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Lower and upper solutions in the theory of phi-Laplacian problems.

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The ϕ-Laplacian equations have appeared in the literature on the basis of the p-Laplacian equations, which follow, for some $p > 1$, a formulation of the type

$$-\frac{\mathrm{d}}{\mathrm{d}t}\,(|u'(t)|^{p-2}\,u'(t) = f(t, u(t), u'(t)), \quad t \in [a, b].$$

This kind of problems models some physical phenomena in non-Newtonian fluid mechanics. Following the qualitative properties of the function $\phi_p(y) = |y|^{p-2}\,y$, the p-Laplacian equations can be studied under the more general formulation

$$-\frac{\mathrm{d}}{\mathrm{d}t}\,(\phi(u'(t)) = f(t, u(t), u'(t)), \quad t \in [a, b],$$

where $\phi : \mathbb{R} \to \mathbb{R}$ is an increasing homeomorphism. Such problems have been recently studied by several authors from various points of view. One of the most useful techniques to approach such problems is given by the method of lower and upper solutions. This method allows us to ensure the existence of at least one solution (in some case, extremal solutions) of the considered problem.

In our talk we present several existence results for such equations, considering general nonlinear boundary conditions under the assumption that the lower and the upper solutions are well-ordered, i.e. the lower solution is less than the upper one. The less common case that the lower and the upper solution are reversely ordered, is also considered for periodic and Neumann boundary conditions and f independent of u'. In such a case optimal estimates on f and ϕ are obtained to ensure the existence of solutions lying between the lower and the upper solution.

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