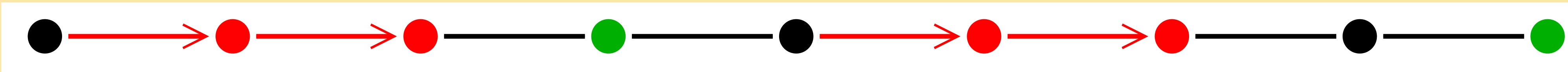

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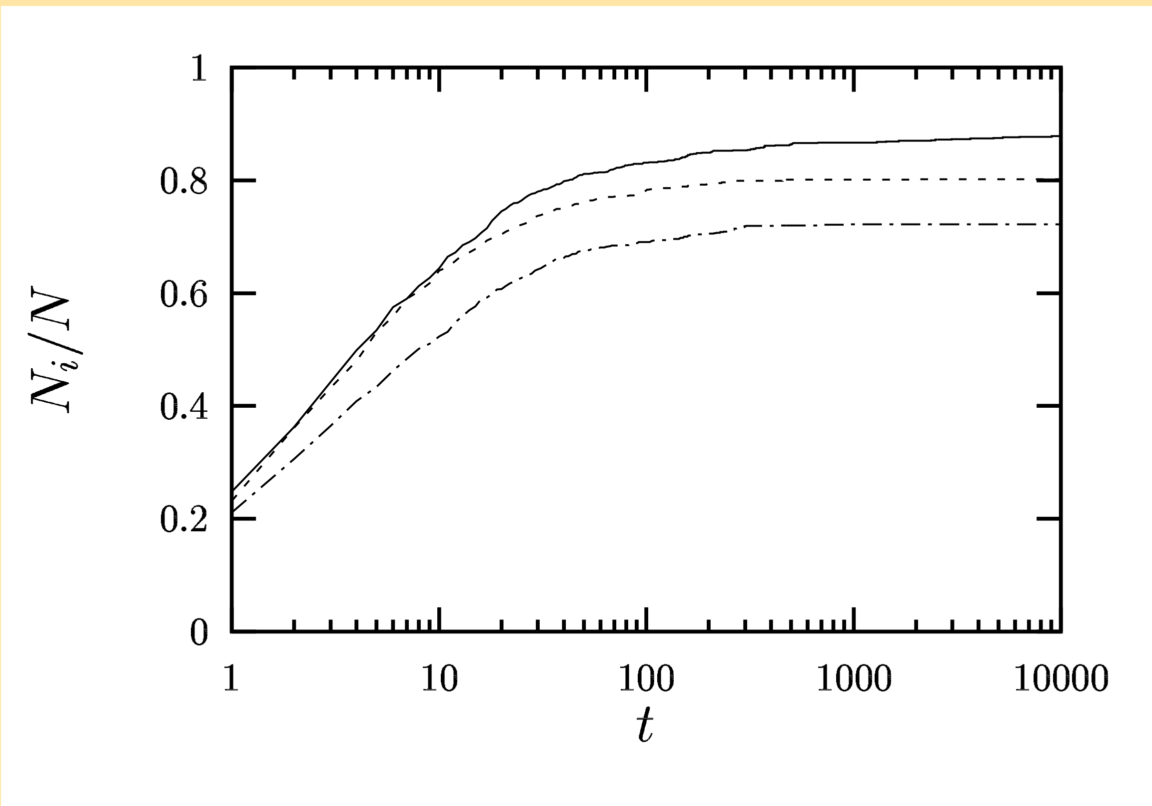
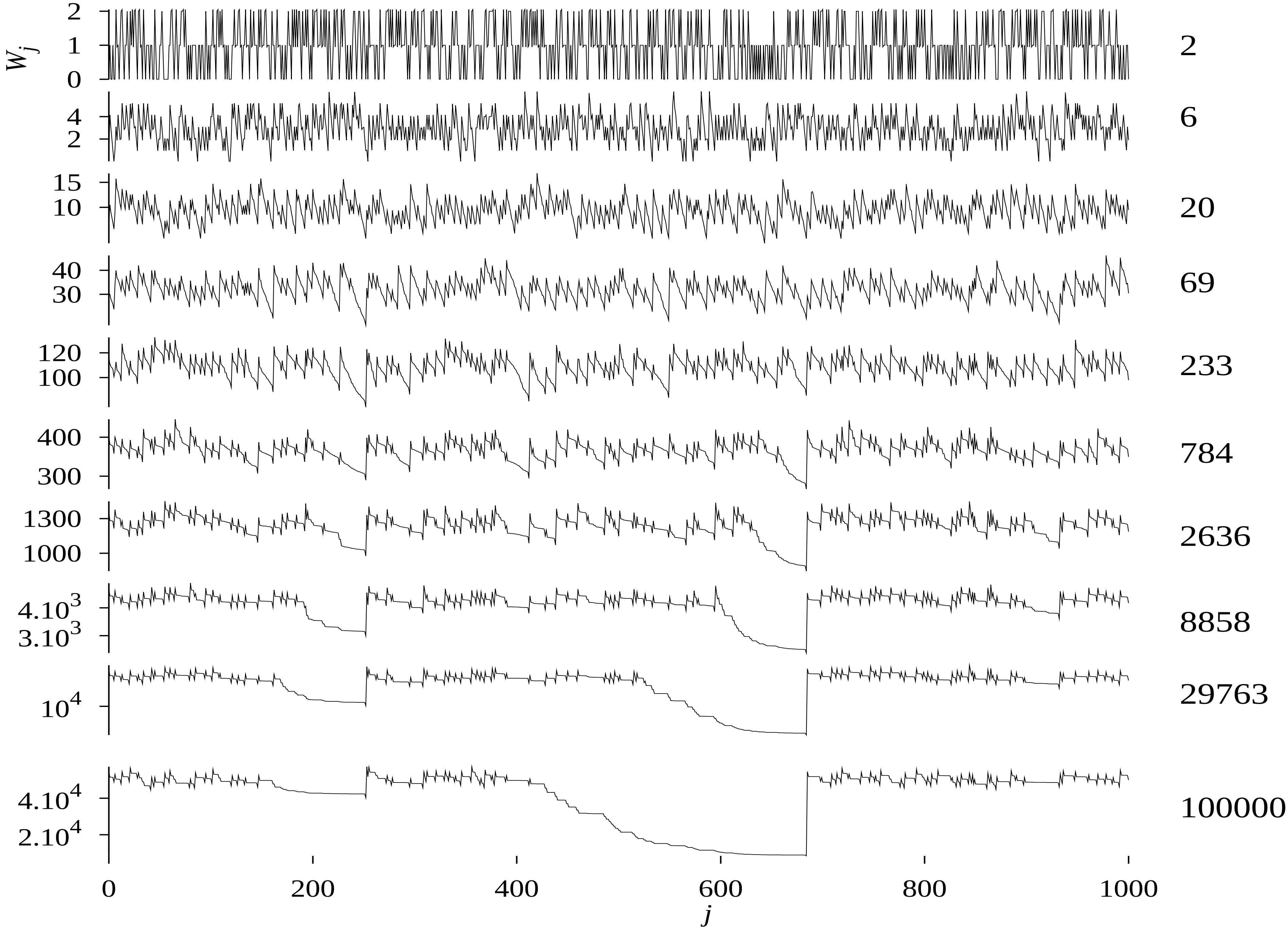
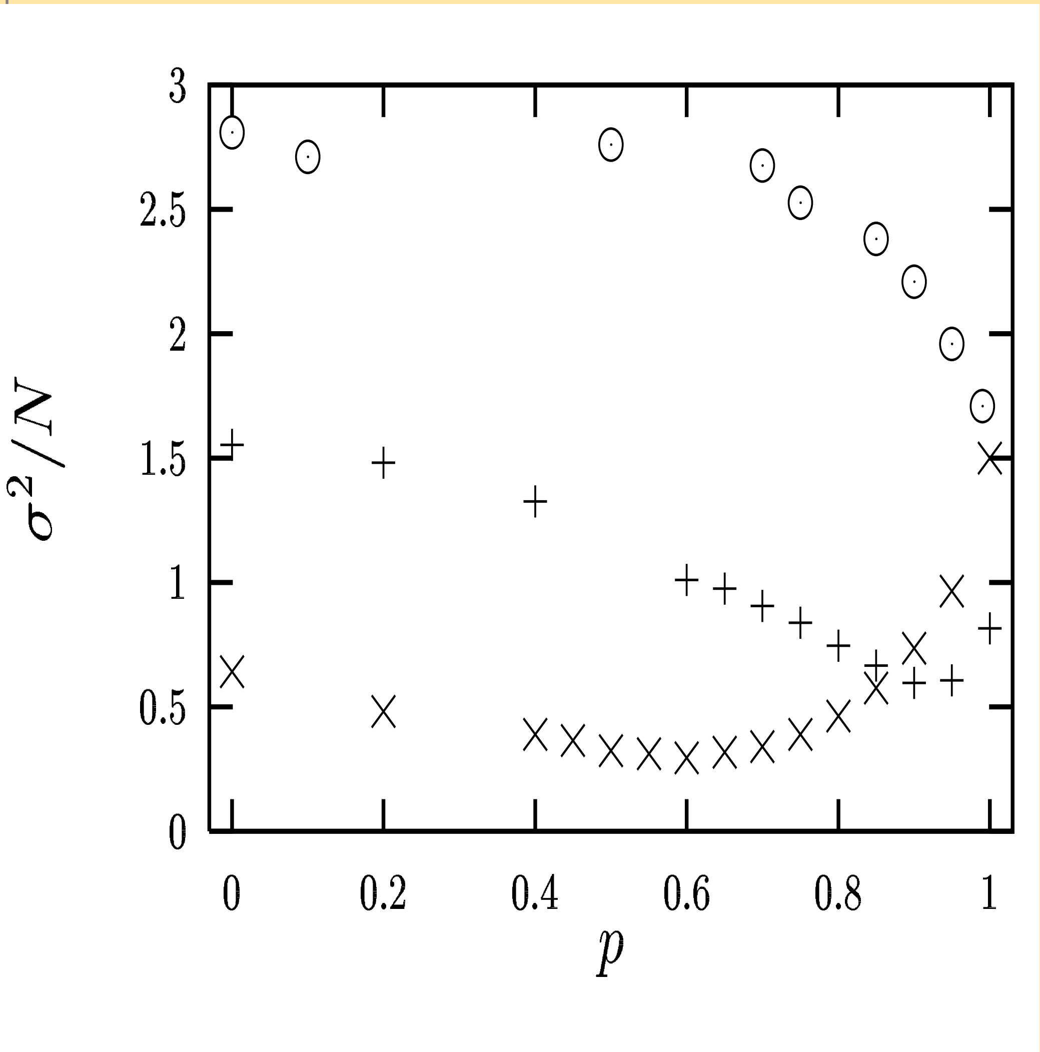


Time dependence of imitation. $p = 0.99$ (full line) 0.95 (dashed), 0.8 (dash-dotted)

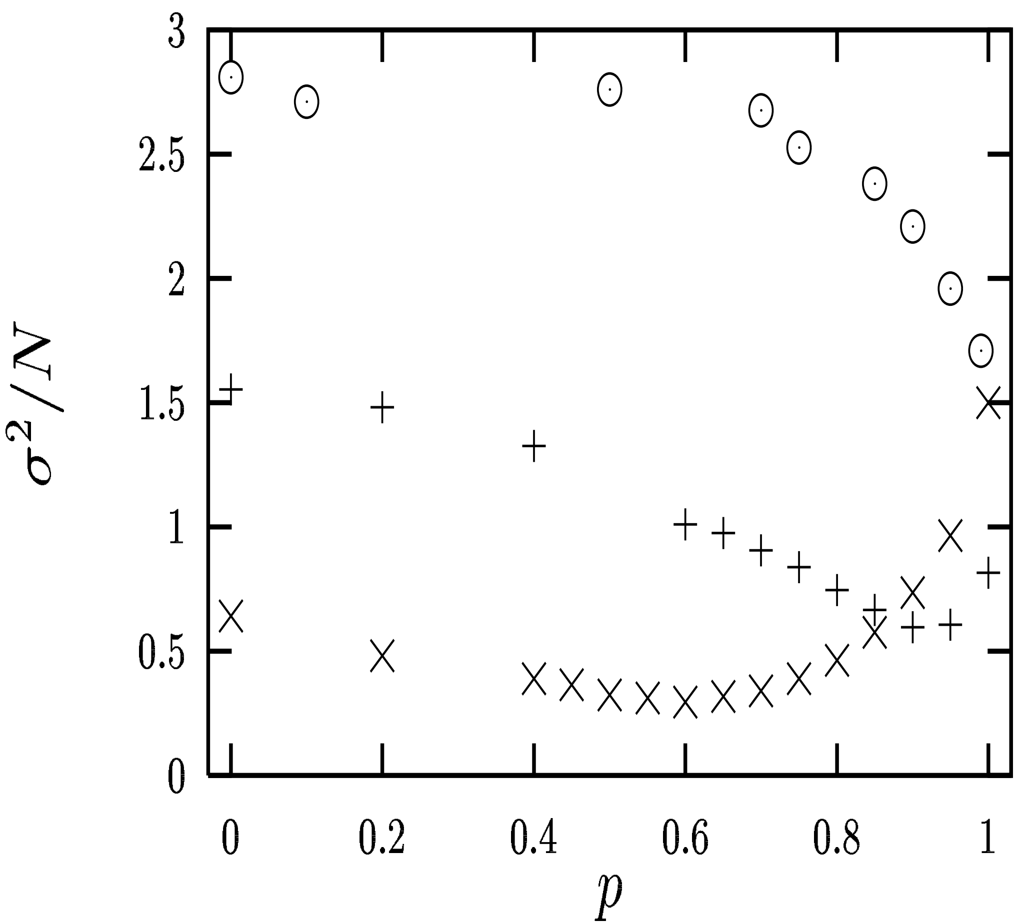
- ● Leaders
- ● Imitators
- ● Potential imitators
- → Information flow

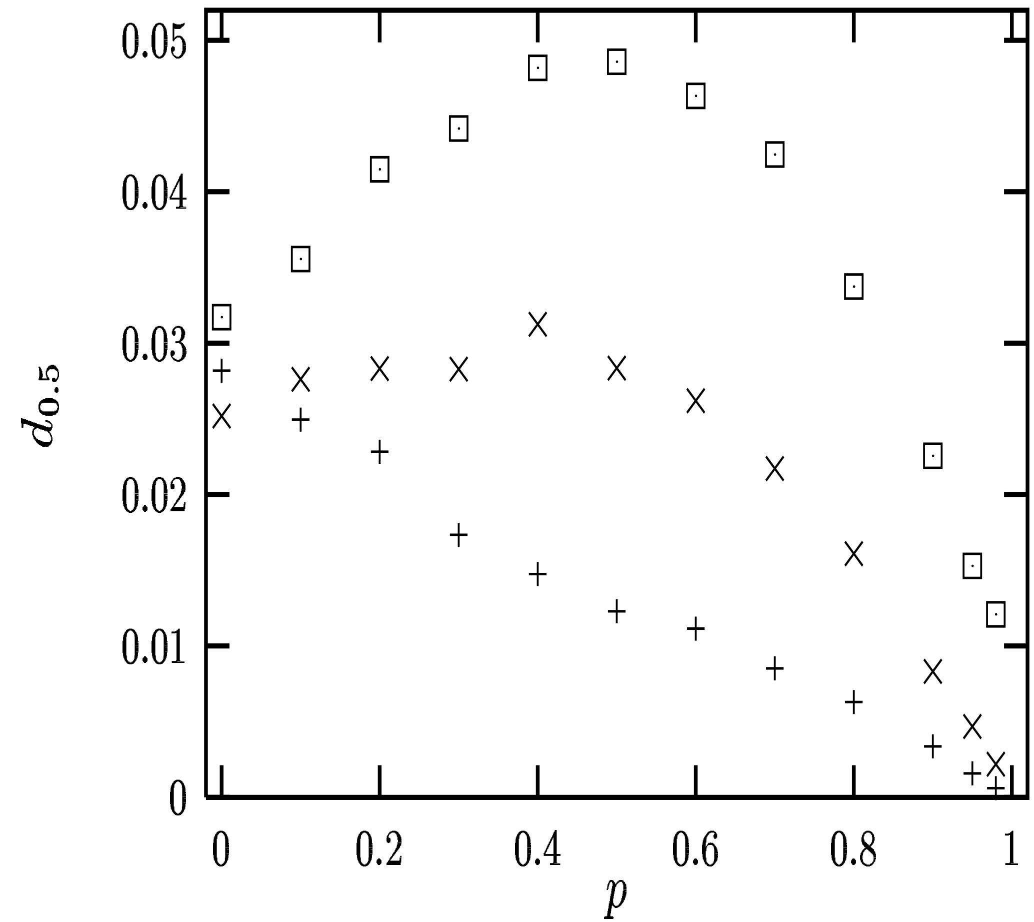


poverty islands created



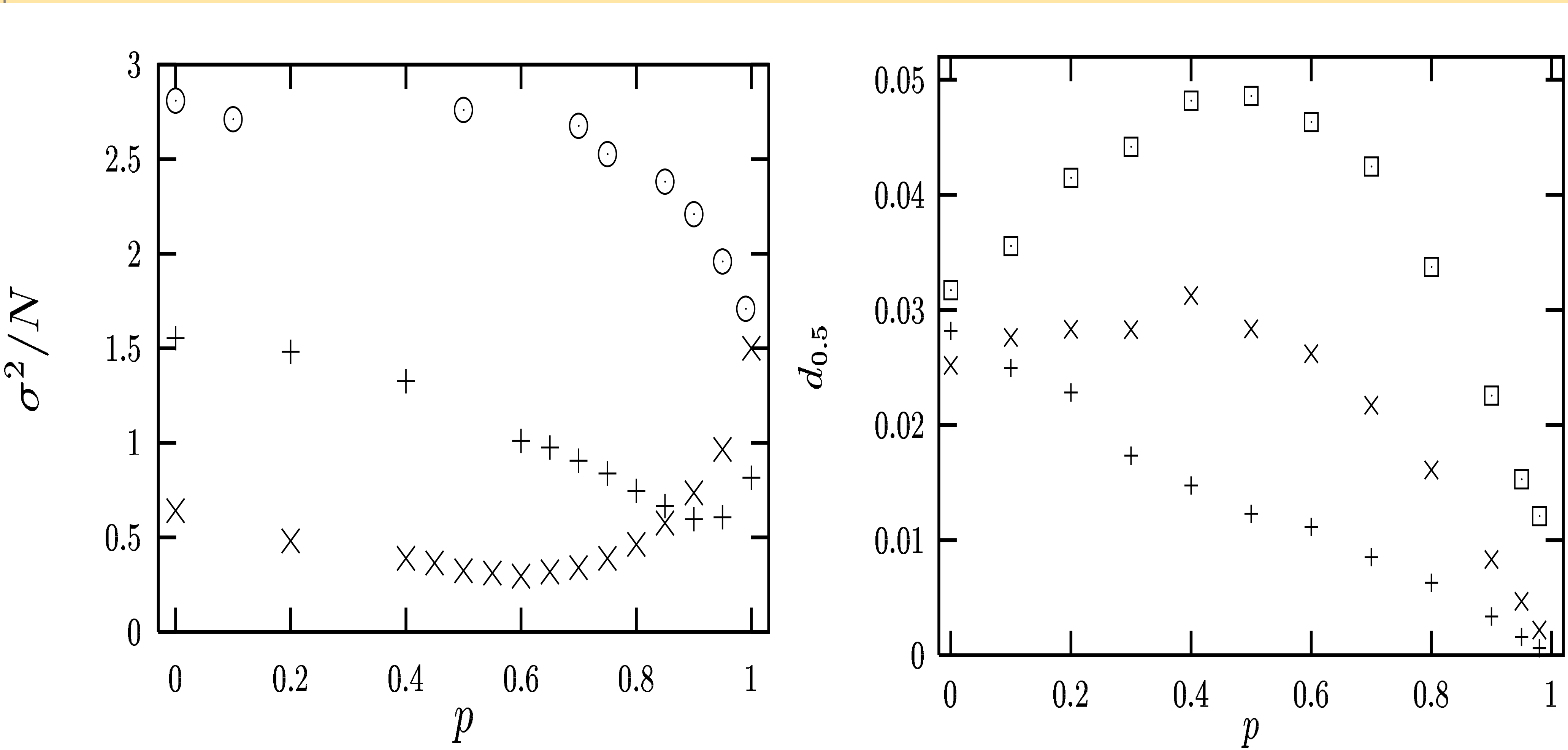
Effectivity optimized vs. imitation $N = 1001$, $M = 5$ ($\odot$), 6 ($+$), 7 ($\times$).





Effectivity optimized vs. imitation $N = 1001$, $M = 5$ ($\odot$), 6 ($+$), 7 ($\times$).

Tensions: information cost is essential. Comission $\varepsilon = 0.05$ ($\boxdot$), 0.03 ($\times$), 0.01 ($+$).



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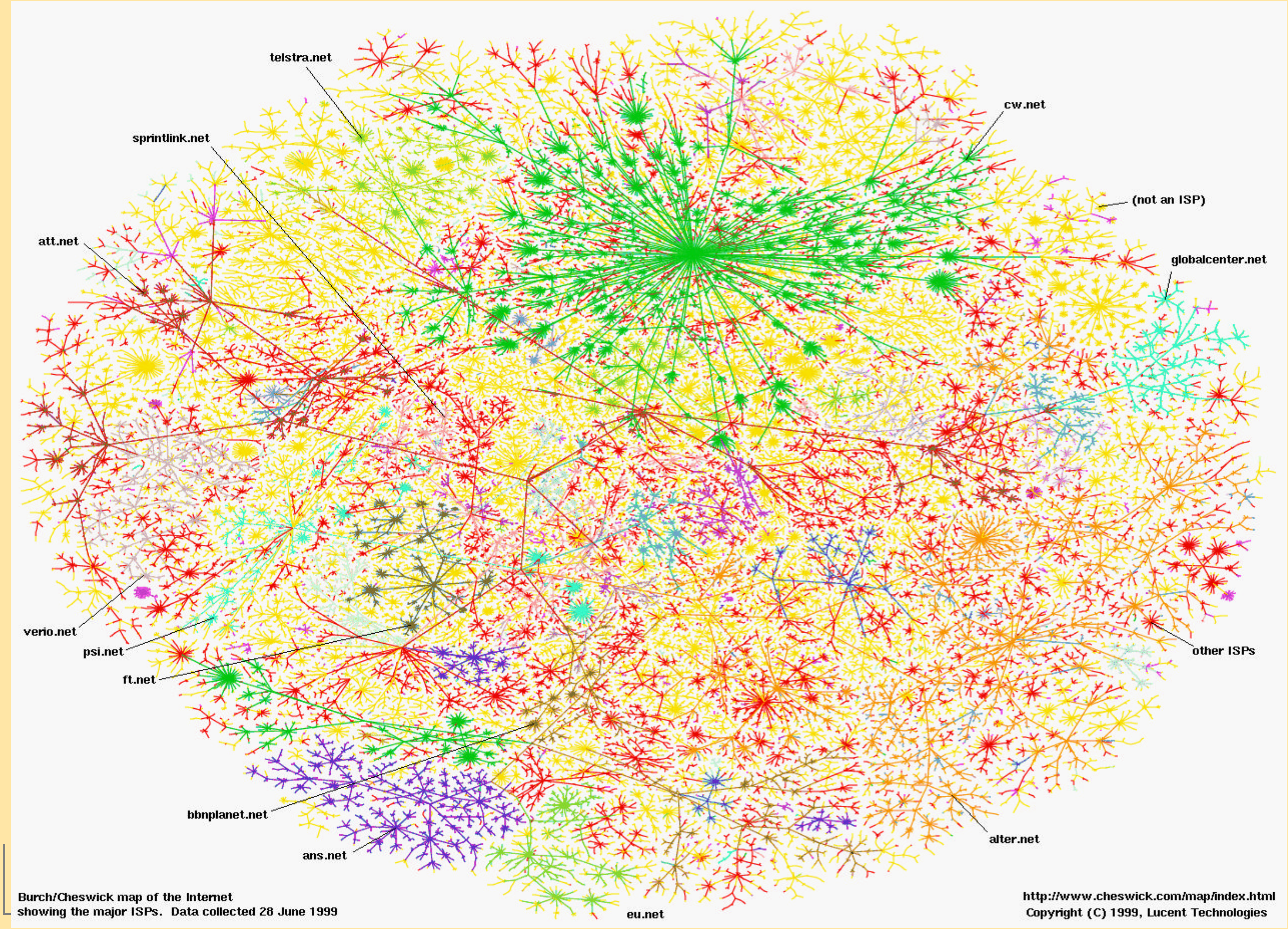
$$d_{0.5} = \frac{1}{\langle W \rangle} \left(\frac{1}{N} \sum_{j=1}^{N-1} |W_j - W_{j+1}|^{1/2} \right)^2 \qquad \langle W \rangle = \frac{1}{N} \sum_{j=1}^{N} W_j$$

Sociophysics

Opinion spreading: Sznajd model

[K. Sznajd-Weron and J. Sznajd, Int. J. Mod. Phys. C 11, 1157 (2000).]

N agents on a social network



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Opinions $\sigma_i \in \{1, -1\}$. The state of the system $\Sigma = [\sigma_1, \sigma_2, ..., \sigma_N]$. performs a discrete-time Markov process.

I. original model: two against one

$$+ + - \longrightarrow + + + \ , \ - - + \longrightarrow - - -$$

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$+ + - \longrightarrow + + +$, $- - + \longrightarrow - - -$

II. Ochrombel simplification: one against one (Voter model)

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Two absorbing states: uniform $\Sigma_+ = \{1\}^N$ and $\Sigma_- = \{-1\}^N$.

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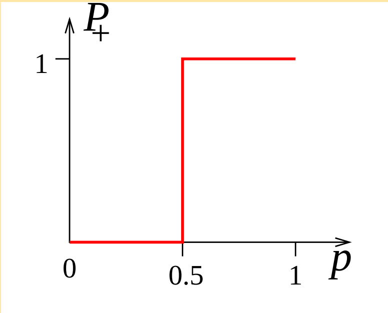
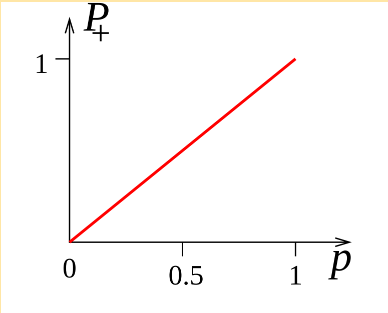
Two absorbing states: uniform $\Sigma_+ = \{1\}^N$ and $\Sigma_- = \{-1\}^N$.

$P_+(p)$ probability of hitting Σ_+ for initial condition $p = N_+/N$:

I. transition: $P_+(p) = \theta(p - 1/2)$

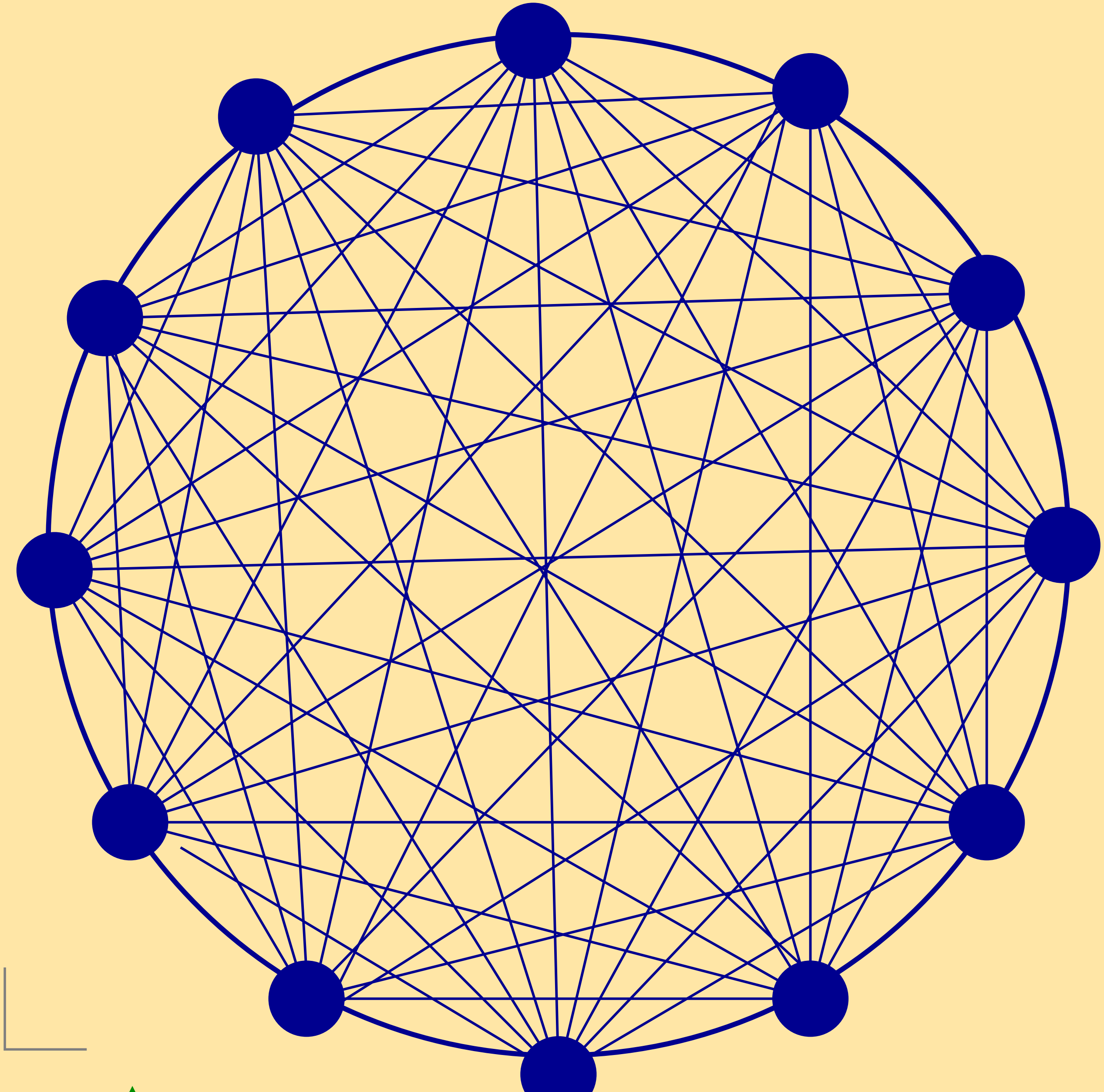


II. no transition $P_+(p) = p$



Solution of Sznajd model [F.S. and H. Lavička, Eur. Phys. J. B 35, 279 (2003).]

Approximation: complete graph ("mean-field")



State fully described by "magnetization", $m = \frac{N_+ - N_-}{N}$.

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Fokker-Planck equation:

I. $$\frac{\partial}{\partial \tau} P_m(m,\tau) = -\frac{\partial}{\partial m}\left[(1-m^2) m\, P_m(m,\tau)\right]$$

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Solution:

I. $P(x,\tau) = [(1-x^2)x]^{-1} \, f(\mathrm{e}^{-\tau} \, \frac{x}{\sqrt{1-x^2}})$

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l	c_l	$\Phi_c(x)$
0	2	1
1	6	x
2	12	$1-5x^2$
3	20	$x-\frac{7}{3}x^3$
4	30	$1-14x^2+21x^4$
$\vdots$	$\vdots$	$\vdots$

Solution:

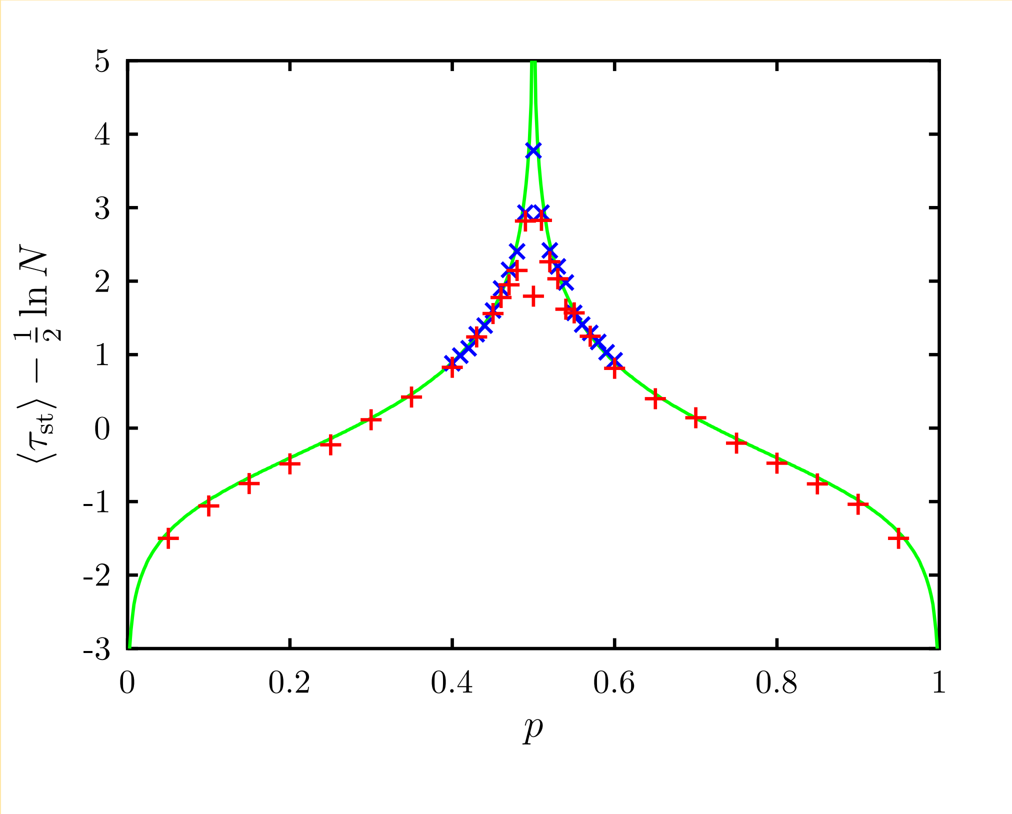
I. $P(x,\tau) = [(1-x^2)x]^{-1} f(\mathrm{e}^{-\tau}\frac{x}{\sqrt{1-x^2}})$

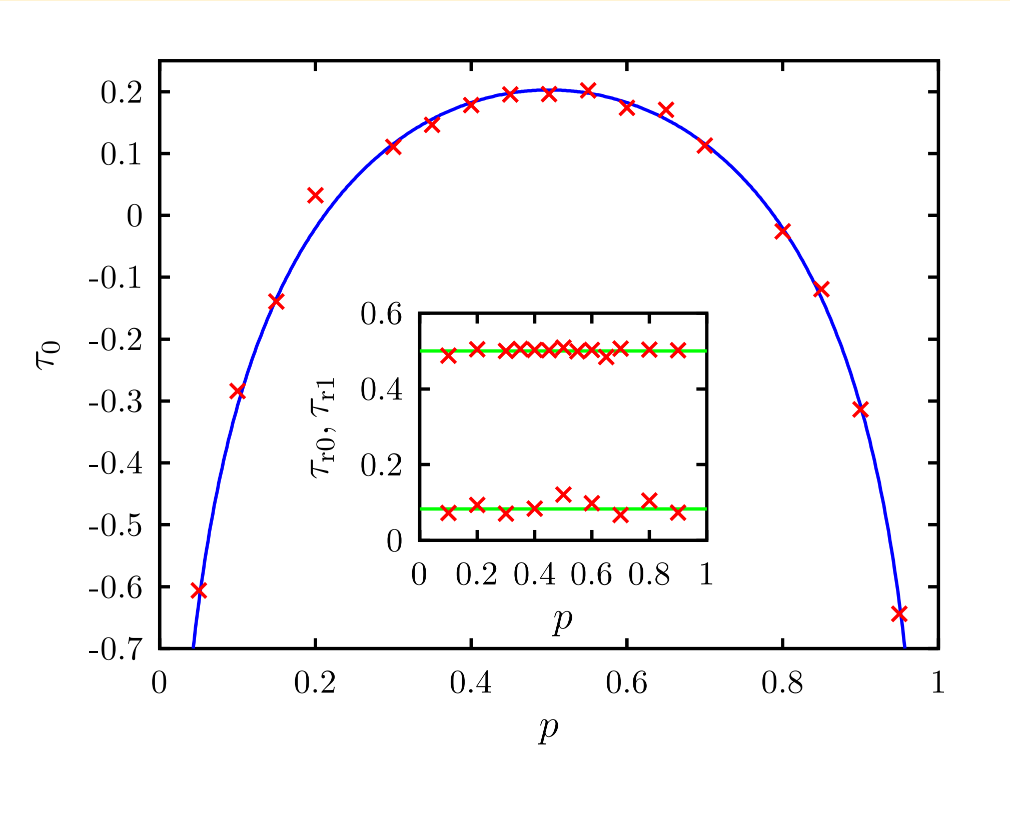
II. Gegenbauer polynomials:

$P(x,\tau) = \sum_c A_c \mathrm{e}^{-c\tau}\,\Phi_c(x)$

$$\langle \tau_{\rm st} \rangle \simeq -\ln\left(\frac{|2p-1|}{\sqrt{p(1-p)}}\, \frac{1}{\sqrt{N}} \right)$$

$$\tau_0 \simeq \ln\sqrt{6\,p\,(1-p)}$$



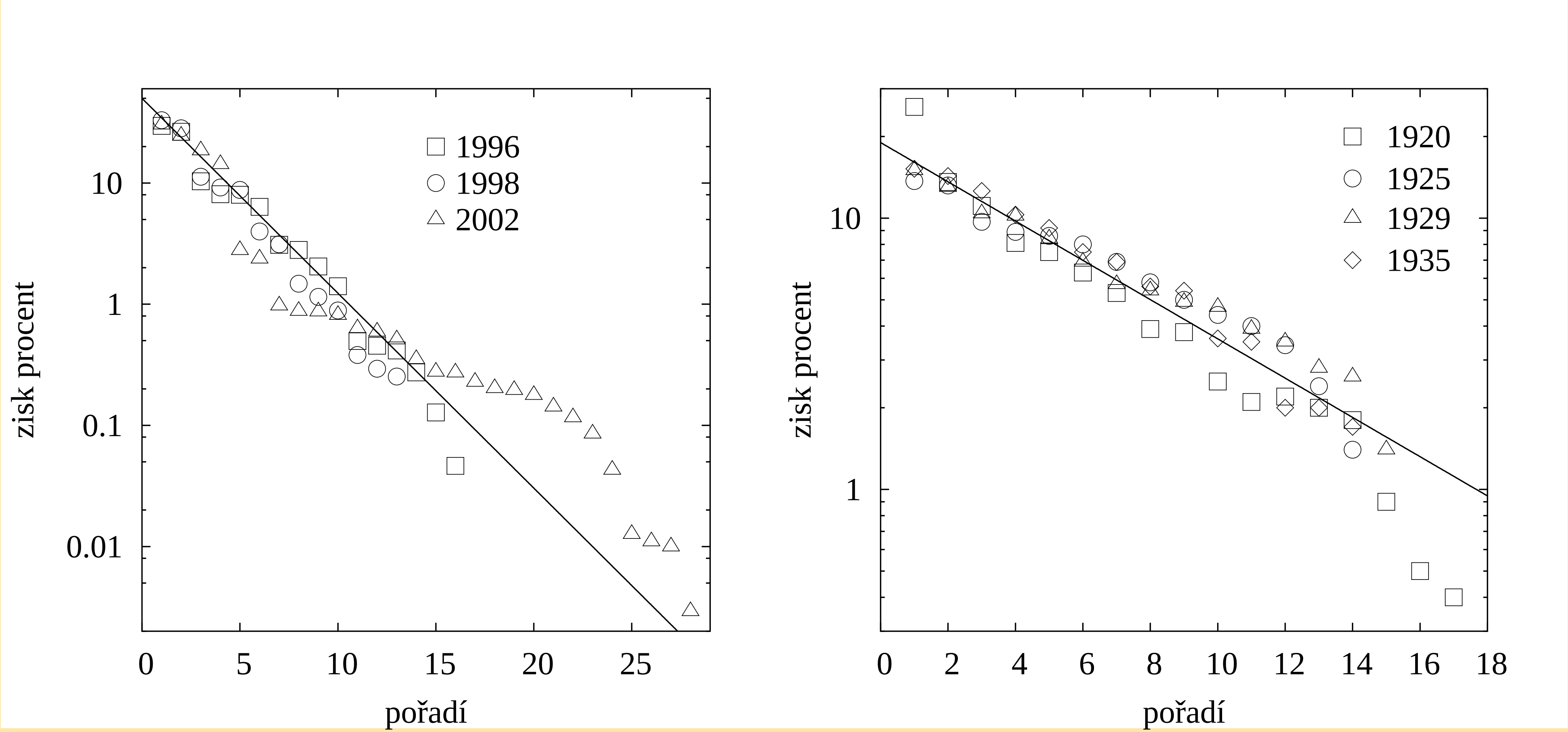


Average time of reaching the absorbing state, case I.

Average time to reach absorbing state, case II.

Elections

Stable patterns of election results:



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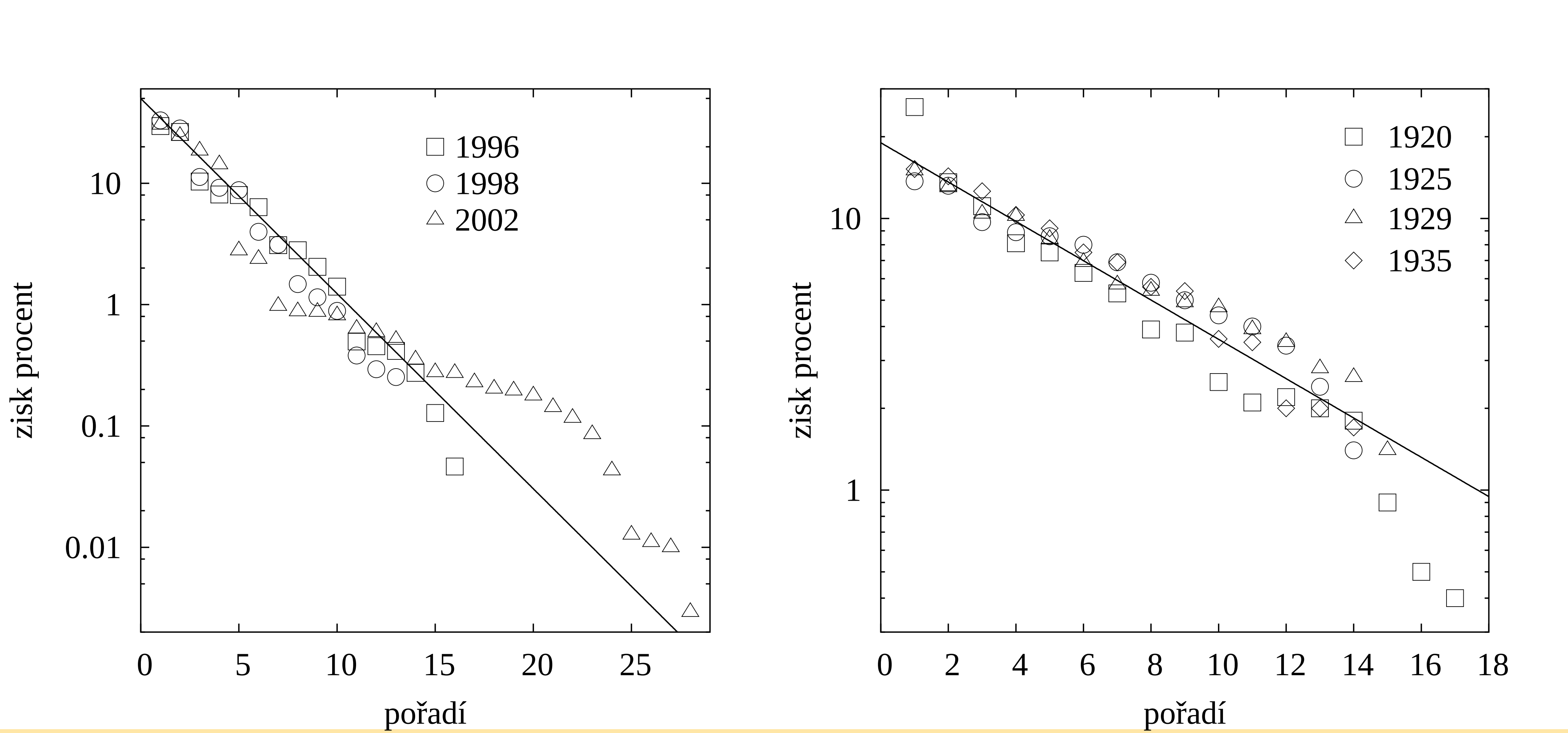

Distribution of votes: $P(n) \propto n^{-1}$

Sznajd model for elections

Choice from q parties: Potts variables $\sigma_i \in \{1, 2, ..., q\}$

Distribution of votes: $D(n) = \frac{N}{q} \sum_{\sigma=1}^{q} \delta\left(n - n_\sigma\right)$ $\quad n_\sigma = \frac{1}{N} \sum_i \delta(\sigma_i - \sigma)$

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Neglect fluctuations: $D(n) \rightarrow P_n(n) \equiv \langle D(n) \rangle$

Fokker-Planck equation: $x = 2n - 1$

$$\frac{\partial}{\partial \tau} P_n(x, \tau) = \frac{\partial^2}{\partial x^2} \left[\left(1 - x^2\right) P_n(x, \tau)\right]$$

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$$\frac{\partial}{\partial \tau} P_n(x, \tau) = \frac{\partial^2}{\partial x^2}\left[\left(1 - x^2\right) P_n(x, \tau)\right]$$

Open system:

Non-normalizable stationary solutions:

$$\tilde{\Phi}(x) = \frac{1}{1 \pm x} \qquad \Rightarrow \qquad P_n(n) \simeq \frac{1}{n}$$

...as in Brazil or Czech Republic...

...let us tune election system now!

Conclusions I: what we learned?

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- Microscopic models explain power-laws

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- Transfer of know-how
- Provide human and machine calculation capacities

Conclusions III: what is Economics good for?

- New ideas, new Hamiltonians
- Access to funding

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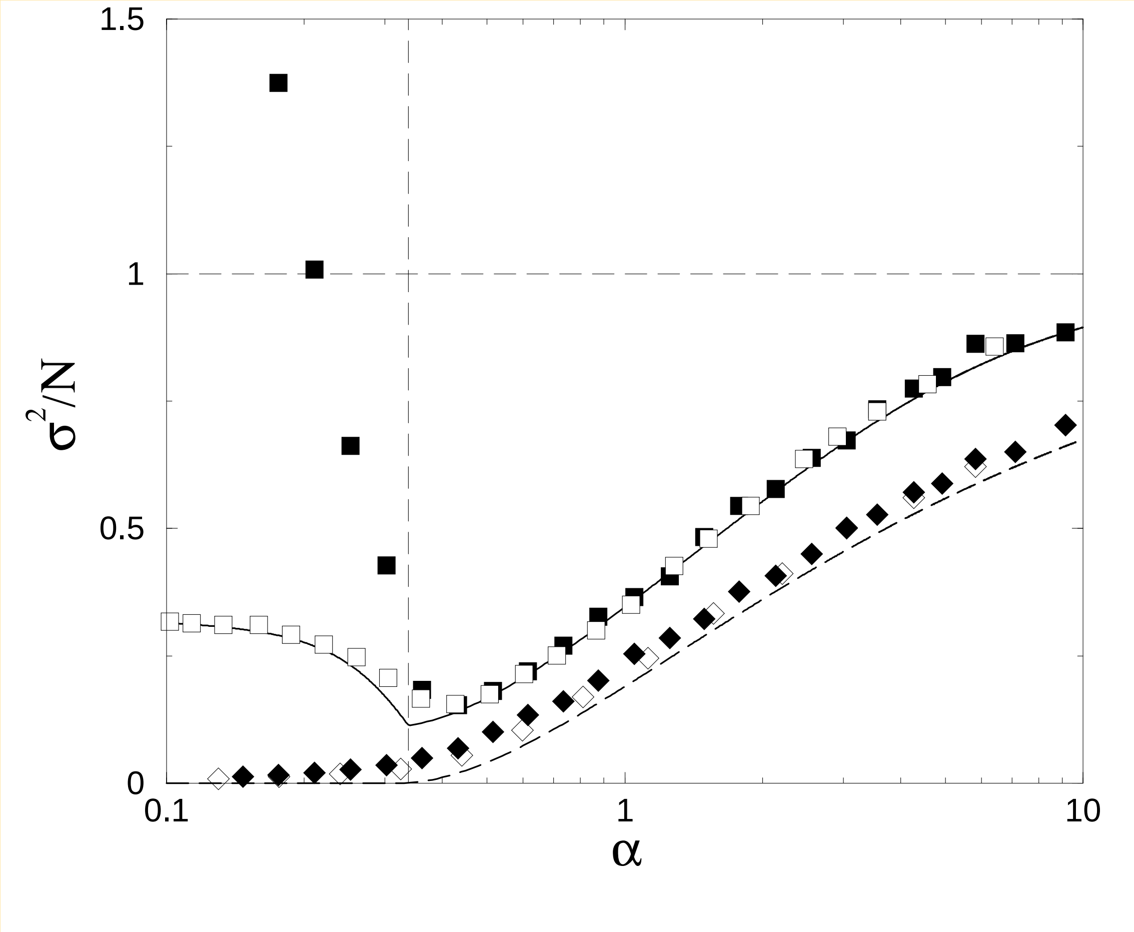
Analytical solution:

- Irrelevance of memory: one of P randomly chosen states ($P = 2^M$) labelled by an integer $\mu = 1, \ldots, P$. $\overline{\bullet} = \frac{1}{P}\sum_\mu \bullet$
- Strategies $a^\mu_{i,s} = \omega^\mu_i + s\xi^\mu_i$, $s \in \{-1, 1\}$, $m_i = \langle s_i \rangle$
- Hamiltonian: $H = \overline{\Omega^2} + 2\sum_i \overline{\Omega\xi_i} m_i + \sum_{ij} \overline{\xi_i\xi_j} m_i m_j$
- Replica method [D. Challet, M. Marsili, and R. Zecchina, Phys. Rev. Lett. 84, 1824 (2000).]



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