Anomalous Hall transport in metallic spin-orbit coupled systems: Spin-Injection Hall effect

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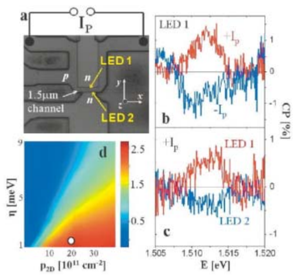
Institute of Physics, ASRC, Prague, November 18th 2008

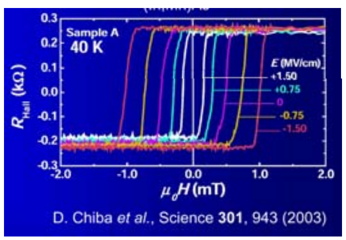
Research fueled by:

The family of spintronic Hall effects



SHE
B=0
charge current gives spin current

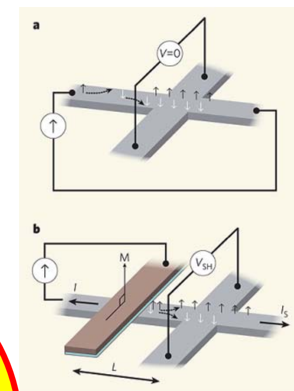
Optical detection



Electrical detection

AHE
B=0
polarized charge current gives charge-spin current

SHE^{-1}
B=0
spin current gives charge current



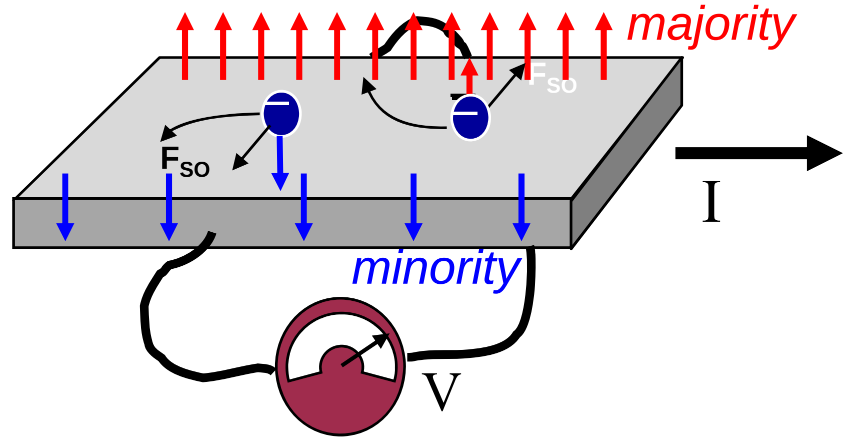
Electrical detection

OUTLINE

1) The basic phenomena of AHE
2) Cartoon of mechanisms contributing to the AHE
3) Semiclassical theory of AHE:
    a) Wave-packets of Bloch electrons: birth of Berry's connection
    b) Dynamics of Bloch electron wave-packets: birth of Berry's curvature
    c) Boltzmann Eq. for Bloch wave-packets
    d) Back to the three main mechanisms: clarifying/correcting popular believes
4) Microscopic theory of AHE (Kubo approach)
    a) Kubo formula microscopic approach to transport
    b) Does it match the semiclassical approach?
    c) Other microscopic approaches
5) Spin-injection Hall effect: a new tool to explore spintronic Hall effects
    a) Device schematics
    b) Experimental observation
    c) Theory Modeling

Anomalous Hall effect

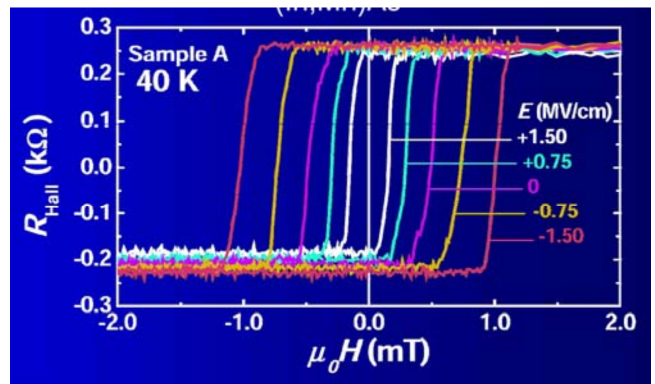
*Spin dependent "force" deflects **like-spin** particles*



$$\rho_H = R_0 B_\perp + 4\pi R_s M_\perp$$

$$R_0 << R_s$$

Simple electrical measurement of magnetization



D. Chiba *et al.*, Science **301**, 943 (2003)

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \approx \frac{1}{\sigma_{xx}}$$

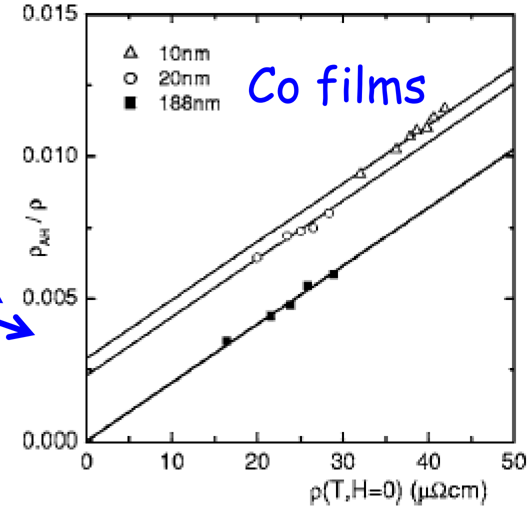
$$\rho_{xy} = \frac{-\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} \approx \frac{-\sigma_{xy}}{\sigma_{xx}^2} \approx -\sigma_{xy}\rho_{xx}^2 \approx -A\rho_{xx} - B\rho_{xx}^2$$

$$\sigma_{xy} \approx B + A\sigma_{xx}$$

Anomalous Hall effect (scaling with ρ)

$$\rho_{xy} = -A\rho_{xx} - B\rho_{xx}^2$$

$$\sigma_{xy} \approx B + A\sigma_{xx}$$



Kotzler and Gil PRB 2005

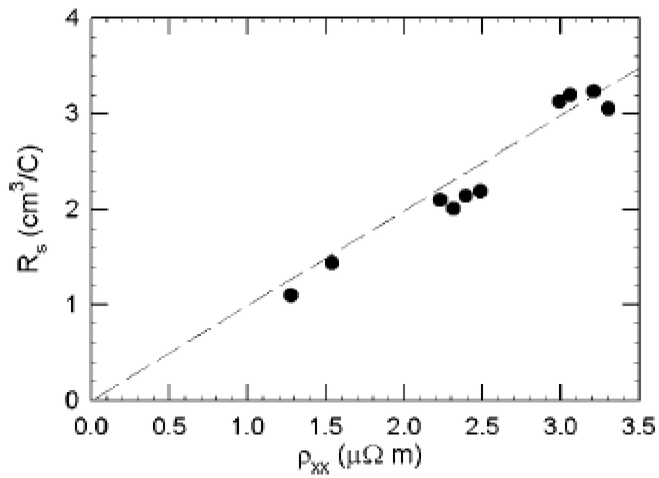


FIG. 5. Anomalous Hall coefficient R_S as a function of electrical resistivity ρ for $Sb_{2-x}Cr_xTe_3$. Data are taken from all samples with $x \geqslant 0.031$ and at temperatures ranging from 2 K up to the respective Curie temperatures. The dashed line illustrates the relation $R_S = c\rho^1$, which is consistent with AHE due to skew scattering.

Dyck et al PRB 2005

Weak SO coupled regime

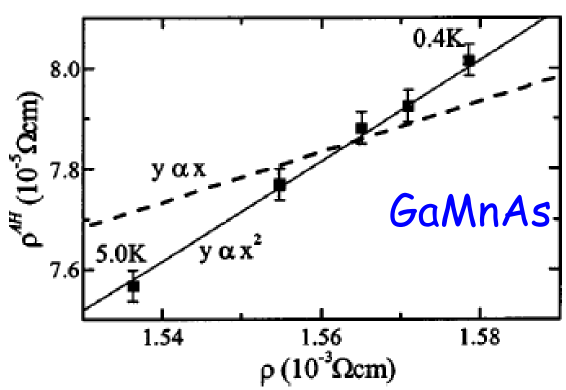


FIG. 3. Anomalous Hall resistivity ρ_{xy}^{AH} vs longitudinal resistivity ρ at fixed $B = 10$ T and varying T between 0.4 and 5.0 K for the $x = 0.06$ sample (squares); best fit lines assuming $R^{AH} \propto \rho$ (broken line) and $R^{AH} \propto \rho^2$ (full line).

Edmonds et al APL 2003

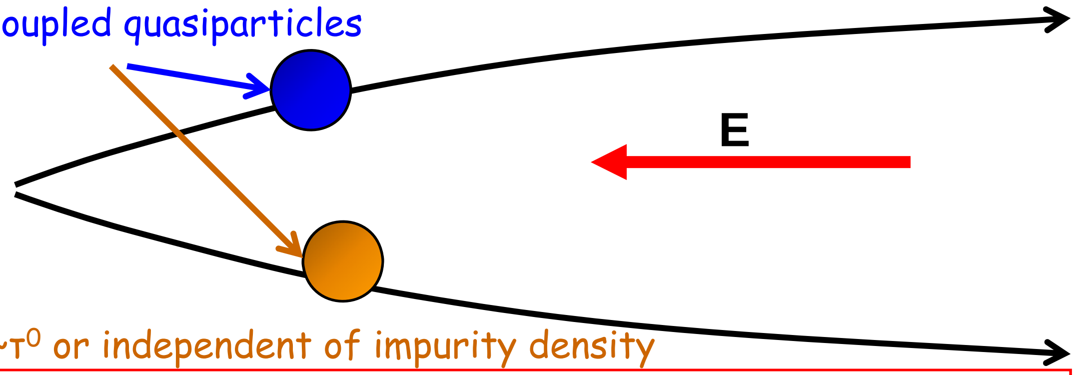
Strong SO coupled regime

STRONG SPIN-ORBIT COUPLED REGIME ($\Delta_{so} > \hbar/\tau$)

Intrinsic deflection

Electrons deflect to the right or to the left as they are accelerated by an electric field ONLY because of the spin-orbit coupling in the periodic potential (electronics structure)

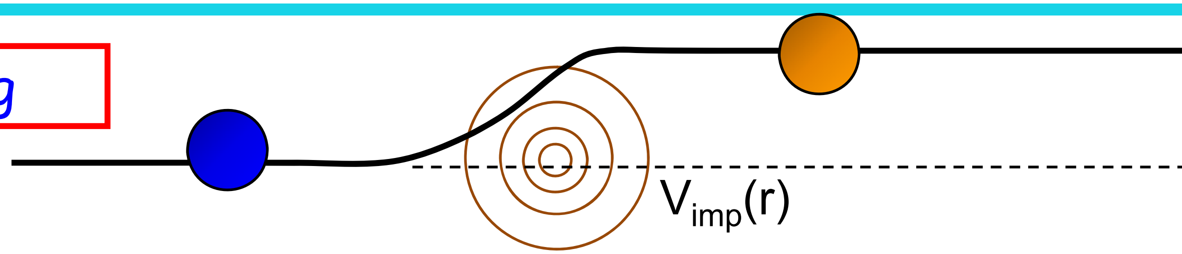
$$\dot{x}_c = \frac{\partial \epsilon}{\hbar \partial \vec{k}} + (e/\hbar)\vec{E} \times \vec{\Omega}.$$



$\sim\tau^0$ or independent of impurity density

Electrons have an "anomalous" velocity perpendicular to the electric field related to their Berry's phase curvature which is nonzero when they have spin-orbit coupling.

Side jump scattering

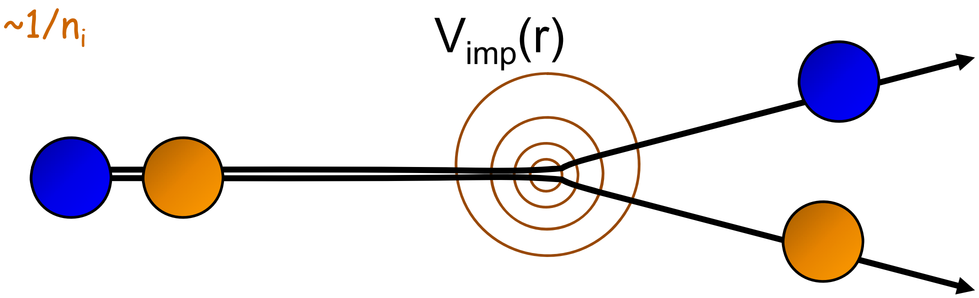


independent of impurity density

Electrons deflect first to one side due to the field created by the impurity and deflect back when they leave the impurity since the field is opposite resulting in a side step. They however come out in a different band so this gives rise to an anomalous velocity through scattering rates times side jump.

Skew scattering

$\sim 1/n_i$

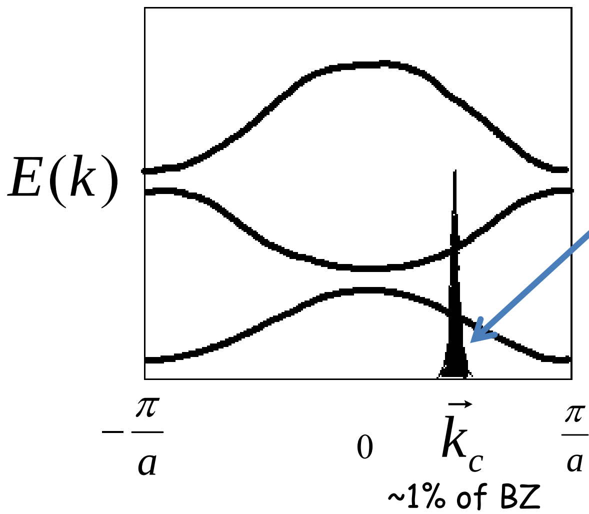


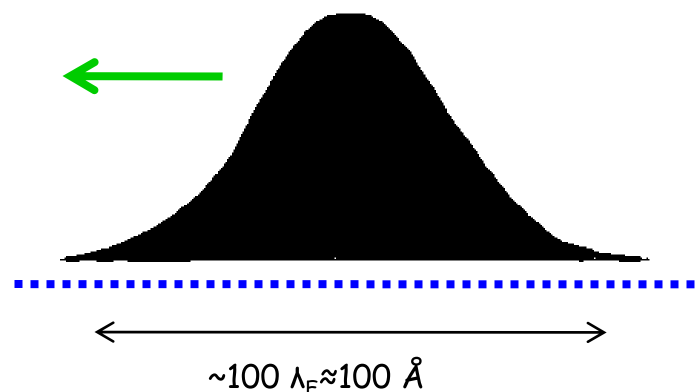
Asymmetric scattering due to the spin-orbit coupling of the electron or the impurity. This is also known as Mott scattering used to polarize beams of particles in accelerators.

Semi-classical theory of transport for Bloch electrons: the dynamics of wave-packets KNOW about the band structure

Idea I: to interpret things in terms of classical particles transform description from plane waves to wave-packets (which is a good basis too)

$$\Psi_{\vec{k}_c \vec{r}_c}(\vec{r},t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} w_{\vec{k}_c}(\vec{k}) e^{ik(\vec{r}-\vec{r}_c)} e^{-i\frac{E_n(\vec{k})}{\hbar}t} u_{\vec{k},n}(\vec{r})$$





Valid for fields and disorder that vary in scales larger than the wavepackets

Are the Eqs of motion

$$\dot{\vec{r}}_c = \left.\frac{\partial E(\vec{k})}{\partial \hbar \vec{k}}\right|_{k_c} \quad ? \quad \text{No}$$

$$\hbar \dot{\vec{k}}_c = -e\vec{E} \quad ? \quad \text{Yes}$$

Once the information of the periodic potential is imbedded into the wave-packet the choice of the phase factors are important to have it center at $\mathbf{r}_c$

Building a wave-packet from Bloch electrons: the birth of the Berry's connection

$$\Psi_{\vec{k}_c\vec{r}_c}(\vec{r},t) = \frac{1}{\sqrt{N}}\sum_{\vec{k}} w_{\vec{k}_c}(\vec{k})e^{ik(\vec{r}-\vec{r}_c)}u_{\vec{k},n}(\vec{r})$$

We want to have $w_{kc}(\mathbf{k})$ such that $\left\langle \Psi_{\vec{k}_c\vec{r}_c} \middle| \vec{r}-\vec{r}_c \middle| \Psi_{\vec{k}_c\vec{r}_c} \right\rangle = 0$

$$\Rightarrow \quad w_{\vec{k}_c}(\vec{k}) = \left| w(\vec{k}-\vec{k}_c) \right| e^{i(\vec{k}-\vec{k}_c)\cdot\left\langle u_{\vec{k}_c,n} \middle| i\frac{\partial}{\partial \vec{k}_c} u_{\vec{k}_c,n} \right\rangle}$$

$$\vec{A}_{\vec{k}_c} \equiv \left\langle u_{\vec{k}_c,n} \middle| i\frac{\partial}{\partial \vec{k}_c} u_{\vec{k}_c,n} \right\rangle$$

Berry's phase connection

Influences dynamics of wave-packets
(velocities: 2nd step of semiclassical approach)

Influences scattering of wave-packets
(collision integral term of Boltzmann eqn. in several parts)

Dynamics of wave-packets of Bloch electrons: the birth of the Berry's curvature and the anomalous velocity

Task: build a Lagrangian from the new dynamic variables $\mathbf{r}_c$ and $\mathbf{k}_c$

$$\Psi_{\vec{k}_c\vec{r}_c}(\vec{r},t)=\frac{1}{\sqrt{V}}\sum_{\vec{k}}\left|w(\vec{k}-\vec{k}_c)\right|e^{i(\vec{k}-\vec{k}_c)\cdot\vec{A}_{\vec{k}_c}}e^{i\vec{k}\cdot(\vec{r}-\vec{r}_c)}e^{-i\frac{E_n(\vec{k})}{\hbar}t}u_{\vec{k},n}(\vec{r})$$

where $\vec{A}_{\vec{k}_c}\equiv\left\langle u_{\vec{k}_c,n}\middle|i\frac{\partial}{\partial\vec{k}_c}u_{\vec{k}_c,n}\right\rangle$

Berry's connection (gauge dependent)

$$L=\left\langle\Psi_{\vec{k}_c\vec{r}_c}\middle|i\hbar\frac{\partial}{\partial t}-\hat{H}_0-eV(\vec{r})\middle|\Psi_{\vec{k}_c\vec{r}_c}\right\rangle=\hbar\vec{k}_c\cdot\dot{\vec{r}}_c+\hbar\dot{\vec{k}}_c\cdot\vec{A}_{\vec{k}_c}-E_n(\vec{k}_c)+eV(\vec{r}_c)$$

Applying Lagrange's equations on the above Lagrangian:

$$\dot{\vec{k}}_c=-e\vec{E}$$

"Anomalous velocity"

$$\dot{\vec{r}}_c=\frac{1}{\hbar}\frac{\partial E_n(\vec{k}_c)}{\partial\vec{k}_c}-\dot{\vec{k}}_c\times\vec{\Omega}_{\vec{k}_c}$$

$$\vec{\Omega}_{\vec{k}_c}=\frac{\partial}{\partial\vec{k}_c}\times\left\langle u_{\vec{k}_c,n}\middle|i\frac{\partial}{\partial\vec{k}_c}u_{\vec{k}_c,n}\right\rangle$$

Berry's curvature (gauge invariant)

motion perpendicular to **E**, Hall type motion!

already LINEAR **E** → The whole Fermi sea participates on this current contribution !!

$$J_x^{\text{int}}=\frac{-e}{V}\sum_k f\vec{v}_c=\frac{-e^2}{V}\sum_k f_0(E_{\vec{k}})\Omega_{\vec{k}_c z}E_y$$

Dynamics of wave-packets of Bloch electrons:
How do the Berry's curvature dynamics affect scattering?

Early theories (Berge,Smit) noticed that Bloch electron wave-packets seem to experience a side-step upon scattering: (a dangerous way of doing dynamics)

$$\dot{\vec{r}}_c = \frac{d}{dt}\left\langle \Psi_{\vec{r}_c\vec{k}_c} \middle| \vec{r} \middle| \Psi_{\vec{r}_c\vec{k}_c} \right\rangle = \frac{d}{dt}\int \frac{d\vec{r}}{V}\int d\vec{k}'\int d\vec{k}\, w^*(\vec{k}')w(\vec{k})e^{-i\vec{k}'\cdot\vec{r}}e^{-iE_{\vec{k}'}t/\hbar}\left(re^{i\vec{k}\cdot\vec{r}}\right)e^{-iE_{\vec{k}}t/\hbar} =$$

Tried to interpret it physically BUT it is **gauge dependent (i.e. only gauge invariant quantities have measurable physical meaning)** !

Dynamics of wave-packets of Bloch electrons:
How do the Berry's curvature dynamics affect scattering?

Early theories (Berge,Smit) noticed that Bloch electron wave-packets seem to experience a side-step upon scattering:

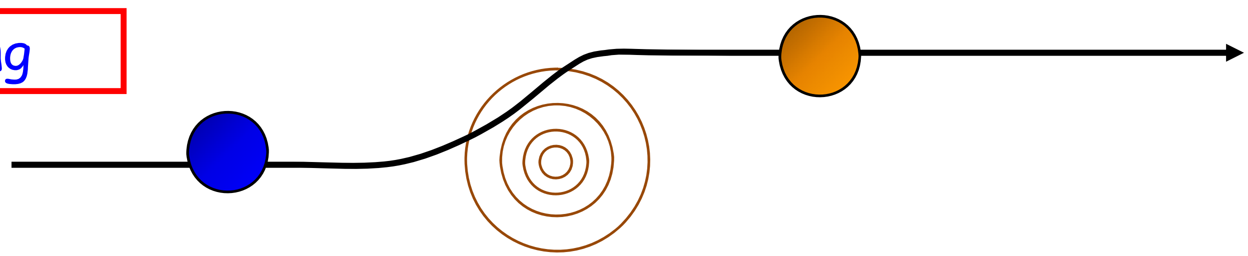
$$\delta\vec{r}_{\vec{k}n,\vec{k}'n'} = \left\langle u_{n\vec{k}} \middle| i\frac{\partial}{\partial\vec{k}} u_{n\vec{k}} \right\rangle - \left\langle u_{n'\vec{k}'} \middle| i\frac{\partial}{\partial\vec{k}'} u_{n\vec{k}'} \right\rangle$$

Tried to interpret it physically BUT it is **gauge dependent (i.e. only gauge invariant quantities have measurable physical meaning)** !

The gauge invariant expressions can be derived using the gauge invariant Lagrangian dynamics shown earlier (Sinitsyn et al 2006) l=(n,**k**)

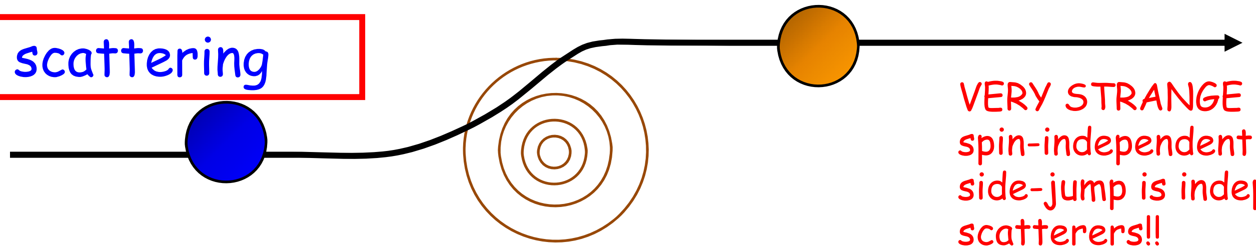
$$\delta\vec{r}_{l,l'} = \left\langle u_l \middle| i\frac{\partial}{\partial\vec{k}} u_{l'} \right\rangle - \left\langle u_l \middle| i\frac{\partial}{\partial\vec{k}'} u_{l'} \right\rangle - \left(\frac{\partial}{\partial\vec{k}} + \frac{\partial}{\partial\vec{k}'}\right)\arg\left[\langle u_l | u_{l'}\rangle\right]$$

Side jump scattering



How does side-jump affect transport?

Side jump scattering



VERY STRANGE THING: for spin-independent scatterers side-jump is independent of scatterers!!

$$\delta\vec{r}_{l,l'} = \left\langle u_l \middle| i\frac{\partial}{\partial \vec{k}} u_l \right\rangle - \left\langle u_{l'} \middle| i\frac{\partial}{\partial \vec{k}'} u_{l'} \right\rangle - \left(\frac{\partial}{\partial \vec{k}} + \frac{\partial}{\partial \vec{k}'}\right)\arg\left[\langle u_l | u_{l'}\rangle\right]$$

The side-jump comes into play through an additional current and influencing the Boltzmann equation and through it the non-equilibrium distribution function

1st-It creates a side-jump current:

$$\vec{v}_l^{\,s-j} = \sum_{l'} \omega_{l,l'} \delta\vec{r}_{l,l'}$$

2nd-An extra term has to be added to the collision term of the Boltzmann eq. to account because upon elastic scattering some kinetic energy is transferred to potential energy.

$$I = -\sum_{l'} \omega_{l,l'} (f_l - f_{l'}) \quad \Rightarrow \quad I = -\sum_{l'} \omega^T_{l,l'} \left(f_l - f_{l'} - \frac{\partial f_0(E_l)}{\partial E_l} e\vec{E}\cdot\delta\vec{r}_{ll'}\right)$$

full $\omega_{ll'}$ does not assume KE conserved, T-matrix approximation of $\omega_{ll'}$ ($\omega^T_{ll'}$) does.

$$\omega^T_{l,l'} = \frac{2\pi}{\hbar} |T_{l,l'}|^2 \, \delta(E_l - E_{l'})$$

$$E_l = E_{l'} + e\vec{E}\cdot\delta\vec{r}_{ll'}$$

Semiclassical transport of spin-orbit coupled Bloch electrons: Boltzmann Eq. and Hall current

We do this in two steps: first calculate steady state non-equilibrium distribution function and then use it to compute the current.

Set to 0 for steady state solution

Only the normal velocity term, $\vec{v}_{0l} = \frac{\partial E_l}{\partial \hbar \vec{k}}$ since we are looking for linear in E equation

$$\frac{\partial f_l}{\partial t} - e\vec{E}\cdot\vec{v}_{0l}\frac{\partial f_0(E_l)}{\partial E_l} = -\sum_{l'}\omega^T_{l,l'}\left(f_l - f_{l'} - \frac{\partial f_0(E_l)}{\partial E_l} e\vec{E}\cdot\delta\vec{r}_{ll'}\right)$$

order of the disorder potential strength and symmetric and anti-symmetric components

$$\omega^T_{l,l'} = \tfrac{2\pi}{\hbar}|T_{l,l'}|^2\,\delta(E_l - E_{l'}) = \omega_{l,l'}^{(2)} + \omega_{l,l'}^{(3a)} + \omega_{l,l'}^{(3s)} + \omega_{l,l'}^{(4a)} + \cdots$$

$$\omega^{(2)}_{l,l'} = \tfrac{2\pi}{\hbar}|V_{l,l'}|^2\,\delta(E_l - E_{l'})$$ 1st Born approximation

$$\omega^{(3a)}_{l,l'} = -(2\pi)^2\sum_{l''}\mathrm{Im}\left[V_{l,l'}V_{l',l''}V_{l'',l}\right]_{dis}\delta(E_l - E_{l'})\delta(E_{l''} - E_{l'})$$ 2nd Born approximation (usual skew scattering contribution)

To solve this equation we write the non-equilibrium component in various components that correspond to solving parts of the equation the corresponding order of disorder

$$f_{\vec{k}} = f_{eq}(E_l) + g_l^{s} + g_l^{3a} + g_l^{4a} + g_l^{adis}$$

Semiclassical transport of spin-orbit coupled Bloch electrons: Boltzmann Eq. and Hall current

$$-e\vec{E}\cdot\vec{v}_{0l}\frac{\partial f_0(E_l)}{\partial E_l} = -\sum_{l'}\omega^T_{l,l'}\left(f_l - f_{l'} - \frac{\partial f_0(E_l)}{\partial E_l}e\vec{E}\cdot\delta\vec{r}_{ll'}\right)$$

$$f_{\vec{k}} = f_{eq}(E_l) + g^s_l + g^{3a}_l + g^{4a}_l + g^{adis}_l$$

$$-e\vec{E}\cdot\vec{v}_{0l}\frac{\partial f_0(E_l)}{\partial E_l} = -\sum_{l'}\omega^{(2)}_{l,l'}(g^s_l - g^s_{l'}) \qquad \sim V^0 \rightarrow g^s_l \propto n_i^{-1}$$

$$0 = -\sum_{l'}\omega^{(2)}_{l,l'}(g^{3a}_l - g^{3a}_{l'}) - \sum_{l'}\omega^{(3a)}_{l,l'}(g^s_l - g^s_{l'}) \qquad \sim V \rightarrow g^{3a}_l \propto n_i^{-1}$$

$$0 = -\sum_{l'}\omega^{(2)}_{l,l'}(g^{4a}_l - g^{4a}_{l'}) - \sum_{l'}\omega^{(4a)}_{l,l'}(g^s_l - g^s_{l'}) \qquad \sim V^2 \rightarrow g^{4a}_l \propto n_i^{0}$$

$$0 = -\sum_{l'}\omega^{(2)}_{l,l'}\left(g^{adis}_l - g^{adis}_{l'} - \frac{\partial f_0(E_l)}{\partial E_l}e\vec{E}\cdot\delta\vec{r}_{ll'}\right) \qquad \sim V^2 \rightarrow g^{adis}_l \propto n_i^{0}$$

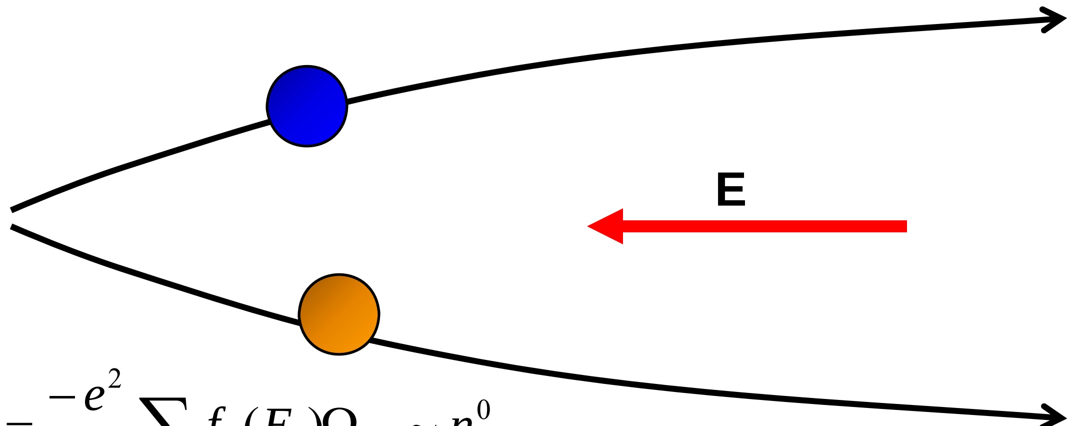
2nd step: (after solving them) we put them into the equation for the current and identify from there the different contributions to the AHE using the full expression for the velocity

$$\vec{v}_l = \frac{1}{\hbar}\frac{\partial E_l}{\partial \vec{k}} + \frac{e\vec{E}}{\hbar}\times\vec{\Omega}_l + \sum_{l'}\omega_{l,l'}\delta\vec{r}_{l,l'}$$

$$\sigma^{int}_{xy} = \frac{-e^2}{V}\sum_l f_0(E_l)\Omega_{l,z} \sim n_i^0 \qquad \sigma^{sk1}_{xy} = \frac{-e}{V}\sum_l \frac{g^{3a}_l}{E_y}v_{0lx} \sim n^{-1}_i \qquad \sigma^{sk2}_{xy} = \frac{-e}{V}\sum_l \frac{g^{4a}_l}{E_y}v_{0lx} \sim n_i^0$$

$$\sigma^{adis}_{xy} = \frac{-e}{V}\sum_l \frac{g^{adis}_l}{E_y}v_{0lx} \sim n_i^0 \qquad \sigma^{s-j}_{xy} = \frac{-e}{V}\sum_l \frac{g^s_l}{E_y}\left(\sum_{l'}\omega_{l,l'}\delta r_{l,l'}\right) \sim n_i^0$$

Intrinsic deflection

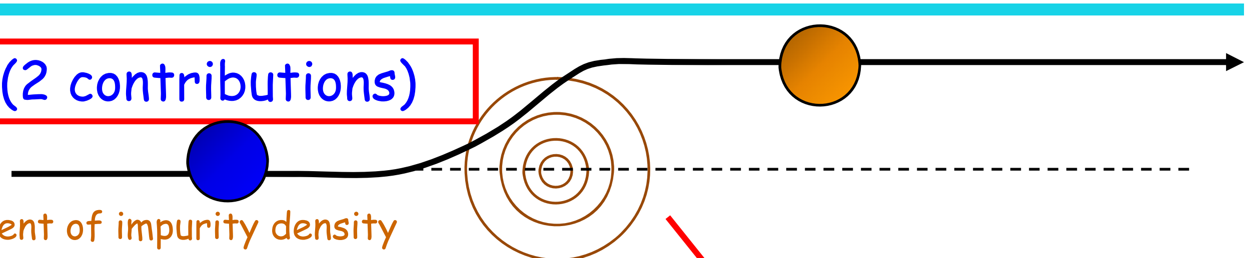


~n_i^0 or independent of impurity density

$$\dot{x}_c = \frac{\partial \epsilon}{\hbar \partial \vec{k}} + (e/\hbar)\vec{E} \times \vec{\Omega}.$$

$$\sigma_{xy}^{\text{int}} = \frac{-e^2}{V} \sum_l f_0(E_l)\Omega_{l,z} \sim n_i^0$$

Side jump scattering (2 contributions)



Popular believe: ~n_i^0 or independent of impurity density

$$\sigma_{xy}^{adis} = \frac{-e}{V} \sum_l \frac{g_l^{adis}}{E_y} v_{0lx} \sim n_i^0$$

Origin is on its effect on the distribution function

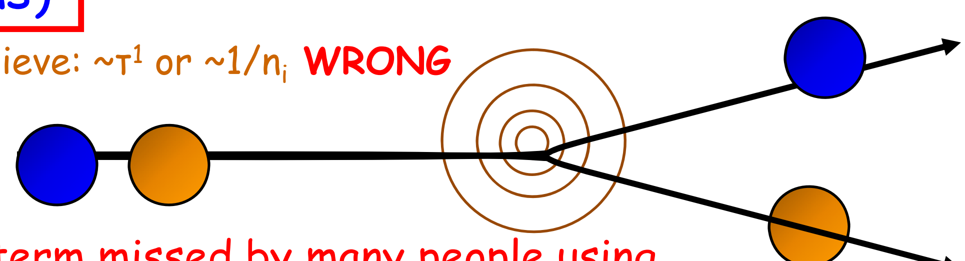
$$\sigma_{xy}^{s-j} = \frac{-e}{V} \sum_l \frac{g_l^{s}}{E_y} \left(\sum_{l'} \omega_{l,l'} \delta r_{l,l'} \right) \sim n_i^0$$

Skew scattering (2 contributions)

Popular believe: ~τ^1 or ~$1/n_i$ **WRONG**

$$\sigma_{xy}^{sk1} = \frac{-e}{V} \sum_l \frac{g_l^{3a}}{E_y} v_{0lx} \sim n^{-1}{}_i$$

$$\sigma_{xy}^{sk2} = \frac{-e}{V} \sum_l \frac{g_l^{4a}}{E_y} v_{0lx} \sim n_i^0$$



term missed by many people using semiclassical approach

Microscopic vs. Semiclassical

- Boltzmann semiclassical approach: easy physical interpretation of different contributions (used to define them) but very easy to miss terms and make mistakes. **MUST BE CONFIRMED MICROSCOPICALLY!** How one understands but not necessarily computes the effect.

- Kubo approach: systematic formalism but not very transparent.

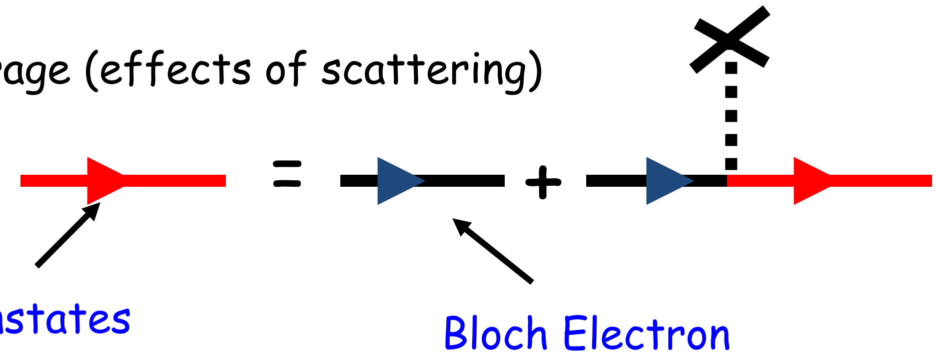
- Keldysh approach: also a systematic kinetic equation approach (equivalent to Kubo in the linear regime). In the quasi-particle limit it must yield Boltzmann semiclassical treatment.

Kubo microscopic approach to transport: diagrammatic perturbation theory

$$\sigma_{ij} \sim \frac{e^2}{V}\sum_l \mathrm{Tr}\left[\hat{G}^R(E_F)\hat{v}_{\vec{k}i}\hat{G}^A(E_F)\hat{v}_{\vec{k}j}\right]$$

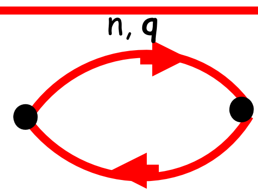
Need to perform disorder average (effects of scattering)

$$\hat{G}^R(E_F) = \frac{1}{E_F - \hat{H}_0 - \hat{V}_{dis} + i\delta}$$



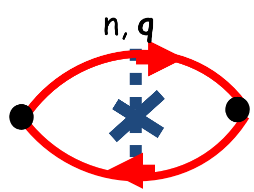
Averaging procedures:  $= 1/\tau\nu_0$  $= \nu_0\tau$

Perturbation Theory: conductivity



→ Drude Conductivity

$\sigma = ne^2\,\tau\,/m^* \sim 1/n_i$



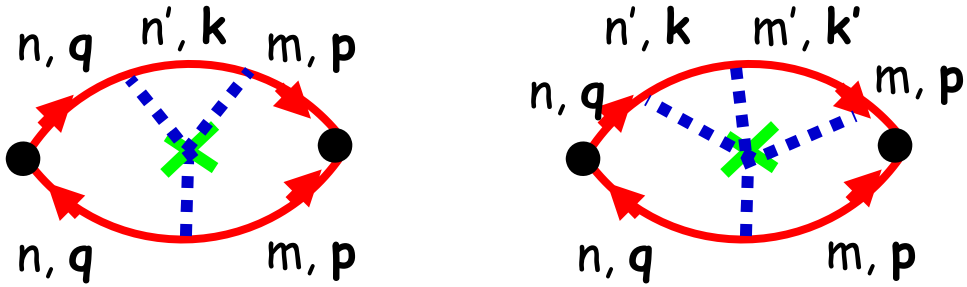
→ Vertex Corrections

$\sim$ 1-cos(θ)

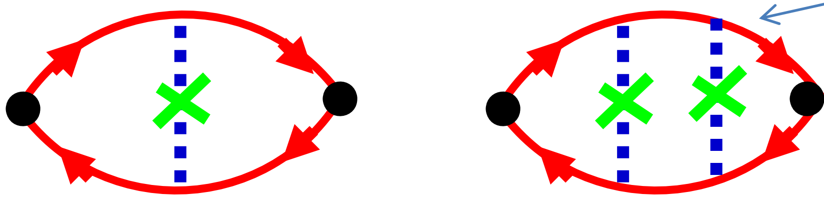
Kubo microscopic approach to AHE

Early identifications of the contributions

“Skew scattering”



“Side-jump scattering”

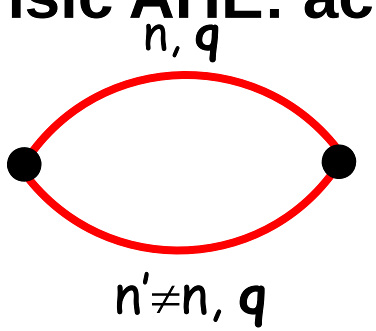


matrix in band index

Vertex Corrections

$\sim \sigma_{Intrinsic} \sim \tau^0$ or n^0_i

Intrinsic AHE: accelerating between scatterings



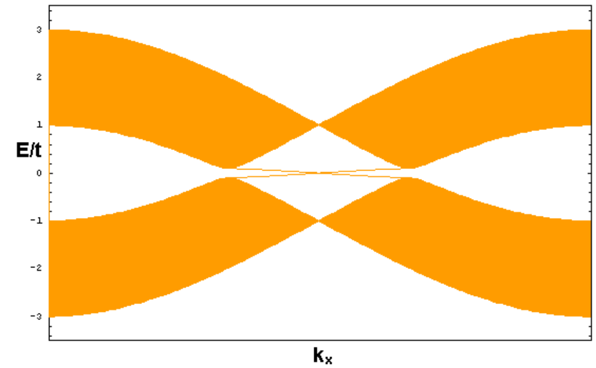
Intrinsic

$\sim \sigma_0 / \varepsilon_F \tau \sim \tau^0$ or n^0_i

"AHE" in graphene: linking microscopic and semiclassical theories

Single K-band with spin up

$$H_{K\uparrow}=v(k_x\sigma_x+k_y\sigma_y)+\Delta_{so}\sigma_z$$



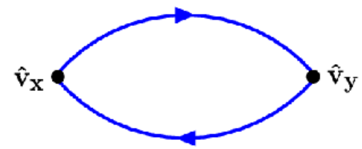
A. Crépieux and P. Bruno (2001)

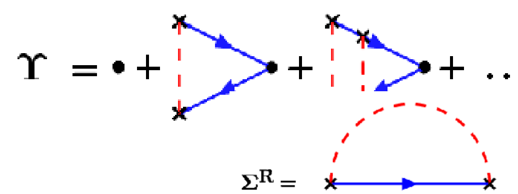
Kubo-Streda formula: $$\sigma_{xy}=\sigma^{I}_{xy}+\sigma^{II}_{xy}$$

$$\sigma^{I}_{xy}=-\frac{e^2}{4\pi}\int_{-\infty}^{+\infty}d\varepsilon\frac{df(\varepsilon)}{d\varepsilon}Tr[v_x(G^R-G^A)v_yG^A-\\-v_xG^Rv_y(G^R-G^A)]$$

$$\sigma^{II}_{xy}=\frac{e^2}{4\pi}\int_{-\infty}^{+\infty}d\varepsilon f(\varepsilon)Tr[v_xG^Rv_y\frac{dG^R}{d\varepsilon}-\\-v_x\frac{dG^R}{d\varepsilon}v_yG^R-v_xG^Av_y\frac{dG^A}{d\varepsilon}+v_x\frac{dG^A}{d\varepsilon}v_yG^A]$$

In metallic regime: $$\sigma^{II}_{xy}=0$$





$$\sigma^{I}_{xy}=\frac{-e^2\Delta_{so}}{4\pi\hbar\sqrt{(vk_F)^2+\Delta_{so}^2}}\left(1+\frac{4(vk_F)^2}{(vk_F)^2+4\Delta_{so}^2}+\frac{3(vk_F)^4}{\left((vk_F)^2+4\Delta_{so}^2\right)^2}\right)-\frac{e^2\langle V^3\rangle}{2\pi n\langle V^2\rangle}\frac{\Delta_{so}(vk_F)^4}{\left((vk_F)^2+4\Delta_{so}^2\right)^2}$$

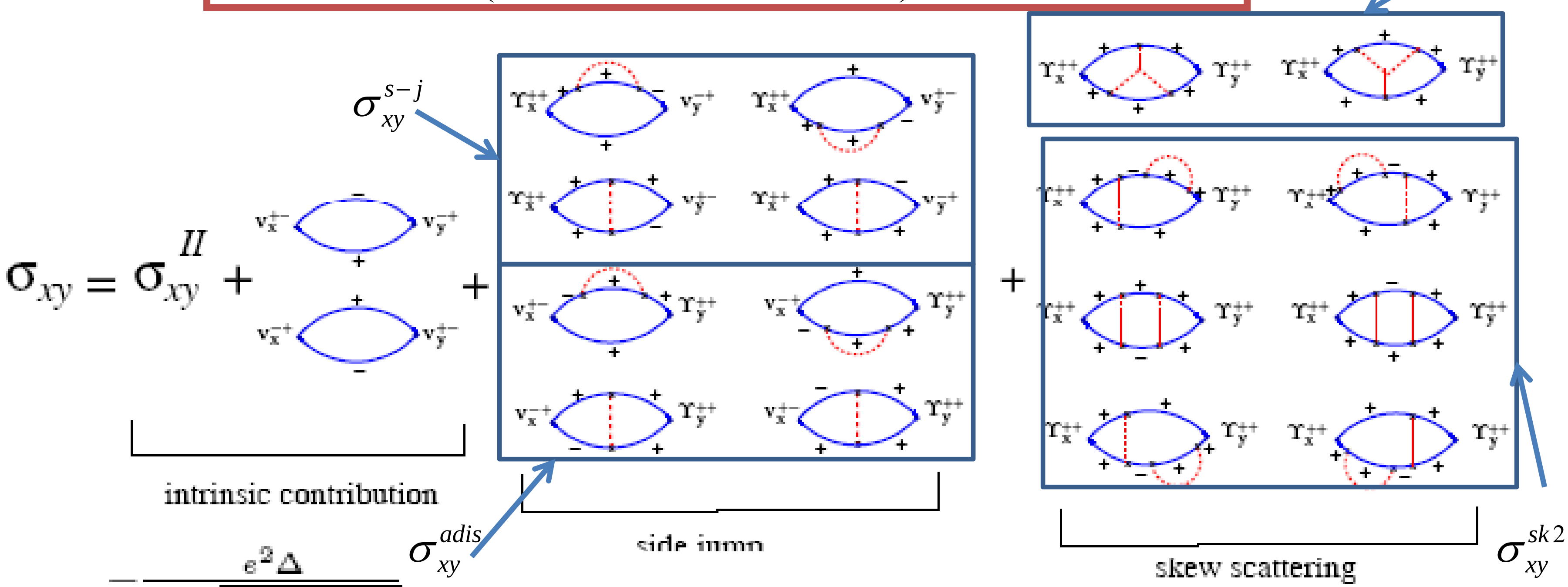
Sinitsyn et al PRB 07

Comparing Boltzmann to Kubo (chiral basis)

$$\hat{H}_0 = v(k_x\sigma_x + k_y\sigma_y) + \Delta\sigma_z.$$

Sinitsyn et al 2007

$$\sigma^{I}_{xy} = \frac{-e^2\Delta_{so}}{4\pi\hbar\sqrt{(vk_F)^2+\Delta_{so}^2}}\left(1+\frac{4(vk_F)^2}{(vk_F)^2+4\Delta_{so}^2}+\frac{3(vk_F)^4}{\left((vk_F)^2+4\Delta_{so}^2\right)^2}\right)-\frac{e^2\langle V^3\rangle}{2\pi n\langle V^2\rangle}\frac{\Delta_{so}(vk_F)^4}{\left((vk_F)^2+4\Delta_{so}^2\right)^2}$$



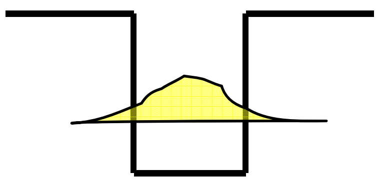
$$-\frac{e^2\Delta}{4\pi\sqrt{\Delta^2+(vk_F)^2}}$$

σ^{int}_{xy}

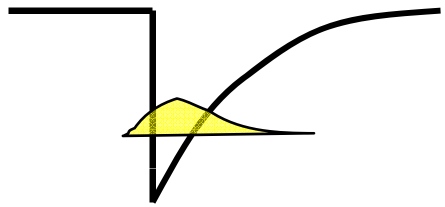
$$-\frac{e^2\Delta k_F^2}{\pi\sqrt{k_F^2+\Delta^2}\,(k_F^2+4\Delta^2)}. \quad -\frac{e^2 3\Delta (vk_F)^4}{4\pi\sqrt{(vk_F)^2+\Delta^2}[4\Delta^2+(vk_F)^2]^2} - \frac{e^2V_1^3}{2\pi n_i V_0^4}\frac{(vk_F)^4\Delta}{(4\Delta^2+(vk_F)^2)^2}$$

Kubo identifies, without a lot of effort, the order in n_i of the diagrams BUT not so much their physical interpretation according to semiclassical theory

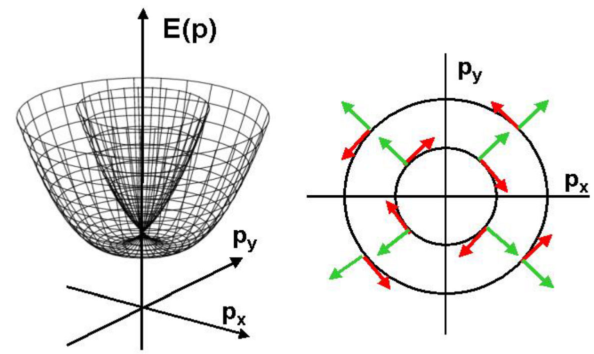
Another simple example: AHE in Rashba 2D system



Inversion symmetry
$\Rightarrow$ no R-SO



Broken inversion symmetry
$\Rightarrow$ R-SO



$$H_k = \frac{\hbar^2 k^2}{2m}\sigma_0 + \lambda(k_x\sigma_y - k_y\sigma_x) + h\sigma_z = \frac{\hbar^2 k^2}{2m}\sigma_0 + \lambda\vec{\sigma}\times\vec{k} + h\sigma_z$$

Bychkov and Rashba (1984)

Kubo and semiclassical approach approach:
(Nuner et al PRB08, Borunda et al PRL 07)

Only when ONE both sub-band
there is a significant contribution

$$\sigma_{xx} = \frac{e^2}{\pi\hbar n_i V_0^2}\left(\frac{\lambda_- k_-}{\nu^-}\right)^2 \frac{1}{3h^2+\lambda_-^2},$$

$$\sigma_{xy}^{\rm skew} = -\frac{e^2 V_1^3}{2\pi\hbar n_i V_0^4}\frac{h\lambda_-\alpha_1^2 k_-^4}{\nu_-(3h^2+\lambda_-^2)^2}.$$

When both subbands are occupied there is additional vertex corrections that contribute

Recent progress: full understanding of simple models in each approach

Semi-classical approach:
Gauge invariant formulation; shown to match microscopic approach in 2DEG+Rashba, Graphene

Sinitsyn et al PRB 05, PRL 06, PRB 07
Borunda et al PRL 07, Nunner et al PRB 08
Sinitsyn JP:C-M 08

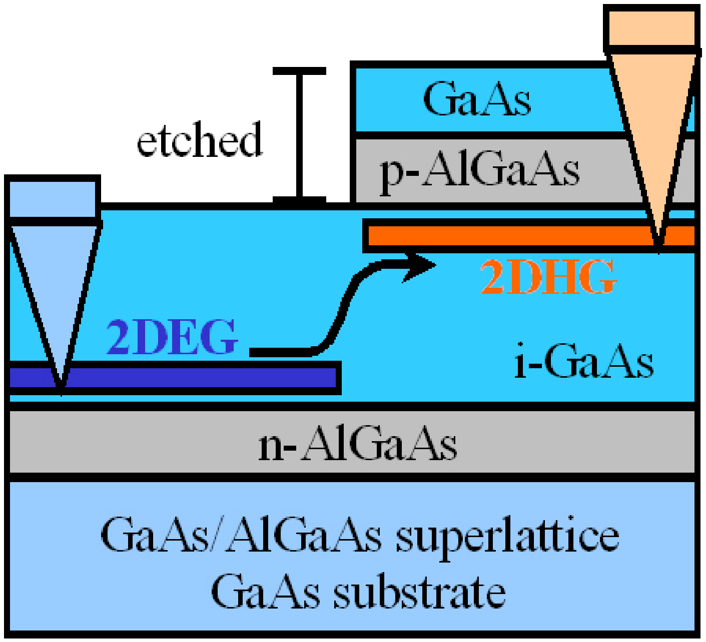
Kubo microscopic approach:
Results in agreement with semiclassical calculations 2DEG+Rashba, Graphene

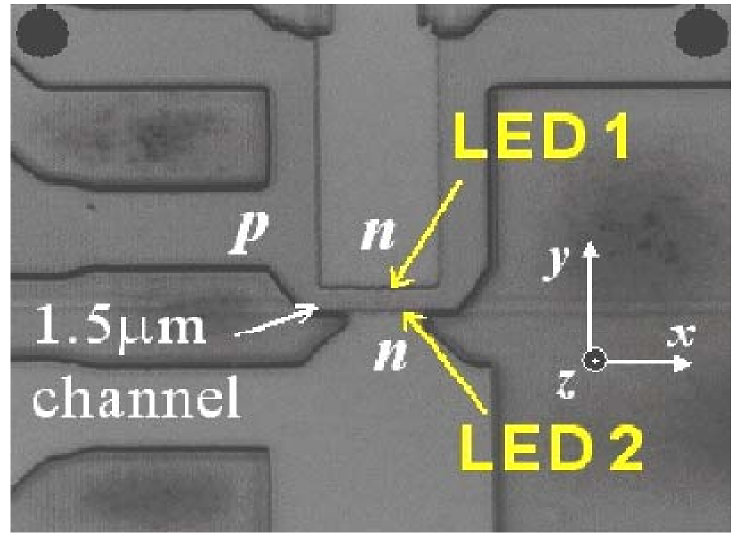
Sinitsyn et al PRL 06, PRB 07, Nunner PRB 08, Inoue PRL 06, Dugaev PRB 05

NEGF/Keldysh microscopic approach:
Numerical/analytical results in agreement in the metallic regime with semiclassical calculations 2DEG+Rashba, Graphene

Kovalev et al PRB 08, Onoda PRL 06, PRB 08

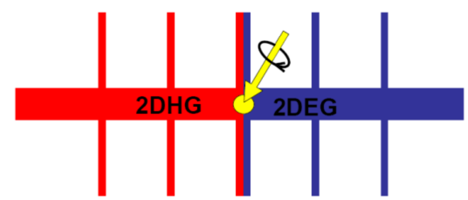
How to test the simple models? utilize technology developed to detect SHE in 2DHG



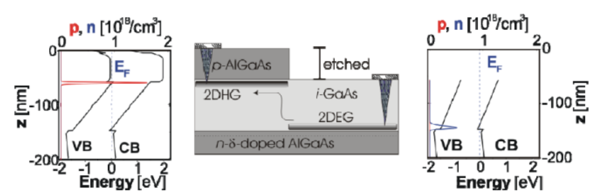


J. Wunderlich, B. Kaestner, J. Sinova and T. Jungwirth, Phys. Rev. Lett. 94 047204 (2005)

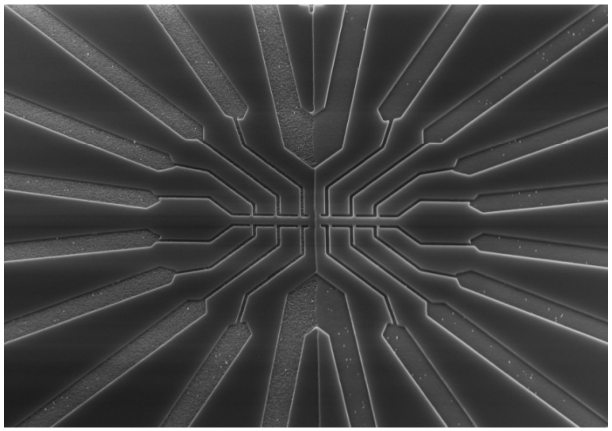
B. Kaestner, et al, JPL 02; B. Kaestner, et al Microelec. J. 03; Xiulai Xu, et al APL 04, Wunderlich et al PRL 05

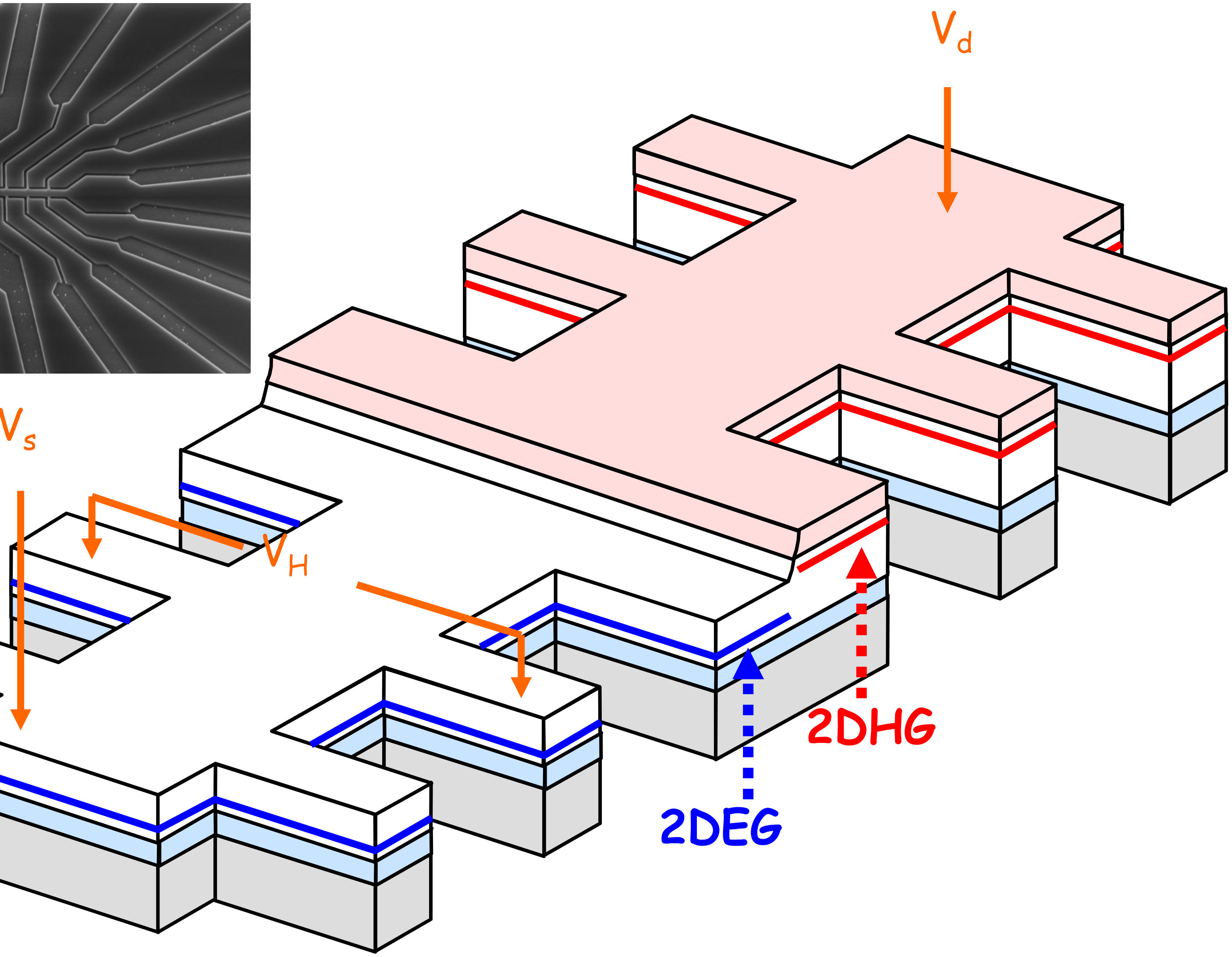


Proposed experiment/device: Coplanar photocell in reverse bias with Hall probes along the 2DEG channel
Borunda, Wunderlich, Jungwirth, Sinova et al PRL 07

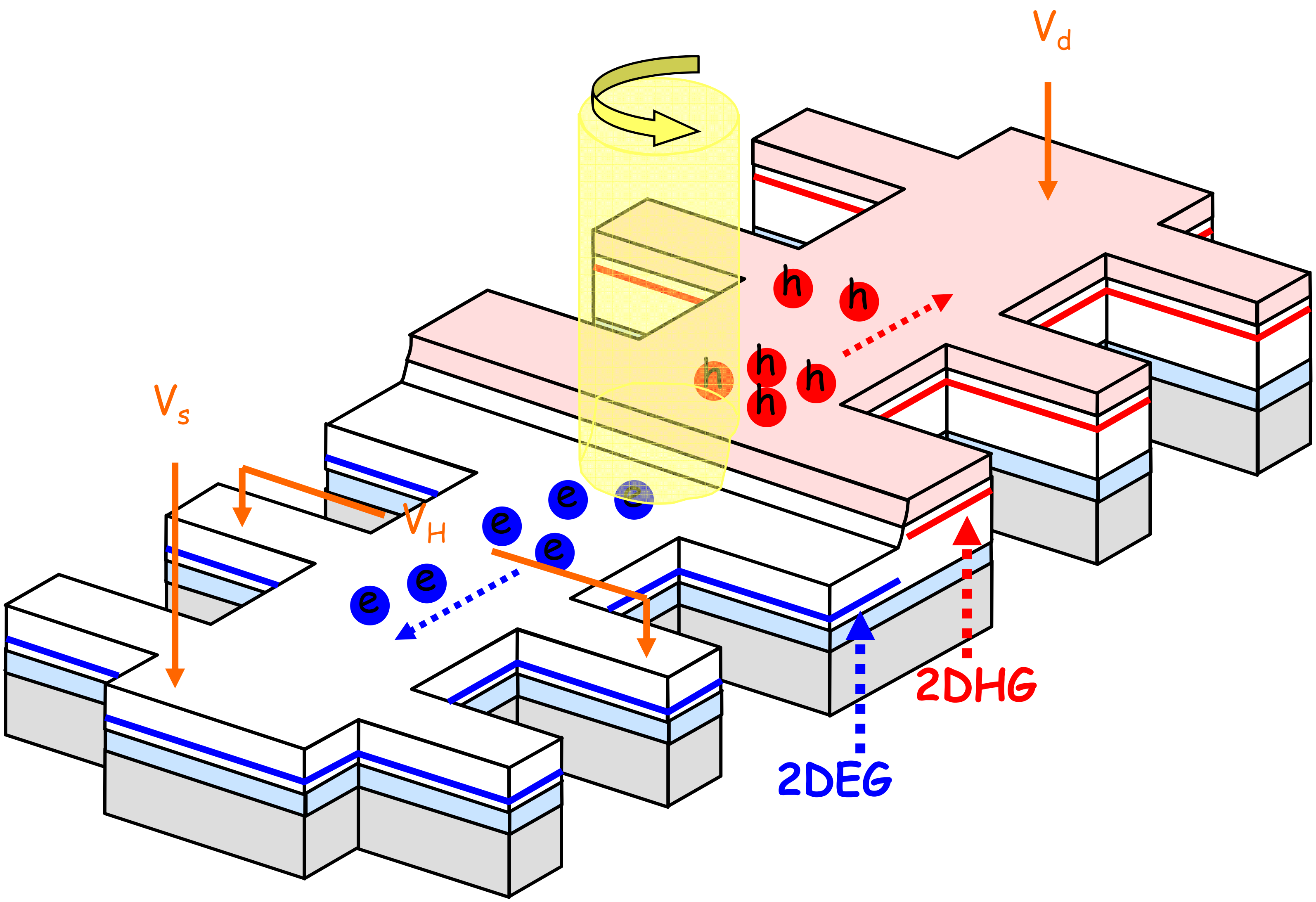


Device schematic – Hall measurement



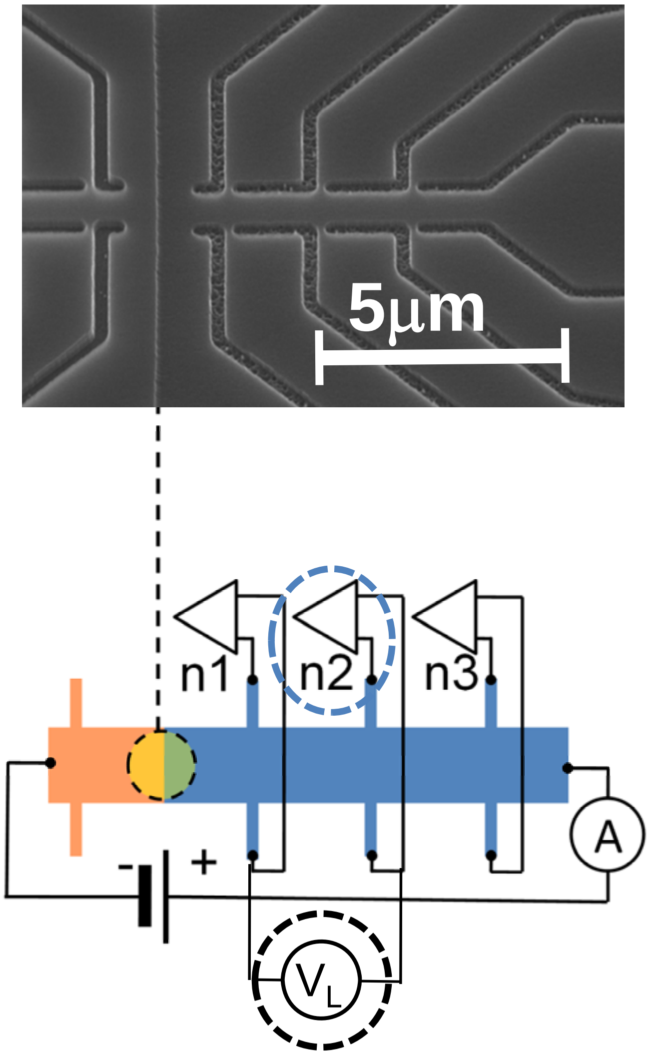


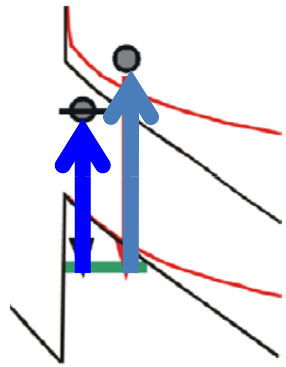
Device schematic - SIHE measurement

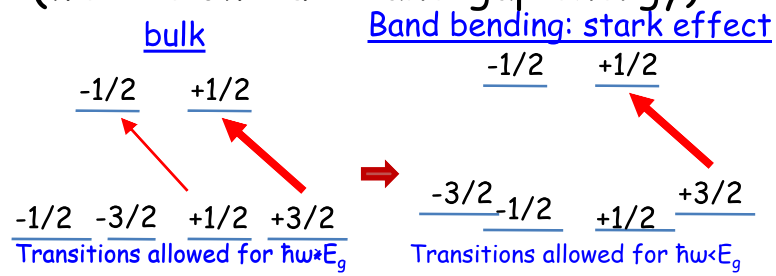


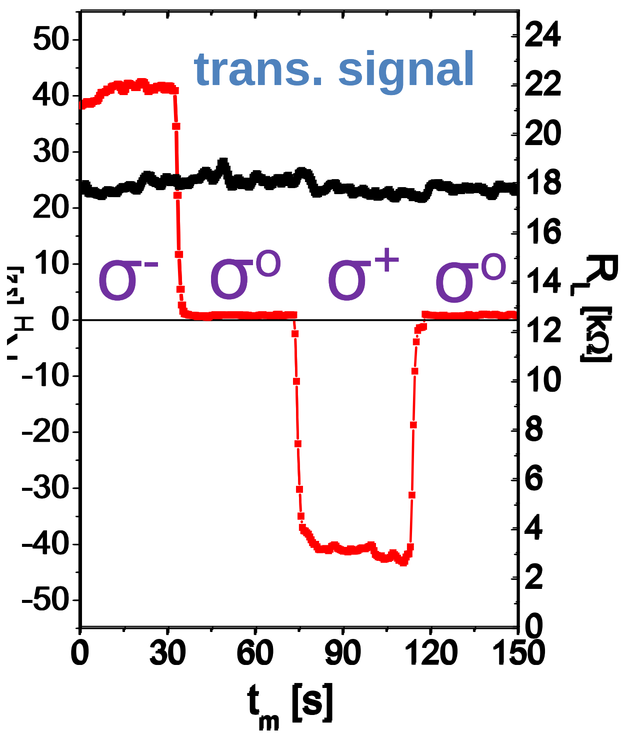
Reverse- or zero-biased: Photovoltaic Cell

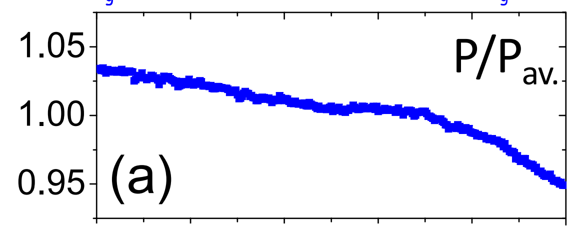
Red-shift of confined 2D hole → free electron trans.
due to built in field and reverse bias
light excitation with λ = **850nm**
(well below bulk band-gap energy)



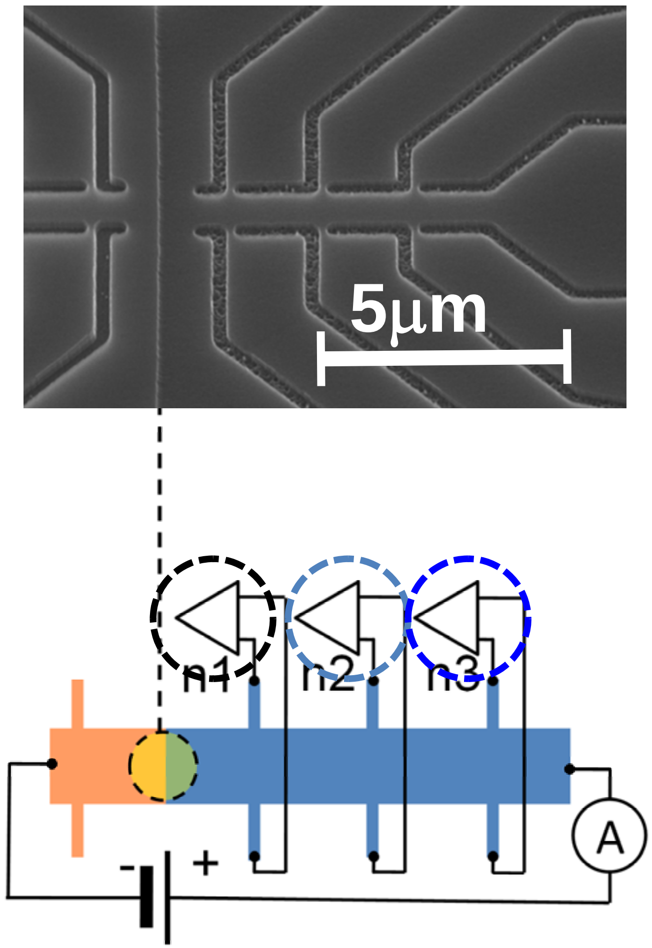


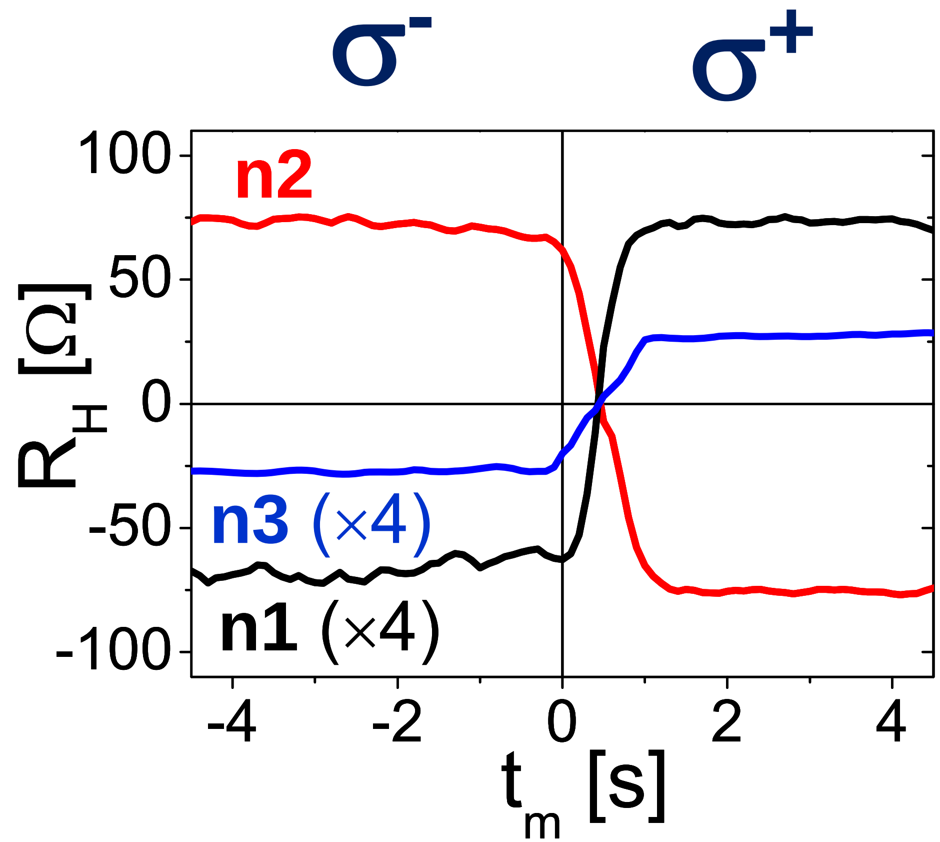






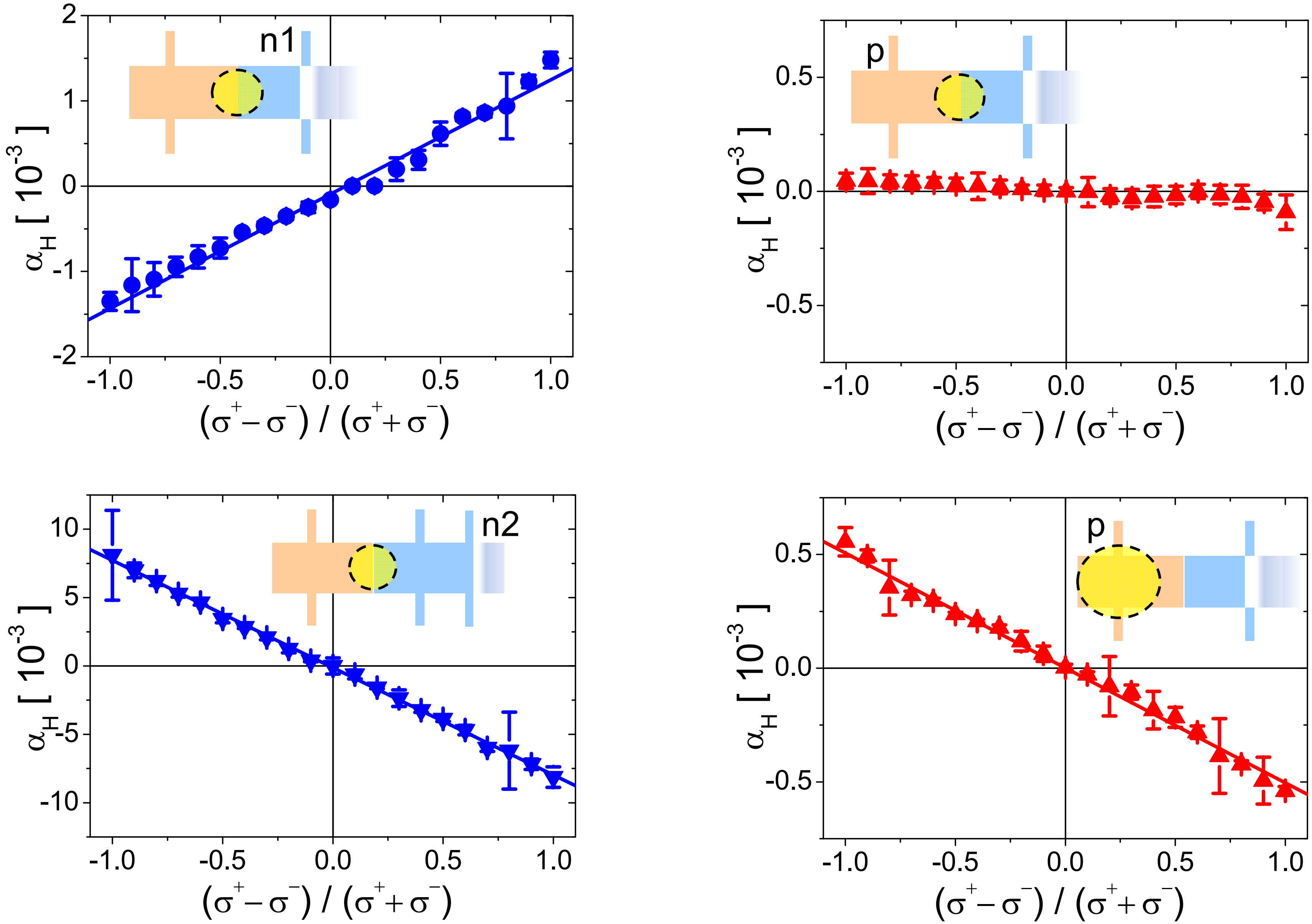
Spin injection Hall effect: experimental observation





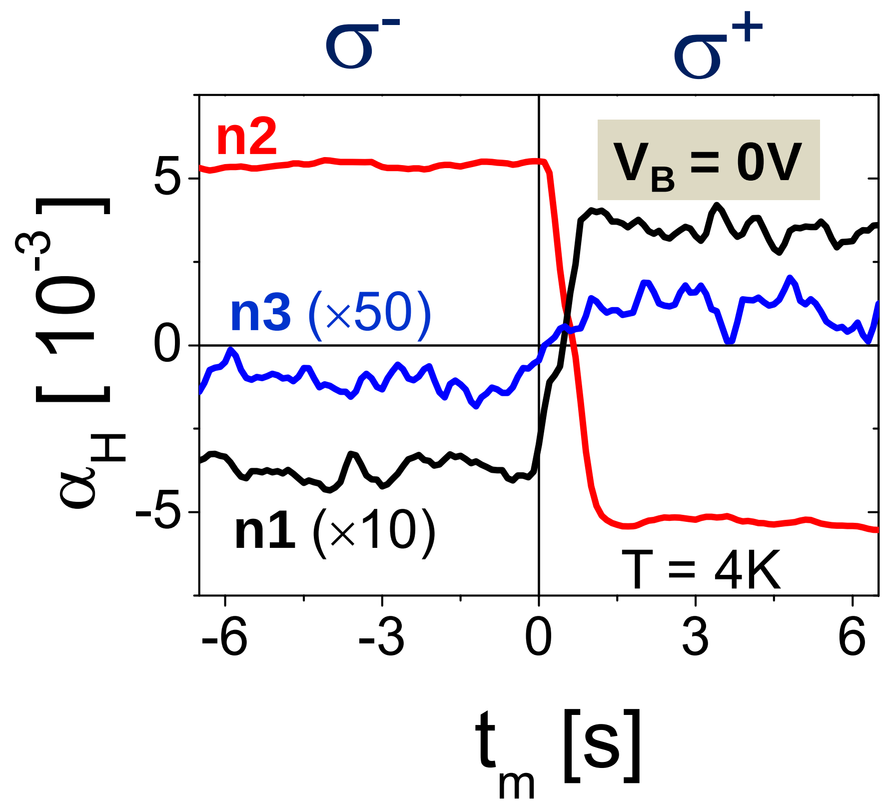
Local Hall voltage changes sign and magnitude along the stripe

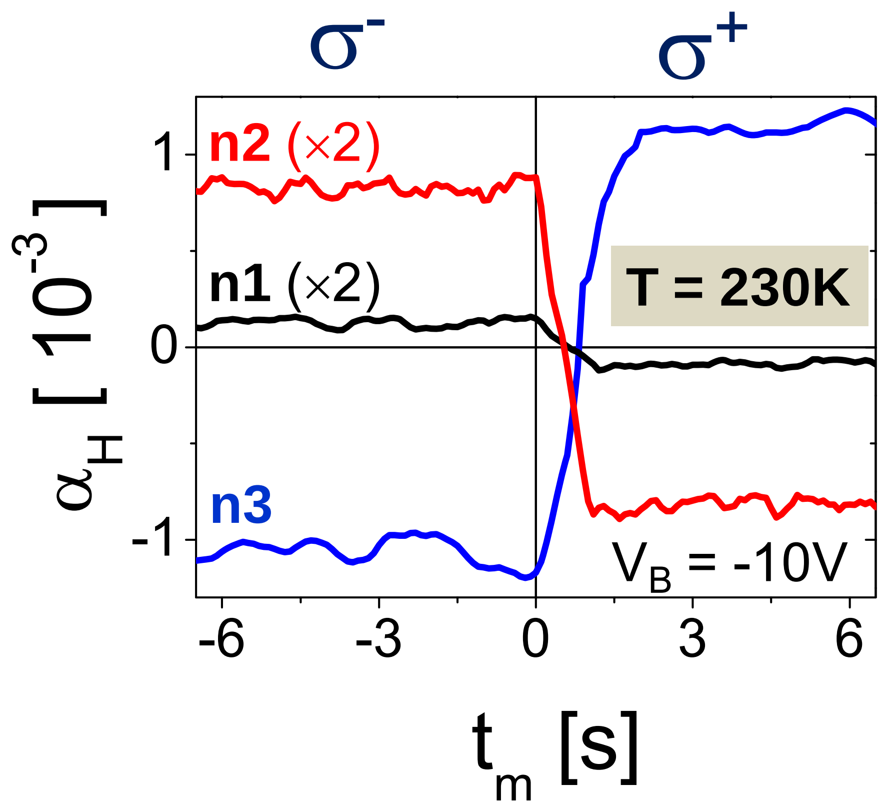
Spin injection Hall effect ←→ Anomalous Hall effect

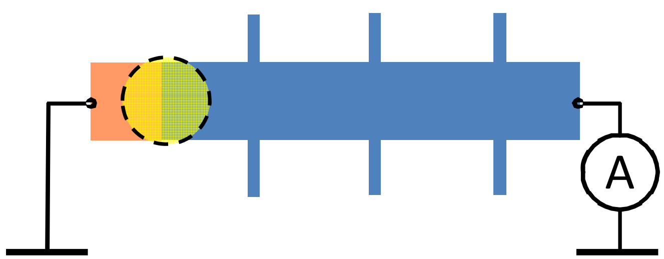


Persistent Spin injection Hall effect

Zero bias-and high temperature operation







THEORY CONSIDERATIONS
Spin transport in a 2DEG with Rashba+Dresselhaus SO

The 2DEG is well described by the effective Hamiltonian:

$$H_{\text{2DEG}} = \frac{\hbar^2 k^2}{2m} + \alpha\left(k_y\sigma_x - k_x\sigma_y\right) + \beta\left(k_x\sigma_x - k_y\sigma_y\right) + \lambda^* \vec{\sigma}\cdot(\vec{k}\times\nabla \mathrm{V}_{\text{dis}}(\vec{r}))$$

$\lambda^* = \frac{P^2}{3}\left(\frac{1}{E_g^2} - \frac{1}{(E_g+\Delta_{so})^2}\right) \approx 5.3\ \mathring{\mathrm{A}}^2$ for GaAs, $\beta = -B\left\langle k_z^2 \right\rangle$ with $B = 10\ \mathrm{eV}\,\mathring{\mathrm{A}}^3$ for GaAs, $\alpha = \lambda^* E_z$

For our 2DEG system: $\beta \approx -0.02\ \mathrm{eV}\,\mathring{\mathrm{A}}$, $m = 0.067 m_e$

$\alpha \approx 0.01 - 0.03\ \mathrm{eV}\,\mathring{\mathrm{A}}$ (for $E_Z \approx 0.01 - 0.03\ \mathrm{eV}/\mathring{\mathrm{A}}$)

Hence $\alpha \sim -\beta$

What is special about $\alpha \sim -\beta$?

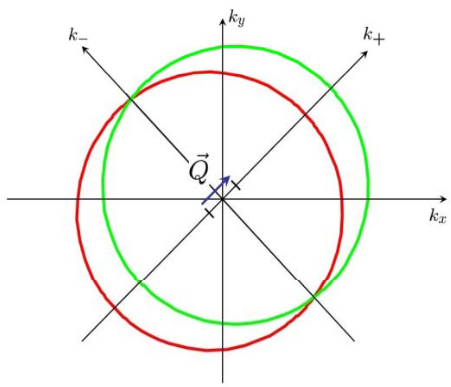
$$H_{\text{2DEG}} \approx \frac{\hbar^2 k^2}{2m} + \alpha(k_y - k_x)(\sigma_x + \sigma_y)$$

Ignoring the term
$\lambda^* \vec{\sigma} \cdot (\vec{k} \times \nabla \mathrm{V_{dis}}(\vec{r}))$
for now

- spin along the [110] direction is conserved
- long lived precessing spin wave for spin perpendicular to [110]
- The nesting property of the Fermi surface:

$$\varepsilon_{\downarrow}\left(\vec{k}\right) = \varepsilon_{\uparrow}\left(\vec{k} + \vec{Q}\right)$$

$$Q = \frac{4m\alpha}{\hbar^2}$$



The long lived spin-excitation: "spin-helix"

- Finite wave-vector spin components

$$S_Q^- = \sum_{\vec{k}} c_{\vec{k}\downarrow}^+ c_{\vec{k}+\vec{Q}\uparrow}, \quad S_Q^+ = \sum_{\vec{k}} c_{\vec{k}+\vec{Q}\uparrow}^+ c_{\vec{k}\downarrow}, \quad S_0^z = \sum_{\vec{k}} c_{\vec{k}\uparrow}^+ c_{\vec{k}\uparrow} - c_{\vec{k}\downarrow}^+ c_{\vec{k}\downarrow}$$

$$\left[S_0^z, S_Q^\pm\right] = \pm 2 S_Q^\pm, \quad \left[S_Q^+, S_Q^-\right] = S_0^z$$

- Shifting property essential

$$\left[H_{\mathrm{ReD}}, c_{\vec{k}+\vec{Q}\uparrow}^+ c_{\vec{k}\downarrow}\right] = \left(\varepsilon_\uparrow\left(\vec{k}+\vec{Q}\right) - \varepsilon_\downarrow\left(\vec{k}\right)\right) c_{\vec{k}+\vec{Q}\uparrow}^+ c_{\vec{k}\downarrow} = 0$$

➡ An exact SU(2) symmetry

Only Sz, zero wavevector U(1) symmetry previously known:

J. Schliemann, J. C. Egues, and D. Loss, Phys. Rev. Lett. **90**, 146801 (2003).

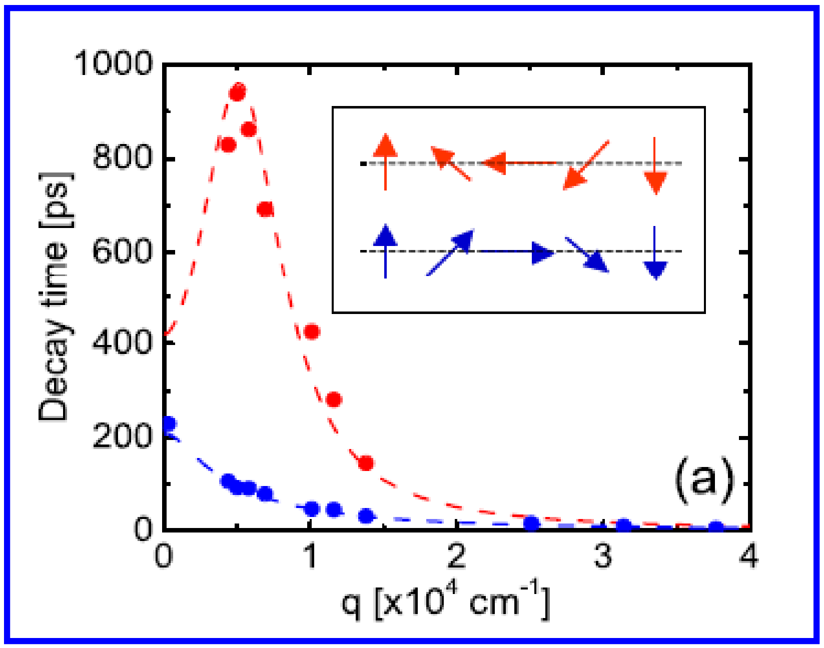
K. C. Hall *et. al.*, Appl. Phys. Lett **83**, 2937 (2003).

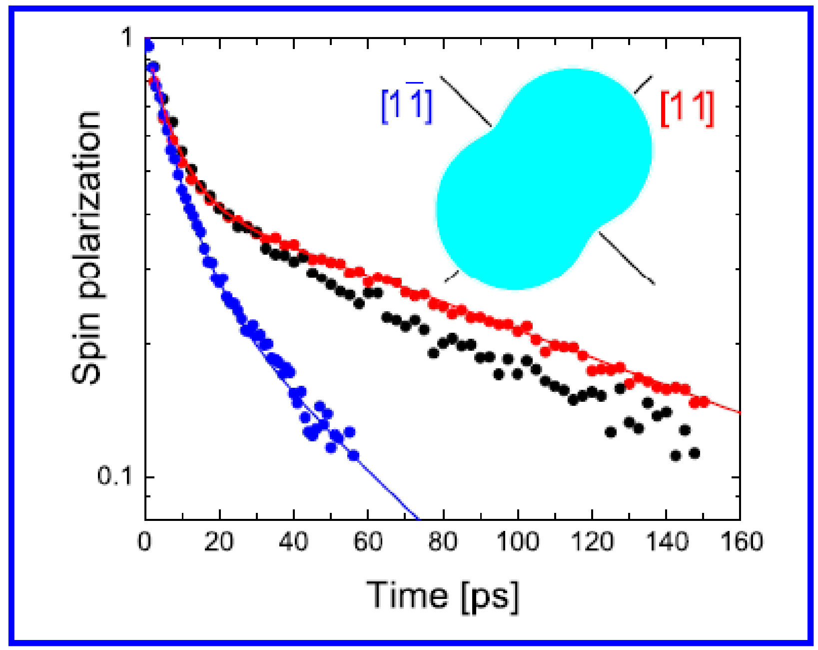
Persistent state spin helix verified by pump-probe experiments

Nondiffusive Spin Dynamics in a Two-Dimensional Electron Gas

C. P. Weber,[1] J. Orenstein,[1] B. Andrei Bernevig,[2] Shou-Cheng Zhang,[2] Jason Stephens,[3] and D. D. Awschalom[3]

PRL 98, 076604 (2007) PHYSICAL REVIEW LETTERS week ending 16 FEBRUARY 2007





Similar wafer parameters to ours

The Spin-Charge Drift-Diffusion Transport Equations

For arbitrary α,β spin-charge transport equation is obtained for diffusive regime

$$\partial_t n = D\nabla^2 n + B_1 \partial_{x+} S_{x-} - B_2 \partial_{x-} S_{x+}$$

$$\partial_t S_{x+} = D\nabla^2 S_{x+} - B_2 \partial_{x-} n - C_1 \partial_{x+} S_z - T_1 S_{x+}$$

$$\partial_t S_{x-} = D\nabla^2 S_{x-} - B_1 \partial_{x+} n - C_2 \partial_{x-} S_z - T_2 S_{x-}$$

$$\partial_t S_z = D\nabla^2 S_z + C_2 \partial_{x-} S_{x-} + C_2 \partial_{x+} S_{x+} - (T_1 + T_2) S_z$$

$$B_{1/2} = 2(\alpha \mp \beta)^2 (\alpha \pm \beta) k_F^2 \tau^2, \quad T_{1/2} = \frac{2}{m} (\alpha \pm \beta)^2 \frac{k_F^2 \tau}{\hbar^2}$$

$$D = v_F^2 \tau / 2, \quad \text{and } C_{1/2}^2 = 4DT_{1/2}$$

For propagation on [1-10], the equations decouple in two blocks.
Focus on the one coupling S_{x+} and S_z:

$$\partial_t S_{x-} = D\nabla^2 S_{x-} - C_2 \partial_{x-} S_z - T_2 S_{x-}$$

$$\partial_t S_z = D\nabla^2 S_z + C_2 \partial_{x-} S_{x-} - (T_1 + T_2) S_z$$

For Dresselhauss = 0, the equations reduce to **Burkov, Nunez and MacDonald,** PRB 70, 155308 (2004);
Mishchenko, Shytov, Halperin, PRL 93, 226602 (2004)

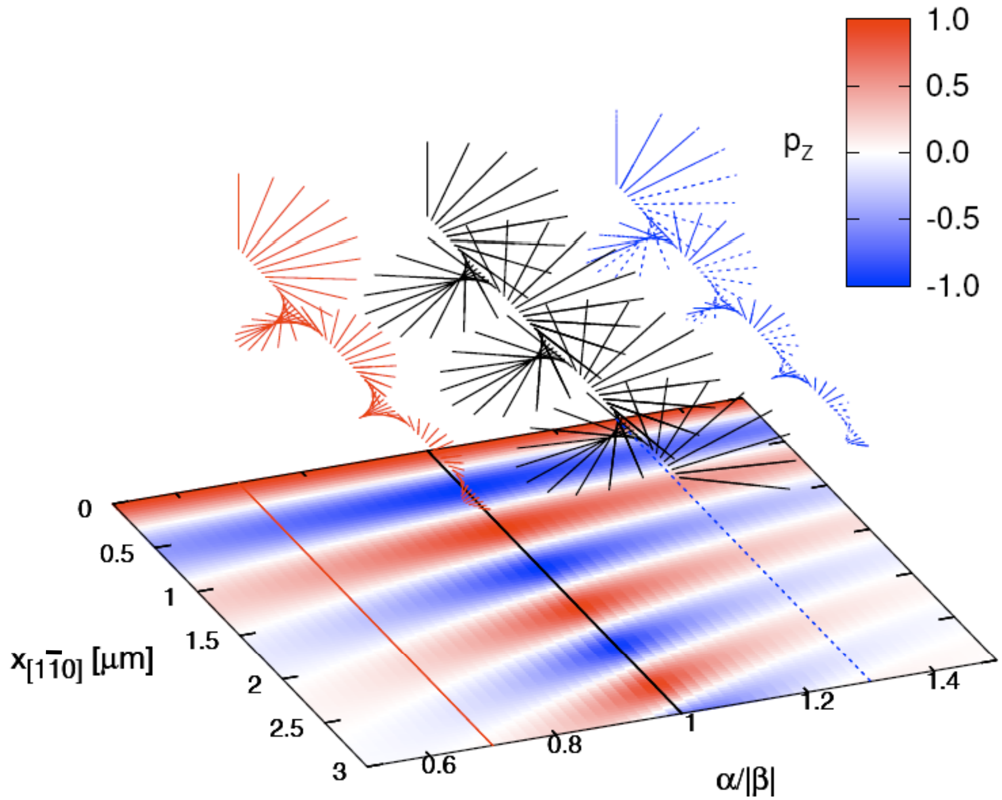
Steady state spin transport in diffusive regime

Steady state solution for the spin-polarization component if propagating along the [1-10] orientation

$$S_{z/x-}(x_{[1\bar{1}0]}) = S^0_{z/x-}\exp[q\,x_{[1\bar{1}0]}]$$

$$(Dq^2+T_2)(Dq^2+T_1+T_2)-C_2^2q^2 = 0$$

$$q = |q|\exp(i\theta),\ |q| = (\tilde{L}_1^{\,2}\tilde{L}_2^{\,2}+\tilde{L}_2^{\,4})^{1/4},\ \theta = \tfrac{1}{2}\arctan\left(\frac{\sqrt{\tilde{L}_1^{\,2}\tilde{L}_2^{\,2}-\tilde{L}_1^{\,4}/4}}{\tilde{L}_2^{\,2}-\tilde{L}_1^{\,2}/2}\right) \qquad \tilde{L}_{1/2} = 2m|\alpha\pm\beta|/\hbar^2$$



Spatial variation scale consistent with the one observed in SIHE

AHE contribution

$$H_{2DEG} = \frac{\hbar^2 k^2}{2m} + \alpha\left(k_y\sigma_x - k_x\sigma_y\right) + \beta\left(k_x\sigma_x - k_y\sigma_y\right) + \lambda^* \vec{\sigma}\cdot(\vec{k}\times\nabla \mathrm{V_{dis}}(\vec{r}))$$

Two types of contributions:

i) S.O. from band structure interacting with the field (external and internal)

ii) Bloch electrons interacting with S.O. part of the disorder

Type (i) contribution much smaller in the weak SO coupled regime where the SO-coupled bands are not resolved, dominant contribution from type (ii)

$$\left|\sigma_{xy}\right|^{\text{skew}} = \frac{2\pi e^2 \lambda^*}{\hbar^2} V_0 \tau\; n(n_\uparrow - n_\downarrow)$$

$$\left|\sigma_{xy}\right|^{\text{side-jump}} = \frac{2e^2\lambda^*}{\hbar}(n_\uparrow - n_\downarrow)$$

Crepieux et al PRB 01
Nozier et al J. Phys. 79

$$\left|\alpha_H\right|^{\text{side-jump}} \approx 5.3\times 10^{-4}$$

$$\alpha_H(x_{[1\bar{1}0]}) = 2\pi\lambda^* \sqrt{\frac{e}{\hbar n_i \mu}}\; n\, p_z(x_{[1\bar{1}0]}) \approx 1.1\times 10^{-3}\, p_z$$

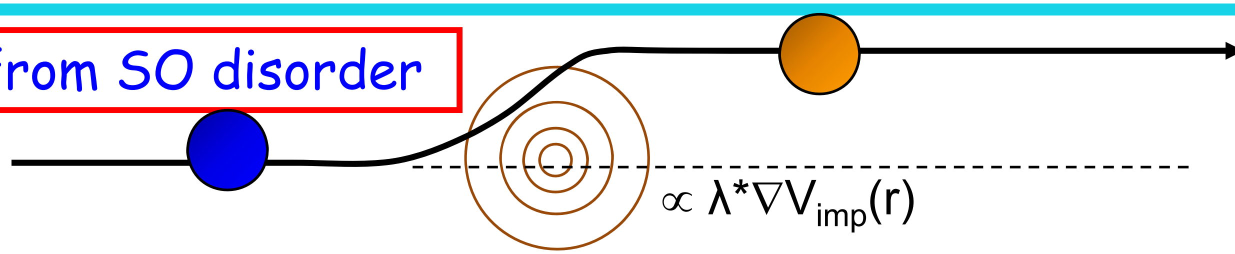
Lower bound estimate of skew scatt. contribution

WEAK SPIN-ORBIT COUPLED REGIME (Δ_{so}<ħ/τ)

Better understood than the strongly SO couple regime

The terms/contributions dominant in the strong SO couple regime are strongly reduced (quasiparticles not well defined due to strong disorder broadening). Other terms, originating from the interaction of the quasiparticles with the SO-coupled part of the disorder potential dominate.

Side jump scattering from SO disorder



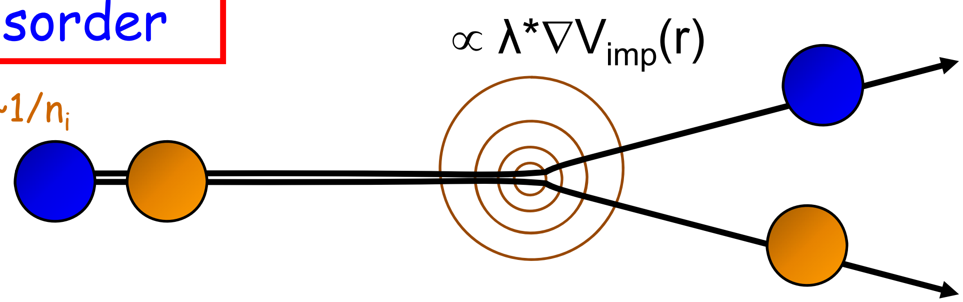
independent of impurity density

Electrons deflect first to one side due to the field created by the impurity and deflect back when they leave the impurity since the field is opposite resulting in a side step. They however come out in a different band so this gives rise to an anomalous velocity through scattering rates times side jump.

Skew scattering from SO disorder

Asymmetric scattering due to the spin-orbit coupling of the electron or the impurity. This is also known as Mott scattering used to polarize beams of particles in accelerators.

~1/n_i



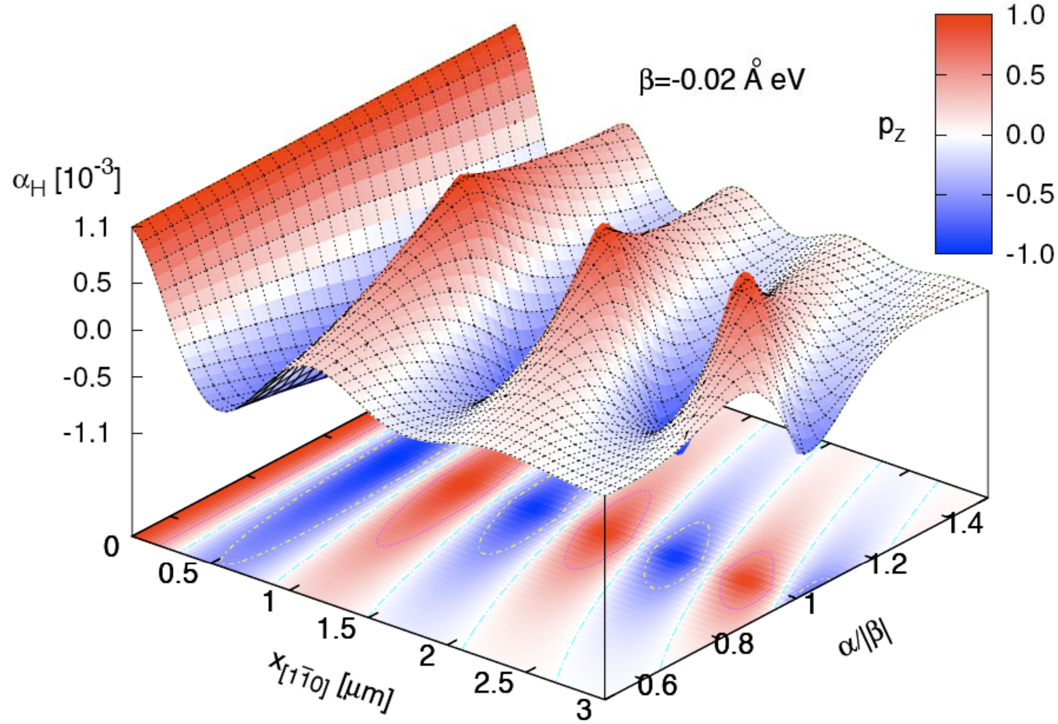
Spin injection Hall effect: Theoretical consideration

Local spin polarization → calculation of the Hall signal

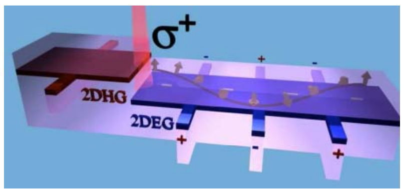
Weak SO coupling regime → extrinsic skew-scattering term is dominant

$$\alpha_H(x_{[1\bar{1}0]}) = 2\pi\lambda^* \sqrt{\frac{e}{\hbar n_i \mu}}\; n\, p_z(x_{[1\bar{1}0]})$$

Lower bound estimate

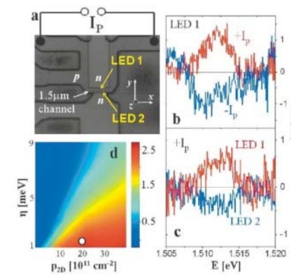


The family of spintronics Hall effects



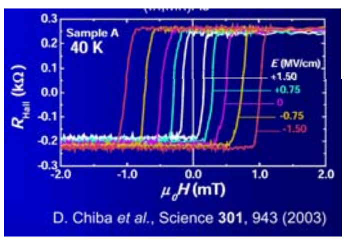
SIHE
B=0
Optical injected polarized current gives charge current

Electrical detection



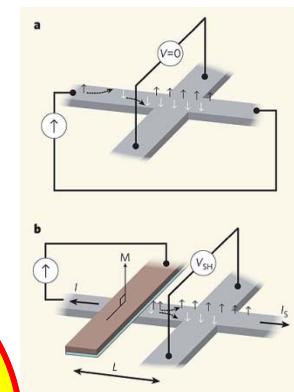
SHE
B=0
charge current gives spin current

Optical detection



Electrical detection

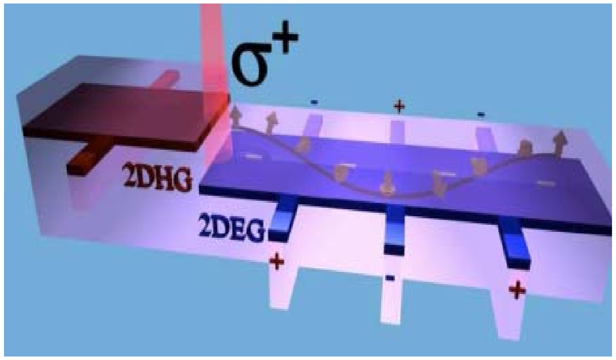
AHE
B=0
polarized charge current gives charge-spin current



SHE^{-1}
B=0
spin current gives charge current

Electrical detection

SIHE: a new tool to explore spintronics



•nondestructive <u>electric</u> probing tool of spin propagation <u>without magnetic elements</u>

•all electrical spin-polarimeter in the optical range

•Gating (tunes α/β ratio) allows for FET type devices (high T operation)

•New tool to explore the AHE in the strong SO coupled regime

CONCLUSIONS (SIHE)

Spin-injection Hall effect observed in a conventional 2DEG

- nondestructive electrical probing tool of spin propagation
- indication of precession of spin-polarization
- observations in qualitative agreement with theoretical expectations
- optical spin-injection in a reverse biased coplanar pn-junction: large and persistent Hall signal (applications !!!)