

Production of Hypernuclei

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Topics:

1. Introduction
 2. Aspects of YN Interactions
 3. Strangeness Production on the Nucleon
 4. Strangeness Production by Resonance Excitation
 5. Hadro-Production of Hypernuclei
 6. Photo- and Electroproduction of Hypernuclei
 7. Production of Hypernuclei in Relativistic Fragmentation Reactions
 8. Outlook
- 

Production of Hypernuclei

...we are at the point to discuss:

- dynamics of the hyperon in nuclear matter
- dynamics of the projectile and the ejectile
- description of the production dynamics

Field Theoretical Lagrangian Approach:

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_M + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{int}}^{\text{mR}} .$$

$$\mathcal{L}_B = \sum_b \bar{\psi}_b (i\not{\partial} - m_b) \psi_b ,$$

$$\mathcal{L}_M = \frac{1}{2} \sum_m \left(\partial_\mu \phi_m \partial^\mu \phi_m - m_m^2 \phi_m^2 \right) - \frac{1}{2} \sum_n \left[\frac{1}{2} F_{\mu\nu}^{(n)} F^{(n)\mu\nu} - m_n^2 V_\mu^{(n)} V^{(n)\mu} \right]$$

$$\begin{aligned} \mathcal{L}_{\text{int}} = & g_\sigma \bar{\psi} \phi_\sigma \psi + g_\delta \bar{\psi} (\boldsymbol{\tau} \cdot \boldsymbol{\phi}_\delta) \psi - i \frac{g_\pi}{2m} \bar{\psi} \gamma^5 \gamma^\mu \partial_\mu (\boldsymbol{\tau} \cdot \boldsymbol{\phi}_\pi) \psi \\ & - i \frac{g_\eta}{2m} \bar{\psi} \gamma^5 \gamma^\mu \partial_\mu \phi_\eta \psi - g_\omega \bar{\psi} \gamma^\mu A_\mu^{(\omega)} \psi - i \frac{f_\omega}{2m} \bar{\psi} \sigma^{\mu\nu} \partial_\mu A_\nu^{(\omega)} \psi \\ & - g_\rho \bar{\psi} \gamma^\mu (\boldsymbol{\tau} \cdot \mathbf{A}_\mu^{(\rho)}) \psi - i \frac{f_\rho}{2m} \bar{\psi} \sigma^{\mu\nu} \partial_\mu (\boldsymbol{\tau} \cdot \mathbf{A}_\nu^{(\rho)}) \psi . \end{aligned}$$

Non-relativistic ΛN Interaction:

$$g_\sigma \Psi \bar{\Psi} \Phi_\sigma = \frac{g_\sigma^2}{4\pi} m_\sigma \Psi \bar{\Psi} D_\sigma(1,2) \Psi \bar{\Psi} \rightarrow \sum_{abcd} \varphi_a(1) \varphi_b^*(2) V_\sigma(1,2) \varphi_d(2) \varphi_c^*(1)$$

$$V_\sigma(1,2) \propto V_\sigma^{(c)}(1,2) + V_\sigma^{(\ell s)}(1,2) \vec{\ell} \cdot \vec{S}$$

$$\vec{S} = \vec{\sigma}_1 + \vec{\sigma}_2$$

and correspondingly :

$$V_\omega(1,2) \propto V_\omega^{(c)}(1,2) + V_\omega^{(\ell s)}(1,2) \vec{\ell} \cdot \vec{S} + V_\omega^{(ss)}(1,2) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_\omega^{(T)}(1,2) S_{12}$$

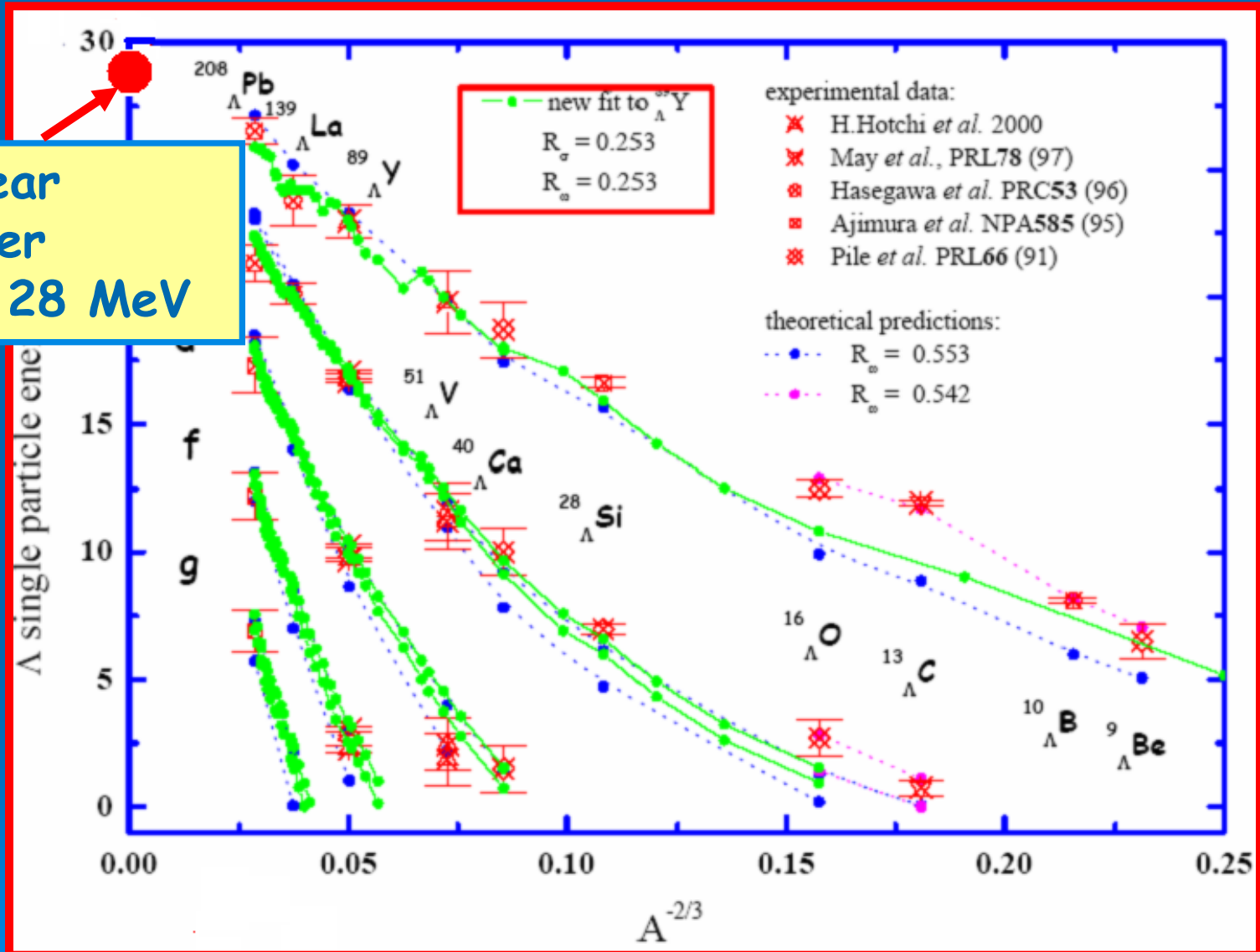
$$V_\eta(1,2) \propto V_\eta^{(ss)}(1,2) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_\eta^{(T)}(1,2)$$

$$S_{12} = \frac{3}{r^2} \vec{\sigma}_1 \cdot \vec{r} \vec{\sigma}_2 \cdot \vec{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

...plus momentum dependent higher order terms!

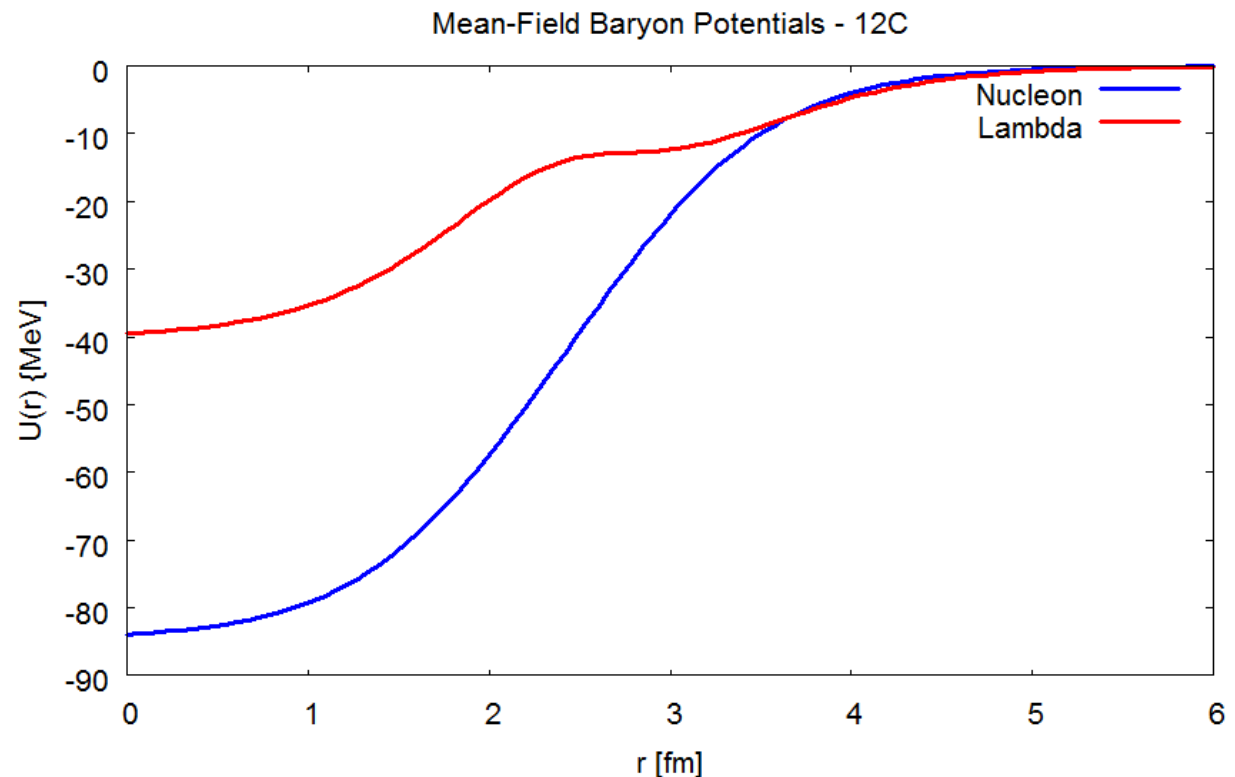
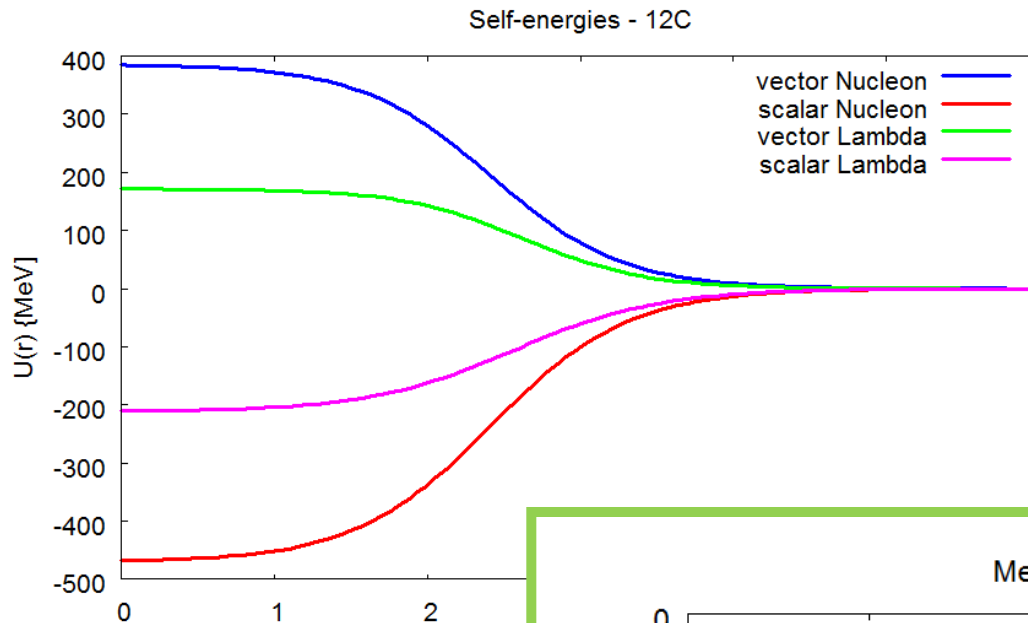
DDRH Flavour Dynamics: Λ Single Particle Energies

Nuclear
Matter
 $S_{\Lambda} = 28 \text{ MeV}$



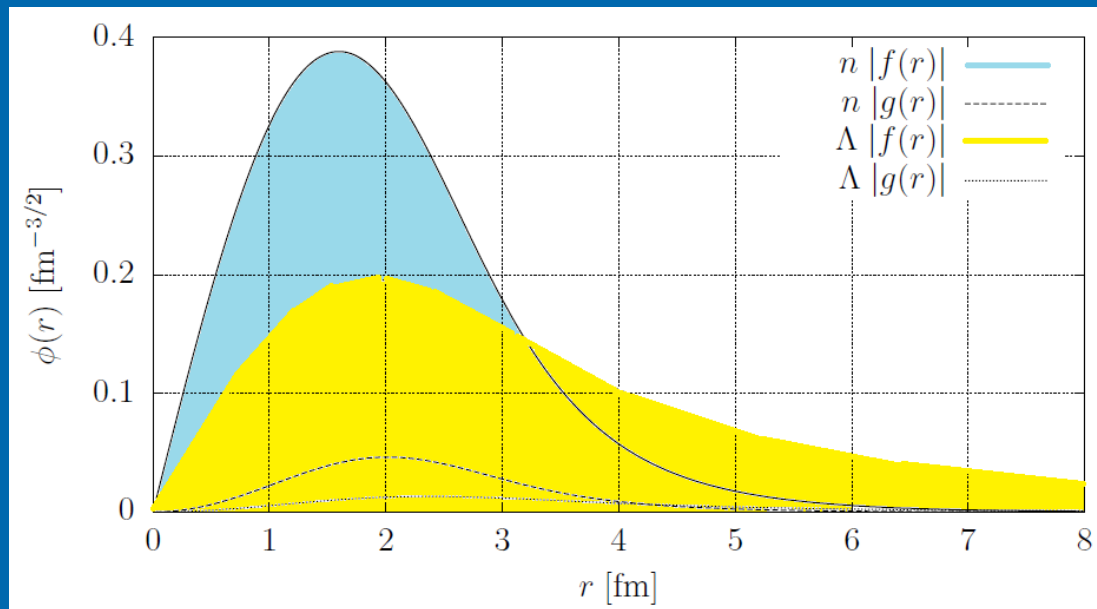
Density Dependent NN and $N\Lambda$ Dirac-Brueckner Vertices

Relativistic Self-Energies and Schroedinger- type Potentials in ^{12}C



state	E_{bind} [MeV]
$s_{1/2}$	10.79 ± 0.11
$p_{3/2}$	0.10 ± 0.04
$p_{1/2}$	0.10 ± 0.20

Size of a hypernucleus:



$1p_{3/2}$
orbits in
 ^{12}C
($S=100\text{keV}$)

bound state	rms radius [fm]	
	neutron	Λ
$s_{1/2}$	1.89	2.43
$p_{3/2}$	2.50	7.18

...**halo-type** configurations
due to weak binding in a shallow potential

Scaling Factors from Potentials:

$$I_a[U_a] = \int d^3r U_a(r)$$

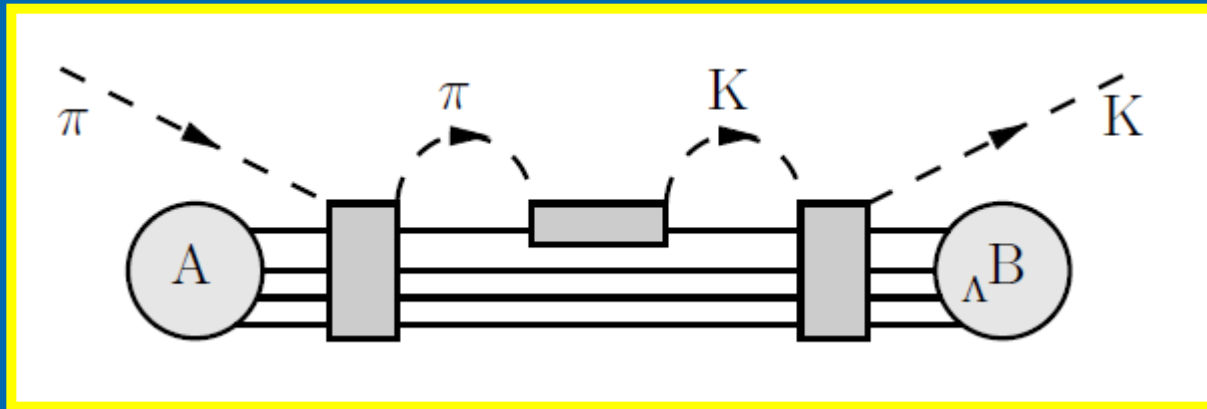
$$R_{\Lambda}^{(s,v)} := \frac{I_{\Lambda}^{(s,v)}}{I_N^{(s,v)}} .$$

$$R_{\Lambda}^s = 0.539 , \quad R_{\Lambda}^v = 0.536$$

V. Hadro-Production of Hypernuclei



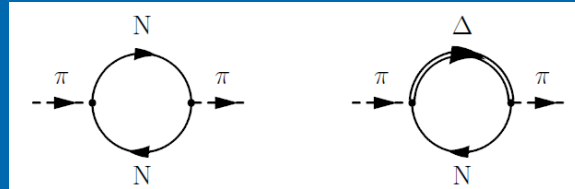
5.1 Reaction Dynamics:



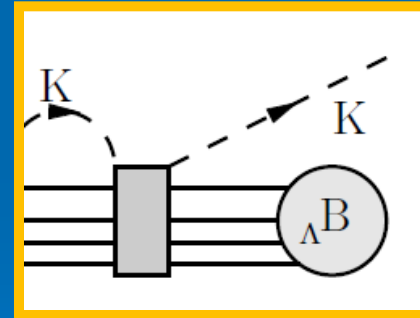
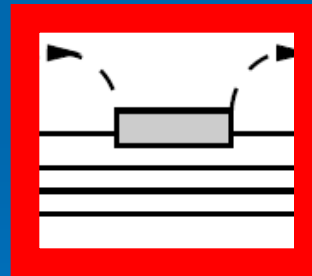
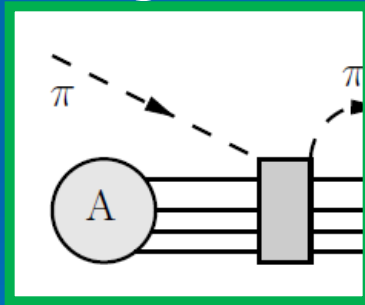
Schematic picture of the $\pi + A \rightarrow K + {}_{\Lambda}B$

...DWIA - Distorted Wave Impulse Approximation:

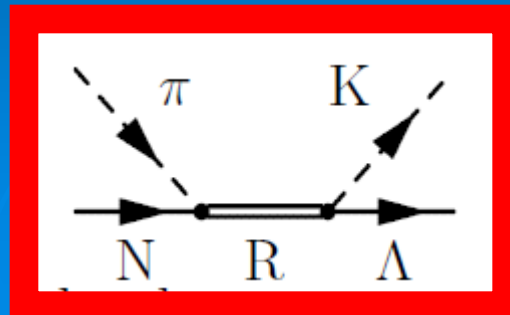
- derive/obtain pion-nucleus and kaon-nucleus self-energies (optical potentials)



- solve Klein-Gordon wave equations for the π^+A and K^+B_Λ systems
- high energies: eikonal approximation: $\Phi \sim e^{iS(z,b)}$



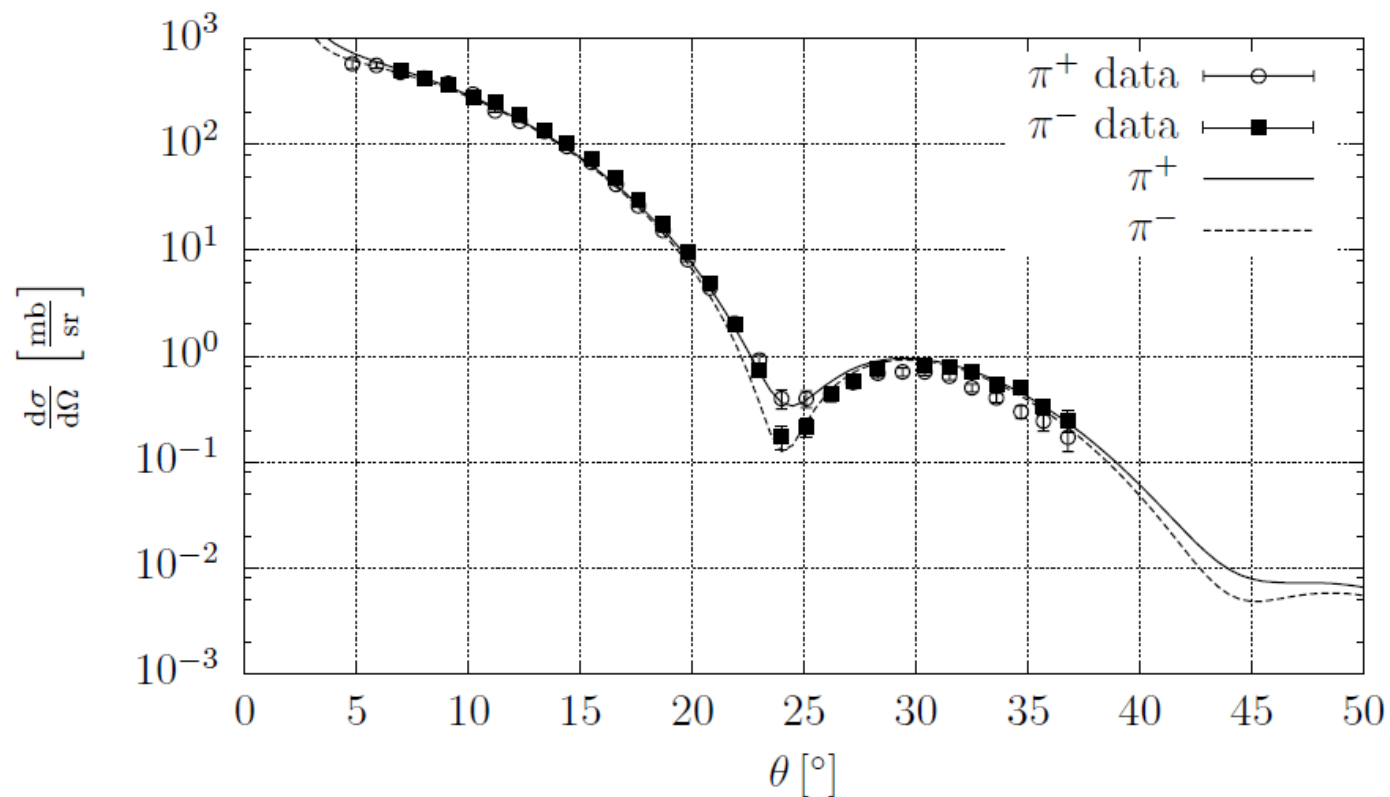
- evaluate the production vertex



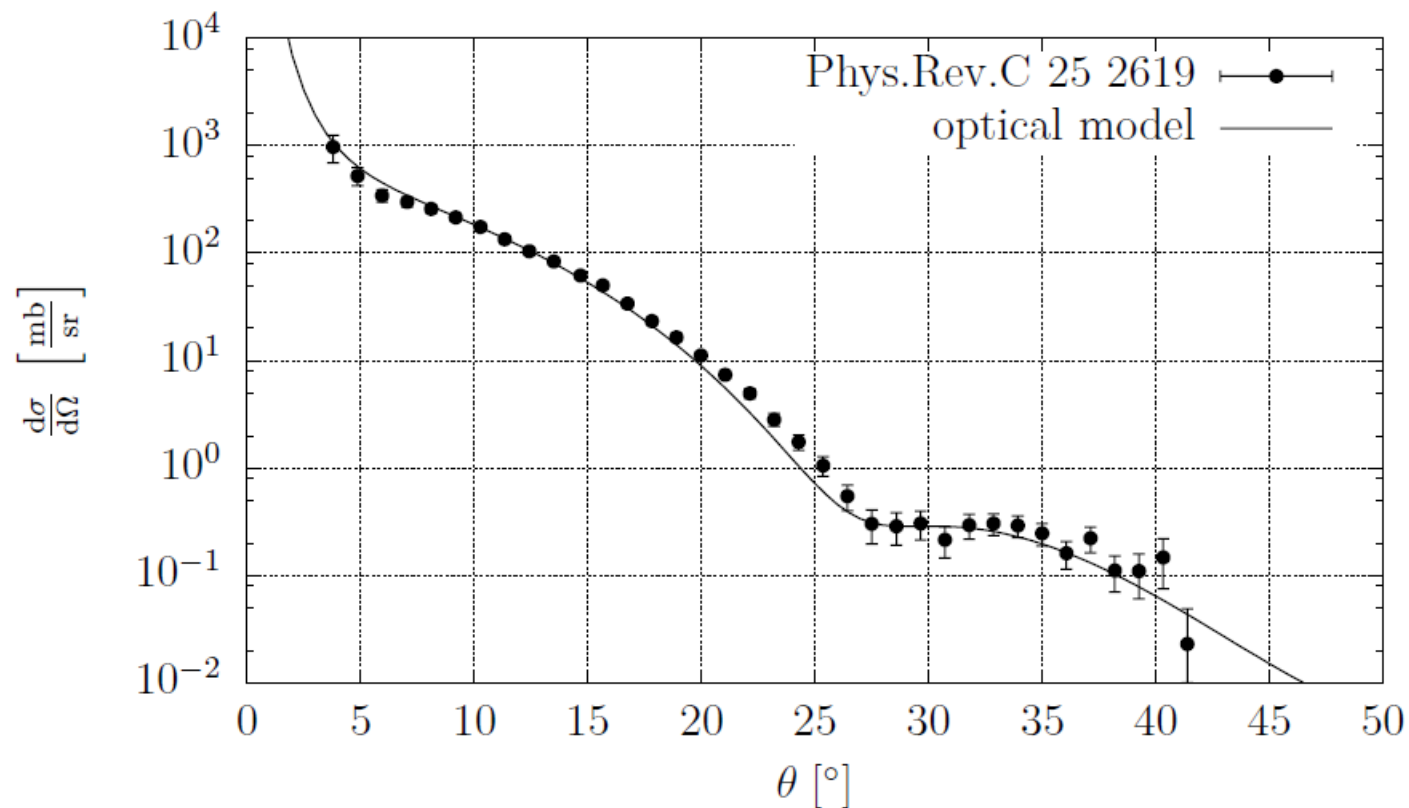
5.2 Meson-nucleus Interactions at high Energies ($\sim 300\text{MeV} \dots 2\text{GeV}$):

$$\left(-\Delta + m^2 + 2EV_{\text{opt}}\right) \phi = (E - V_{\text{coul}})^2 \phi$$

$$2EV_{\text{opt}}(r) = -Ak^2b_0\varrho(r) + Ab_1\nabla\varrho(r)\cdot\nabla$$



Differential cross section for π^+ and π^- elastic scattering on ^{12}C at $p_{\text{lab}} = 800$ MeV. Shown are the calculations using the solution of the Klein-Gordon equation with an optical potential, with the only parameter $b_0 = -0.16 + i0.90 \text{ fm}^3$.



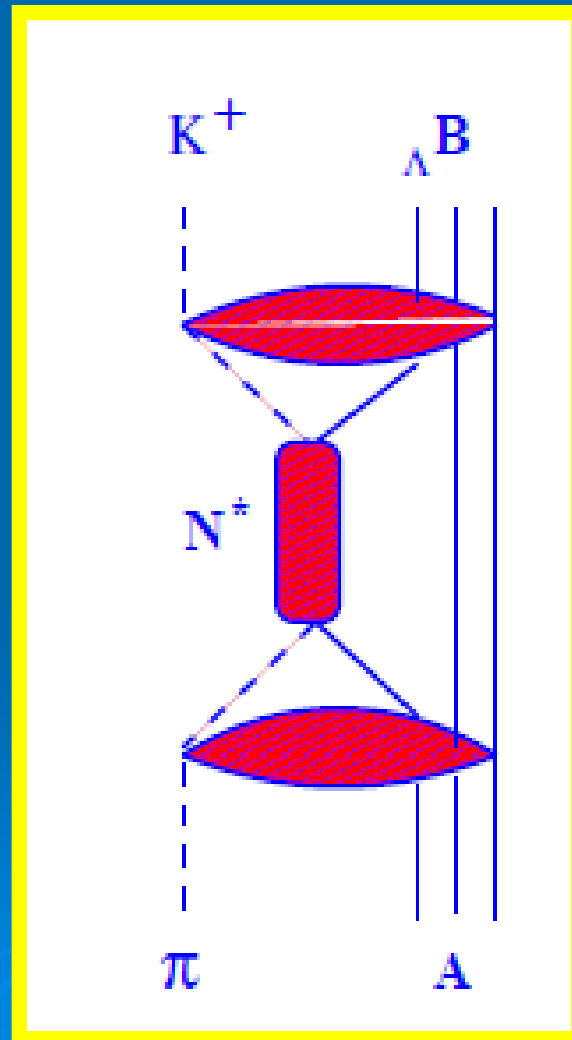
Differential cross section for K^+ elastic scattering on ^{12}C at $p_{\text{lab}} = 800$ MeV. Shown is the calculations using the optical potential with the parameter $b_0 = -0.3960 + i0.3506 \text{ fm}^3$.

$p_{\text{lab}} [\text{MeV}]$	$\Re b_0 [\text{fm}^3]$	$\Im b_0 [\text{fm}^3]$
635	-0.5937	0.4417
715	-0.3433	0.3923
800	-0.396	0.3506
692	-0.3862	0.4057

5.3 Resonances and their Interactions



...remind our model:



Spin-1/2 Resonances

Pseudo-vector coupling:

$$\mathcal{L}_{\pi\text{NR}}^{\text{PV}} = -\frac{g_{\pi\text{NR}}}{m_{\text{R}} \pm m_{\text{N}}} \bar{\psi}_{\text{R}} \gamma^{\mu} \Gamma \partial_{\mu} (\boldsymbol{\tau} \cdot \boldsymbol{\phi}_{\pi}) \psi_{\text{N}} + \text{h. c.} ,$$

$$\mathcal{L}_{\text{RK}\Lambda}^{\text{PV}} = -\frac{g_{\text{RK}\Lambda}}{m_{\text{R}} \pm m_{\Lambda}} \bar{\psi}_{\text{R}} \gamma^{\mu} \Gamma \partial_{\mu} \phi_{\text{K}} \psi_{\Lambda} + \text{h. c.} ,$$

$$\Gamma = \begin{cases} i & \text{for odd parity} \\ \gamma^5 & \text{for even parity} \end{cases}$$

Spin-3/2 Resonances

$$(i\not{D} - m)\psi^\mu = 0$$

with

$$\gamma_\mu \psi^\mu = 0 .$$

Rarita-Schwinger equation(s)

$$\begin{aligned}\mathcal{L}_{\pi\text{NR}} &= \frac{g_{\pi\text{NR}}}{m_\pi} \bar{\psi}_\text{R}^\mu \partial_\mu (\boldsymbol{\tau} \cdot \boldsymbol{\phi}_\pi) \psi_\text{N} + \text{h. c.} , \\ \mathcal{L}_{\text{RK}\Lambda} &= \frac{g_{\text{RK}\Lambda}}{m_\text{K}} \bar{\psi}_\text{R}^\mu \partial_\mu \phi_\text{K} \psi_\Lambda + \text{h. c.} .\end{aligned}$$

Gauge-invariant Pascalsutsa-choice:

Elimination of non-physical $s=1/2$ degrees of freedom
(„off-shell parameter“ $z=-1/2$)

The Giessen Resonance Model: Nucleon Resonances, Branching, and Coupling Constants

resonance	width [MeV]	decay channel	branching ratio	g
N(1650) S_{11} $\frac{1}{2} \left(\frac{1}{2}^- \right)$	150	$N\pi$	0.700	0.8096
		$N\rho$	0.080	2.6163
		$N\omega$		1.8013
		$N\sigma$	0.025	2.5032
		$K\Lambda$	0.070	0.7658
N(1710) P_{11} $\frac{1}{2} \left(\frac{1}{2}^+ \right)$	100	$N\pi$	0.150	1.0414
		$N\rho$	0.150	4.1421
		$N\omega$	0.130	1.2224
		$N\sigma$	0.170	0.6737
		$K\Lambda$	0.150	6.1155
N(1720) P_{13} $\frac{1}{2} \left(\frac{3}{2}^+ \right)$	150	$N\pi$	0.150	0.1469
		$N\rho$	0.700	19.483
		$N\omega$		16.766
		$N\sigma$	0.120	1.5557
		$K\Lambda$	0.080	1.0132

The invariant Matrix Element:

$$\frac{d\sigma}{d\Omega_K} = \frac{1}{16\pi^2} \frac{m_A m_B}{s} \frac{|\mathbf{p}_K(\tilde{E}_K)|}{|\mathbf{p}_\pi|} |\mathcal{M}(E_K = \tilde{E}_K)|^2$$

$$\mathcal{M} = \int d^4x d^4y \bar{\psi}_\Lambda(x) \phi_K^*(x) \Gamma_1 G_R(x, y) \Gamma_2 \phi_\pi(y) \psi_N(y)$$

$$G_R(x, y) = \int_{\mathbb{R}^4} \frac{d^4p}{(2\pi)^4} i \frac{\not{p} + m_R}{p^2 - m_R^2 + i\epsilon} e^{-ip \cdot (x-y)}$$

5.4 Results: Hadro-Production of Hypernuclei and Spectroscopy

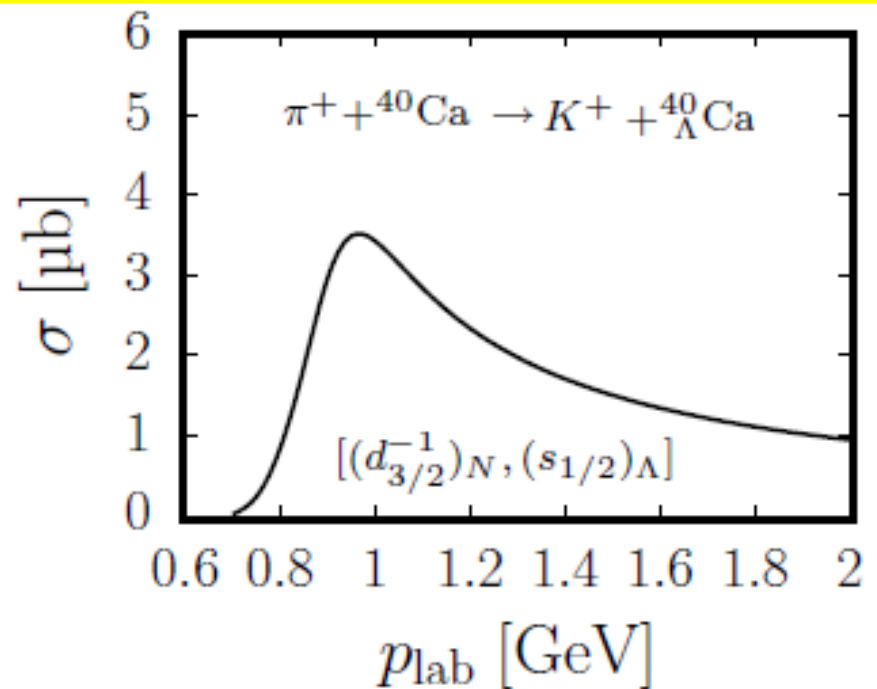
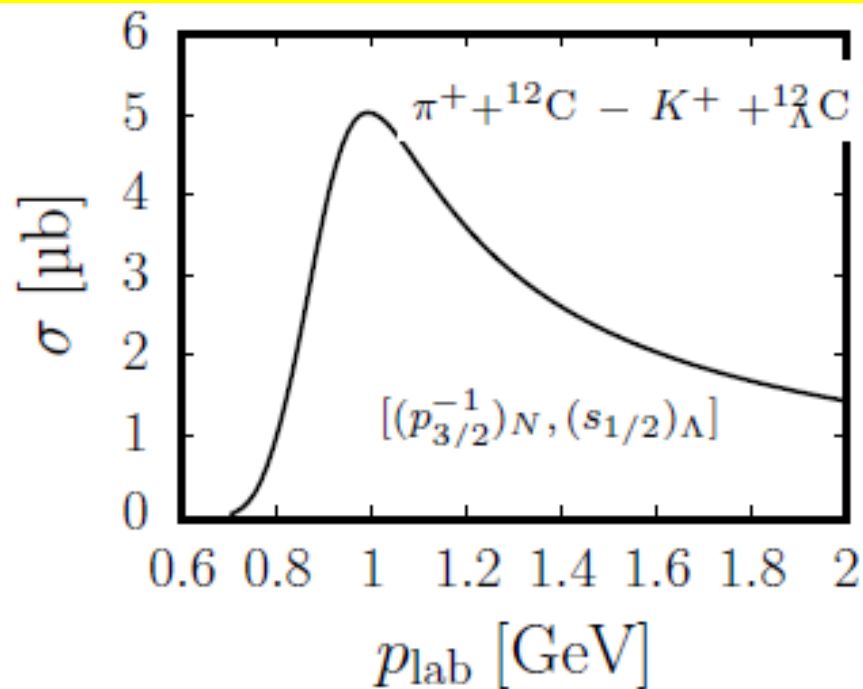


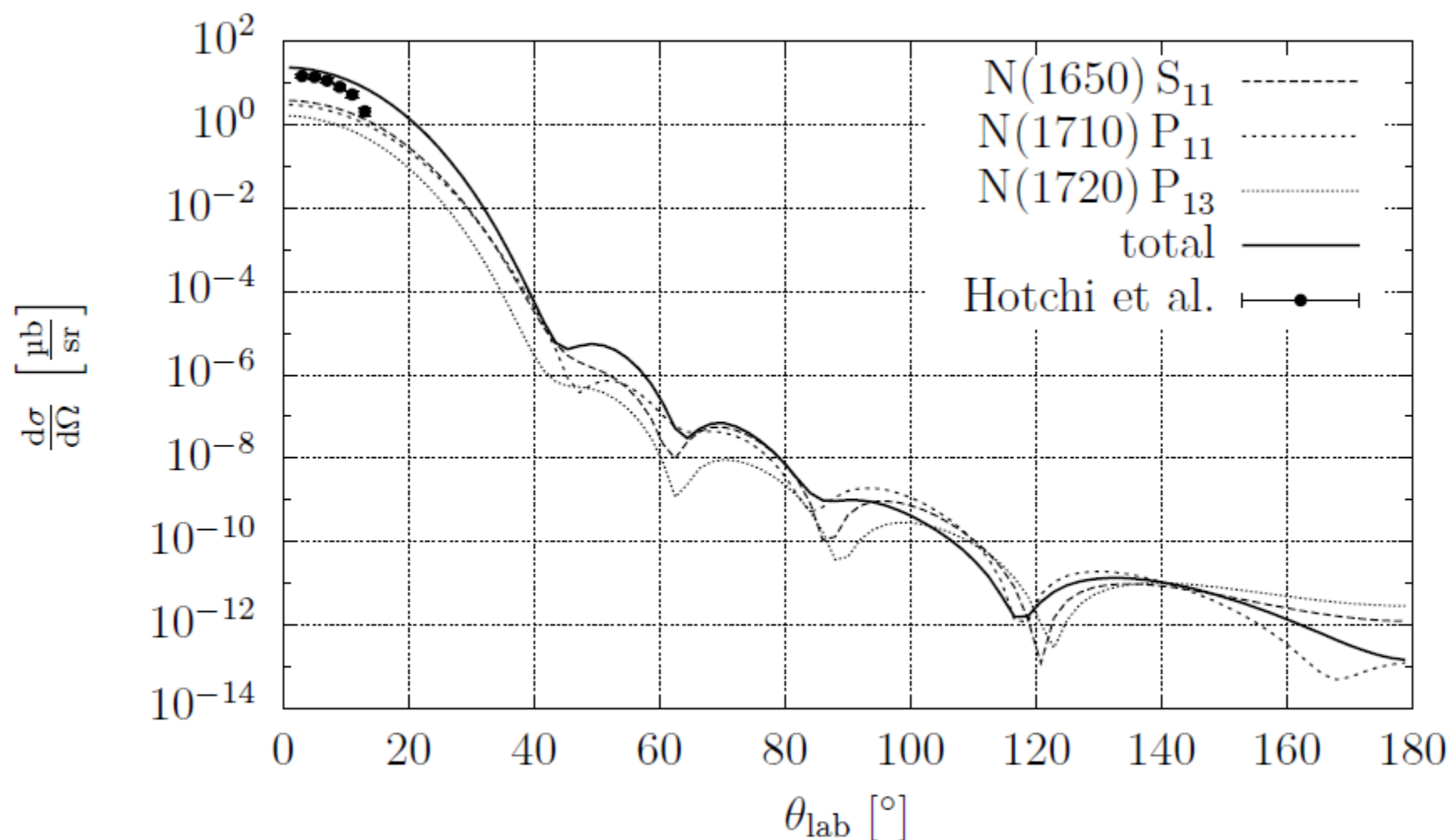
DWIA Matrix Element in Momentum Space:

$$\mathcal{M} = \int \frac{d^4 k_N}{(2\pi)^4} \int \frac{d^4 k_\Lambda}{(2\pi)^4} \int \frac{d^4 p}{(2\pi)^4} \hat{\phi}_K^*(p - k_\Lambda) \bar{\psi}_\Lambda(k_\Lambda) \Gamma_1 i \frac{\gamma \cdot p + m_R}{p^2 - m_R^2 + i\epsilon} \Gamma_2 \hat{\phi}_\pi(p - k_N) \psi_N(k_N)$$

...numerically quite involved: 12-D integration!!
(Monte-Carlo Integration)

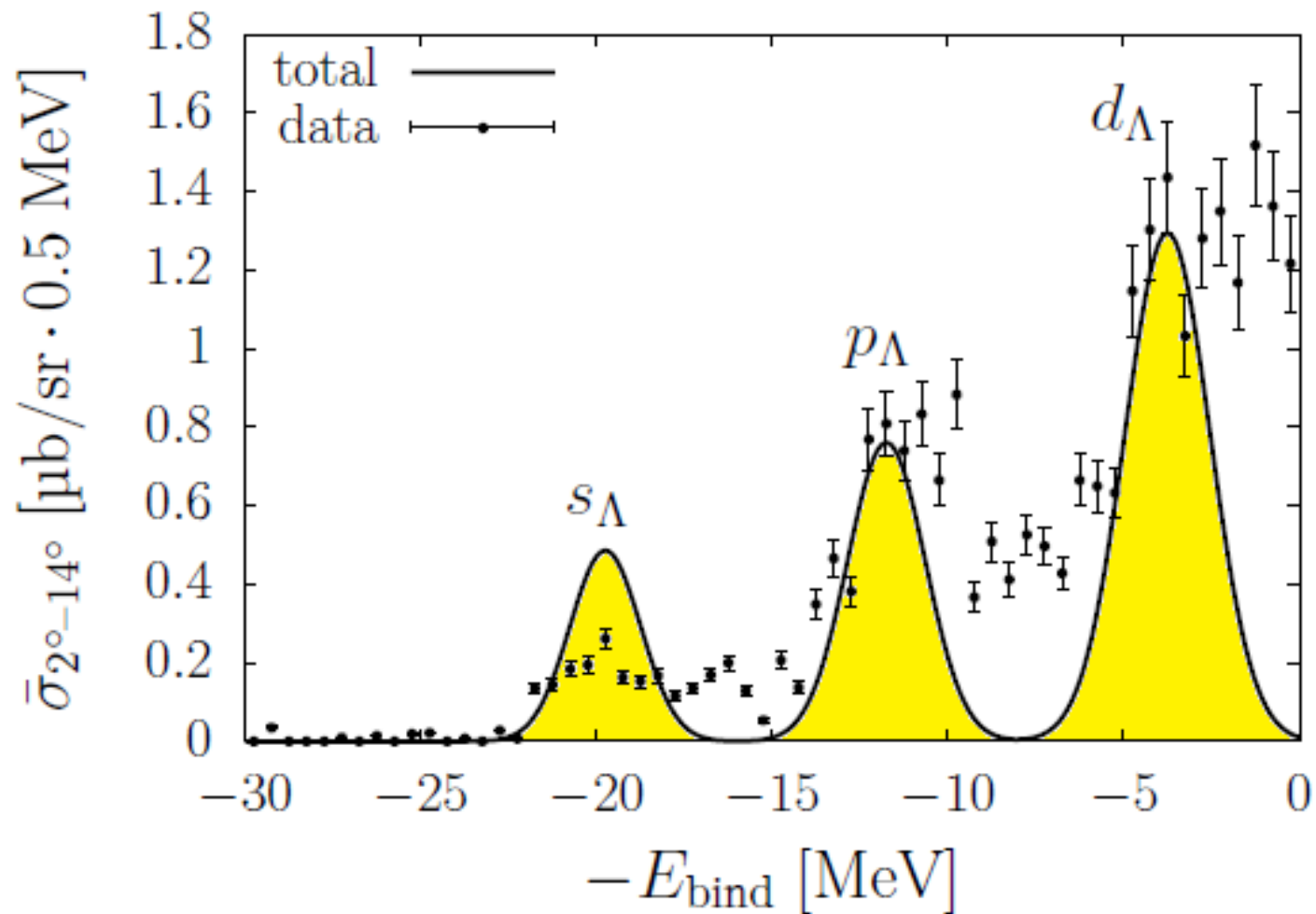
Total (angle integrated) production cross sections





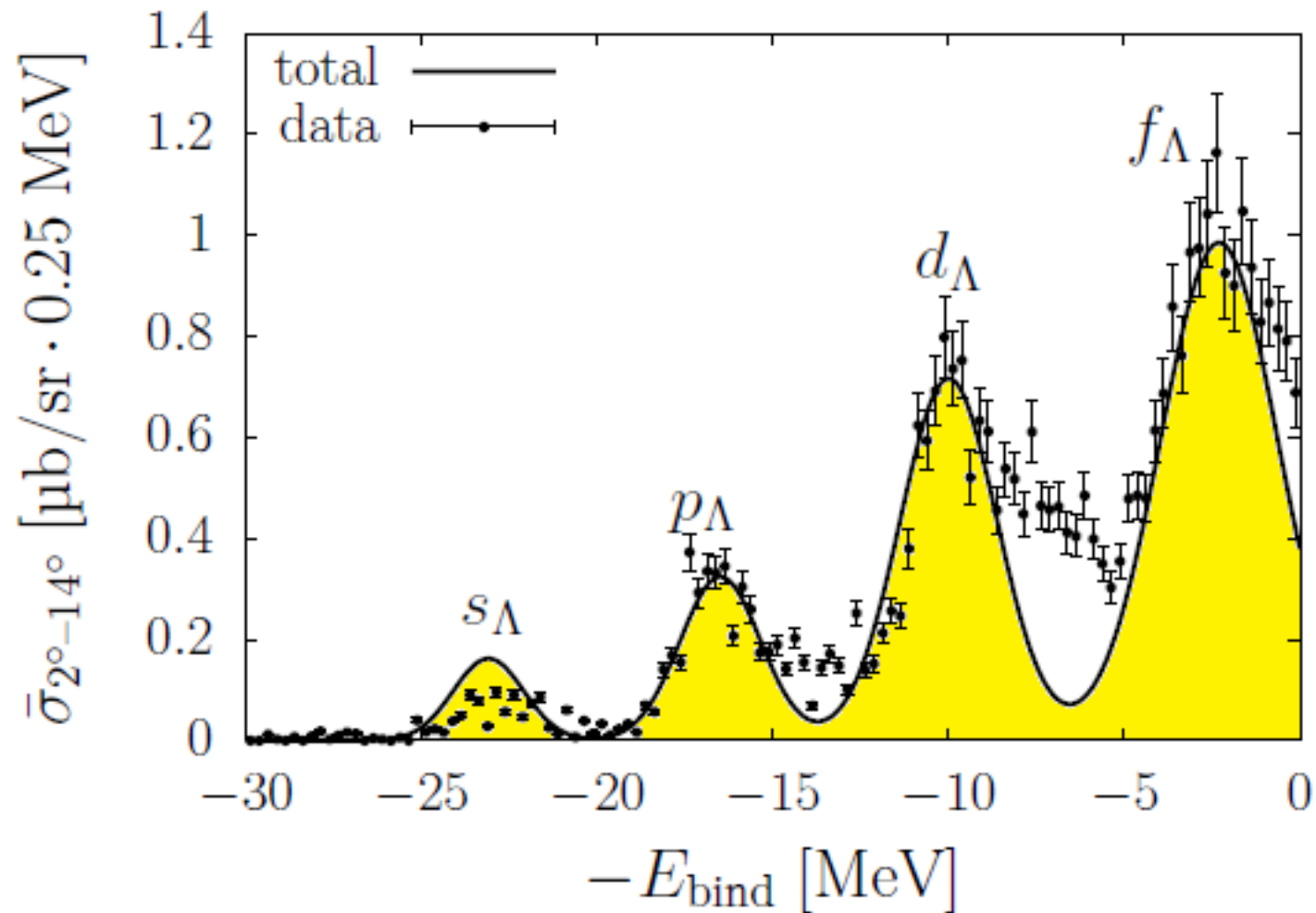
The differential cross section for $\pi^+ + {}^{12}\text{C} \rightarrow \text{K}^+ + {}^{12}_{\Lambda}\text{C}$ at a pion incoming momentum of 1050 MeV over the complete angular range from 0° to 180° .

Results for $^{51}\text{V}_\Lambda$



The spectral distribution for the $^{51}_\Lambda\text{V}$ hypernucleus.

Results for $^{89}\text{Y}_\Lambda$



The total averaged cross section for the $^{89}\text{Y}_\Lambda$ hypernucleus.

Λ -States in ^{89}Y and ^{41}V

Including:

- relativistic tensor int.
- core polarization

$$H_j = H_{\ell s} \vec{\ell} \cdot \vec{s} + H_{Ij} \vec{I} \cdot \vec{j}$$



$$\langle E_j \rangle = E_{\ell s} \langle l \cdot s \rangle + E_{Ij} \langle I \cdot j \rangle$$

$$\vec{J} = \vec{I} + \vec{j}$$

$$|I - j| \leq J \leq I + j$$

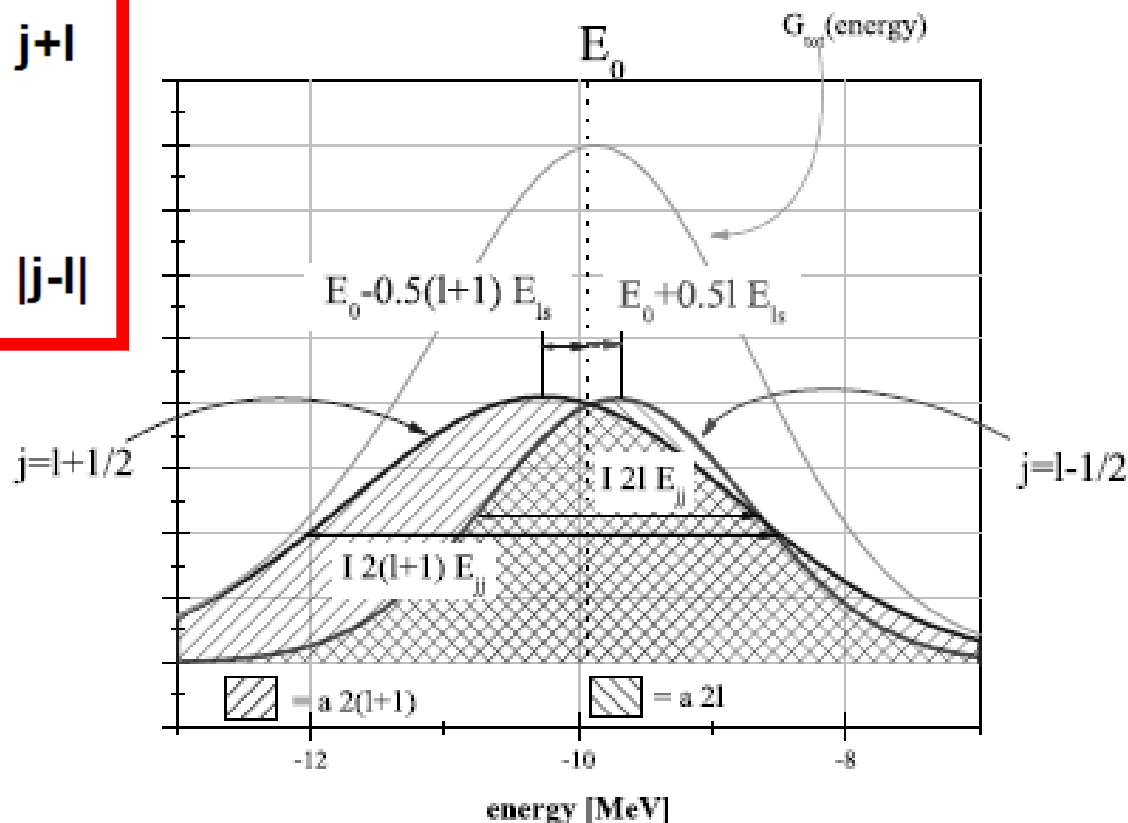
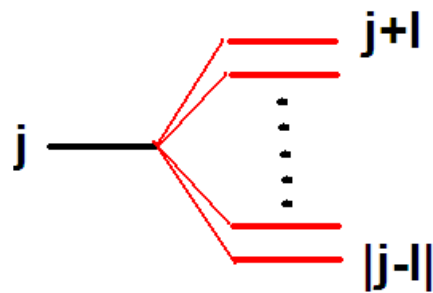
	^{89}Y			^{51}V		
	Λ			Λ		
E_s	-22.94	± 0.66	MeV	-19.79	± 1.22	MeV
E_p	-16.79	± 0.66	MeV	-11.48	± 1.11	MeV
E_d	-9.93	± 0.66	MeV	-2.22	± 1.06	MeV
E_f	-2.60	± 0.66	MeV	-		
$E_{\ell s}$	222.7	± 152.7	keV	283.0		keV
E_{Ij}	61.3	± 2.5	keV	106.0		keV
E_c	2.83	± 0.12	MeV	1.36		MeV
w_c	0.50			0.30		

Λ -States in High Spin Core Nuclei:

$$|^{89}\text{Y}\rangle = [|^{88}\text{Y}(4-)\rangle |\Phi_{\Lambda}\rangle]_j$$

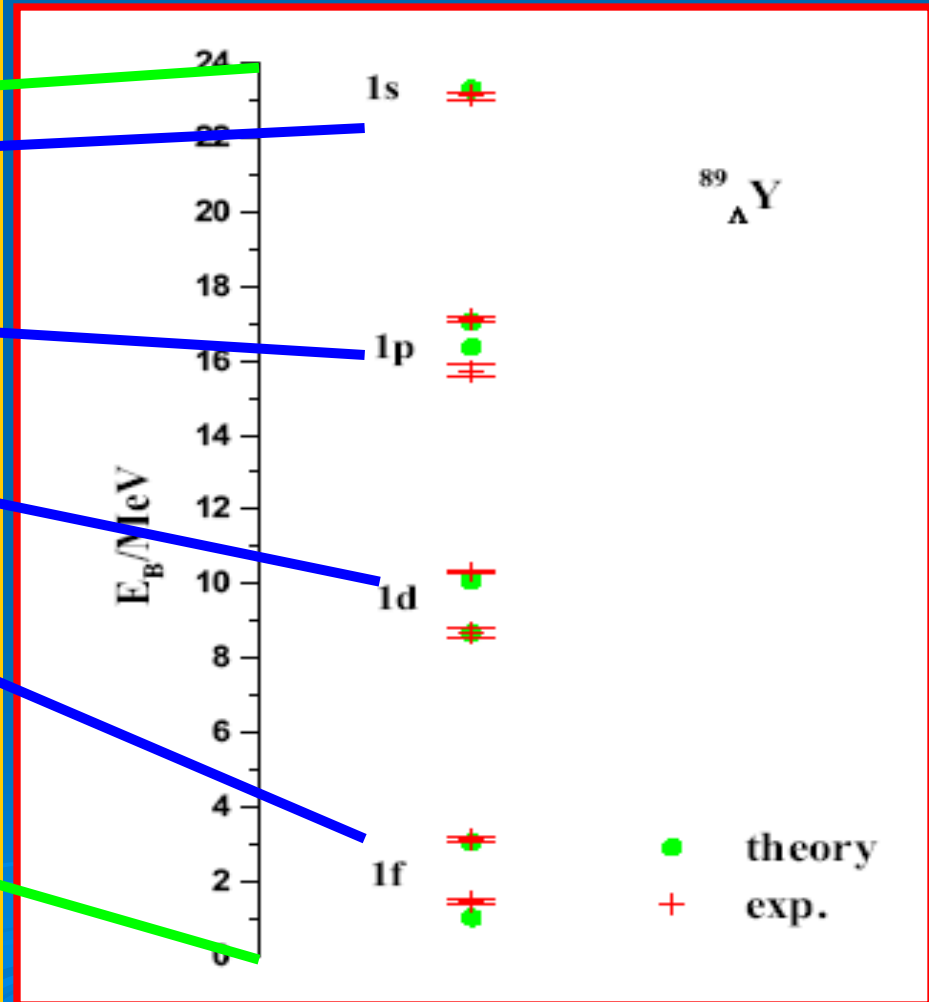
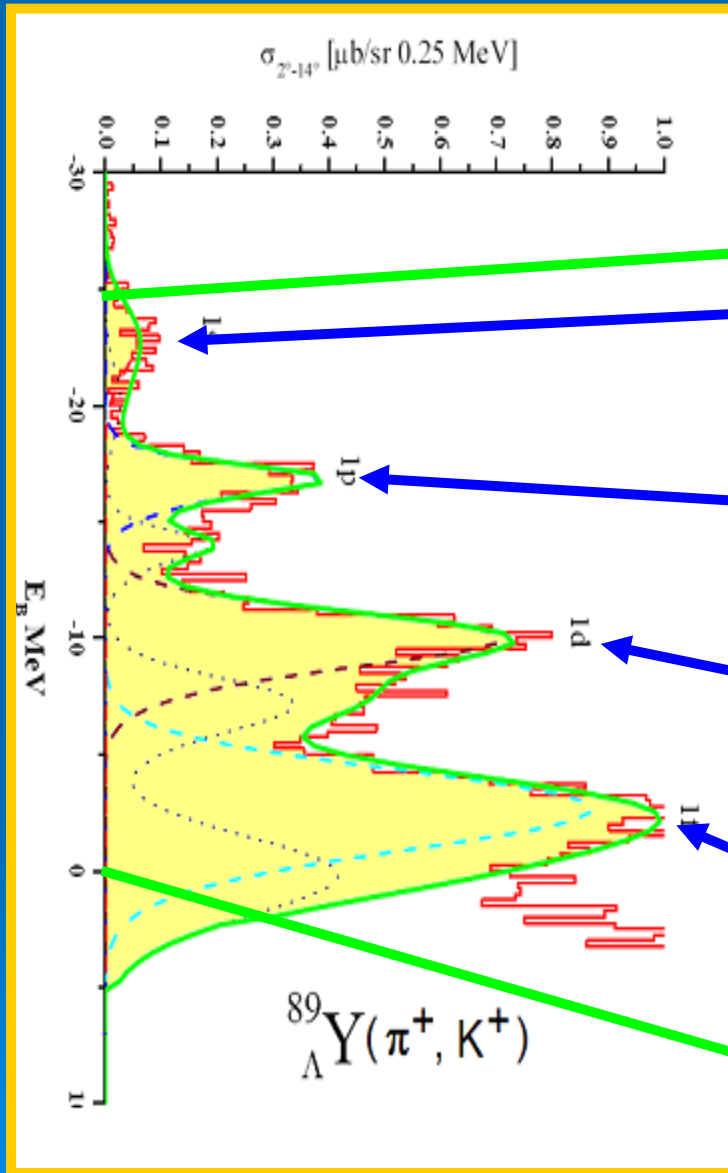
and

$$|^{41}\text{V}\rangle = [|^{40}\text{V}(6-)\rangle |\Phi_{\Lambda}\rangle]_j$$

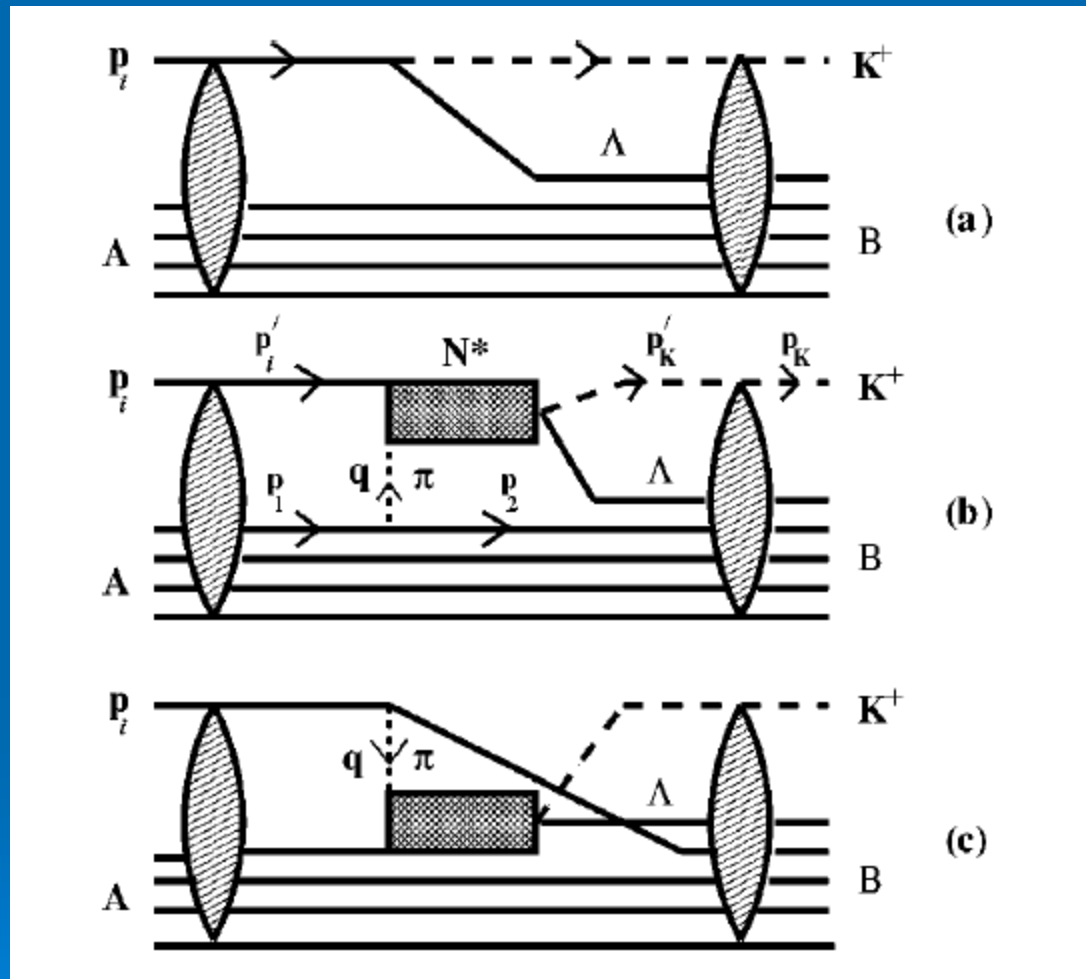


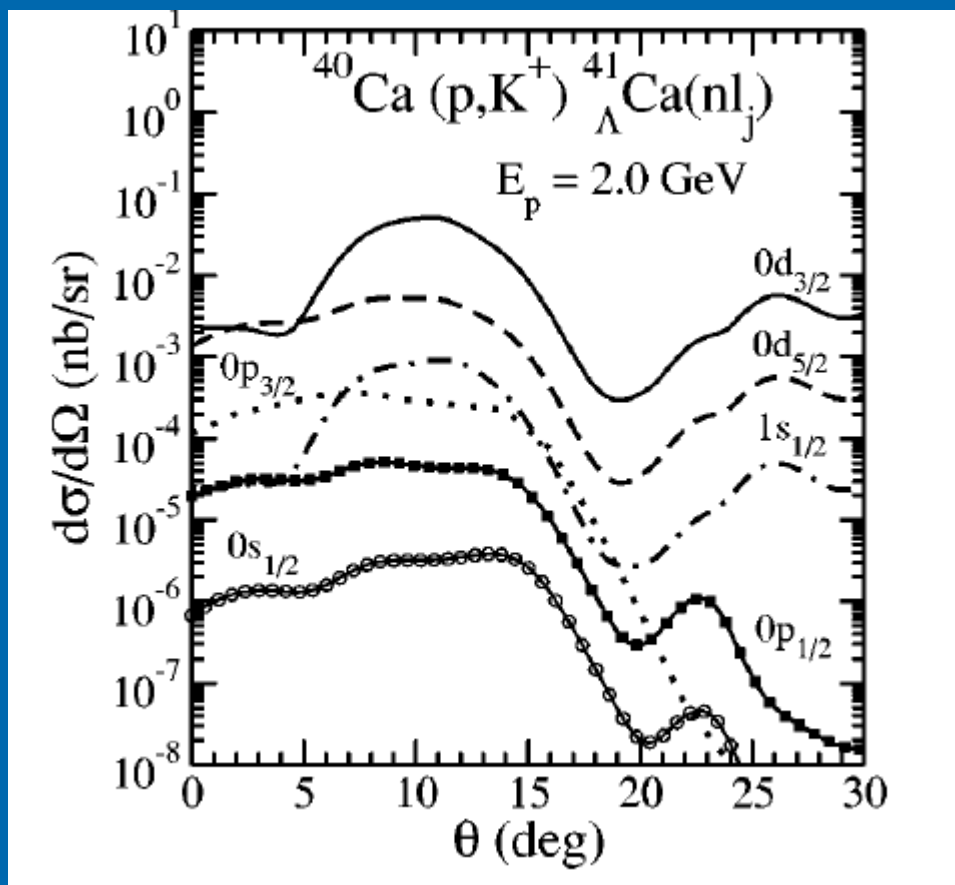
DDRH Spectrum of ^{89}Y :

- $^{89}\text{Y} = \Lambda + ^{88}\text{Y}(4-, \text{g.s.})$
- Λ -Core Interactions
- Tensor Interaction



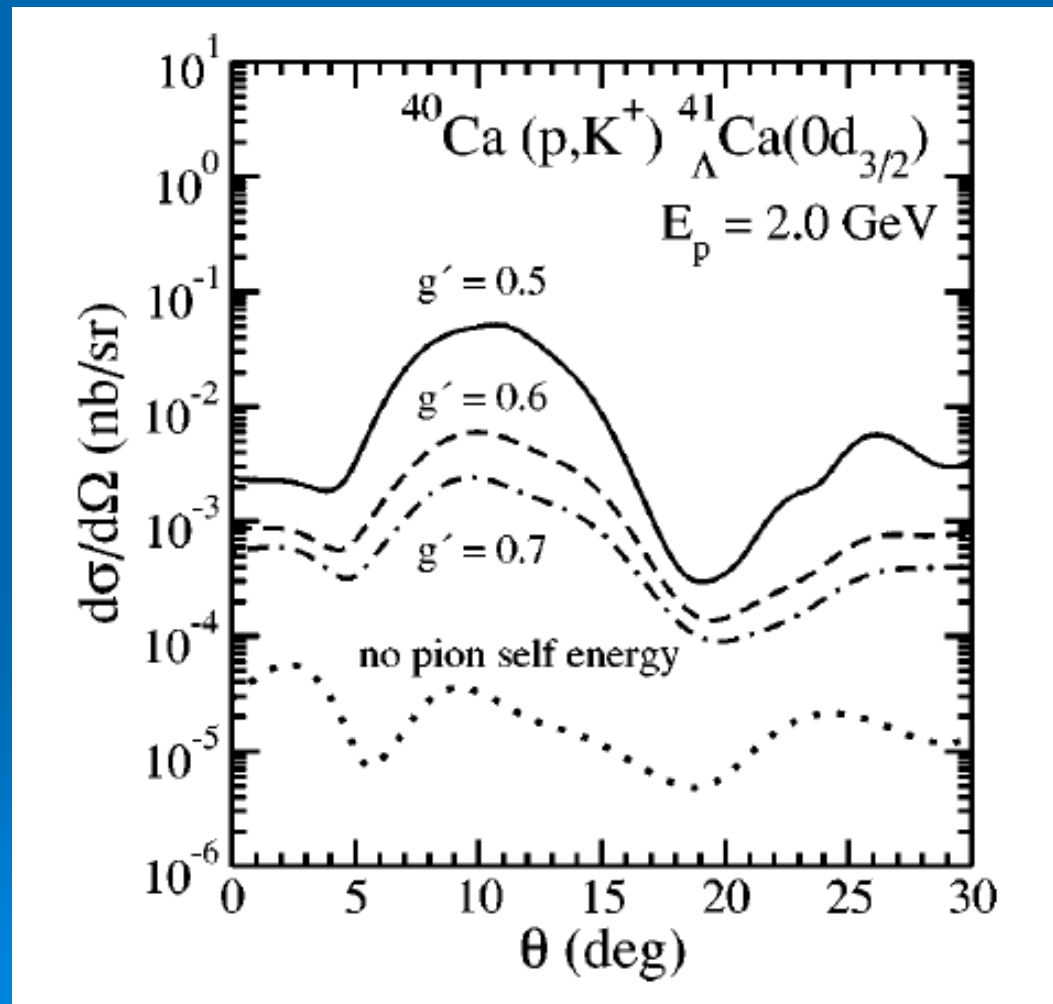
5.5 Production of Hypernuclei by Protons





Differential cross section for the $^{40}\text{Ca}(p, K^+)^{41}_{\Lambda}\text{Ca}$ reaction for the incident proton energy of 2.0 GeV for various bound states of final hypernucleus as indicated in the figure. The Λ separation energies for $0d_{3/2}$, $0d_{5/2}$, $0p_{3/2}$, $0p_{1/2}$, $1s_{1/2}$, and $0s_{1/2}$ states were taken to be 0.7529 MeV, 1.5435 MeV, 9.6768 MeV, 9.1400 MeV, 17.8820 MeV, and 1.1081 MeV, respectively. The quantum number and the binding energy of the two intermediate nucleon states were $0d_{3/2}$ and 8.3282 MeV, respectively.

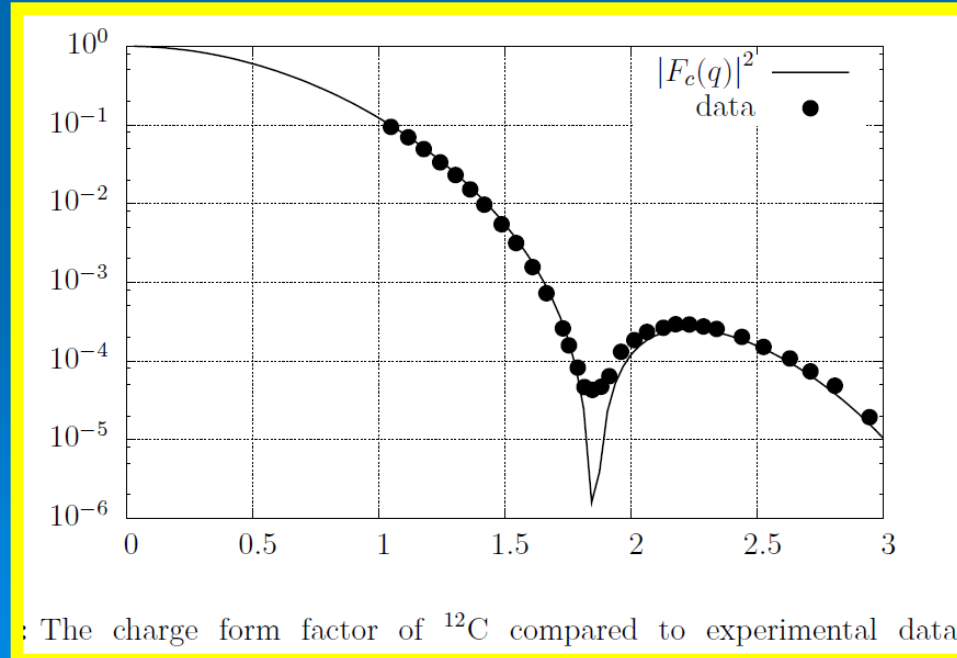
...dependence on Pion-short range Correlations



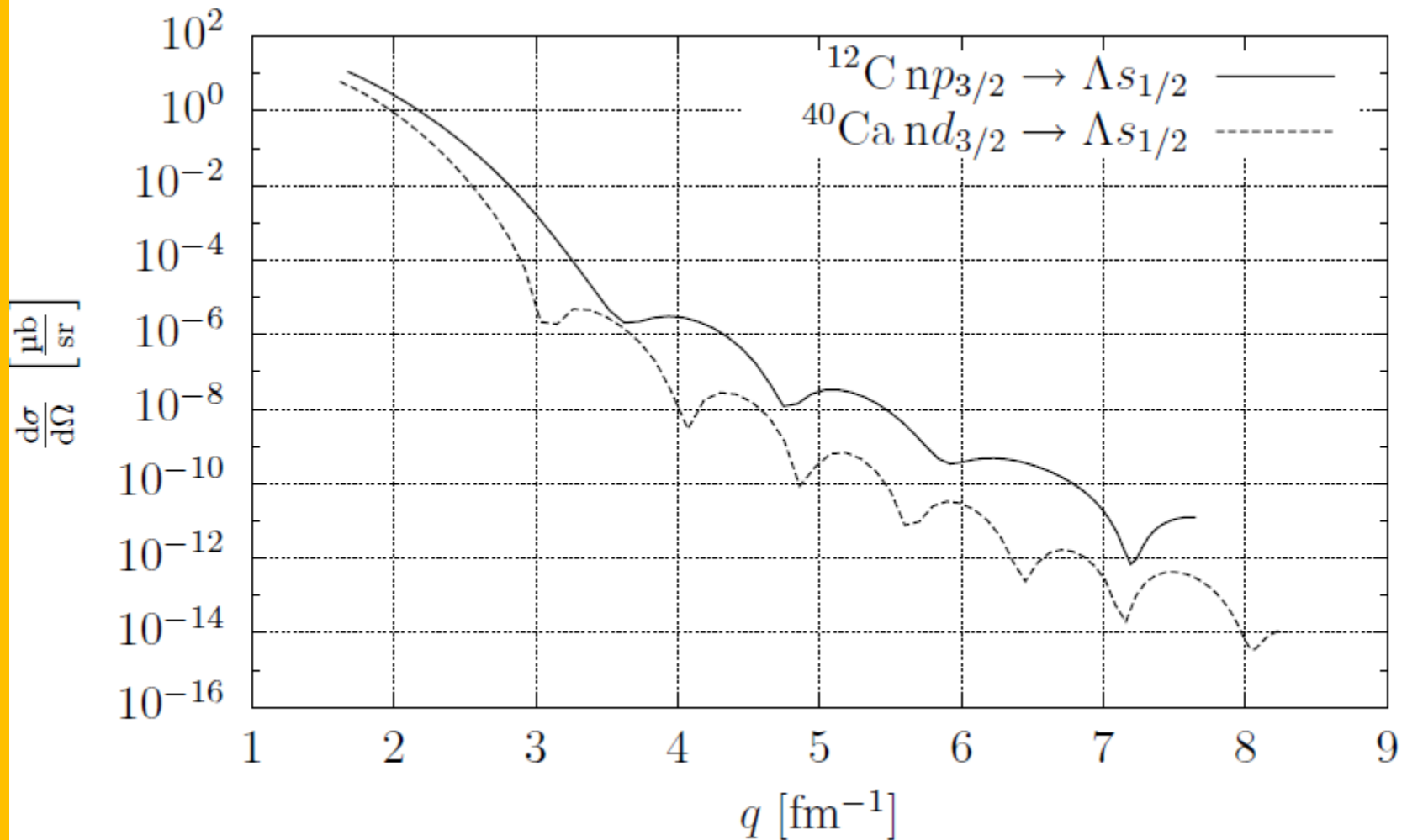
What is controlling the size of the production cross sections?

$$\mathcal{M} = \int \frac{d^4 k_N}{(2\pi)^4} \int \frac{d^4 k_\Lambda}{(2\pi)^4} \int \frac{d^4 p}{(2\pi)^4} \hat{\phi}_K^*(p-k_\Lambda) \bar{\psi}_\Lambda(k_\Lambda) \Gamma_1 i \frac{\gamma \cdot p + m_R}{p^2 - m_R^2 + i\epsilon} \Gamma_2 \hat{\phi}_\pi(p-k_N) \psi_N(k_N)$$

...it's the Nuclear Form Factor!!



Form Factor Effects in $(\pi, K)@1\text{GeV}$



VII. Production of Hypernuclei in Relativistic Fragmentation Reactions



Sketch of covariant Transport Theory (GiBUU)...

$$\mathcal{L} = \bar{\psi}[\gamma(i\partial - g_{\omega}\omega) - m_{\text{nuc}} - g_{\sigma}\sigma]\psi + \frac{1}{2}(\partial\sigma)^2 - U(\sigma) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega^2$$

$$U(\sigma) = \frac{1}{2}m_{\sigma}^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4.$$

The one-body density matrix :

$$\rho(x_1, x_2) \sim \Psi^+(x_1)\Psi(x_2) \rightarrow f(x, \vec{p}) = \int \frac{d^3s}{(2\pi)^3} e^{i\vec{p}\vec{s}} \rho(\vec{x} + \vec{s}/2, \vec{x} - \vec{s}/2, t)$$

Equation of Motion for the Wigner distribution $f(\mathbf{x}, \mathbf{p})$
(Kadanoff and Baym)

Sketch of covariant Transport Theory...

$f(x, \mathbf{p}) \frac{g d^3 r d^3 p}{(2\pi)^3} =$ (number of particles of that species in the phase space element $d^3 r d^3 p$), where g is the spin-isospin degeneracy.

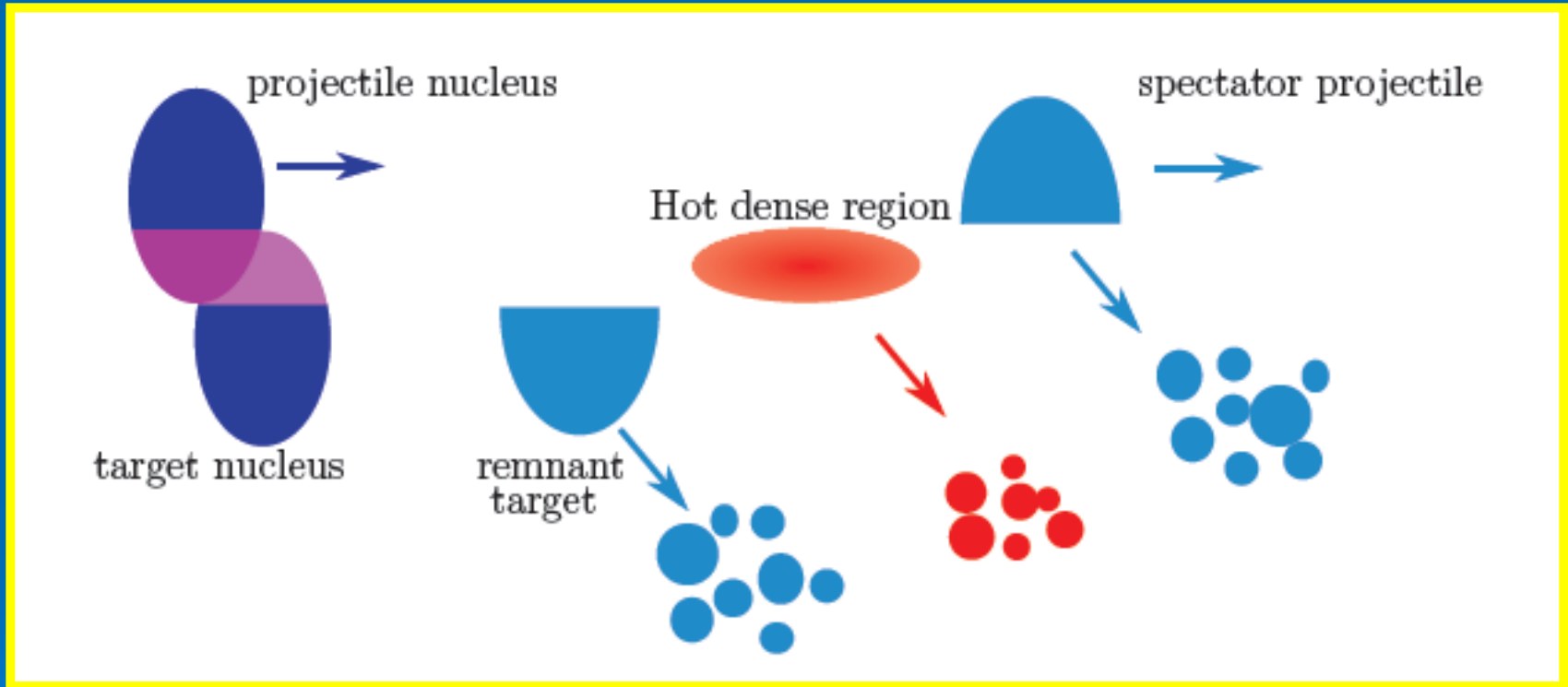
$$\begin{aligned} & [k^{*\mu} \partial_\mu^x + (k_\nu^* F^{\mu\nu} + M^* \partial_x^\mu M^*) \partial_\mu^{k^*}] f(x, k^*) \\ &= \frac{1}{2(2\pi)^9} \int \frac{d^3 k_2}{E_{\mathbf{k}_2}^*} \frac{d^3 k_3}{E_{\mathbf{k}_3}^*} \frac{d^3 k_4}{E_{\mathbf{k}_4}^*} W(k k_2 | k_3 k_4) [f_3 f_4 \bar{f} \bar{f}_2 - f f_2 \bar{f}_3 \bar{f}_4], \end{aligned}$$

$$M^* = M - \Sigma_s$$

$$p_\mu^* = p_\mu - \Sigma_\mu$$

$$F^{\mu\nu} = \partial^\mu \Sigma^\nu - \partial^\nu \Sigma^\mu$$

Scenario of a fragmentation reaction ($T_{\text{lab}} > 2A\text{GeV}$)



The Fragmentation scenario in practice...

- Initial stage of the Collision: Transport Theory
- strangeness production by resonance excitation and direct production
- Fragment formation by grand canonical statistics
- SMM: Statistical Multi-fragmentation Model
- SMM for the formation of the pure nucleonic fragments: formation of core nuclei
- hypernuclei by capture (coalescence) of hyperons from the fireball

The SMM: grand canonical approach to fragmentation

A.S. Botvina, J. Pochodzalla, Phys. Rev. C 76 (2007) 024909

$$Y_{AZH} = g_{AZH} V_f \frac{A^{3/2}}{\lambda_T^3} \exp \left[-\frac{1}{T} (F_{AZH} - \mu_{AZH}) \right],$$
$$\mu_{AZH} = A\mu + Z\nu + H\xi,$$

$$\lambda_T = (2\pi\hbar^2/m_N T)^{1/2}$$

$$F_{AZH}(T, V) = F_A^B + F_A^S + F_{AZH}^{\text{sym}} + F_{AZ}^C + F_{AH}^{\text{hyp}}$$

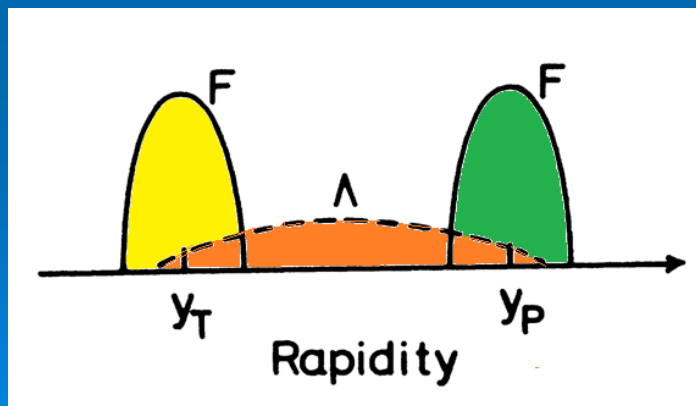
$$F_A^B(T) = (-w_0 - \frac{T^2}{\varepsilon_0})A, \quad F_A^S(T) = \beta_0 \left(\frac{T_c^2 - T^2}{T_c^2 + T^2} \right)^{5/4} A^{2/3}$$

$$F_{AZH}^{\text{sym}} = \gamma \cdot (A - H - 2Z)^2 / (A - H)$$

$$F_{AH}^{\text{hyp}} = (H/A) \cdot (-10.68A + 21.27A^{2/3})$$

Formation of a Hypernucleus by capturing a Λ to a pre-formed fragment F:

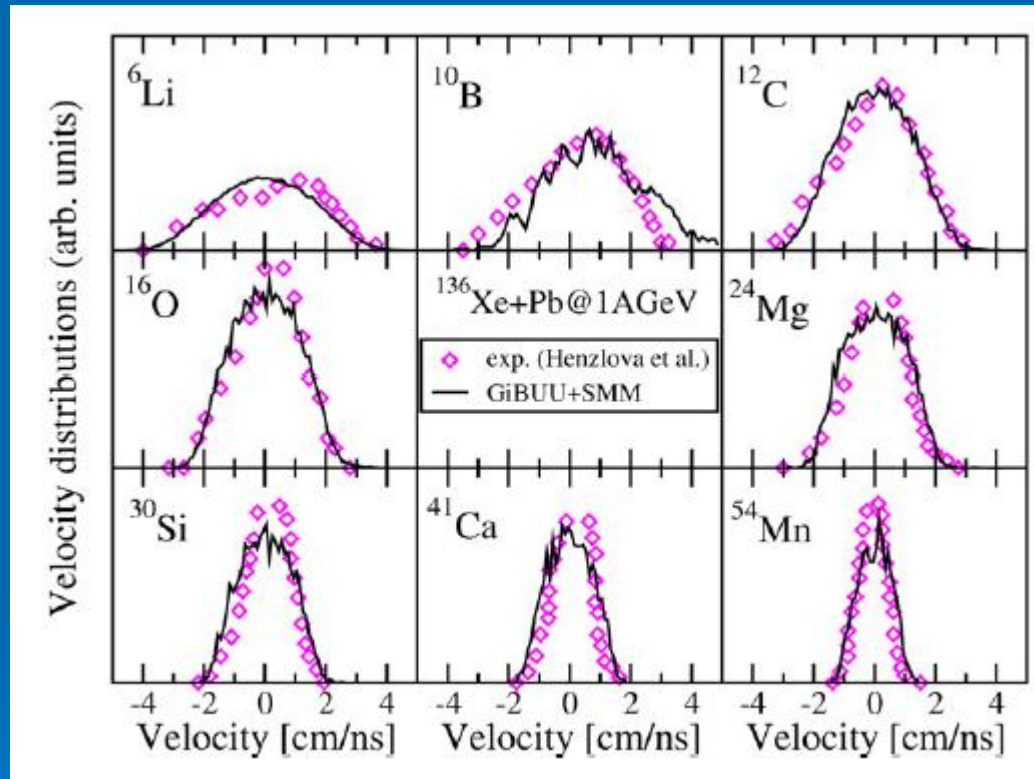
$$\frac{\gamma}{\sigma_r} \frac{d^3\sigma^{(\Lambda F)}}{dk_c^2} = \left[\frac{m_\Lambda + m_F}{m_\Lambda m_F} \right]^3 \boxed{S_{\Lambda F}} \left[\frac{\gamma}{\sigma_r} \frac{d^3\sigma^{(\Lambda)}}{dk_c^3} \right] \left[\frac{\gamma}{\sigma_r} \frac{d^3\sigma^{(F)}}{dk_c^3} \right]$$



Λ
Production
X-section

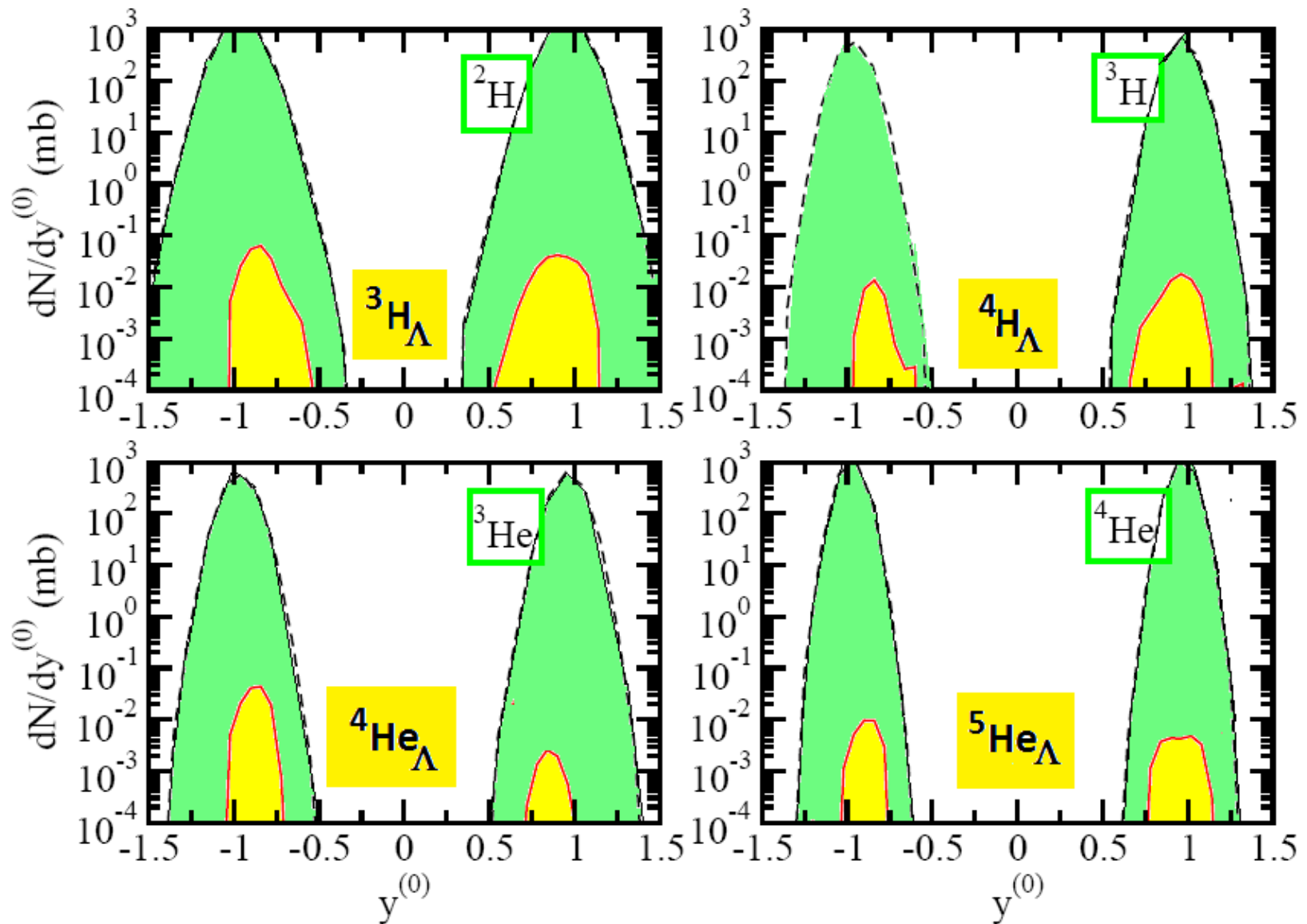
Fragment
Production
X-section

Production of Light Nuclei by GiBUU+SMM (FOPI data)



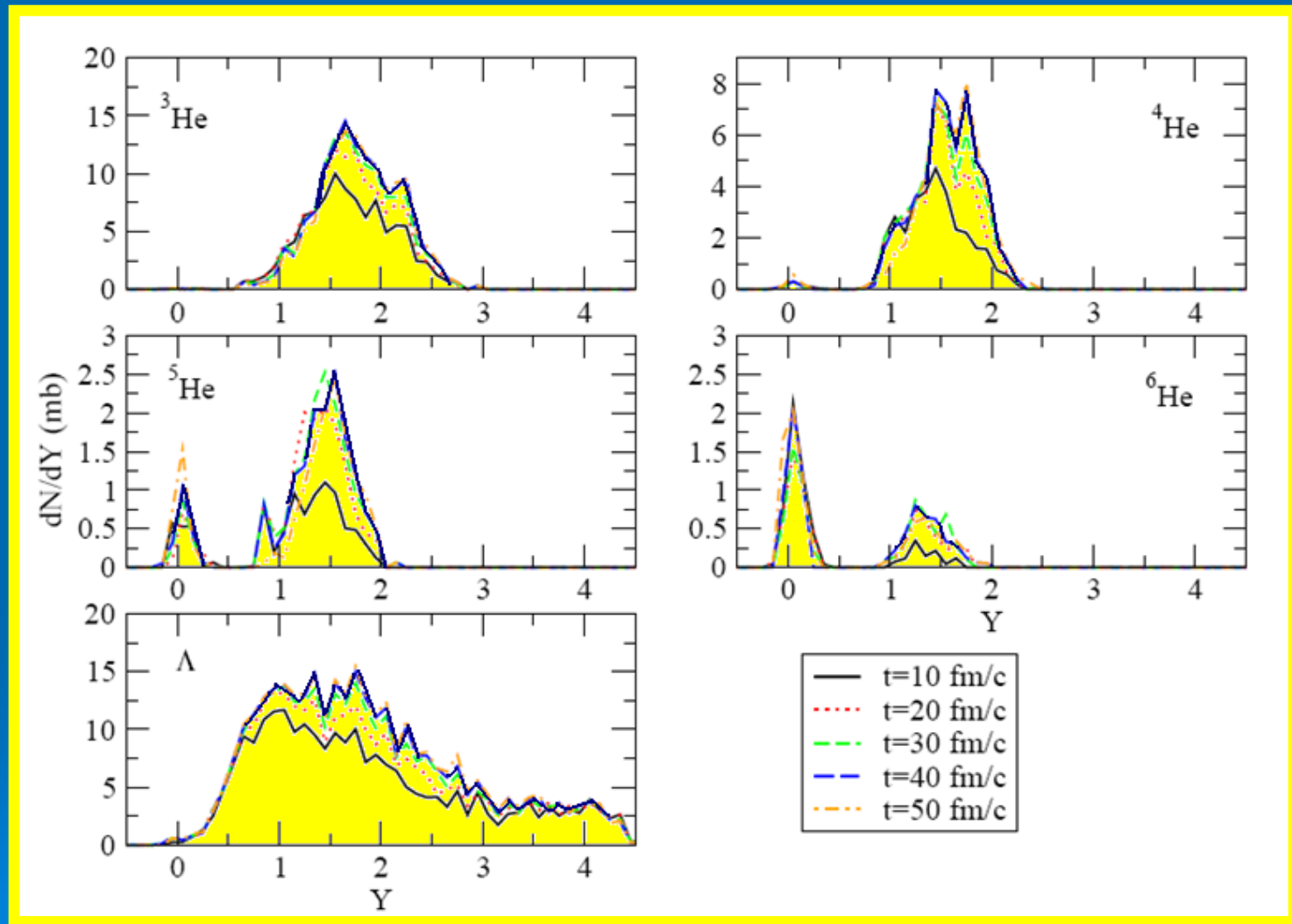
Longitudinal velocity distributions in the projectile frame

Production of Hypernuclei in $^{12}\text{C}+^{12}\text{C}@2\text{AGeV}$



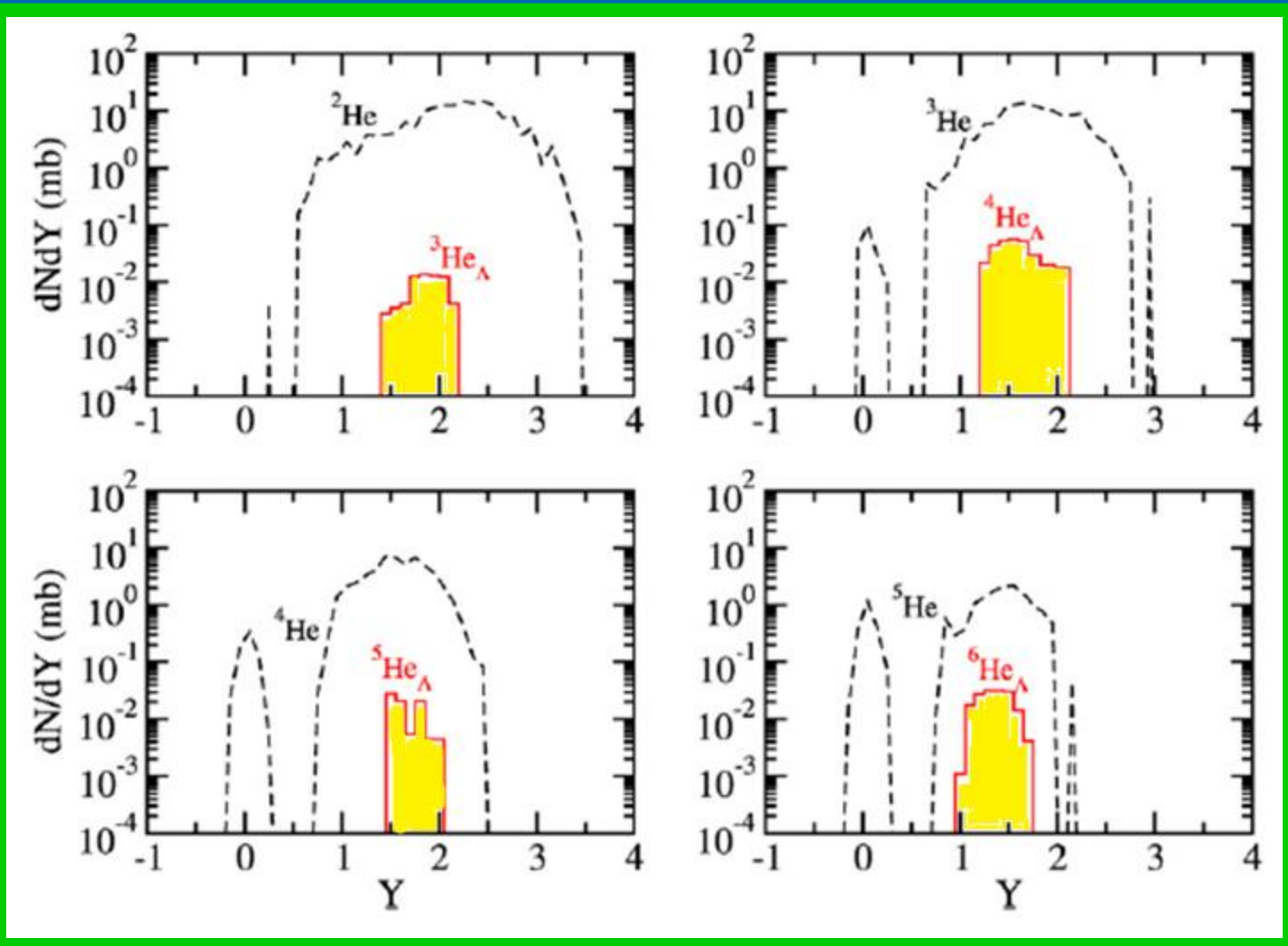
(Th. Gaitanos, HL, U. Mosel, Phys. Lett. B
675, 297 (2009))

Production of Nuclei and Lambda's in $p+^{12}\text{C}@50\text{GeV}$ (J-PARC)



(Th. Gaitanos, HL, U. Mosel, Phys. Lett. B
675, 297 (2009))

Hypernuclei from $p+^{12}\text{C}@50\text{GeV}$ (J-PARC)



Production of Hypernuclei in HI-Collisions: where do the hyperons come from?

- direct (resonance) production: $N+N \rightarrow N+N^* \rightarrow N+Y+K$
- indirect (mesonic) production : $\pi+N \rightarrow N^* \rightarrow Y+K$

	${}^4_{\Lambda}H$	${}^4_{\Lambda}He$	${}^5_{\Lambda}He$
Total yield (μb)	2.2	4	1.4
Pionic contribution (μb)	0.3	0.2	0.03

${}^{12}C+{}^{12}C@$
2AGEV

	$\sigma_{tot} [\mu b]$	$\sigma_{pionic} [\mu b]$		$\sigma_{tot} [\mu b]$	$\sigma_{pionic} [\mu b]$
${}^3_{\Lambda}H$	43	33	${}^3_{\Lambda}He$	0.9	0.8
${}^4_{\Lambda}H$	28	14	${}^4_{\Lambda}He$	35	16
${}^5_{\Lambda}H$	9.5	2.6	${}^5_{\Lambda}He$	9.8	4.3
${}^6_{\Lambda}H$	0.8	0.2	${}^6_{\Lambda}He$	16	1.3

$p+{}^{12}C@$
50GEV

(Th. Gaitanos, HL, U. Mosel, Phys. Lett. B 675, 297 (2009))

Summary:

- Aspects of YN Interactions
- Structure and Spectroscopy of Hypernuclei
- Strangeness Production on the Nucleon
- Hadro-Production of Hypernuclei
- Photo- and Electroproduction of Hypernuclei
- Production of Hypernuclei in Relativistic Fragmentation Reactions → HypHI
- Production by Antiprotons → PANDA@FAIR

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