Equilibrium configurations of epitaxially strained elastic films: qualitative properties of solutions

I present some recent results on the equilibrium configurations of a variational model for the epitaxial growth of a thin film with planar symmetries on a thick substrate obtained in colllaboration with M.Morini. In our model the total energy is given by the sum of an elastic energy and a surface term of the type

$$\int_{\Omega_\Gamma} W(E(u))dx + \sigma_f \mathcal{H}^1(\Gamma)\,,$$

where Ω_Γ is the region occupied by the film (bounded from above by the graph Γ), W is a quadratic form depending on the symmetric part $E(u)$ of the gradient of the displacement u and $\mathcal{H}^1$ denotes the 1-dimensional Hausdorff measure.
Though many properties have been well understood since long by means of numerical or asintotic expansion arguments, rigorous analytical proofs of the existence and regularity of minimal configurations have been obtained only in the last few years.
In the talk I will discuss a second variation approach that led us to determine analytically the critical volume thresholds for the local and global minimality of the flat configuration and to understand under which conditions cusp singularities do not form, once the flat configuration becomes unstable.