





Continuous time quantum Monte Carlo method for multiorbital strongly correlated systems

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## Motivation: STM





www.physnet.uni-hamburg.de/home/vms/pascal/stm.htm

www.almaden.ibm.com

Theoretical problem is: how to treat realistic multiorbital systems with full atomic Coulomb interaction

## Outline

- CT-QMC formalism
- Kondo model
- Multi-orbital impurity problem, Co in Cu
- Correlated solids, Sr<sub>2</sub>RuO<sub>4</sub>
- Conclusions

## **I. Theory. Impurity problem** Multiorbital Anderson impurity model:

$$H = \sum_{ij\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \frac{1}{2} \sum_{ijkl} U_{ijkl} c_{i\sigma}^{\dagger} c_{j\sigma'}^{\dagger} c_{l\sigma'} c_{k\sigma}$$

$$+ \sum_{jk\sigma} [\epsilon_j(\mathbf{k}) - \mu] d_{j\sigma}^{\dagger} d_{j\sigma} + \sum_{ik\sigma} V_{ik} (d_{k\sigma}^{\dagger} c_{i\sigma} + c_{i\sigma}^{\dagger} d_{k\sigma})$$
On-site Coulomb repulsion:  $H_{int} = Un_{\uparrow}n_{\downarrow}$ 
Impurity action:
$$S = S_0 + S_{int}$$

$$= -\int_0^\beta d\tau d\tau' \sum_{\sigma} c_{\sigma}^{\dagger}(\tau) \mathcal{G}_{\sigma}^{-1}(\tau - \tau') c_{\sigma}(\tau') + \int_0^\beta d\tau Un_{\uparrow}(\tau) n_{\downarrow}(\tau)$$

$$\mathcal{G}_{\sigma}^{-1}(i\omega) = i\omega + \mu - \Delta_{\sigma}(i\omega)$$

$$G(\tau - \tau') = \langle T_{\tau}c^{\dagger}(\tau)c(\tau') \rangle_S = \frac{Tr \left[T_{\tau}e^{-S}c^{\dagger}(\tau)c(\tau')\right]}{Tr \left[T_{\tau}e^{-S}\right]}$$

#### **Perturbation expansion**

Perturbation-series expansion for the partition function:

$$Z = Tr \left[ T_{\tau} e^{-(S_0 + S_{int})} \right]$$
  
=  $\sum_k \frac{(-1)^k}{k!} \int_0^\beta d\tau_1 \dots d\tau_k Tr \left[ T_{\tau} e^{-S_0} U^k n_{\uparrow}(\tau_1) n_{\downarrow}(\tau_1) \dots n_{\uparrow}(\tau_k) n_{\downarrow}(\tau_k) \right]$   
=  $\sum_k \frac{(-U)^k}{k!} \int_0^\beta d\tau_1 \dots d\tau_k \left\langle T_{\tau} c^{\dagger}(\tau_1) c(\tau_1) \dots c^{\dagger}(\tau_k) c(\tau_k) \right\rangle_{S_0}$ 

According to Wick's theorem:

$$\left\langle T_{\tau}c^{\dagger}(\tau_1)c(\tau_1)...c^{\dagger}(\tau_k)c(\tau_k)\right\rangle_{S_0} = \det |\mathcal{G}(\tau_l,\tau_m)| = D^k$$

#### Random walks in the space of $\Omega_k$

$$Z = \sum_{k} Z_{0} \int_{0}^{\beta} d\tau_{1} \dots d\tau_{k} \frac{(-U)^{k}}{k!} D^{k} = \sum_{k} \int_{0}^{\beta} d\tau_{1} \dots d\tau_{k} \Omega_{k}$$

$$\Omega_{k} = Z_{0} \frac{(-U)^{k}}{k!} D^{k}$$
random walks over expansion terms:
$$Z = \dots + \qquad +\Omega_{k-1} \qquad +\Omega_{k} \qquad +\Omega_{k+1} \qquad +\dots$$
An example of  $\Omega_{k}$  distribution
$$Acceptance ratio$$

$$Acceptance ratio$$

$$Step k-1 \qquad Step k+1$$

$$\frac{k}{|U|} \frac{D^{k+1}}{D^{k}} \qquad \frac{|U|}{k+1} \frac{D^{k+1}}{D^{k}}$$
Rubtsov et al., JETP Lett **80** 61 (2004)

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ov et al., JETP Lett **80** 61 (2004), PRB **72** 035122 (2005)

#### **Measuring of Green's functions**

Perturbation-series expansion for the Green function:

Perturbation-series expansion for the four point correlator:

$$\begin{split} \chi(\tau,\tau',\tau'',\tau''') &\equiv Z^{-1} < Tc^{\dagger}(\tau)c(\tau')c^{\dagger}(\tau'')c(\tau''')e^{-S_{int}} > \\ &= Z^{-1}\sum_{k}\int d\tau_{1}\int d\tau_{1}'...\int d\tau_{2k}'\chi^{(2k)}(\tau,\tau',\tau'',\tau''')\Omega_{k}(\tau_{1},\tau_{1}',...,\tau_{2k}') \\ \chi^{(2k)}(\tau,\tau',\tau'',\tau''') &= \frac{< Tc^{\dagger}(\tau)c(\tau')c^{\dagger}(\tau'')c(\tau''')c^{\dagger}(\tau_{1})c(\tau_{1}')...c^{\dagger}(\tau_{2k})c(\tau_{2k}') >}{< Tc^{\dagger}(\tau_{1})c(\tau_{1}')...c^{\dagger}(\tau_{2k})c(\tau_{2k}') >} \\ &= \frac{D^{(2k+2)}}{D^{(2k)}} \end{split}$$

#### **Computational details**



#### **Computational details, multiorbital formalism**

Interaction of the general form:

$$\Omega_k = Z_0 \frac{(-U)^k}{k!} D^k \implies \Omega_k = Z_0 \frac{(-1)^k}{k!} U_{r_1 r_2}^{r'_1 r'_2} \dots U_{r_{2k-1} r'_{2k}}^{r'_{2k-1} r'_{2k}} D_{r'_1 r'_2 \dots r'_{2k}}^{r_1 r_2 \dots r_{2k}}$$

$$\int_{0}^{\beta} d\tau \quad \Longrightarrow \quad \int dr = \sum_{\{ijml\sigma\sigma'\}} \int_{0}^{\beta} d\tau_{1} \dots \int_{0}^{\beta} d\tau_{2k}, \qquad \qquad r = \{0, 0\} = 0$$

Green's functions calculation:

$$G_{ij}(\tau,\tau') = \mathcal{G}_{ij}(\tau,\tau') - \sum_{n,m}^{K} \mathcal{G}_{ijn}(\tau,\tau_n) D_{n,m}^{-1} \mathcal{G}_{imj}(\tau_m,\tau')$$

$$r = \{\tau, i, \sigma\}$$
  
 $D = [\mathcal{G}(\tau_p, \tau_q)]$ 

$$G_{ij}(\omega) = \mathcal{G}_{ij}(\omega) - \frac{1}{\beta} \sum_{n,m}^{K} \mathcal{G}_{ij_n}(\omega) D_{n,m}^{-1} e^{i\omega(\tau_n - \tau_m)} \mathcal{G}_{i_m j}(\omega),$$

#### Sign problem: introduction of $\alpha$ parameters

$$Z = \sum_{k} \sum_{\{ijml\sigma\sigma'\}} Z_0 \int_0^\beta d\tau_1 \dots d\tau_k \frac{(-1)^k}{k!} U_{i_1 j_1 m_1 l_1} \dots U_{i_k j_k m_k l_k} D_{\tau'_1 \tau'_2 \dots \tau'_k}^{\tau_1 \tau_2 \dots \tau_k}$$

$$S_{0} = \sum_{ij\sigma} \int_{0}^{\beta} \int_{0}^{\beta} \left( -\mathcal{G}_{ij}^{-1}(\tau - \tau') + \frac{1}{2} \sum_{\{ml\sigma'\}} \alpha_{ml}^{\sigma'}(U_{ilmj} + U_{lijm}) \delta_{\tau\tau'} \right) c_{i\sigma}^{\dagger} c_{j\sigma} d\tau d\tau'$$

$$S_{int} = \frac{1}{2} \sum_{\{ijml\sigma\sigma'\}} \int_{0}^{\beta} U_{ijml} (c_{i\sigma}^{\dagger} c_{l\sigma} - \alpha_{il}^{\sigma}) (c_{j\sigma'}^{\dagger} c_{m\sigma'} - \alpha_{jm}^{\sigma'}) d\tau.$$
  

$$\alpha_{ii}^{\sigma} + \alpha_{jj}^{\sigma'} = \bar{\alpha}$$
  

$$D_{\tau_{1}'\tau_{2}'...\tau_{k}'}^{\tau_{1}\tau_{2}...\tau_{k}} = \det |\mathcal{G}_{iplq}(\tau_{p}, \tau_{q}) - \alpha_{iplq}^{\sigma} \delta_{pq}| \qquad \qquad \alpha_{ij}^{\sigma} + \alpha_{kl}^{\sigma'} = 0$$

Rubtsov et al., JETP Lett **80** 61 (2004), PRB **72** 035122 (2005)

#### **Susceptibility**

$$\chi^{+-}(\tau) = \left\langle S^{+}(\tau)S^{-}(0) \right\rangle = \left\langle c_{\uparrow}^{\dagger}(\tau)c_{\downarrow}(\tau)c_{\downarrow}^{\dagger}(0)c_{\uparrow}(0) \right\rangle$$
$$\chi^{+-}(\Omega) = \frac{1}{\beta} \sum_{\omega_{1},\omega_{2}=-\infty}^{\infty} G_{\uparrow}(\omega_{1}+\Omega,\omega_{2}+\Omega)G_{\downarrow}(\omega_{2},\omega_{1})$$
$$G(\omega,\omega') = \mathcal{G}(\omega)\delta_{\omega,\omega'} - \mathcal{G}(\omega) \left[ \frac{1}{\beta} \sum_{i,j} e^{i\omega\tau_{i}} D_{i,j}^{-1} e^{-i\omega'\tau_{j}} \right] \mathcal{G}(\omega')$$







#### **II.** Kondo effect, introduction



Adapted from L. Kouwenhoven and L. Glazman, Revival of the Kondo effect, Physics World (2001)

#### **Abrikosov's representation**

$$H_{sd} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma}$$

$$- \sum_{\mathbf{k}\mathbf{k}'} J_{kk'} \left[ S^{+} c_{\mathbf{k}\downarrow}^{\dagger} c_{\mathbf{k}'\uparrow} + S^{-} c_{\mathbf{k}\uparrow}^{\dagger} c_{\mathbf{k}'\downarrow} + S_{z} \left( c_{\mathbf{k}\uparrow}^{\dagger} c_{\mathbf{k}'\uparrow} - c_{\mathbf{k}\downarrow}^{\dagger} c_{\mathbf{k}'\downarrow} \right) \right]$$

$$= \frac{\mu}{1 - 2} \int Ss \int DOS \int DOS \int DOS \int DOS \int S^{+} = a_{\uparrow}^{\dagger} a_{\downarrow}, \quad S_{z} = \frac{1}{2} (a_{\uparrow}^{\dagger} a_{\uparrow} - a_{\downarrow}^{\dagger} a_{\downarrow}), \quad n_{\sigma}^{a} = a_{\sigma}^{\dagger} a_{\sigma}, \quad n_{\sigma}^{c} = c_{\sigma}^{\dagger} c_{\sigma}$$

$$H_{int} = -2J(S^{z}s^{z} + 1/2(S^{+}s^{-} + S^{-}s^{+}))$$

$$= -J/2 \left[ (n_{\uparrow}^{a}n_{\uparrow}^{c} + n_{\downarrow}^{a}n_{\downarrow}^{c}) + (n_{\uparrow}^{a}n_{\downarrow}^{c} + n_{\downarrow}^{a}n_{\uparrow}^{c}) + 2(a_{\uparrow}^{\dagger} c_{\uparrow} c_{\uparrow}^{\dagger} a_{\downarrow} + c_{\uparrow}^{\dagger} a_{\uparrow} a_{\downarrow}^{\dagger} c_{\downarrow}) \right]$$

A. A. Abrikosov. Electron scattering on magnetic impurities in metals and anomalous resistivity effects. *Physics* 2, 5 (1965)

#### Kondo impurity in non-uniformal bath DOS



 $\rho(0) = 1/3$ 





#### Kondo impurity in realistic bath DOS



A. Zhuravlev, I. Zharekeshev, E. Gorelov, A. I. Lichtenstein, E. R.Mucciolo, S. Kettemann, "Nonperturbative Scaling Theory of FreeMagneticMoment Phases in DisorderedMetals", Phys. Rev. Lett. 99, 247202 (2007)

#### **III.** Multiorbital impurity model

Interaction of general form:

$$H_{loc} = \sum_{ij\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \frac{1}{2} \sum_{\substack{ijkl\\\sigma\sigma'}} U_{ijkl} c_{i\sigma}^{\dagger} c_{j\sigma'}^{\dagger} c_{l\sigma'} c_{k\sigma}$$

#### Rotationally invariant Hamiltonian:

$$H_{loc} = -\sum_{\langle ij \rangle m\sigma} t_m c^{\dagger}_{im\sigma} c_{jm\sigma} + U \sum_{im} n_{im\uparrow} n_{im\downarrow} + U' \sum_{\substack{i \\ \sigma \neq \sigma' \\ m \neq m'}} n^{\dagger}_{im\sigma} n_{im'\sigma'} + J \sum_{\substack{i \\ \sigma \neq \sigma' \\ m \neq m'}} c^{\dagger}_{im\sigma} c^{\dagger}_{im'\sigma'} c_{im\sigma'} c_{im'\sigma'} + c^{\dagger}_{im\sigma} c^{\dagger}_{im\sigma'} c_{im'\sigma'} c_{im'\sigma'} + c^{\dagger}_{im\sigma'} c_{im'\sigma'} c_{im$$

C. Castellani et al., PRB, 18, 4945 (1978)

5-band model: general U vertex  $H_{loc} = \sum_{ij\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \frac{1}{2} \sum_{ijkl\sigma\sigma'} U_{ijkl} c_{i\sigma}^{\dagger} c_{j\sigma'}^{\dagger} c_{l\sigma'} c_{k\sigma'}$ Slater parametrization of U for atom Multipole expansion:  $\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \sum_{ka} \frac{4\pi}{2k+1} \cdot \frac{r^k_{<}}{r^{k+1}_{>}} Y^*_{kq}(\hat{r}) Y_{kq}(\hat{r}')$ Coulomb matrix elements in  $Y_{\text{Im}}$  basis:  $< mm' |U|m''m''' > = \sum a_k(m, m'', m', m'')F^k$ Angular part – 3j symbols  $a_k(m,m',m''',m''') = \sum_{q=-k}^k (2l+1)^2 (-1)^{m+q+m'} \begin{pmatrix} l & k & l \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{pmatrix} l & k & l \\ -m & -q & m' \end{pmatrix} \begin{pmatrix} l & k & l \\ -m'' & q & m''' \end{pmatrix}$ Slater integrals:  $F^{k} = e^{2} \int_{0}^{\infty} r^{2} dr |\varphi_{d}(r)|^{2} \int_{0}^{\infty} (r')^{2} dr' |\varphi_{d}(r')|^{2} \frac{r_{<}^{k}}{r^{k+1}}$ 

## Constrain LDA: U and J

Average Coulomb parameter:

$$U = \frac{1}{(2l+1)^2} \sum_{mm'} U_{mm'} = F^0$$

Average exchange parameter:

$$J = \frac{1}{(2l+1)^2} \sum_{mm'} J_{mm'} = \sum_{k \neq 0} \left( \begin{array}{ccc} l & k & l \\ 0 & 0 & 0 \end{array} \right)^2 F^k$$

For d-electrons:  $J = \frac{1}{14}(F^2 + F^4), \quad \frac{F^2}{F^4} = 0.625$ Coulomb and exchange interactions:

$$U_{mm'} = < mm' |U| mm' >$$
  
$$J_{mm'} = < mm' |U| m'm >$$

Anisimov et al., J. Phys.: Condens. Matter 9, 767 (1997)

#### **Effects of spin-flip interaction**



#### 5-orbital impurity model in the atomic limit

#### High temperature



#### Low temperature, d<sup>5</sup> configuration



#### Spin-flip interaction vs. coupling to bath

#### Three-orbital fully degenerated impurity



## 5-band realistic model, Co atom in bulk Cu

- Comparison of d<sup>7</sup> and d<sup>8</sup> configuration
- Comparison of diagonal and full U in d<sup>7</sup> configuration



Quasiparticle weight:

$$Z = \left[1 - \frac{\partial Im\Sigma(i\omega)}{\partial(i\omega)}\Big|_{\omega=\omega_1}\right]^{-1} \approx 0.5$$

#### **IV.** Lattice model, DMFT approximation



$$G_{loc}(i\omega_n) = \sum_{\mathbf{k}} (G_0^{-1}(\mathbf{k}, i\omega_n) - \Sigma(i\omega_n))^{-1}$$
$$\mathcal{G}_{imp}^{-1}(i\omega_n) = G_{loc}^{-1}(i\omega_n) + \Sigma(i\omega_n).$$
$$G_{imp}(\tau) = -\left\langle T_{\tau}c(\tau)c^{\dagger}(0)\right\rangle_{S_{imp}}$$
$$\Sigma(i\omega_n) = \mathcal{G}_{imp}^{-1}(i\omega_n) - G_{imp}^{-1}(i\omega_n)$$

### 2-band lattice model





#### 3-band lattice model, non-integer occupancy



# Sr<sub>2</sub>RuO<sub>4</sub>: 3-band lattice model within LDA+DMFT scheme

• 4d t<sub>2g</sub> Ru orbitals

• LDA + DMFT scheme



John Passaneau, Penn State



V. I. Anisimov et al., J. Phys.: Condens. Matter **9**, 7359 (1997) A. I. Lichtenstein et al., Phys. Rev. B **57**, 6884 (1998)

## Sr<sub>2</sub>RuO<sub>4</sub>: 3-band lattice model within LDA+DMFT scheme



## Sr<sub>2</sub>RuO<sub>4</sub>: 3-band lattice model within LDA+DMFT scheme

![](_page_28_Figure_1.jpeg)

## Conclusions

- We developed general and efficient CT-QMC formalism for multi-orbital problems
- A general scheme for investigation a susceptibilities of general impurity model coupled to bath with arbitrary DOS is proposed
- The 3-orbital impurity model with rotationally invariant interaction including spin-flip terms has no sign problem in CT-QMC
- For a first time realistic 5-orbital impurity model for Co in Cu have been investigated
- We show a possibility of LDA+DMFT calculation for rotationally invariant 3-band lattice problem in CT-QMC

## 5-band realistic model, Co atom in bulk Cu

- Comparison of d<sup>7</sup> and d<sup>8</sup> configuration
- · Comparison of diagonal and full U
- ---- LDA U = 4 CT-QMC, diagoval vertex J = 0.7CT-QMC, full vertex  $\beta = 10$ DOS -2 Ó 2 -3 -1 3 5 E (eV) -0.1 full U -0.2 diagonal U -0.3 U = 3G(τ) -0.4 J = 0.7b = 10 -0.5 -0.6 -0.7 10 Ó 2 3 5 8 9 τ

•  $d^7$  configuration, U = 3

![](_page_31_Figure_0.jpeg)

#### 5-band realistic model, Co adatom on Cu (111) or Pt (111) surface. Spin-orbital coupling is taken into account

![](_page_32_Figure_1.jpeg)

#### Distribution of U-fields: computational effort

![](_page_34_Figure_0.jpeg)

![](_page_34_Figure_1.jpeg)

![](_page_35_Figure_0.jpeg)

![](_page_36_Figure_0.jpeg)

![](_page_37_Figure_0.jpeg)