Hypernuclear Spectroscopy

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Decay of hypernuclei   recall $m_\Lambda = 1115.68$ MeV

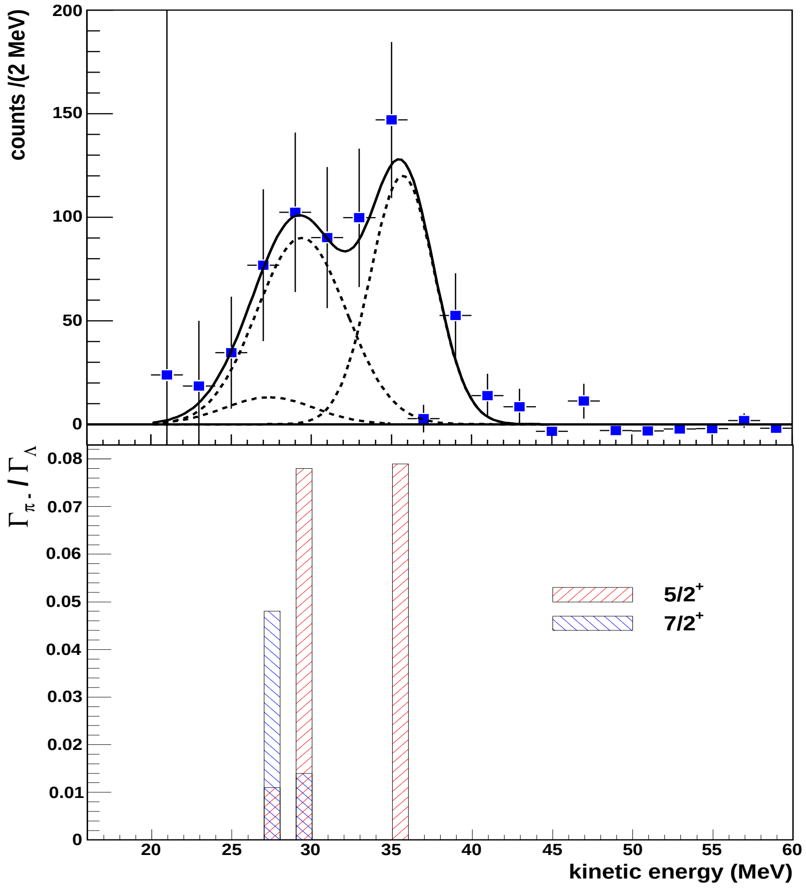
- Weak decay from ground state with $\tau \sim 200$ ps ($\tau_\Lambda = 263 \pm 2$ ps)

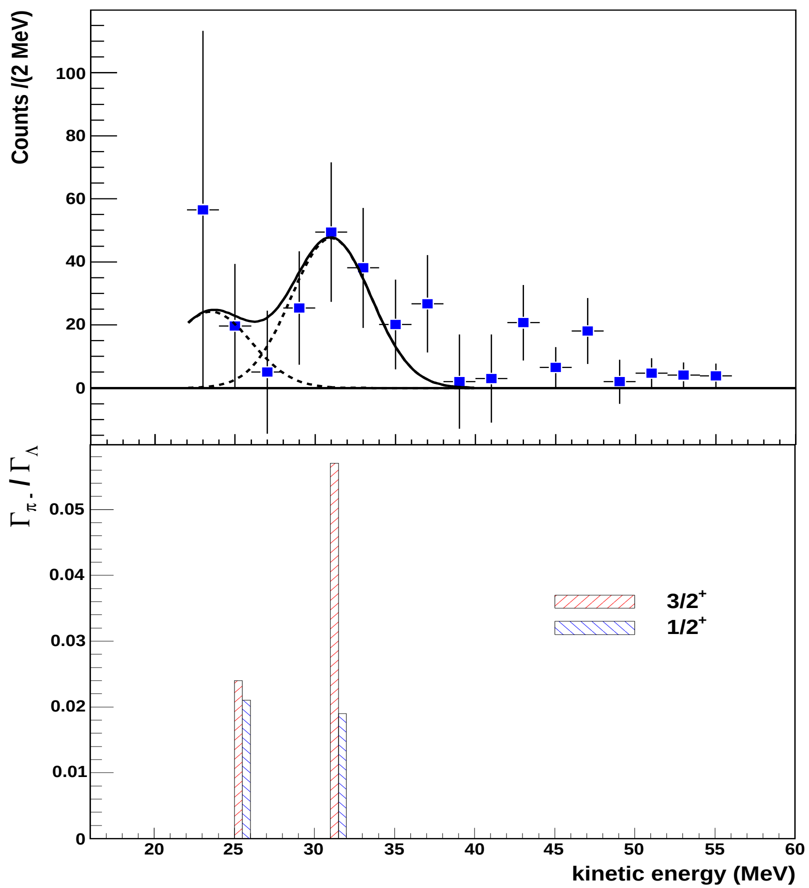
Γ_{π^-}	$\Lambda \to p + \pi^-$	free 63.9(5)%
Γ_{π^0}	$\Lambda \to n + \pi^0$	free 35.8(5)%
Γ_N	$\Lambda N \to nN$	in medium $\sim 80\%$
Γ_{NN}	$\Lambda NN \to nNN$	in medium $\sim 20\%$

- γ decay from bound excited states or particle emission followed by γ decay from daughter hypernucleus

[a] assisted by John Millener

Recent mesonic weak-decay spectra from FINUDA, PLB 681 (2009) 139





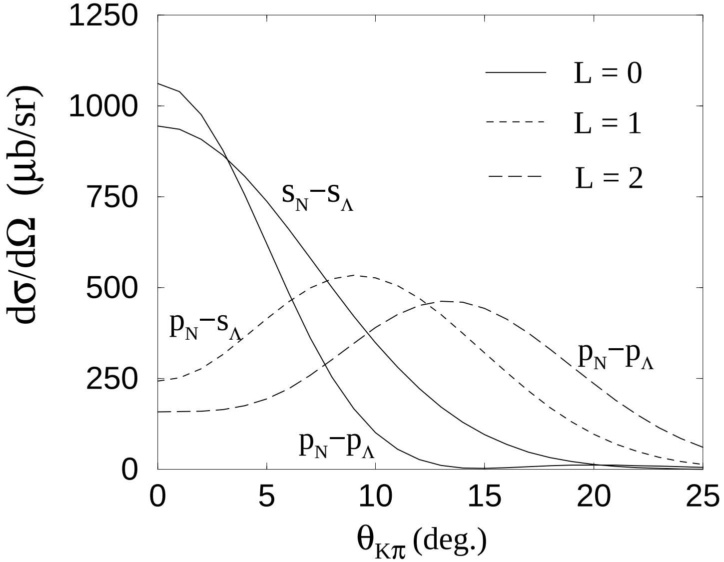
${}^{11}_{\Lambda}\mathrm{B} \rightarrow \pi^- + {}^{11}\mathrm{C}$, spin $5/2^+$ favored ${}^{15}_{\Lambda}\mathrm{N} \rightarrow \pi^- + {}^{15}\mathrm{O}$, spin $3/2^+$ favored

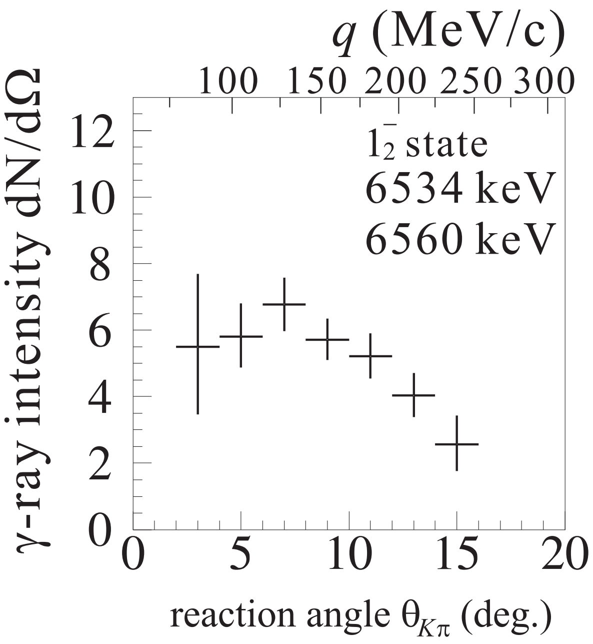
Motoba, Itonaga, Bando, NPA 489 (1988) 683; update Gal, NPA 828 (2009) 72

Studies of Λ hypernuclei

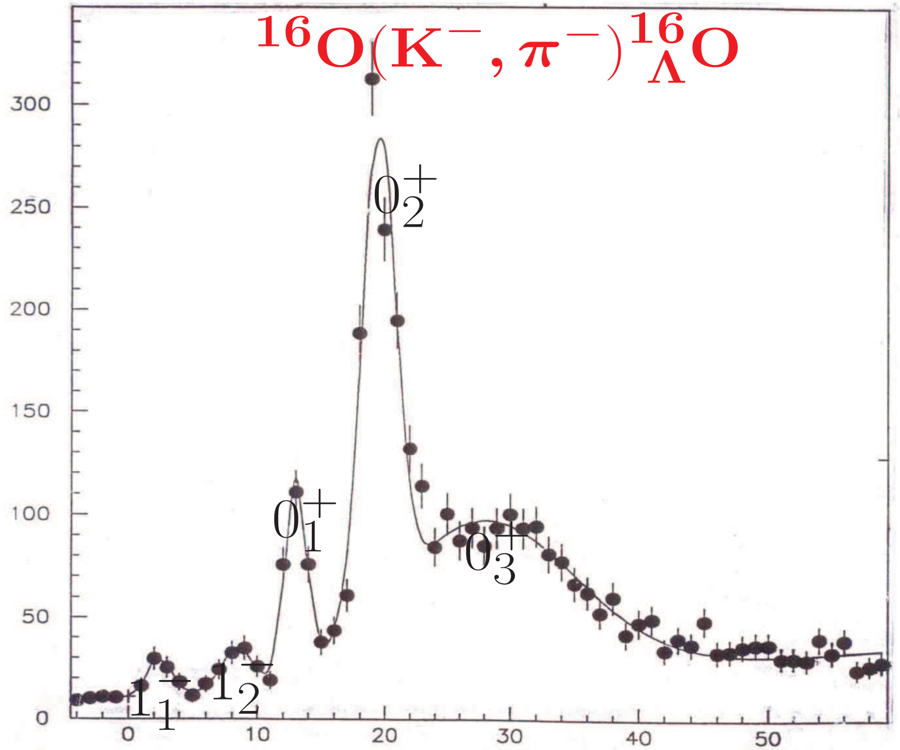
- $n(K^-, \pi^-)\Lambda$ – **emulsions, CERN, BNL, KEK, Frascati**
- $n(\pi^+, K^+)\Lambda$ – **BNL, KEK**
- $(\pi^+, K^+\gamma)$ **at KEK and** $(K^-, \pi^-\gamma)$ **at BNL, with Hyperball**
- $p(e, e'K^+)\Lambda$ – **JLab, Hall A and Hall C**

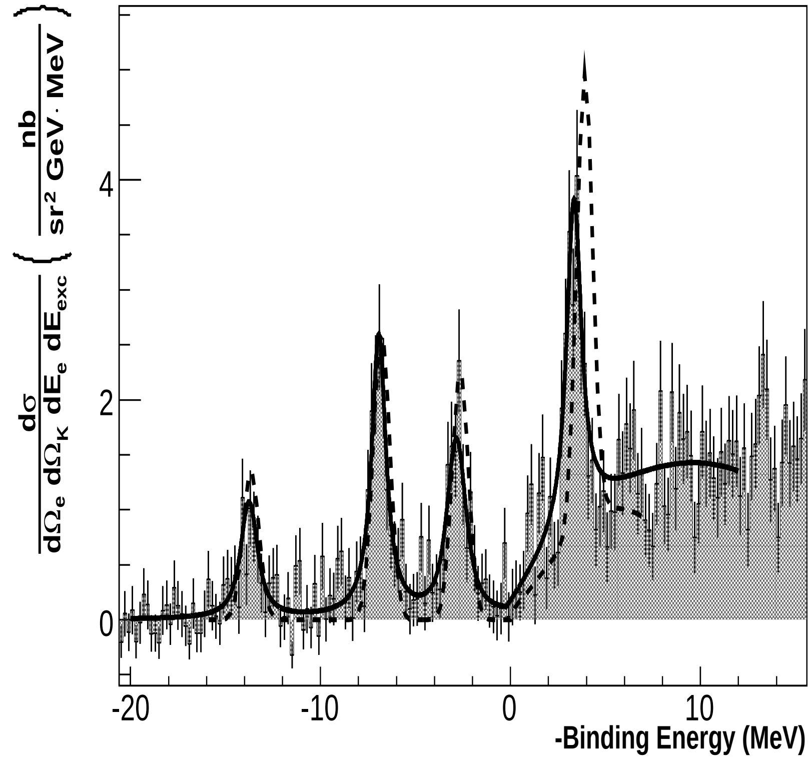
^{16}O$(K^-, \pi^-\gamma)^{16}_{\Lambda}$O at $p_K = 900$ MeV/c





M. Ukai et al., Phys. Rev. C 77 (2008) 054315 [BNL-E930]





$^{16}\mathrm{O}(K^-,\pi^-)^{16}_{\Lambda}\mathrm{O}$ from CERN

1_1^- $p_{1/2}^{-1}s_{1/2\Lambda}$ 1_2^- $p_{3/2}^{-1}s_{1/2\Lambda}$

0_1^+ $p_{1/2}^{-1}p_{1/2\Lambda}$ 0_2^+ $p_{3/2}^{-1}p_{3/2\Lambda}$

0_3^+ $s_{1/2}^{-1}s_{1/2\Lambda}$

$^{16}\mathrm{O}(e,e'K^+)^{16}_{\Lambda}\mathrm{N}$ from Jlab Hall A

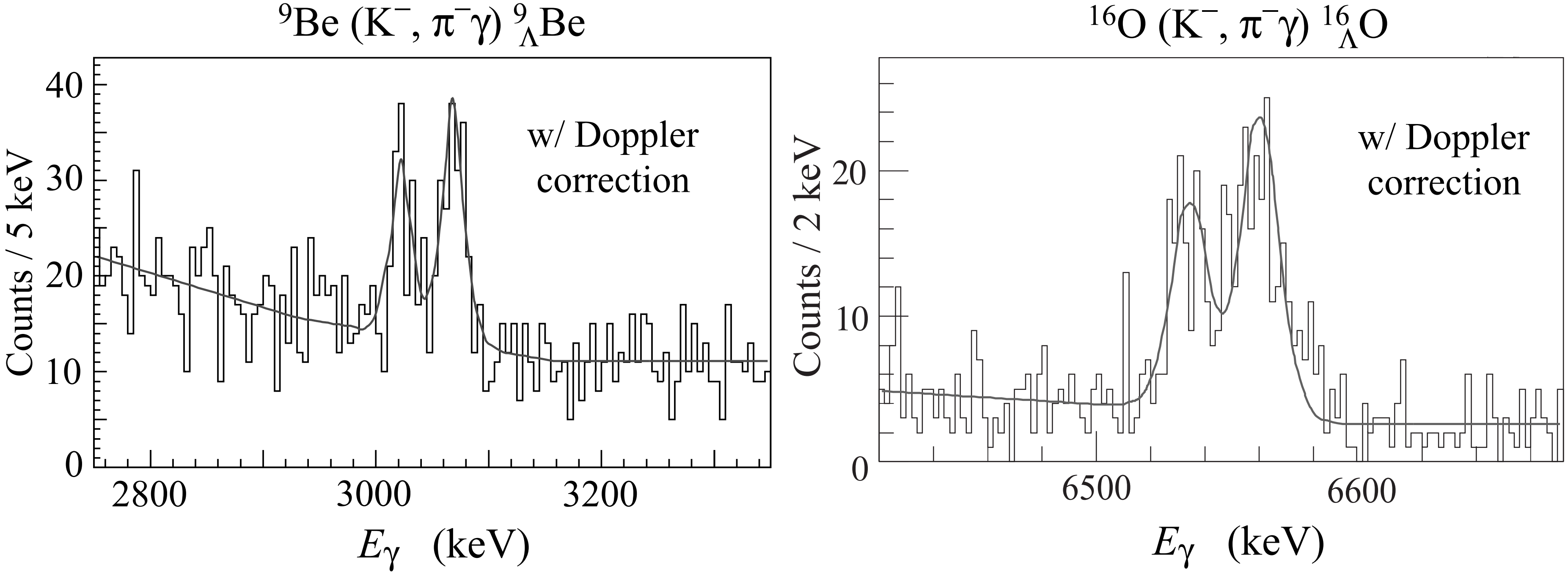
PRL 103 (2009) 202501

$B_\Lambda = 13.76 \pm 0.16$ MeV for $^{16}_{\Lambda}\mathrm{N}(1_1^-)$

Levels and cross sections obtained by fitting the $^{16}O(e, e'K^+)^{16}_{\Lambda}N$ spectrum compared with theoretical predictions.

E_x (MeV)	Width (FWHM, MeV)	Cross section ($nb/sr^2/GeV$)	E_x (MeV)	Wave function	J^π	Cross section ($nb/sr^2/GeV$)
0.00	1.71	1.45 ± 0.26	0.00	$p_{1/2}^{-1} \otimes s_{1/2\Lambda}$	0^-	0.002
			0.03	$p_{1/2}^{-1} \otimes s_{1/2\Lambda}$	1^-	1.45
6.83 ± 0.06	0.88	3.16 ± 0.35	6.71	$p_{3/2}^{-1} \otimes s_{1/2\Lambda}$	1^-	0.80
			6.93	$p_{3/2}^{-1} \otimes s_{1/2\Lambda}$	2^-	2.11
10.92 ± 0.07	0.99	2.11 ± 0.37	11.00	$p_{1/2}^{-1} \otimes p_{3/2\Lambda}$	2^+	1.82
			11.07	$p_{1/2}^{-1} \otimes p_{1/2\Lambda}$	1^+	0.62
17.10 ± 0.07	1.00	3.44 ± 0.52	17.56	$p_{3/2}^{-1} \otimes p_{1/2\Lambda}$	2^+	2.10
			17.57	$p_{3/2}^{-1} \otimes p_{3/2\Lambda}$	3^+	2.26

Note $p_{1/2\Lambda} - p_{3/2\Lambda}$ degeneracy within curve fitting

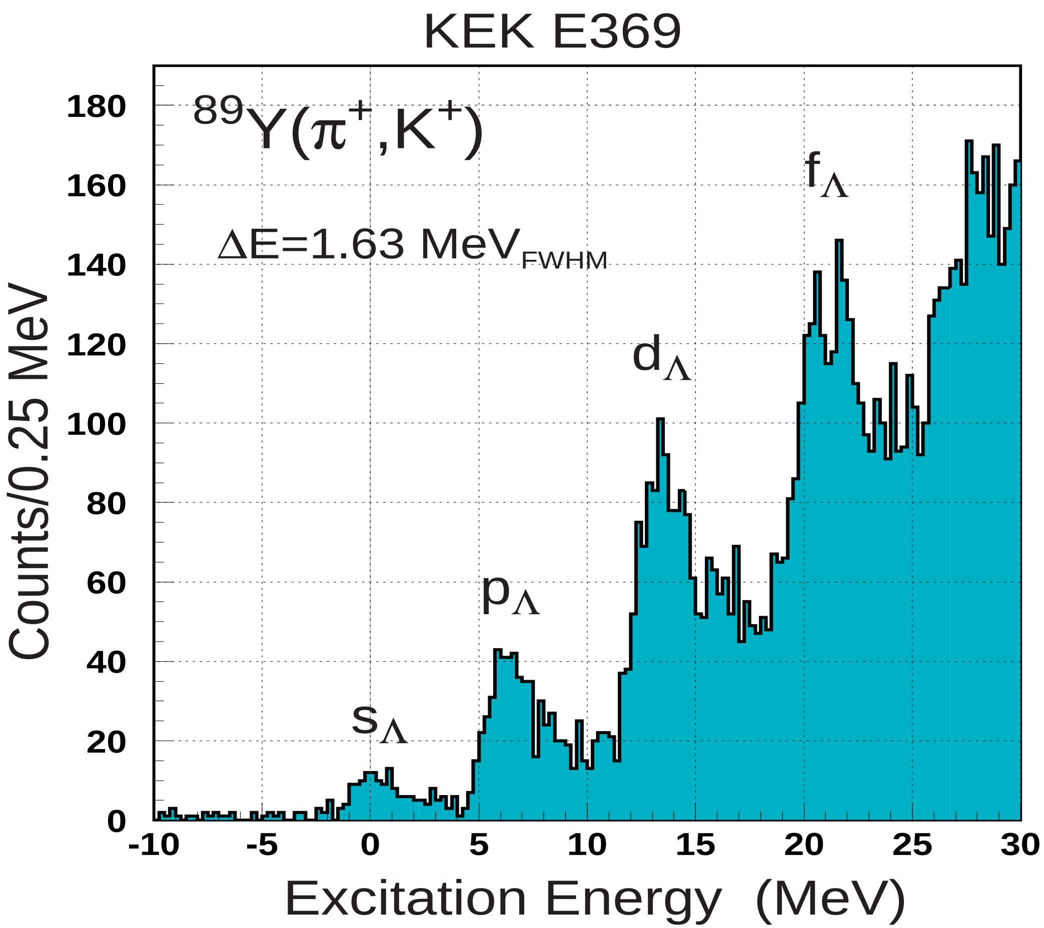


H. Tamura, Hyperball @ BNL: NPA 804 (2008) 73; 827 (2009) 153c [PANIC08]

Left: $^{9}_{\Lambda}\mathrm{Be}(5/2^+, 3/2^+ \rightarrow 1/2^+)$ Right: $^{16}_{\Lambda}\mathrm{O}(1^- \rightarrow 1^-, 0^-)$

Hypernuclear 'fine structure' in γ-ray spectra

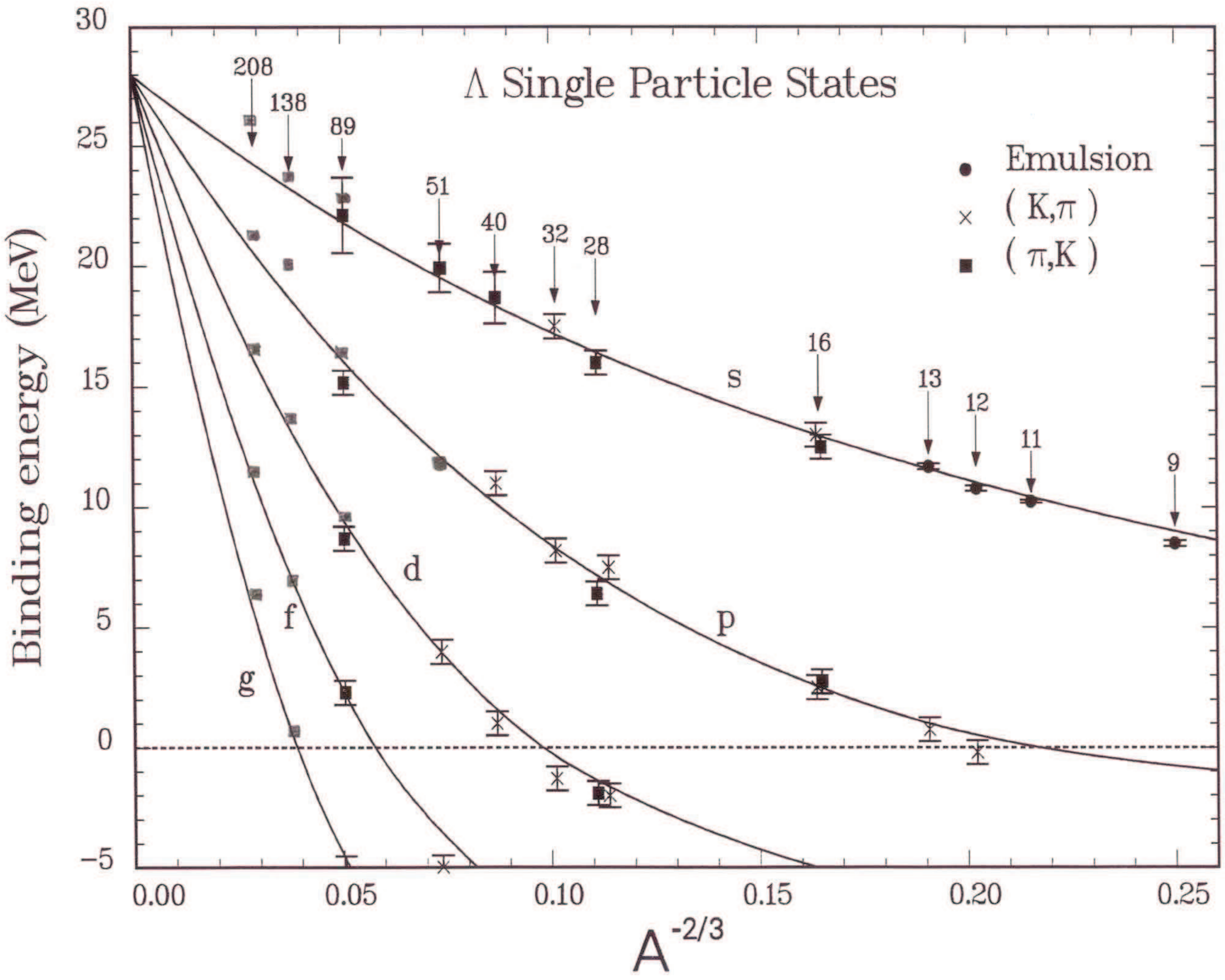
Sensitivity to 'spin-orbit' ($^{9}_{\Lambda}$Be) and to 'tensor' ($^{16}_{\Lambda}$O)



Hotchi et al., Phys. Rev. C 64 (2001) 044302 $B_\Lambda = 23.11 \pm 0.10$ MeV

T. Motoba, D.E. Lanskoy, D.J. Millener, Y. Yamamoto, NPA 804 (2008) 99

deduce negligible Λ spin-orbit splittings, 0.2 MeV for $1f_\Lambda$



Update: Millener, Dover, Gal, PRC 38, 2700 (1988)

Textbook demonstration of the shell model for Λ hypernuclei

Contents

References

- D.J. Millener, in *Topics in Strangeness Nuclear Physics*
  Eds. P. Bydžovský, A. Gal, J. Mareš
  Lecture Notes in Physics **724** (Springer 2007) pp. 31-79
- D.J. Millener, NPA **804** (2008) 84, NPA **835** (2010) 11 [HYP-X]

Shell model techniques

Shell-model calculations

- Both $|p^n\alpha_c J_c T \times s_\Lambda\rangle$ and $|p^n\alpha_c J_c T_c \times s_\Sigma\rangle$ configurations included. In general, T_c can take three values. E.g. for ${}^{10}_{\Lambda}$Li, $T_c = 1/2, 3/2, 5/2$.

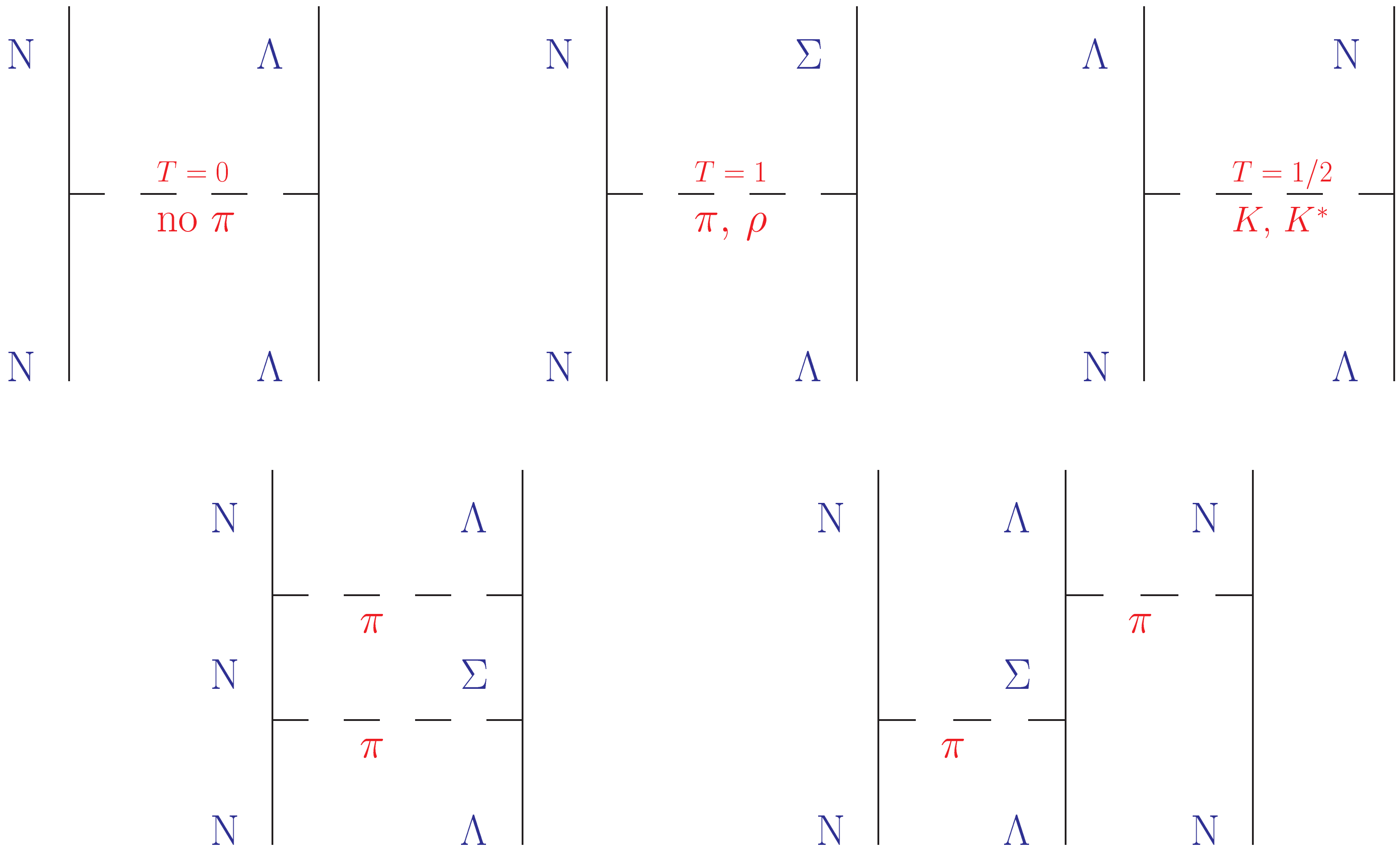
- Supermultiplet basis $|p^n[f_c]\beta_c(L_cS_c)J_cT_c\rangle$ is very good for p shell $\Rightarrow$ states with different $[f_c]$ (often T_c) well separated. E.g. $\sim$ 15 MeV for lowest $T_c\!=\!1/2 \times \Sigma$ and $T_c\!=\!3/2 \times \Sigma$ in ${}^{10}_{\Lambda}$Li example.

- p-shell interactions fitted with tensor interaction constrained to give cancellation in ^{14}C β decay; single-particle LS spacing constrained by data at the beginning of the shell.

- Need $N\Lambda$-$N\Lambda$ (parametrized, Δ,..), $N\Lambda$-$N\Sigma$ (see following slides), and $N\Sigma$-$N\Sigma$ (for T=1/2 and T=3/2; from YNG-type interaction) two-body matrix elements. All can be represented in the same way.

- Diagonal energies of Λ and Σ states differ by $\sim$ 80MeV, plus core energy differences, plus contributions from YN interactions.

$S = -1 \quad T = 1/2 \qquad \Lambda N - \Sigma N$



Effective ΛN (YN) interaction

$$V_{\Lambda N} = V_0(r) + V_\sigma(r)\, s_N \cdot s_\Lambda + V_\Lambda(r)\, l_{N\Lambda} \cdot s_\Lambda + V_N(r)\, l_{N\Lambda} \cdot s_N + V_T(r)\, [3(\sigma_N \cdot \hat{r})(\sigma_\Lambda \cdot \hat{r}) - \sigma_N \cdot \sigma_\Lambda]$$

$$V(r) = \sum_k V_k(r)\; \mathcal{L}^k \cdot \mathcal{S}^k$$

$$\mathcal{L}^k \cdot \mathcal{S}^k = (-)^k \widehat{k} [\mathcal{L}^k, \mathcal{S}^k]^0$$

$$S_{12} = \sqrt{6}\, C^2(\widehat{r}) \cdot [\sigma_1, \sigma_2]^2$$

YNG interactions

$$V(r) = \sum_i v_i\, e^{-r^2/\beta_i^2} \qquad V_T(r) = \sum_i v_i\, r^2\, e^{-r^2/\beta_i^2}$$

Harmonic Oscillator – Wave Functions

$$H = p^2/2m + 1/2\, m\omega^2 r^2, \qquad b^2 = \hbar/m\omega$$

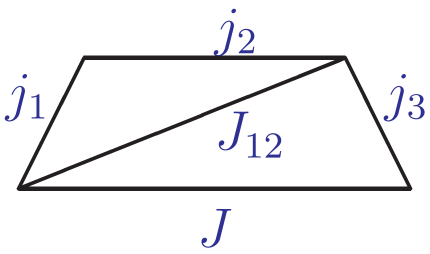
$$r \to r/b \qquad R_{0s} \propto e^{-r^2/2} \quad R_{0p} \propto r e^{-r^2/2} \quad R_{0d} \propto r^2 e^{-r^2/2} \quad R_{1s} \propto (3 - 2r^2) e^{-r^2/2}$$

$$\int_0^\infty r^{2p} e^{-r^2} dr = 1/2\Gamma(p + 1/2) \qquad \Gamma(p+1) = p\Gamma(p) \qquad \Gamma(1/2) = \sqrt{\pi}$$

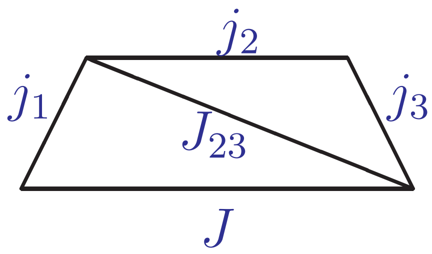
- Symmetry group - SU3
  single-particle transforms as $(q\, 0)$ with $q = 2n + l$
- Coordinate transformations, $r_1,\ r_2 \to (r_1 - r_2)/\sqrt{2},\ (r_1 + r_2)/\sqrt{2}$
  Talmi-Moshinsky transformation
- Generally for A particles $\to$ set of internal coordinates (maybe a number of clusters) plus center of mass

Some Racah Algebra basics for SU2

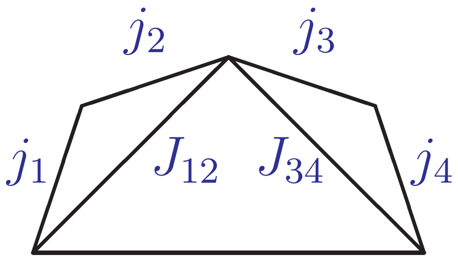
$$\langle J_f M_f | T^{kq} | J_i M_i \rangle = \langle J_i M_i\, kq | J_f M_f \rangle \langle J_f || T^k || J_i \rangle$$



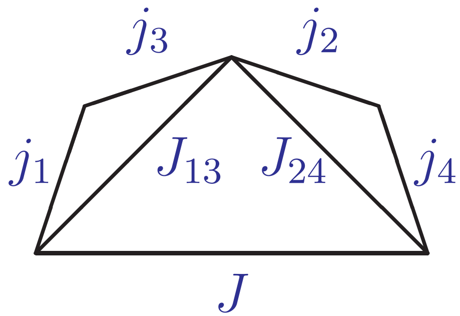
$$= \quad \sum_{J_{23}} U(j_1 j_2 J j_3,\, J_{12} J_{23})$$



$$U(abcd,\, ef) = \hat{e}\hat{f}\, W(abcd,\, ef) = \hat{e}\hat{f}\, (-)^{a+b+c+d} \begin{Bmatrix} a & b & e \\ d & c & f \end{Bmatrix}$$



$$= \quad \sum_{J_{13} J_{24}} \begin{pmatrix} j_1 & j_2 & J_{12} \\ j_3 & j_4 & J_{34} \\ J_{13} & k & J \end{pmatrix}$$



$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \widehat{c}\,\widehat{f}\,\widehat{g}\,\widehat{h} \left\{ \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} \right\}$$

$$\langle J_1 J_2; J || [R^{k_1}, S^{k_2}]^k || J_1' J_2'; J' \rangle = \begin{pmatrix} J_1' & k_1 & J_1 \\ J_2' & k_2 & J_2 \\ J' & k & J \end{pmatrix} \langle J_1 || R^{k_1} || J_1' \rangle \langle J_2 || S^{k_2} || J_2' \rangle$$

$$\langle x\Gamma || [R^\sigma, S^\lambda]^\nu || x'\Gamma' \rangle = (-)^{\sigma+\Lambda-\nu} \sum_{y\Gamma_1} U(\Gamma\sigma\Gamma'\lambda,\, \Gamma_1\nu) \langle x\Gamma || R^\sigma || y\Gamma_1 \rangle \langle y\Gamma_1 || S^\lambda || x'\Gamma' \rangle$$

Two-body matrix elements

$$\langle l_1 l_2 LS|V|l_1' l_2' L' S'\rangle^{JT} = \sum_k (-)^{L'+S+J} \left\{ \begin{array}{ccc} L & L' & k \\ S' & S & J \end{array} \right\} \widehat{L} \langle l_1 l_2 L||V_k(r)\mathcal{L}^k||l_1' l_2' L'\rangle \widehat{S} \langle S||\mathcal{S}^k||S'\rangle$$

$$\langle l_1 l_2 L||V_k(r)\mathcal{L}^k||l_1' l_2' L'\rangle = \sum_{NL_c ll'(nn')} \langle nlNL_c, L|n_1 l_1 n_2 l_2, L\rangle \langle n'l'NL_c, L'|n_1' l_1' n_2' l_2', L'\rangle$$

$$\cdot (-)^{l+L_c+k+L'} \widehat{L'}\widehat{l} \left\{ \begin{array}{ccc} L_c & l' & L' \\ k & L & l \end{array} \right\} \langle nl|V_k(r)|n'l'\rangle \langle l||\mathcal{L}^k||l'\rangle$$

	$s_N \cdot s_\Lambda$	$l_{N\Lambda} \cdot (s_\Lambda + s_N)$	$l_{N\Lambda} \cdot (s_\Lambda - s_N)$	Tensor
$\langle S\|\|\mathcal{S}^k\|\|S'\rangle$	$\delta_{SS'}(\delta_{S1} - 3\delta_{S0})/4$	$\delta_{SS'}\sqrt{S(S+1)}$	$(-)^S\sqrt{3}(1-\delta_{SS'})/\widehat{S}$	$\delta_{SS'}\sqrt{20S/3}$
$\langle l\|\|\mathcal{L}^k\|\|l'\rangle$	$\delta_{ll'}$	$\delta_{ll'}\sqrt{l(l+1)}$	$\delta_{ll'}\sqrt{l(l+1)}$	$\sqrt{6}\langle l020\|l'0\rangle$

$$\langle nl|V(r)|n'l'\rangle = \sum_p \; B(nl, n'l'; p)\, I_p$$

$$I_p = \frac{2}{\Gamma(p+3/2)} \int_0^\infty r^{2p} e^{-r^2} V(\sqrt{2} r b) r^2 dr$$

Gaussian: $V(r) = V_0\, e^{-r^2/\mu^2}$

$$I_p = \frac{V_0}{(1+2\theta^2)^{p+3/2}} \qquad \theta = \frac{b}{\mu}$$

ΛN (YN) interaction parameters

$V_{N\Lambda}$	$s_N s_\Lambda$	$p_N s_\Lambda$	${}^7_\Lambda$Li values (MeV)
V_0	I_0^e	$\bar{V} = \frac{1}{2}(I_0^e + I_1^o)$	(-1.22)
$V_\sigma s_N . s_\Lambda$	I_0^e	$\Delta = \frac{1}{2}(I_0^e + I_1^o)$	0.480
$V_\Lambda l_{N\Lambda} . s_\Lambda$		$S_\Lambda = \frac{1}{2} I_1^o$	-0.015
$V_N l_{N\Lambda} . s_N$		$S_N = \frac{1}{2} I_1^o$	-0.400
$V_T S_{12}$		$T = \frac{1}{3} I_1^o$	0.030

$$V_0 = \frac{1}{4} V_C(S=0) + \frac{3}{4} V_C(S=1) \qquad V_\sigma = V_C(S=1) - V_C(S=0)$$

$$V_{LS} l_{N\Lambda}.(s_\Lambda + s_N) + V_{ALS} l_{N\Lambda}.(s_\Lambda - s_N) = V_\Lambda l_{N\Lambda}.s_\Lambda + V_N l_{N\Lambda}.s_N$$

$$V_\Lambda = V_{LS} + V_{ALS} \qquad V_N = V_{LS} - V_{ALS}$$

$$V_{\Lambda N} = V_0(r) + V_\sigma(r)\, s_N \cdot s_\Lambda + V_{LS}(r)\, l_{N\Lambda} \cdot (s_\Lambda + s_N) + V_{ALS}(r)\, l_{N\Lambda} \cdot (s_\Lambda - s_N) + V_T(r)\, S_{12}$$

$$V_0 = 1/4\, {}^1V_C + 3/4\, {}^3V_C \qquad V_\sigma = {}^3V_C - {}^1V_C$$

For $p_N s_Y$ $\quad V_{\Lambda N} = \bar{V} + \Delta\, s_N \cdot s_\Lambda + S_\Lambda\, l_N \cdot s_\Lambda + S_N\, l_N \cdot s_N + T\, S_{12}$

Parameters in MeV

	$\bar{V}$	Δ	S_Λ	S_N	T
$N\Lambda$-$N\Lambda$ $\;A = 7-$?		0.430	-0.015	-0.390	0.030
$A = 11 - 16$		0.330	-0.015	-0.350	0.024
$N\Lambda$-$N\Sigma$	1.45	3.04	-0.085	-0.085	0.157

Can write the central Λ-Σ coupling interaction as

$$\sqrt{4/3}\; t_N \cdot t_Y\; \overline{V}' + \sqrt{4/3}\; s_N \cdot s_Y\; t_N \cdot t_Y\; \Delta'$$

The factor $\sqrt{4/3}$ arises from defining t_Y as an operator that changes a Λ into a Σ

Diagonal matrix element $\sqrt{4/3}\sqrt{T(T+1)}\overline{V}' + a(J)\langle J_c || \sum_i s_i t_i || J_c \rangle \Delta'$

Off-diagonal matrix element $b(J)\langle J_c' || \sum_i s_i t_i || J_c \rangle \Delta'$

The important Λ-Σ coupling matrix elements involve a Σ coupled to the same core state as the Λ and states connected to this by a large Gamow-Teller matrix element.

Detour to light hypernuclei

S-Shell Λ Hypernuclei

Hypernucleus	$J^\pi(gs)$	B_Λ MeV	J^π	E_x MeV
$^3_\Lambda$H	$1/2^+$	0.13(5)		
$^4_\Lambda$H	0^+	2.04(4)	1^+	1.04(5)
$^4_\Lambda$He	0^+	2.39(3)	1^+	1.15(4)
$^5_\Lambda$He	$1/2^+$	3.12(2)		

Ab Initio Calculations

- $A = 3, 4$ A. Nogga et al., PRL 88 (2002) 172501
  Faddeev and Faddeev-Yakubovsky
- $A = 4$ E. Hiyama et al., PRC 65 (2002) 011301(R)
  Jacobi-coordinate Gaussian basis
- $A = 3, 4, 5$ H. Nemura et al., PRL 89 (2002) 142504
  Stochastic variation with correlated Gaussians

$\Lambda - \Sigma$ coupling for ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He

Y. Akaishi et al., PRL 84 (2000) 3539

$$|{}^{4}_{\Lambda}\mathrm{He}(\mathrm{T}=1/2)\rangle = \alpha s^3 s_\Lambda + \beta s^3 s_\Sigma$$

From ΛN$-\Sigma$N g matrix for 0s orbits

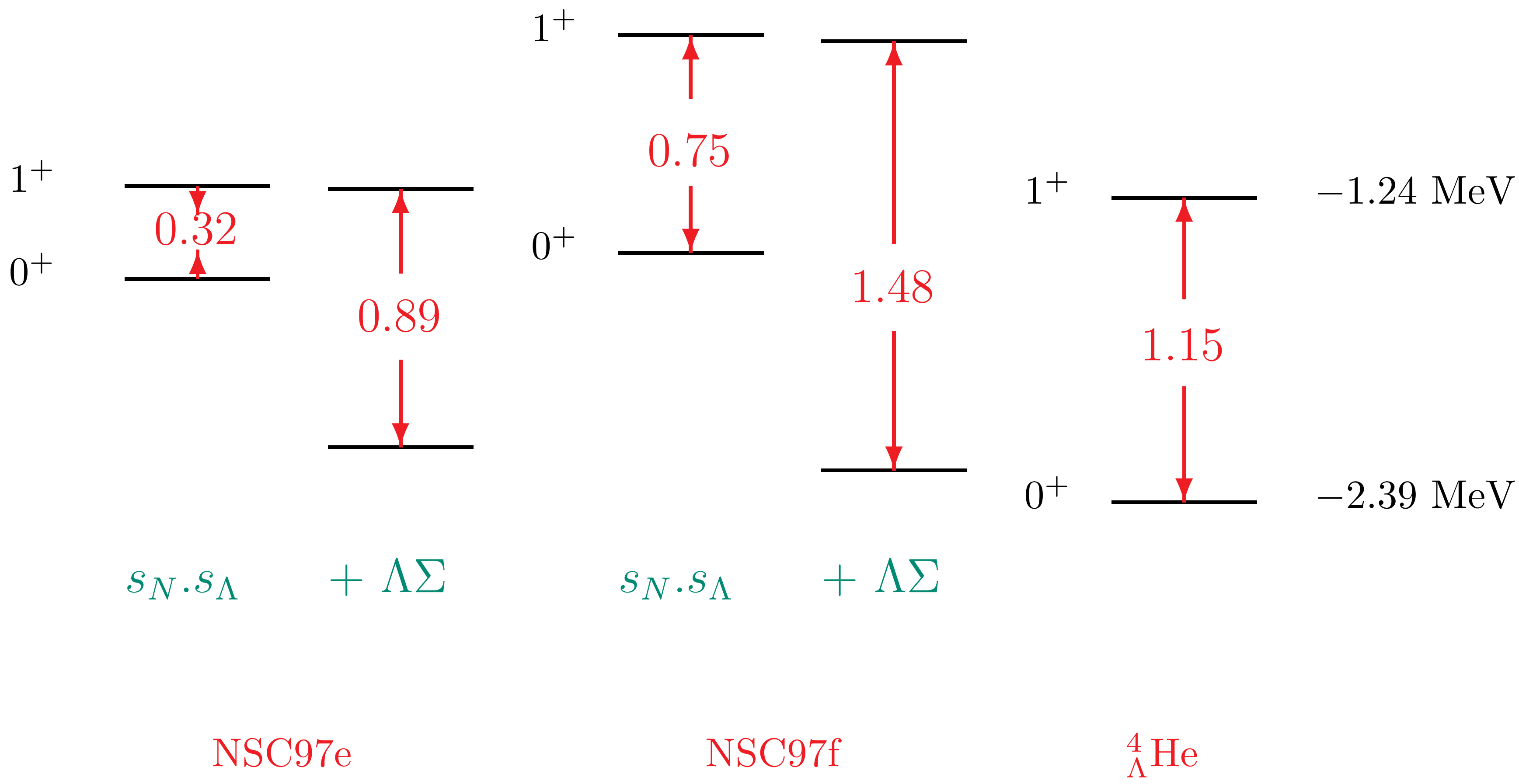
$v = \langle s^3 s_\Lambda | g | s^3 s_\Sigma \rangle$, $\quad \Delta E \sim 80$ MeV $\qquad {}^3g_{ss} = 4.8 \qquad {}^1g_{ss} = -1.0$

$\bar{V} = 3.35 \qquad \Delta = 5.8$

$0^+ \qquad v = \frac{3}{2}\,{}^3g_{ss} - \frac{1}{2}\,{}^1g_{ss} = \bar{V} + \frac{3}{4}\Delta \qquad$ Admixture $\sim -v/\Delta E$

$1^+ \qquad v = \frac{1}{2}\,{}^3g_{ss} + \frac{1}{2}\,{}^1g_{ss} = \bar{V} - \frac{1}{4}\Delta \qquad E^{shift} \sim v^2/\Delta E$

NSC97f: for $0^+ \quad v \sim 7.7$ MeV $\Rightarrow E^{shift} \sim 0.74$ MeV



1+
0+
0.32
0.89
1+
0+
0.75
1.48
1+
0+
1.15
−1.24 MeV
−2.39 MeV
$s_N . s_\Lambda$
$+ \Lambda\Sigma$
$s_N . s_\Lambda$
$+ \Lambda\Sigma$
NSC97e
NSC97f
${}^4_\Lambda$He

$$ME = \langle s^3 s_\Lambda, J | V | s^3 s_\Sigma, J \rangle$$

$$|s^3\rangle = \sum_{S(T)} 1/\sqrt{2}(-)^{1+S} |[s^2(TS), s](1/2 1/2)\rangle \qquad TS = 0\,1,\; 1\,0$$

$$ME = 3/2 \sum_{S\bar{S}} U(S\frac{1}{2}J\frac{1}{2}, \frac{1}{2}\bar{S})^2 U(T\frac{1}{2}\frac{1}{2}0, \frac{1}{2}\frac{1}{2}) U(T\frac{1}{2}\frac{1}{2}1, \frac{1}{2}\frac{1}{2}) \langle ss_\Lambda, \bar{S} | V | ss_\Sigma, \bar{S} \rangle$$

$$J = 0 \qquad ME = 3/2[{}^3V + U(1\frac{1}{2}\frac{1}{2}1, \frac{1}{2}\frac{1}{2})\,{}^1V] = 3/2[{}^3V - 1/3\,{}^1V] = \overline{V} + 3/4\Delta$$

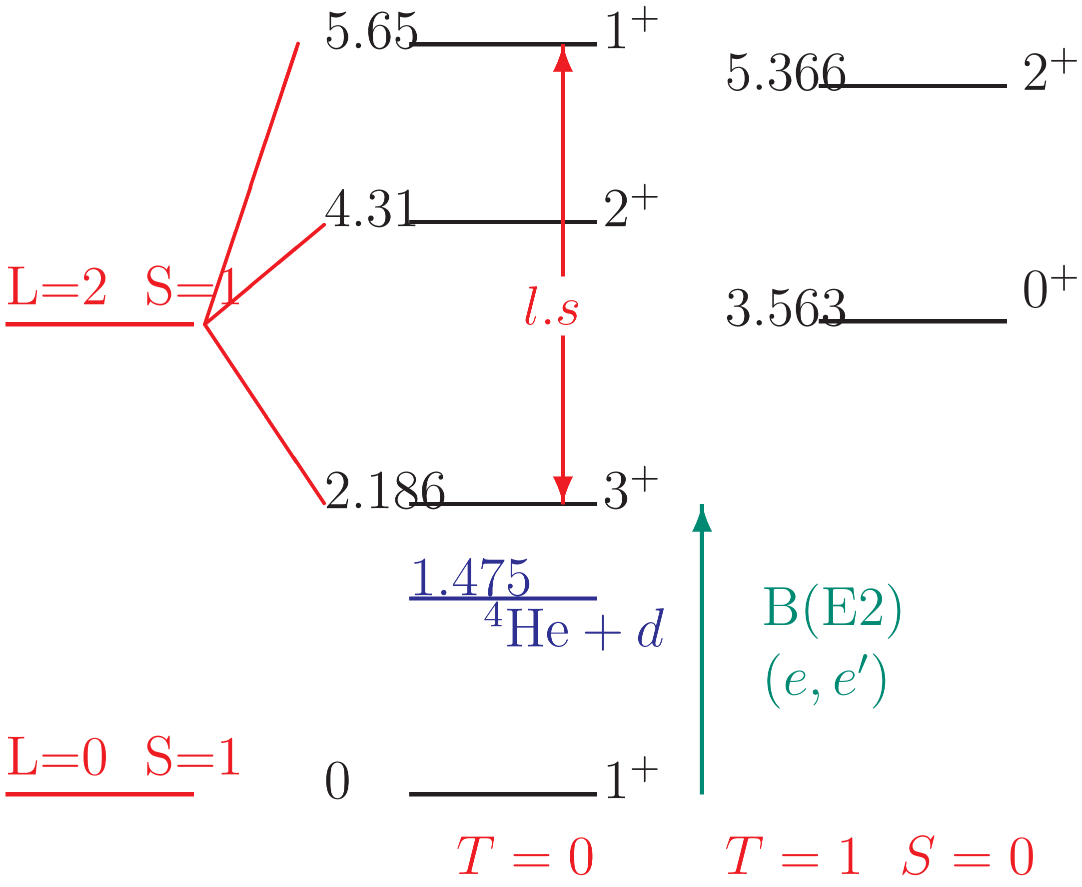
$$^3V = 4.8 \text{ MeV} \qquad {}^1V = -1.0 \text{ MeV}$$

$$\overline{V} = 1/4\,{}^1V + 3/4\,{}^3V = 3.35 \text{ MeV} \qquad \Delta = {}^3V - {}^1V = 5.8 \text{ MeV}$$

$$V(0^+) = 7.7 \text{ MeV} \qquad V(1^+) = 1.9 \text{ MeV}$$

$^{7}_{\Lambda}\mathbf{Li}$ $^{9}_{\Lambda}\mathbf{Be}$ $^{16}_{\Lambda}\mathbf{O}$

The ^{6}Li core for $^7_\Lambda$Li



$$H\Psi = E\Psi \qquad \Psi = \sum_i a_i \Phi_i$$

Φ_i are p^2 states

Single-particle energies

$\varepsilon_{3/2} \quad \varepsilon_{1/2}$

Centroid $\quad \varepsilon_p$

$\varepsilon_{3/2} - \varepsilon_{1/2} = 3/2\xi_p$

where one-body spin-orbit is

$-\xi_p \sum_i l_i \cdot s_i$

Wave functions for $A = 6$

	$A = 6 - 9$	fit5	CK616	CK816	CKPOT
			$1^+_1; 0$		
^{3}S	0.9873	0.9906	0.9576	0.9484	0.9847
^{3}D	−0.0422	−0.0437	−0.2777	−0.3093	−0.1600
^{1}P	−0.1532	−0.1298	−0.0761	−0.0703	−0.0685
			$1^+_2; 0$		
^{3}S	0.0287	−0.0347	−0.2810	−0.3082	−0.1426
^{3}D	−0.9007	−0.9987	−0.9591	−0.9510	−0.9673
^{1}P	0.4334	0.0708	−0.0354	0.0259	0.2096
			$0^+_1; 1$		
^{1}S	0.9560	0.9909	0.9997	0.9999	0.9946
^{3}P	0.2935	0.1348	0.0247	−0.0137	0.1036
			$2^+_1; 1$		
^{1}D	0.8760	0.9827	0.9486	0.9959	0.9839
^{3}P	0.4824	0.3148	0.1854	0.0905	0.1789

Note $|{}^{14}\mathrm{N}(1^+_1; 0)\rangle = -0.1139\ {}^3S + 0.2405\ {}^1P - 0.9639\ {}^3D$

$$|{}^6\mathrm{Li}(gs)\rangle = \alpha\,{}^3S_1 + \beta\,{}^3D_1 + \gamma\,{}^1P_1$$

$$Q({}^6\mathrm{Li}) = e^0 b^2(4/\sqrt{5}\alpha\beta + \gamma^2 - 7/10\beta^2)$$

$Q(expt) = -0.082\ \mathrm{fm}^2$ Spin-orbit vs. Tensor

$$B(M1; 2^+; 1 \rightarrow 1^+; 0) = 8.3 \pm 1.5 \times 10^{-2}\ \mathrm{Wu}$$

$$|{}^6\mathrm{Li}(2^+; 1)\rangle = a({}^1D)\,{}^1D + a({}^3P)\,{}^3P$$

$\langle l\tau\rangle > 0$ but not big enough; need $\langle s\tau\rangle > 0$

$$\langle s\tau\rangle = [a({}^3D)a({}^1D) - \sqrt{5/3}a({}^1P)a({}^3P)]\,g_s^{(1)}$$

${}^{7}_{\Lambda}\mathrm{Li}$

Lowest particle decay threshold for ${}^{7}_{\Lambda}\mathrm{Li}$ is ${}^{5}_{\Lambda}\mathrm{He}+d$

$$\begin{aligned} S_d({}^{7}_{\Lambda}\mathrm{Li}) &= S_d({}^{6}\mathrm{Li}) + B_\Lambda({}^{7}_{\Lambda}\mathrm{Li}) - B_\Lambda({}^{5}_{\Lambda}\mathrm{He}) \\ &= 1.475 + 5.58 - 3.12 \\ &= 3.94 \text{ MeV} \end{aligned}$$

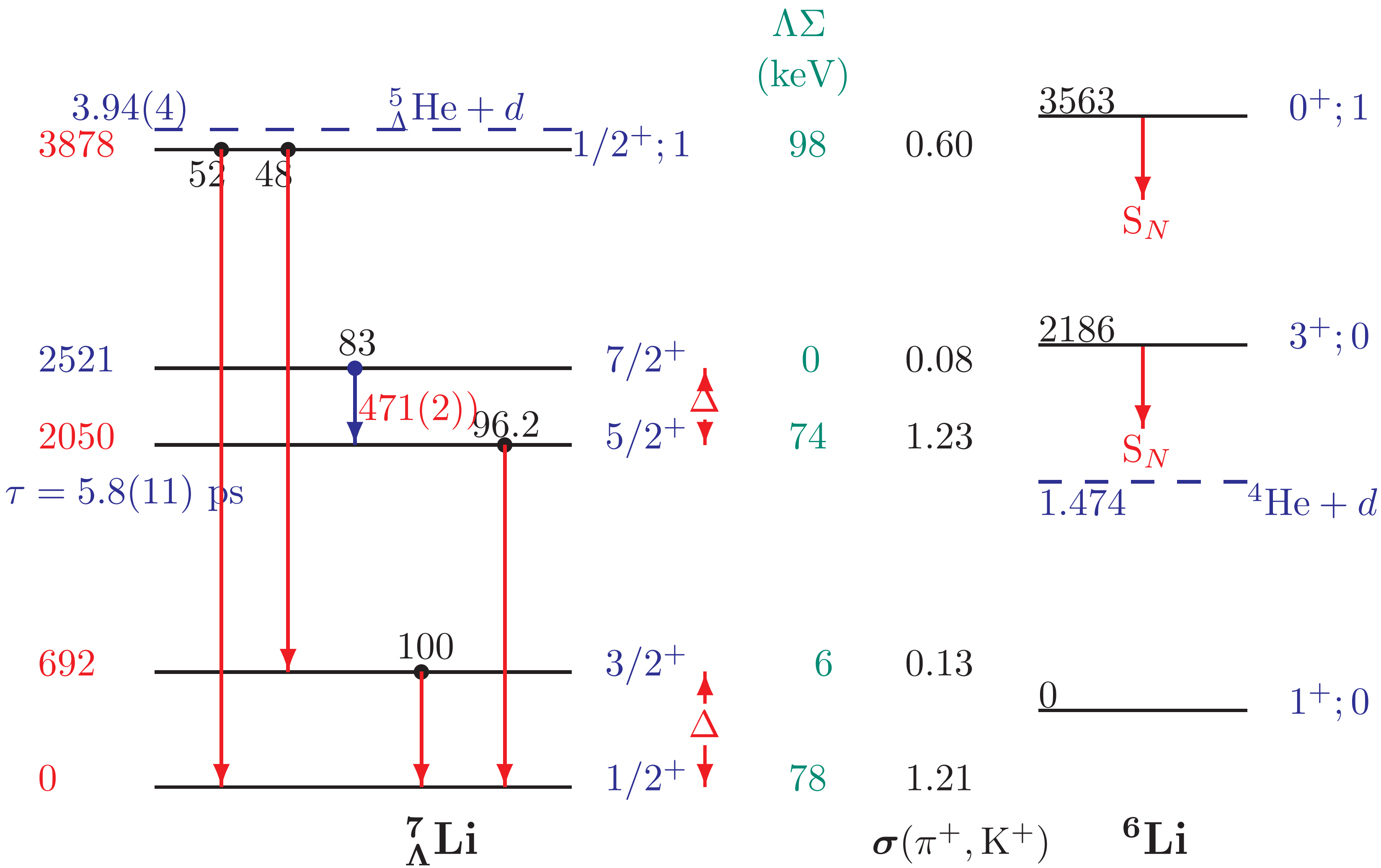
Shell-Model (more later)

$$H = H_N + H_Y + V_{NY}$$

Weak-coupling basis $\quad |(\alpha_c J_c T_c, j_Y Y) JT\rangle$

Take core energies from experiment where possible

$^{7}_{\Lambda}$Li γ rays – Hyperball, KEK E419 and BNL E930



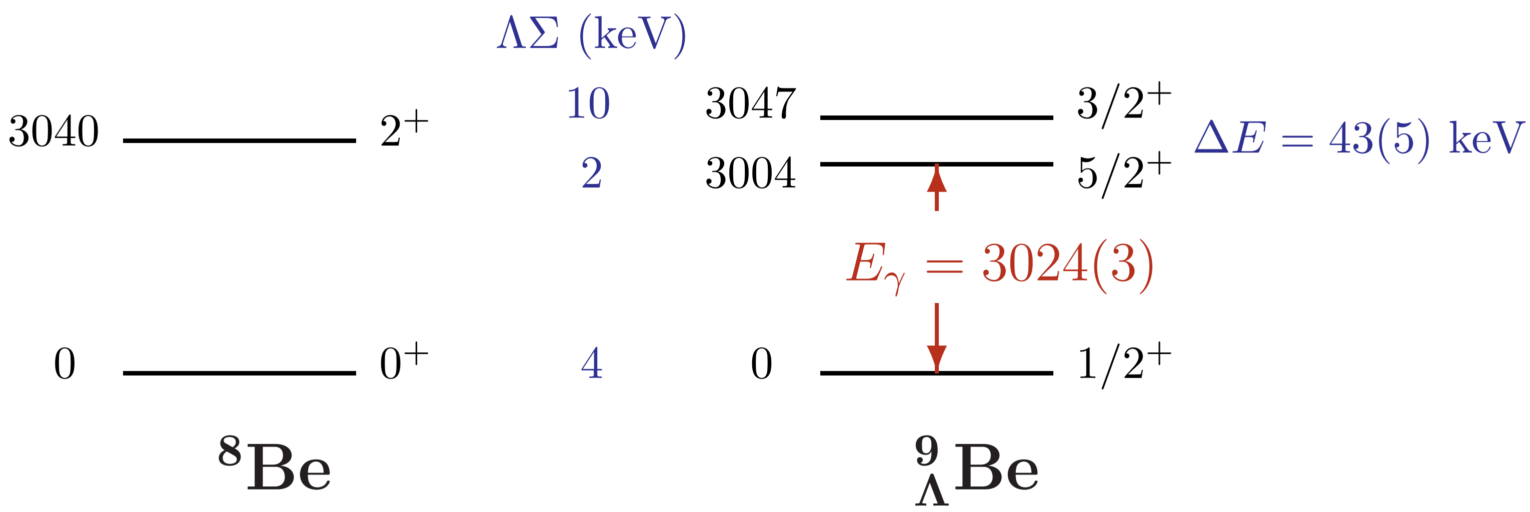
H. Tamura et al., PRL 84 (2000) 5963 K. Tanida et al., PRL 86 (2001) 1982

Energy spacings in $^{7}_{\Lambda}$Li

Energy contributions are in keV $\Delta E = \Delta E_C + \Delta E_{\Lambda N}$ Expt. in green

$\Delta = 0.430 \quad S_\Lambda = -0.015 \quad S_N = -0.390 \quad T = 0.030$

$J_i^\pi - J_f^\pi$	ΔE_C	$\Lambda\Sigma$	Δ	S_Λ	S_N	T	ΔE
$3/2^+ - 1/2^+$			1.461	0.038	0.011	−0.285	692
	0	72	628	−1	−4	−9	693
$5/2^+ - 1/2^+$			0.179	−1.140	0.738	1.097	2050
	2186	4	77	17	−288	33	2047
$1/2^+ - 1/2^+$			0.972	−0.026	0.211	−0.085	3878
	3565	−20	418	0	−82	−3	3886
$7/2^+ - 5/2^+$			1.294	2.166	0.020	−2.380	471
	0	74	557	−32	−8	−71	494



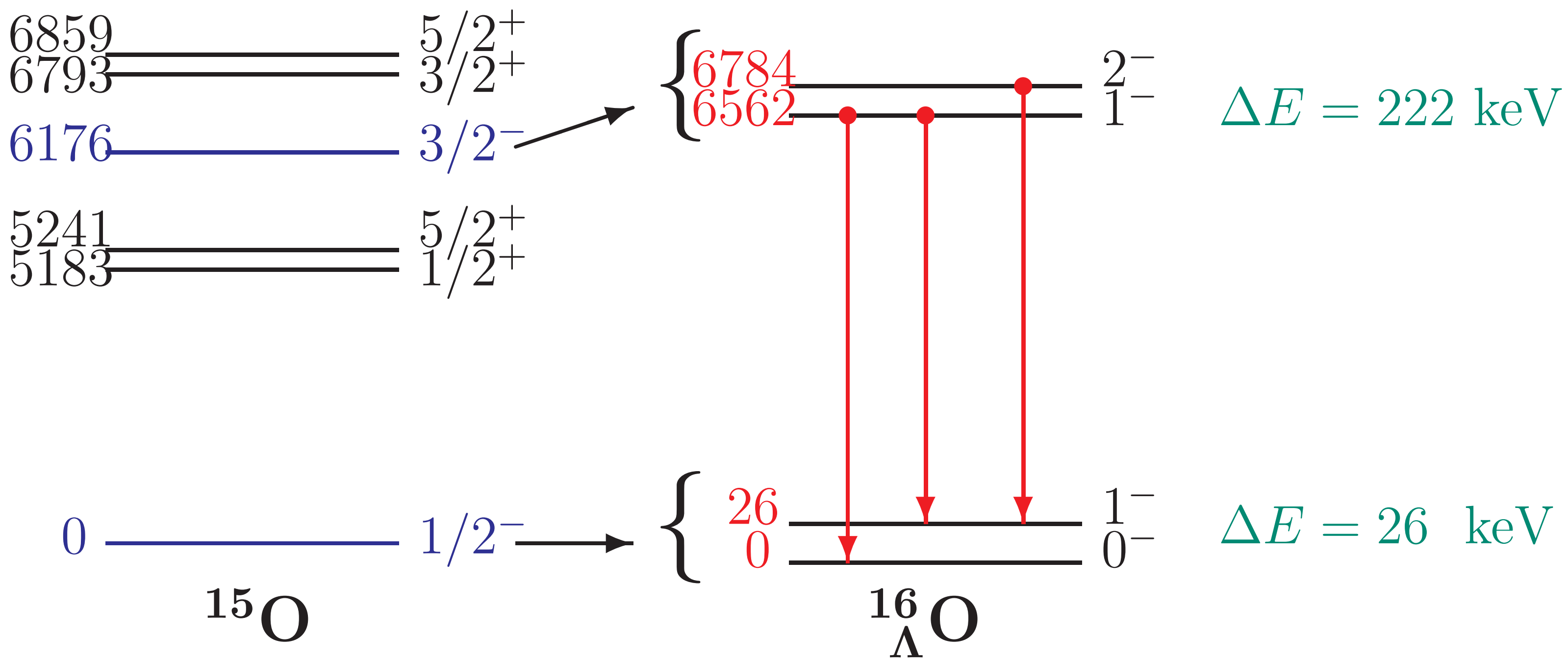
ΛΣ	Δ	S_Λ	S_N	T	ΔE
	−0.033	−2.467	0.000	0.939	
−8	−14	37	0	28	44 keV

- Order of $3/2^+$, $5/2^+$ not determined from this experiment.
- In 2001 run on ^{10}B target, the upper level is seen following ${}^{10}_{\Lambda}\mathrm{B} \to {}^{9}_{\Lambda}\mathrm{Be} + p$ enabling us to deduce $J^\pi_> = 3/2^+$
- Small spin-orbit splitting; note $\Delta(p_{\frac{1}{2}} - p_{\frac{3}{2}})_\Lambda \approx 150$ keV in ${}^{13}_{\Lambda}\mathrm{C}$, compared to $\Delta(p_{\frac{1}{2}} - p_{\frac{3}{2}})_N \sim 4 - 6$ MeV

Tensor Interaction

For pure $p_{1/2}^{-1}s_\Lambda$, the combination of parameters governing the doublet splitting is [R.H. Dalitz, A. Gal, Ann. Phys. 116 (1978) 167]

$$E(1_1^-) - E(0^-) = -\frac{1}{3}\Delta + \frac{4}{3}S_\Lambda + 8T$$



M. Ukai *et al.* – **Phys. Rev. Lett. 93 (2004) 232501**

M. Ukai *et al.* – **Phys. Rev. C 77 (2008) 054315**

Energy spacings in $^{16}_{\Lambda}$O

Energy contributions are in keV $\Delta E = \Delta E_C + \Delta E_{\Lambda N}$ Expt. in green

$\Delta = 0.330 \quad S_\Lambda = -0.015 \quad S_N = -0.350 \quad T = 0.0239$

$J_i^\pi - J_f^\pi$	ΔE_C	$\Lambda\Sigma$	Δ	S_Λ	S_N	T	ΔE
$1^- - 0^-$			−0.372	1.369	−0.003	7.883	26
	0	−29	−123	−21	1	188	27
$1_2^- - 1_1^-$			−0.256	−1.239	−1.494	−0.769	6536
	6176	−32	−84	19	523	−18	6535
$2^- - 1_2^-$			0.627	1.369	−0.003	−1.752	222
	0	82	207	−21	1	−41	238

$E_x(2^-) = 6784$ keV

1. G-Matrix elements from $N\Lambda$-$N\Sigma$ calculation fitted with sums of Gaussians, Yukawas, OBEP forms, ...
2. Hypernuclear two-body matrix elements calculated using Woods-Saxon wave functions.

		p-shell					s-shell	
		$\overline{V}$	Δ	S_Λ	S_N	T	$\overline{V}_s$	Δ_s
fit-djm	${}^{7}_{\Lambda}$Li	−1.142	0.438	−0.008	−0.414	0.031	−1.387	0.497
	${}^{16}_{\Lambda}$O	−1.161	0.441	−0.007	−0.401	0.030		
nsc97f	${}^{7}_{\Lambda}$Li	−1.086	0.421	−0.149	−0.238	0.055	−1.725	0.775
esc04a	${}^{7}_{\Lambda}$Li	−1.287	0.381	−0.108	−0.236	0.013	−1.577	0.850
esc08a	${}^{7}_{\Lambda}$Li	−1.221	0.146	−0.074	−0.241	0.055	−1.796	0.650

- First two lines show that matrix elements are roughly constant with A - same YNG interaction, WS wells have $R = r_0 A^{1/3}$, but rms radii of p-shell nuclei are roughly constant

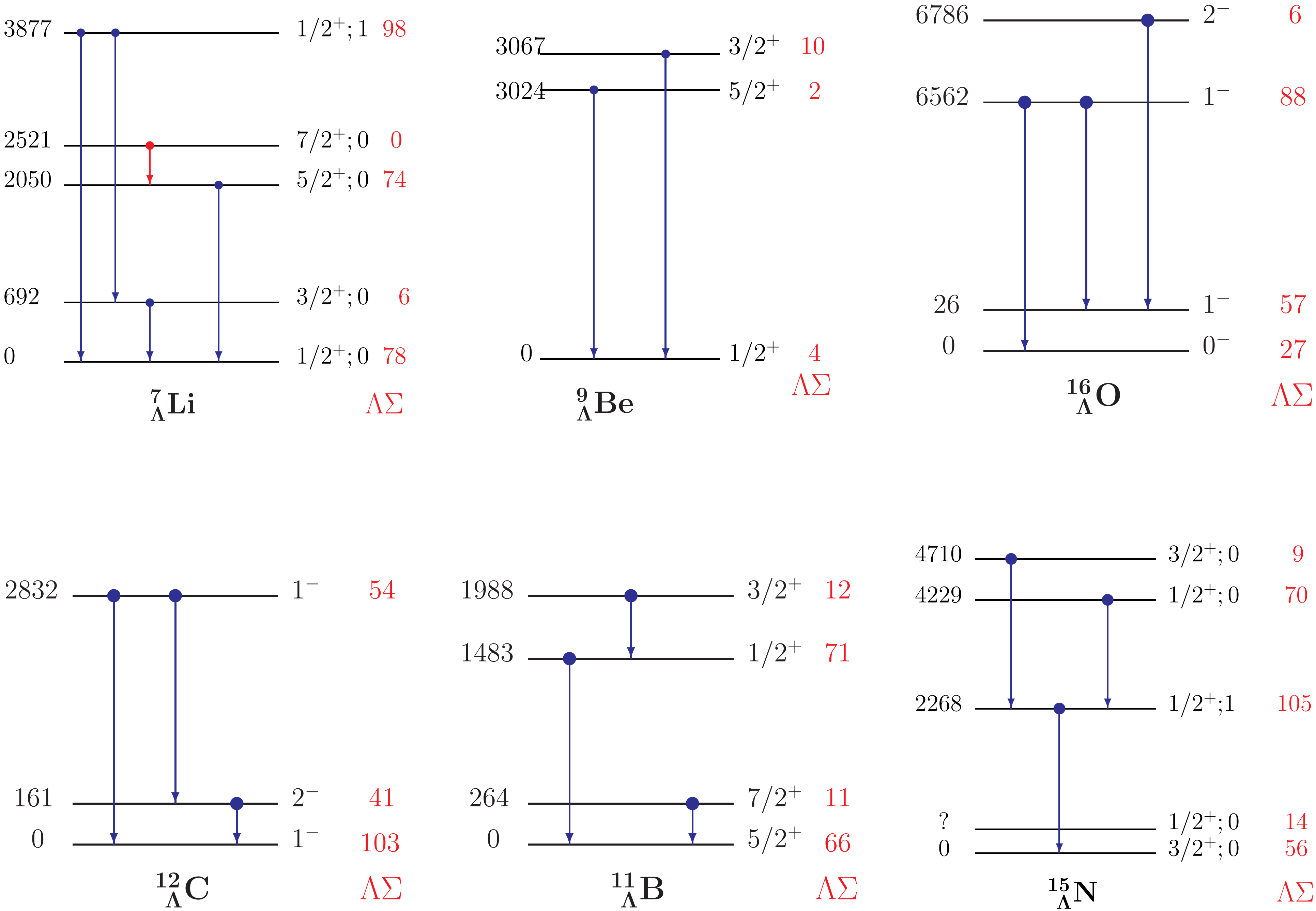
$p_N s_\Lambda$ Λ-Σ coupling parameters from Nijmegen baryon-baryon potentials.

Source	Interaction	$\bar{V}'$	Δ'	S'_Λ	S'_N	T'
Akaishi (s-shell)	NSC97e/f	1.45	3.04	−0.09	−0.09	0.16
Yamamoto	NSC97f	0.96	3.62	−0.07	−0.07	0.31
Halderson	NSC97e	0.75	3.51	−0.45	−0.24	0.31
Halderson	NSC97f	1.10	3.73	−0.45	−0.23	0.30
Halderson ∗	ESC04a	−2.30	−2.59	−0.17	−0.17	0.23
Halderson	ESC08a	1.05	4.71	−0.07	0.02	0.32

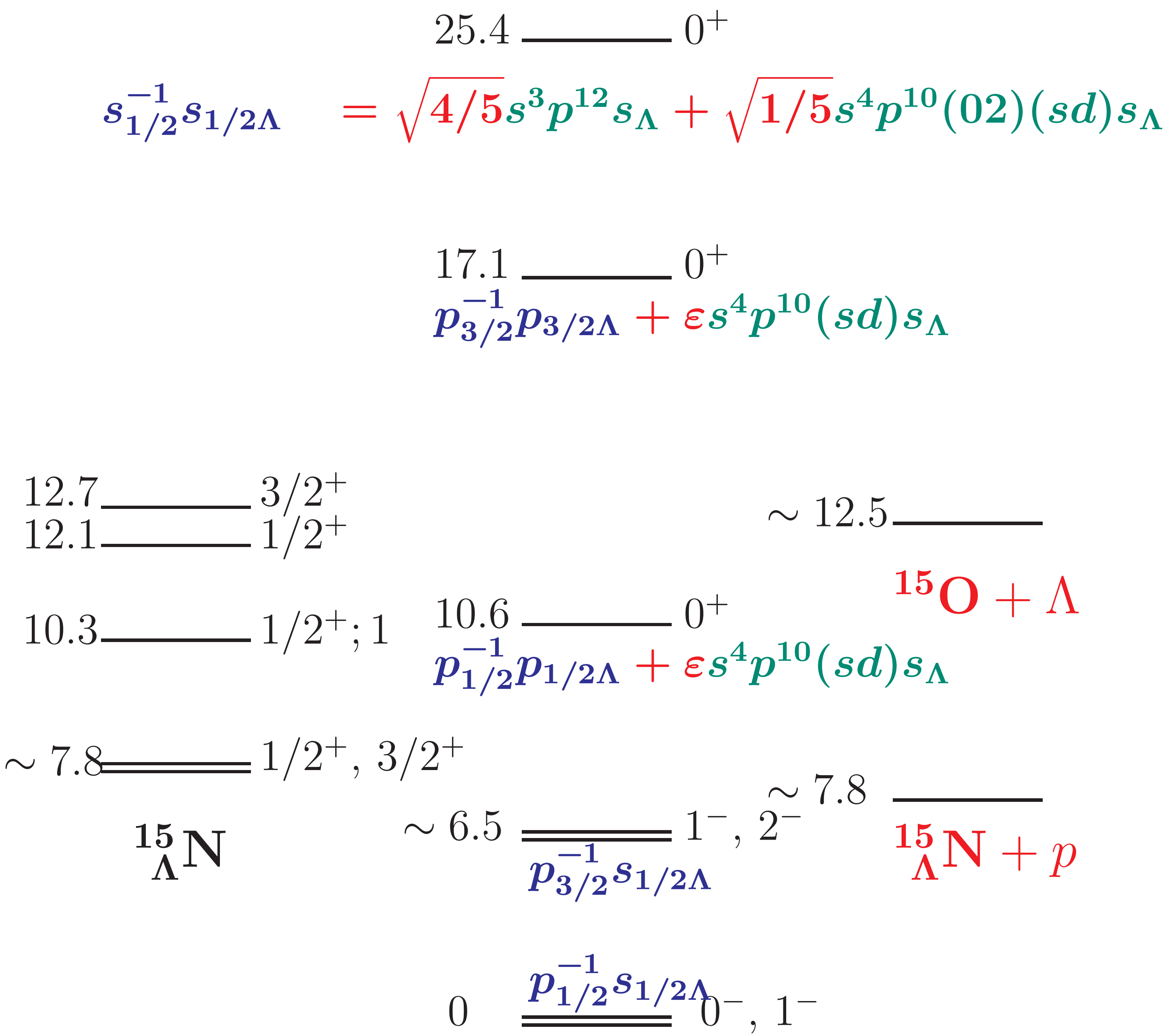
∗ D. Halderson, Phys. Rev. 77, 034304 (2008).

- ${}^4_\Lambda$H/${}^4_\Lambda$He 0^+ $\bar{V}'_s + 3/4\ \Delta'_s$
- ${}^4_\Lambda$H/${}^4_\Lambda$He 1^+ $\bar{V}'_s - 1/4\ \Delta'_s$
- Effective central interaction from second-order tensor; ESC04 interactions have a peculiar radial behavior (see Halderson) but the overall strength is not so different from the other interacions (when resticted to the p shell).

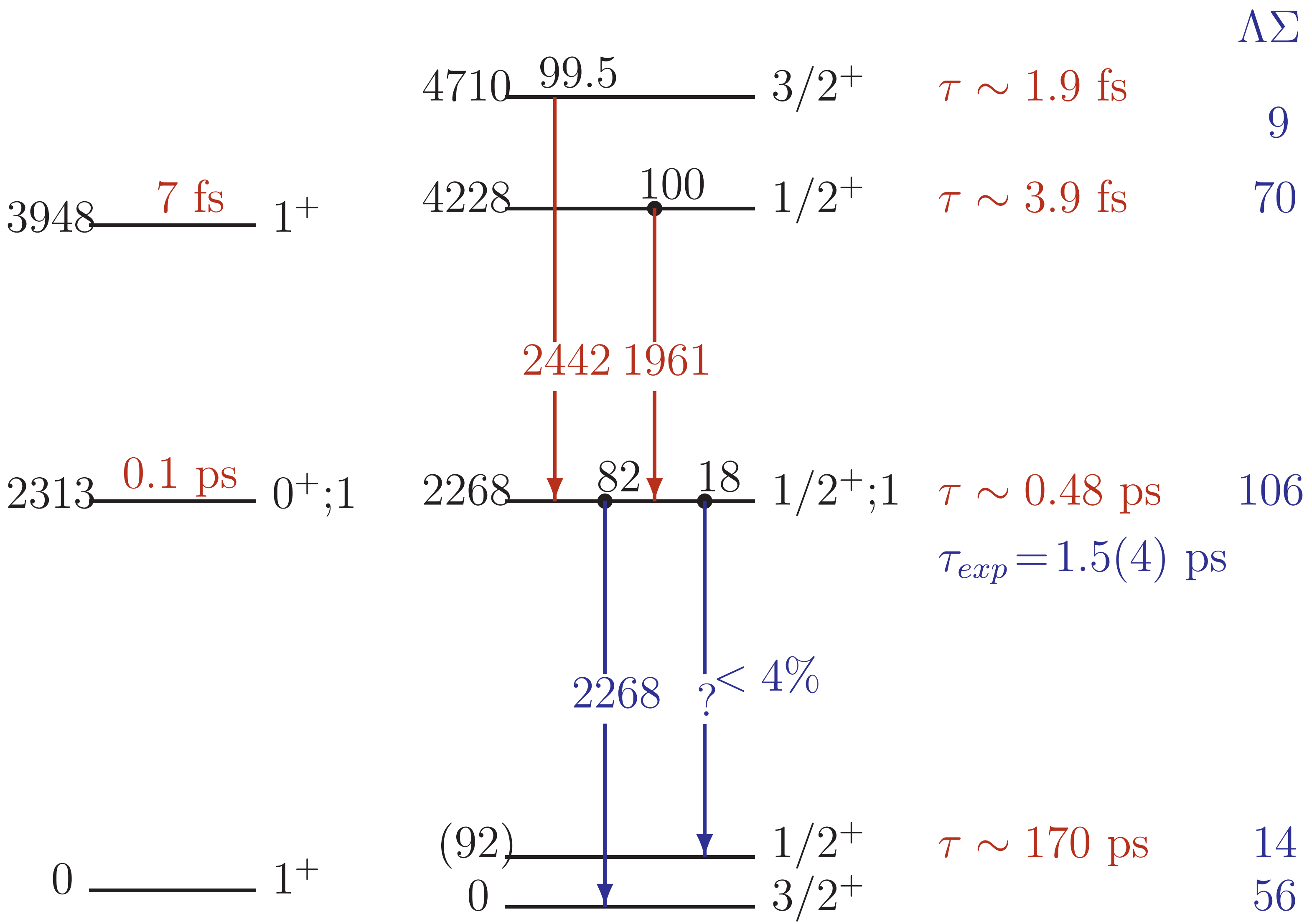
Other p-shell hypernuclei


3877
2521
2050
692
0
1/2+; 1 98
7/2+; 0 0
5/2+; 0 74
3/2+; 0 6
1/2+; 0 78
7ΛLi
ΛΣ
3067
3024
0
3/2+ 10
5/2+ 2
1/2+ 4
9ΛBe
ΛΣ
6786
6562
26
0
2− 6
1− 88
1− 57
0− 27
16ΛO
ΛΣ
2832
161
0
1− 54
2− 41
1− 103
12ΛC
ΛΣ
1988
1483
264
0
3/2+ 12
1/2+ 71
7/2+ 11
5/2+ 66
11ΛB
ΛΣ
4710
4229
2268
?
0
3/2+; 0 9
1/2+; 0 70
1/2+;1 105
1/2+; 0 14
3/2+; 0 56
15ΛN
ΛΣ

Strong 1^- and 0^+ states in $^{16}\mathrm{O}(K^-,\pi^-)^{16}_{\Lambda}\mathrm{O}$



${}^{15}_{\Lambda}$N γ rays – Hyperball, BNL E930 ('01)



Energy spacings in ${}^{15}_{\Lambda}$N

Energy contributions are in keV $\Delta E = \Delta E_C + \Delta E_{\Lambda N}$ Expt. in green

$\Delta = 0.330 \quad S_\Lambda = -0.015 \quad S_N = -0.350 \quad T = 0.0239$

$J_i^\pi - J_f^\pi$	ΔE_C	$\Lambda\Sigma$	Δ	S_Λ	S_N	T	ΔE
$p_{1/2}^{-2}$			0.5	−2.0	0	−12	
$1/2^+ - 3/2^+$			0.740	−2.237	0.024	−8.956	?
	0	42	244	33	−8	−214	96
$1/2^+;1 - 3/2^+$			0.262	−0.752	0.016	−2.966	2268
	2313	−50	86	11	−5	−71	2282
$1/2_2^+ - 3/2_2^+$			1.367	0.130	0.034	−0.424	481
	0	61	451	−2	−12	−10	502
$3/2_2^+ - 1/2^+;1$			0.474	0.025	−1.335	−0.271	2442
	1635	96	156	0	467	−6	2342

Close to perfect fit with $\Delta = 0.31$ and $S_N = -0.40$

Excitation energies and weak-coupling wave functions for ${}^{15}_{\Lambda}$N.

$J^\pi_n;T$	E_x (keV)	Wave function
$3/2^+_1;0$	0	$0.9985\,1^+_1;0 \times s_\Lambda \;+\; 0.0318\,1^+_2;0 \times s_\Lambda \;+\; 0.0378\,2^+_1;0 \times s_\Lambda$
$1/2^+_1;0$	96	$0.9986\,1^+_1;0 \times s_\Lambda \;+\; 0.0503\,1^+_2;0 \times s_\Lambda$
$1/2^+_1;1$	2282	$0.9990\,0^+_1;1 \times s_\Lambda \;+\; 0.0231\,1^+_1;1 \times s_\Lambda \;+\; 0.0206\,0^+_2;1 \times s_\Lambda$
		$-\; 0.0261\,0^+_1;1 \times s_\Sigma$
$1/2^+_2;0$	4122	$-0.0502\,1^+_1;0 \times s_\Lambda \;+\; 0.9984\,1^+_2;0 \times s_\Lambda$
$3/2^+_2;0$	4624	$-0.0333\,1^+_1;0 \times s_\Lambda \;+\; 0.9984\,1^+_2;0 \times s_\Lambda \;+\; 0.0363\,2^+_1;0 \times s_\Lambda$

M1 decays from $1/2^+;1$ in ${}^{15}_{\Lambda}$N

$1/2^+;1 \rightarrow 3/2^+;0$	a_f a_i core M1	
large component	$0.9979 \times 0.9988 \times (-0.251)$	-0.250
$1^+_2, s_\Lambda$ admixture	$0.0318 \times 0.9988 \times (\ \ 2.957)$	$+0.095$
Σ admixture		$+0.011$
		-0.137

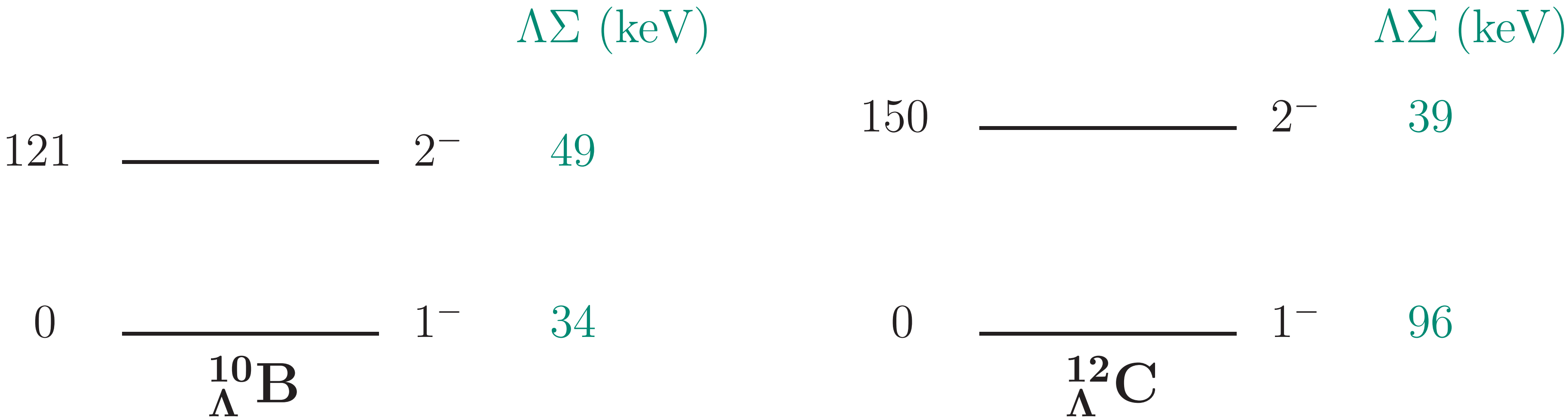
$1/2^+;1 \rightarrow 1/2^+;0$	a_f a_i core M1	
large component	$0.9983 \times 0.9988 \times (-0.251)$	-0.250
$1^+_2, s_\Lambda$ admixture	$0.0545 \times 0.9988 \times (\ \ 2.957)$	$+0.161$
Σ admixture		$+0.008$
		-0.081

$g_\Lambda = -1.226\ \mu_N \qquad g_{\Lambda\Sigma} = 3.22\ \mu_N$

Doublet spacings in p-shell hypernuclei

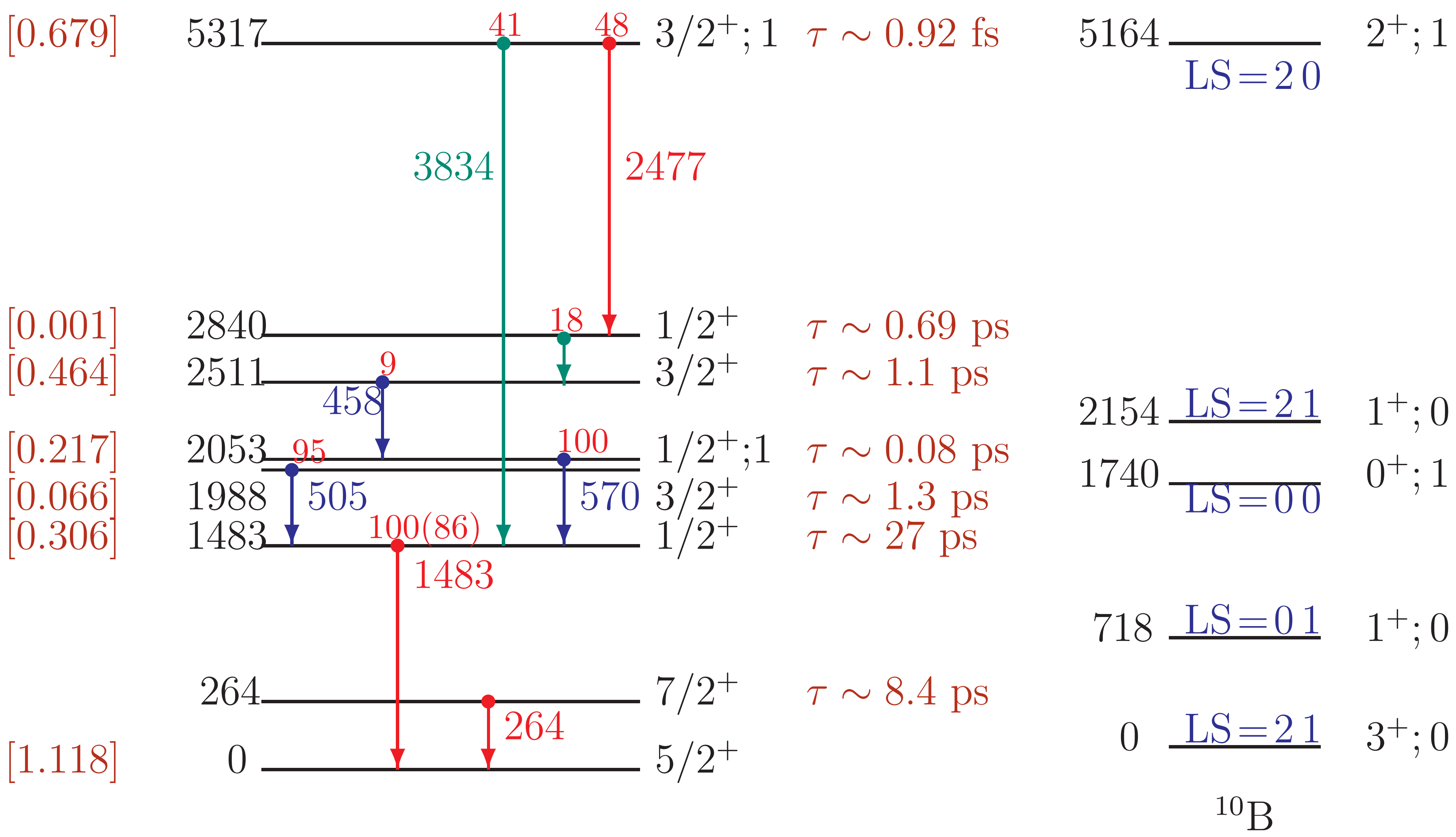
	J_u^π	J_l^π	$\Lambda\Sigma$	Δ	S_Λ	S_N	T	ΔE^{th}	ΔE^{exp}
$^{7}_{\Lambda}$Li	$3/2^+$	$1/2^+$	72	628	−1	−4	−9	693	692
$^{7}_{\Lambda}$Li	$7/2^+$	$5/2^+$	74	557	−32	−8	−71	494	471
$^{8}_{\Lambda}$Li	2^-	1^-	151	396	−14	−16	−24	450	(442)
$^{9}_{\Lambda}$Li	$5/2^+$	$3/2^+$	116	530	−17	−18	−1	589	
$^{9}_{\Lambda}$Li	$3/2^+_2$	$1/2^+$	−80	231	−13	−13	−93	−9	
$^{9}_{\Lambda}$Be	$3/2^+$	$5/2^+$	−8	−14	37	0	28	44	43
$^{11}_{\Lambda}$B	$7/2^+$	$5/2^+$	56	339	−37	−10	−80	267	264
$^{11}_{\Lambda}$B	$3/2^+$	$1/2^+$	61	424	−3	−44	−10	475	505
$^{12}_{\Lambda}$C	2^-	1^-	61	175	−12	−13	−42	153	161
$^{15}_{\Lambda}$N	$3/2^+_2$	$1/2^+_2$	65	451	−2	−16	−10	507	481
$^{16}_{\Lambda}$O	1^-	0^-	−33	−123	−20	1	188	23	26
$^{16}_{\Lambda}$O	2^-	1^-_2	92	207	−21	1	−41	248	224

Ground-state doublet spacings in ${}^{10}_{\Lambda}$B and ${}^{12}_{\Lambda}$C

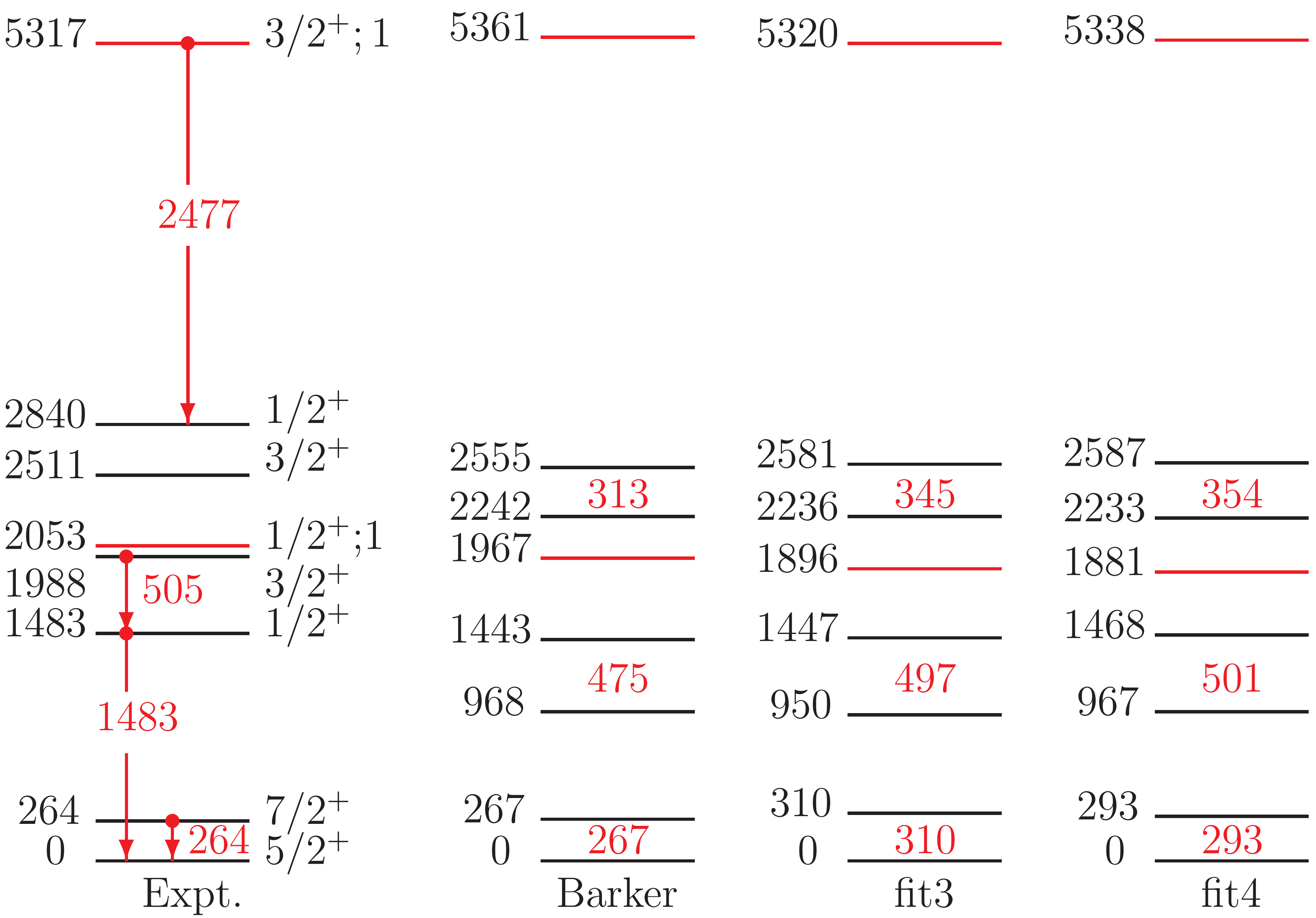


	$\Lambda\Sigma$	Δ	S_Λ	S_N	T	ΔE
${}^{12}_{\Lambda}$C		0.531	1.453	0.038	−1.742	
	57	175	−22	−13	−42	150 keV
${}^{10}_{\Lambda}$B		0.570	1.425	0.008	−1.099	
	−14	188	−21	−3	−26	121 keV

Speculations on the placement of $^{11}_{\Lambda}$B γ rays.



Shell-model calculations for ${}^{11}_{\Lambda}$B



Double one-pion exchange ΛNN interaction

Gal, Soper, and Dalitz: Ann. Phys. (N.Y.) 63, 53 (1971)

Independent of Λ spin. Averaged over s_Λ wave function gives

$$V_{NN}^{eff} = \sum_{klm} Q_{lm}^{k}(r_1, r_2)\,[\sigma_1, \sigma_2]^k \cdot [C_l(\hat{r_1}), C_m(\hat{r_2})]^k \;\; \tau_1 \cdot \tau_2$$

Parameters in MeV

Q_{00}^0	Q_{22}^0	Q_{22}^1	$Q_{02}^2 = Q_{20}^2$	Q_{22}^2
0.026	1.037	−0.531	−0.049	0.245

- Q_{00}^0 and Q_{22}^0 give repulsive contributions to B_Λ that depend quadratically on the number of p-shell nucleons in the core.
- Q_{22}^1 represents an anti-symmetric spin-orbit interaction that behaves rather like S_N

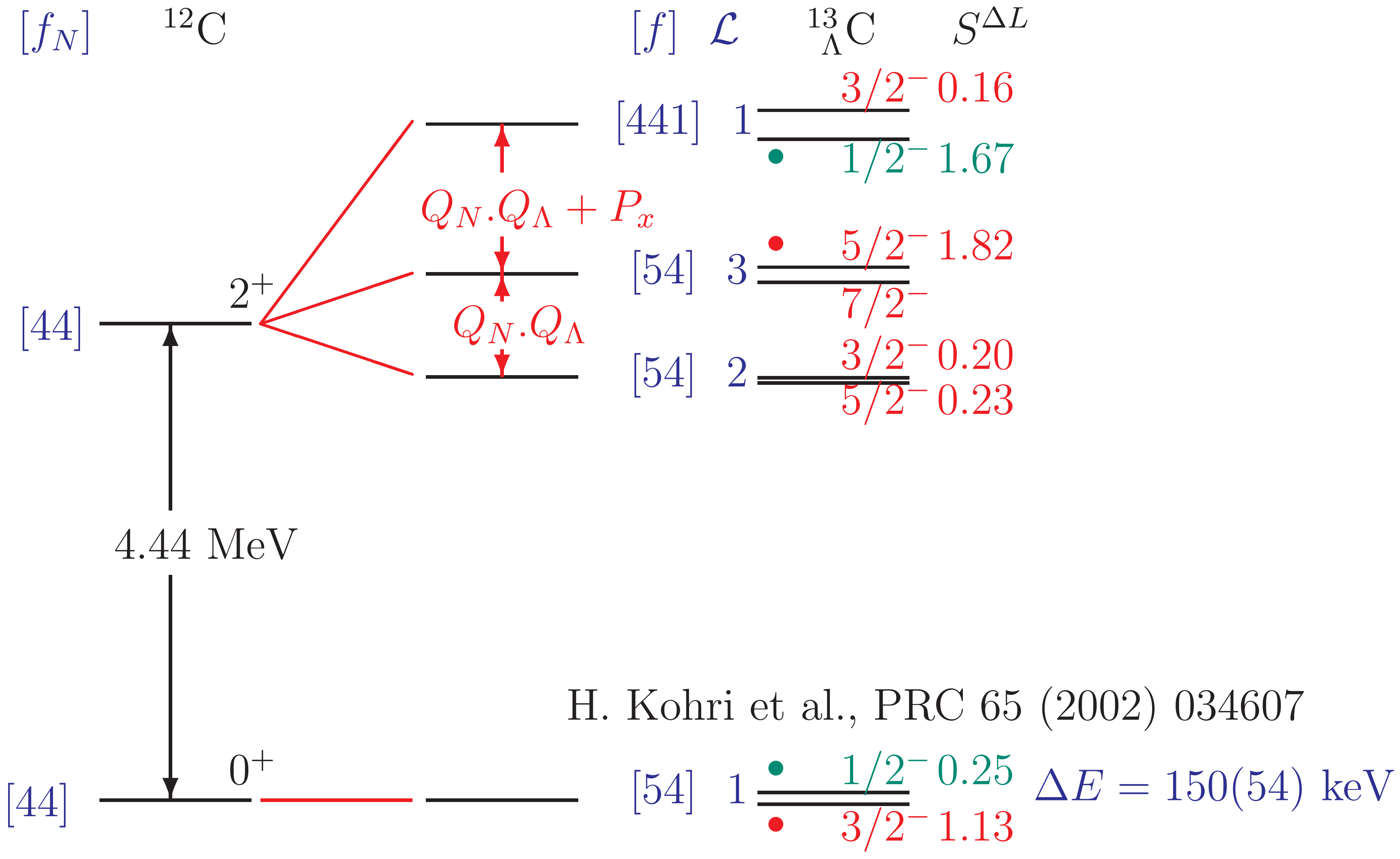
Λ-Σ and spin-dependent contributions to ground-state binding energies

	$^{7}_{\Lambda}$Li	$^{8}_{\Lambda}$Li	$^{9}_{\Lambda}$Li	$^{9}_{\Lambda}$Be	$^{10}_{\Lambda}$B	$^{11}_{\Lambda}$B	$^{12}_{\Lambda}$B	$^{13}_{\Lambda}$C	$^{15}_{\Lambda}$N	$^{16}_{\Lambda}$N
	$1/2^+$	1^-	$3/2^+$	$1/2^+$	1^-	$5/2^+$	1^-	$1/2^+$	$3/2^+$	1^-
Λ-Σ $\widehat{1}03$	28	59	62							
Δ	419	288	350	0	125	203	108	−4	40	94
S_Λ	0	−6	−10	0	−13	−20	−14	0	12	6
S_N	94	192	434	207	386	652	704	841	630	349
T	−2	−9	−6	0	−15	−43	−29	−1	−69	−45
Sum	589	625	952	211	518	858	869	864	726	412
Expt	5.58	6.80	8.50	6.71	8.89	10.24	11.37	11.69		13.76
		6.84	8.29		9.11					*
$\overline{V}$	−0.94	−1.02	−1.06		−1.05	−1.04	−1.05	−0.96		−0.93

* $B_\Lambda = 13.76(16)$ MeV: F. Cusanno et al. PRL 103, 202501 (2009)

To get a rough $\overline{V}$, take $B_\Lambda(^{5}_{\Lambda}\mathrm{He}) = 3.12$ MeV as s_Λ single-particle energy, and subtract sum from experimental B_Λ value.

$^{12}C(0^+, 2^+) \times p_\Lambda$ states of $^{13}_\Lambda C$



$B_\Lambda = 11.69 \pm 0.12$ MeV

$3/2^-/1/2^-$ $E_x = 10.83/10.98$ MeV

$^{13}C(K^-, \pi^-)^{13}_{\Lambda}C$ M. May et al., PRL 47, 1106 (1981)

Theory: E.H. Auerbach et al., PRL 47, 1110 (1981); Ann. Phys. 148, 381 (1983)

Basic data: Separation between 10.4 and 16.4 MeV peaks at 0° and shift in position of upper peak at 15°.

Woods-Saxon: p_Λ bound at 0.8 MeV

$^{13}_{\Lambda}C$	nsc97f	esc04a	djm	Experiment
$1/2^-_2 - 1/2^-_1$	6.94	6.05	6.18	6.0 ± 0.4
$1/2^-_2 - 5/2^-_1$	2.18	1.23	1.37	1.7 ± 0.4

Odd-state tensor, even-state tensor, and Λ-Σ mixing work against one-body and two-body spin-orbit interactions in the "single-particle" p_Λ splitting. Mixing of $2^+ \times p_\Lambda$ into $0^+ \times p_\Lambda$ (typically 5%) also contributes to the spacing.

Contribution to "single-particle" p_Λ splitting in $^{13}_{\Lambda}$C

Source	ΔE (keV)
$Q_N \cdot Q_\Lambda$: mixing with $2^+ \times p_\Lambda$ states	224
One-body spin-orbit	43
Two-body LS + ALS interaction	57
$\Lambda - \Sigma$ mixing	−30
Spin-spin interaction	27
Odd-state tensor interaction	−45
Even-state tensor interaction	−94
	182
Full diagonalization	186
Total LS + ALS	100

Future Hypernuclear γ-ray Spectroscopy

- Hyperball-J at J-PARC (Tamura)
- $(K^-, \pi^-\gamma)$ $\quad p_K = 1.1$ or 1.5 GeV/c
- Larger spin-flip amplitudes - test case ${}^4_\Lambda$He $1^+ \to 0^+$
- p-shell and light sd-shell nuclei as targets
- $(K^-, \pi^0\gamma)$ also possible