Location and multiplicity results for general nonlinear fourth order BVPs and applications

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Abstract

This work concerns to the existence, non-existence, multiplicity and location results for the problem composed by the fourth order fully nonlinear equation

$$u^{(4)}(x) + f\left(x, u(x), u'(x), u''(x), u'''(x)\right) = s\ p(x) \tag{E}$$

for $x \in [0,1]$, $f : [0,1] \times \mathbb{R}^4 \to \mathbb{R}$, $p : [0,1] \to \mathbb{R}^+$ continuous functions and s a real parameter, with the Lidstone boundary conditions

$$u(0) = u(1) = u''(0) = u''(1) = 0. \tag{L}$$

It will be done an Ambrosetti-Prodi type discussion on s. That is, there are $s_0, s_1 \in \mathbb{R}$ such that:

· for $s < s_0$ or $(s > s_0)$ there is no solution of (E)-(L).

· for $s = s_0$ problem (E)-(L) has at least a solution.

· for $s \in]s_0, s_1]$ (or $s \in [s_1, s_0[$) there are at least two solutions of (E)-(L).

The arguments used apply lower and upper solutions technique, *a priori* estimations and topological degree theory.

This method will be applied to more general boundary value problems, which include the equation

$$u^{(4)}(x) = f\left(x, u(x), u'(x), u''(x), u'''(x)\right) \tag{1}$$

and the functional boundary conditions

$$\begin{array}{l} L_0\left(u, u', u'', u''(0)\right) = 0 = L_1\left(u, u', u'', u''(0)\right) \\ L_2\left(u, u', u'', u''(0), u'''(0)\right) = 0 = L_3\left(u, u', u'', u''(1), u'''(1)\right) \end{array} \tag{2}$$

where L_i, $i = 0, 1, 2, 3$, are continuous functions satisfying some monotonicity assumptions.

Some particular cases of problem (1)-(2), such as nonlocal and multipoint problems, will be considered.

An application to a continuous model of the human spine, used in aircraft ejections, vehicle crash situations and some forms of scoliosis, will be presented.