On the limit case $p \longrightarrow 1$ of the power law model (stationary motion)

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Abstract

The stationary motion of an incompressible fluid is governed by the system of PDEs

(1) $$\nabla \cdot \mathbf{u} = 0, \qquad -\nabla \cdot S + \nabla p = \mathbf{f}$$

where $\mathbf{u} = (u_1, \ldots, u_n)$ velocity, $S = \{S_{ij}\}$ deviatoric stress, p pressure, $\mathbf{f}$ external force. We consider the following constitutive law (R. von Mises (1913)):

(2) $$D = 0 \Longrightarrow |S| \leq g, \quad D \neq 0 \Longrightarrow S = \frac{g}{|D|} D$$

($D = D(\mathbf{u}) = \{D_{ij}(\mathbf{u})\}$, $D_{ij}(\mathbf{u}) = \frac{1}{2} \left(\partial_j u_i + \partial_i u_j\right)$ rate of strain, $g = \text{const} > 0$ yield value). The relations (2) model perfect plastic behavior of an incompressible fluid („von Mises solid").

The weak formulation of (1), (2) in a bounded domain $\Omega \subset \mathbb{R}^n$ under Dirichlet boundary conditions on $\mathbf{u}$ leads to the problem

$$\text{minimize} \qquad \mathcal{F}(\mathbf{u}) := g \int_\Omega |D(\mathbf{u})| - \int_\Omega \mathbf{f} \cdot \mathbf{u}.$$

We solve this problem in the space $\mathrm{BD}(\Omega)$ under a physically motivated smallness assumption on $\mathbf{f}$. Our method of proof consists in approximating (2) by the power law

$$S_\varepsilon = g|D|^{\varepsilon - 1} D \qquad (\varepsilon > 0)$$

and carrying out the passage to the limit $\varepsilon \to 0$.