On the partial regularity of suitable weak solutions to the generalized Navier-Stokes equations

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Abstract In our lecture we consider an incompressible fluid with shear rate dependent viscosity which are governed by the following generalized Navier-Stokes equations

$$\mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} - \operatorname{div}(\nu(|\mathbf{Du}|)\mathbf{Du})) = \mathbf{f} - \nabla p, \quad \operatorname{div}\mathbf{u} = 0$$

in a cylindrical domain $Q = \Omega \times (0,T)$, where velocity $\mathbf{u}$ and the pressure p are the unknown quantities.

The special case, where ν_0 is a positive constant represents the well-known Navier-Stokes equations, for which the partial regularity has been proved by Caffarelli-Kohn-Nirenberg. However in case of the power law model

$$\nu(\mathbf{Du}) = |\mathbf{Du}|^{q-2}, \quad 1 < q < \infty$$

the question of partial regularity together with the corresponding estimation of the singular set is still open. In our talk we introduce a generalized notion of suitable weak solution and give the main idea how to prove the partial regularity by using an appropriate rescaling of the system under consideration.