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Generalized mass formula for non-strange, strange and multi-strange nuclei

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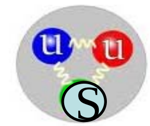
Outline of the Talk

- Introduction
- Generalised Mass formula for Non-strange, Strange and Multi-strange nuclei
- Comparison with relativistic-mean-field (RMF) calculations
- Hyperonic effects on neutron and proton driplines
- Limits of hypernuclei (or, hyper-drip points of normal nuclei of all N, Z)
- Pure hyperonic and exotic nuclear systems
- Summary

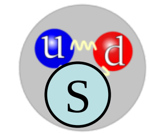
Hyperons: Baryons with Strangeness

❖ **There are three Sigma hyperons, Σ^+, Σ^0 and Σ^-.**

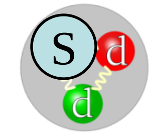
- **m(Σ^+)** =1189.37 +/- 0.07
- **m(Σ^0)** = 1192.642 +/- 0.024
- **m(Σ^-)** = 1197.449 +/- 0.030,
- lifetimes of ~1×10^{-10} s



Σ^+(uus)



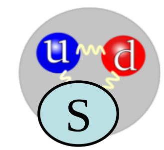
Σ^0=(uds)



Σ^-=(dds)

with the exception of Σ^0 whose lifetime is shorter than 1×10^{-19} s.

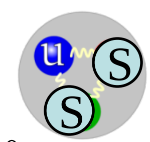
❖ **There is one Lambda hyperon, Λ^0. (Charge = 0, like neutron)**



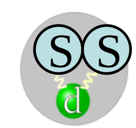
Λ^0=(uds)

- **m(Λ^0)** = 1115.683 +/- 0.006 MeV
- lifetime of ~2.6×10^{-10} s.

❖ **There are two Xi or, Cascade hyperons, Ξ^0 and Ξ^-.**



Ξ^0=(uss)

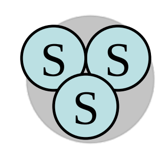


Ξ^-=(dss)

- **m(Ξ^-)** = 1321.71 +/- 0.07
- **m(Ξ^0)** = 1314.86 +/- 0.2
- lifetimes of ~ 2.9×10^{-10} s and ~ 1.6×10^{-10} s.

❖ **There is one Omega hyperon, the last discovered, Ω^-,**

- **m(Ω^-)** = 1672.45 +/- 0.29
- lifetime of ~ 8.2×10^{-11} s.



Ω^- =(sss)

Hypernucleus: A Strange Matter

Normal nucleus: consists of nucleon (neutron, proton)

Hypernucleus: consists of nucleon (n, p) + hyperon (Y)

Hyperon acts like a glue & makes a nucleus more bound.

- ^{10}Li is known to be **unbound**, but
- ^{10}**Li**$_{\Lambda}$ is **bound** *(PRL94,052502'05)*

$A = N_n + N_p + N_Y$

N_n = neutron no.

N_p = proton no.

N_Y = hyperon no.

Net Charge = Z

$= Z_p + (N_Y \,.\, q_Y)$

q_Y = Charge of the hyperon

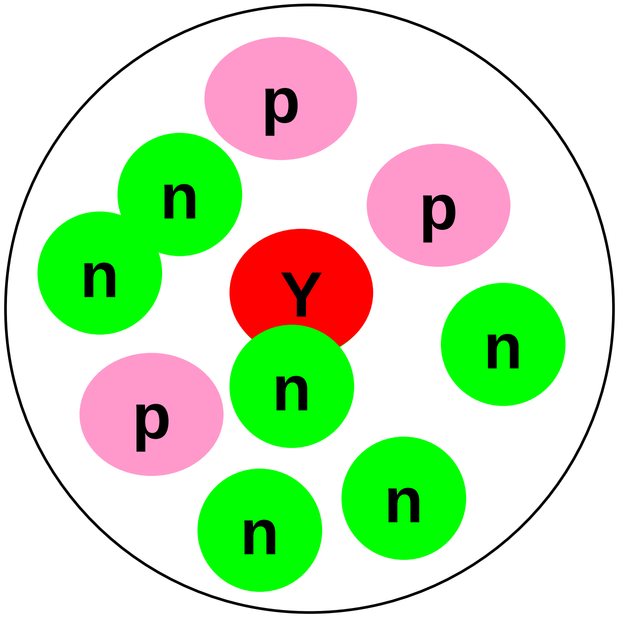
Notation: $_{Y}{}^{A}Z$

$p + p + n = {}^3\mathrm{He}$

$p + p + \Xi^- = {}_{\Xi^-}{}^3\mathrm{H}$

$p + p + \Sigma^+ = {}_{\Sigma^+}{}^3\mathrm{Li}$

Net charge Z denotes the name of the nucleus



Z ≠ no. of protons (Z_p)!

- Periodic Table arranges the elements according to their proton number.
- A Hypernucleus can have same proton number, but the different element name.

How many Strange Hypernuclei are discovered so far?

Hypernuclei with:

Λ^0 (S= -1) ~ Fifty

$\Lambda\Lambda$–hypernuclei (Three)

Σ^+ (S= -1) One

Ξ^- (S= -2) Five

And, one anti-hyper-triton with anti-Lambda.

Can we theoretically suggest mass/binding energy of those Strange-nuclei which have NOT been detected so far?

- At present, some relativistic-mean-field (RMF) calculations have provided results for a limited number of medium heavy and heavy nuclei.

- A properly constructed mass formula can
  - provide a quick check on the RMF calculations
  - extrapolate to a wider mass region from light to heavy.

Liquid Drop Mass Formula for Binding Energy

Bethe-Weizsäcker mass formula (no shell effect):

$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(N-Z)^2}{A} + \delta$$

where $\mathbf{a_v}$=15.777 MeV, $\mathbf{a_s}$ =18.34 MeV**,** $\mathbf{a_c}$=0.71 MeV**,** $\mathbf{a_{sym}}$=23.21 MeV,

and the pairing term δ **= 12 A$^{-1/2}$** for even N and even Z
= -12 A$^{-1/2}$ for odd N and odd Z
= 0 for odd A

It was later extended for light mass nuclei as well as for nuclei away from the valley of stability in which two correction terms were introduced. The parameters were fixed by fitting the available mass data. BE is given by:

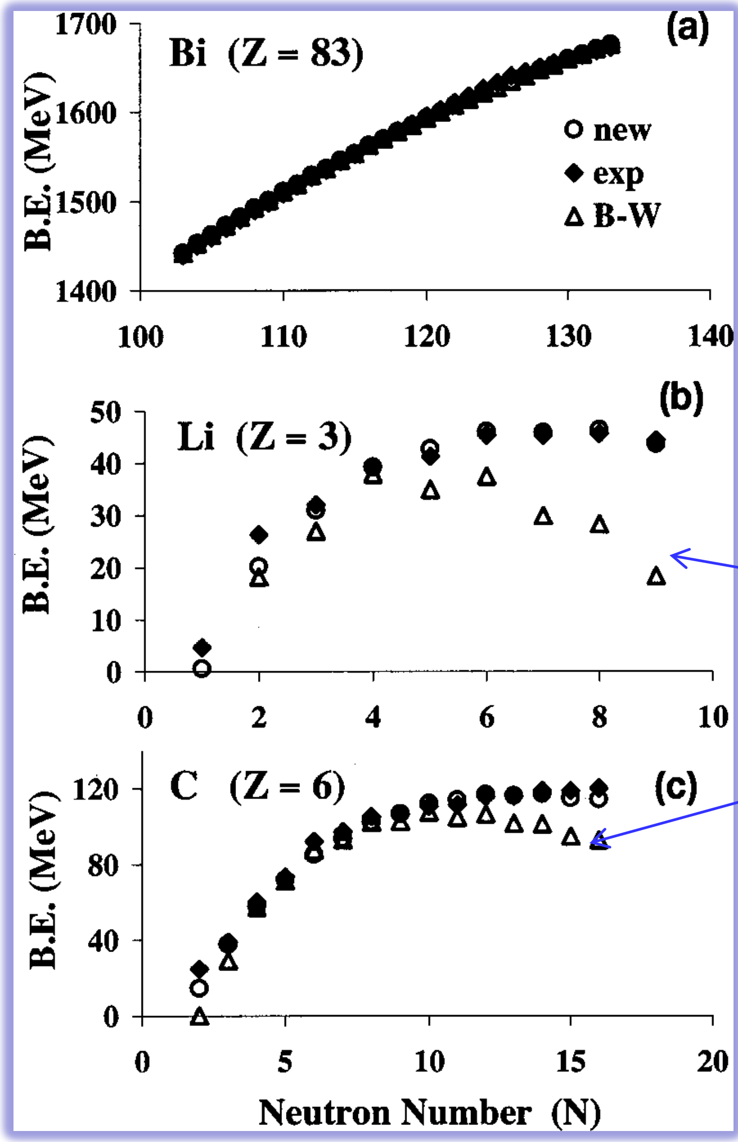
$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(N-Z)^2}{A\left(1+\exp^{-A/17}\right)} + \delta_{new}$$

$$\delta_{new} = \delta\left(1-\exp^{-A/30}\right)$$

C. Samanta & S. Adhikari, PRC 65(2002) 037301

Binding Energies of Non-Strange Lithium Isotopes

C. Samanta & S. Adhikari, PRC 65(2002) 037301



B-W Formula:

$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(N-Z)^2}{A} + \delta$$

Modified B-W Formula (new):

$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(N-Z)^2}{A\left(1+\exp^{-A/17}\right)} + \delta_{new}$$

$$\delta_{new} = \delta\left(1-\exp^{-A/30}\right)$$

Correction terms

Poor agreement of the BW mass formula with the experimental data without the correction terms.

Generalized Mass Formula For Non-Strange & Strange nuclei

C. Samanta et al., JPG32 (2006) 363

A systematic search using experimental separation energy (S_Y) for Λ^o, $\Lambda\Lambda$, Σ^+ and Ξ^- -hypernuclei leads to a generalized mass formula which is valid for normal nuclei (S=0) as well as Hypernuclei (S≠0).

$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(N-Z_c)^2}{A\left(1+\exp^{-A/17}\right)} + \delta_{new} - 48.7 \frac{|S|}{A^{2/3}} + n_Y \left[0.0035 m_Y - 26.7\right]$$

n_Y = no. of hyperons in a nucleus
m_Y = mass of hyperon in MeV
S = strangeness no.of the hyperon,
A = N + Z_c+ n_Y = total no. of baryons
Z_c = no. of protons,
Z = Z_c + n_Y q = net charge no.
q = charge no. of Hyperon with proper sign. (viz., q= -1, 0, 1)

Explicit dependence on Hyperon Mass →Breaks mass symmetry

Hyperon	S	n_Y
Λ^0	-1	1
$\Lambda\Lambda$	-2	2
$\Sigma^{+,-,0}$	-1	1
Ξ^-	-2	1
Θ^+	+1	1
Normal	0	0

The hyperon separation energy S_Y defined as:

$S_Y = BE(A, Z)_{hyper} - BE(A - n_Y, Z_c)_{core}$

Multiply Strange Nuclear Systems

J. Schaffner J, C.B. Dover, A. Gal , C. Greiner, D.J. Millener and H. Stöcker, Ann. Phys. NY 235 (1994) 35

- Multiply strange: System made out of nucleons and different hyperons.

- Stability of (N, Λ, Ξ) was investigated in relativistic mean field (RMF) approach.

- Possibility of production of such systems in heavy-ion collision was predicted

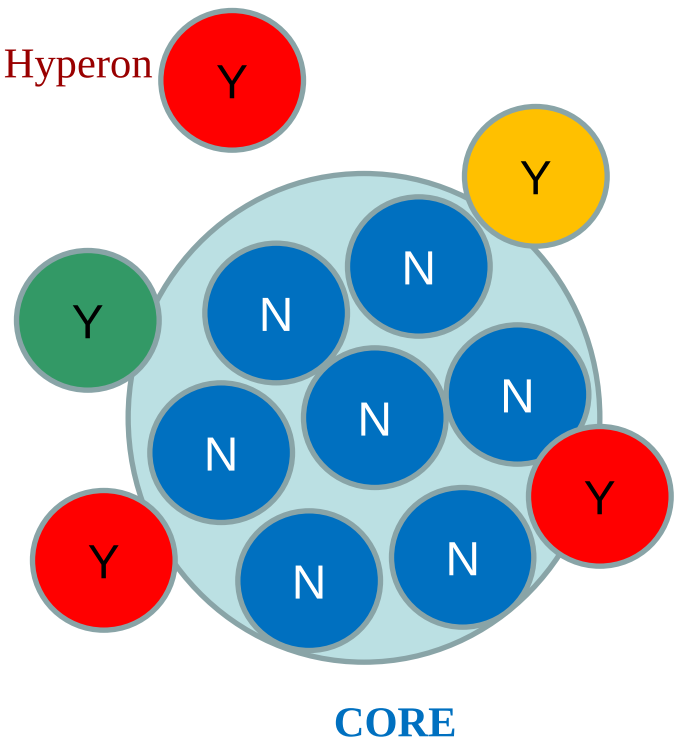
Mass formula for Multi-strange Nuclear Systems

C. Samanta, Jour. Phys. G: Nucl. Part. Phys. 37 (2010)075104

We consider: **Hypernucleus** = **Core** (normal nucleus with nucleons, **N**) + **Hyperons(Y)**

A hypernucleus may have one kind of hyperon(s) or a mixture of different kind of hyperons

$\mathbf{A = n + z_c + \sum n_Y}$: **n**= no. of neutrons, $\mathbf{z_c}$ = no. of protons, $\mathbf{n_Y}$ = no. of hyperons of each kind



The Hyperon-separation energy

$$S_Y = BE(A, Z)_{hyper} - BE(A - n_Y, z_c)_{core}$$

Hyperon-separation energy for multi-strange hypernuclei

$$S_Y = B(A, Z) - B(A - \sum_Y n_Y, Z)$$

For a nucleus to be bound:

B.E. = $m_A - z_c.m_p - n.m_n - n_Y.m_Y$ = Negative

S_Y = Positive

Application of Mass Formulae for Multiply Strange Nuclei

Shmuel BALBERG,* Avraham GAL*,** and Jürgen SCHAFFNER

The generalized BW (GBW) mass formula for SHM is constructed[9] in analogy to the ordinary nuclear BW formula, Eq. (8). One assumes that SHM saturates for roughly equal densities of the various species and that the Fermi momentum of the underlying strange baryonic Fermi gas is about the same as for ordinary nuclear matter. Whereas a single Coulomb term and a single surface term are retained in the GBW formula, there are now several volume and symmetry terms, e.g.

$$E_B(\{p, n\}) = -a_V^{(0)}A + a_S^{(0)}A^{2/3} + a_C^{(0)}\frac{Z^2}{A^{1/3}} + a_x^{(0)}\frac{(N-Z)^2}{A} .$$

$$a_V^{(0)} \to a_V - b_V^{(w)}w - b_V^{(y)}y , \tag{10a}$$

$$a_x^{(0)}x^2 \to a_x x^2 + a_u u^2 + a_w w^2 + a_y y^2 + a_{wy}wy , \tag{10b}$$

where

$$x = (N-Z)/A , \quad u = (\Xi^0 - \Xi^-)/A , \quad w = \left(\frac{N+Z}{2} - \frac{\Xi^0 + \Xi^-}{2}\right)/A ,$$

$$y = [(N+Z+\Xi^0+\Xi^-)/4 - \Lambda]/A . \tag{11}$$

Table I. Parameter sets for use in the GBW formula.[a)]

	a_V	$b_V^{(w)}$	$b_V^{(y)}$	a_x	a_u	a_w	a_y	a_{wy}
set I	10.7	−35.5	−16.75	43	23.7	57.1	45	7.7
set II	28.7	− 5.5	− 4.75	43	23.7	57.1	45	7.7

Generalized Mass Formula for Non-strange, Strange and Multiply-strange Nuclear Systems

C. Samanta, JPG 37 (2010) 075104

A = n + z_c + n_Y : **n** = no. of neutrons, **z_c** = no. of protons, **n_Y** = no. of hyperons

Binding Energy: B(A,Z) = $m_A - z_c.m_p - n.m_n - n_Y.m_Y$ = Negative

$$-B(A,Z) = 15.777A - 18.34A^{2/3} - 0.71Z(Z-1)/A^{1/3} - 23.21(n-z_c)^2/[(1+e^{-A/17})A] + (1-e^{-A/30})\delta$$
$$+\sum_Y n_Y[0.0335(m_Y) - 26.7 - 48.7\,|\,S\,|\,/A^{2/3}$$
$$-a_Y\{(n_\Lambda + n_{\Xi^o} + n_{\Xi^-} - z_c)^2 + (n_\Lambda + n_{\Xi^o} + n_{\Xi^-} - n)^2\}/\{(1+e^{-A/17})A\}].$$

m_Y = mass of hyperon in MeV

S = strangeness no. of the hyperon

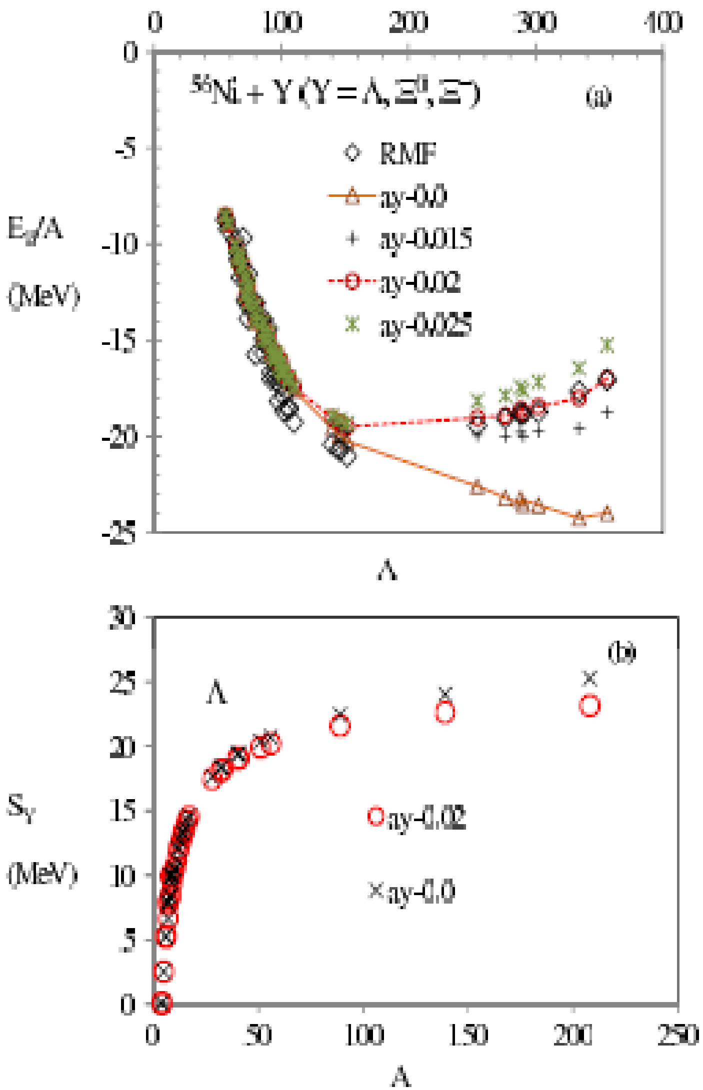
$$Z = \left| z_c + \sum_Y n_Y q_Y \right|$$

Z = Net Charge
= Proton charges + Total hyperon charges

q = -1, 0, 1 depending on the hyperon type

Note: the net charge of a nucleus can be negative if the hyperon number is larger than the proton number and the hyperon has a negative charge!!!!

Multi-Strange Nuclei: Model-2 with Y-Y interaction



C. Samanta, JPG 37 (2010) 075104

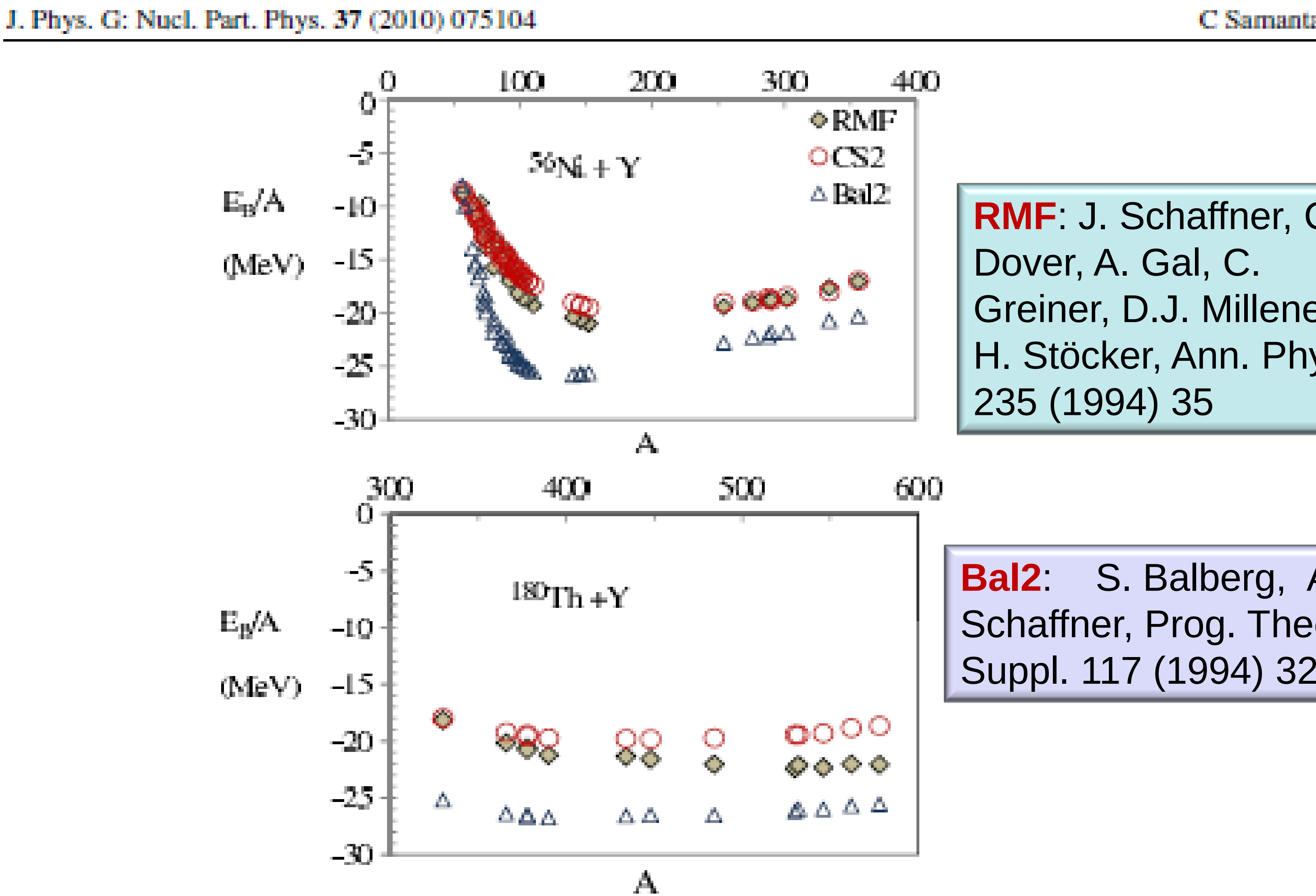
Effect of the new hyperon-asymmetry term: choice of a_y value

(a) E_B/A vs. A plots for stable multi-strange systems in relativistic mean field (RMF) calculations based on ^{56}Ni nuclear cores for model 2 (wih YY interaction) of Schaffner et al.(Ref.1).

(b) Single lambda-hyperon separation energy S_Y vs. A for different elements. With a_Y=0.0 and 0.2.

Ref.1: J. Schaffner J, C.B. Dover, A. Gal , C. Greiner, D.J. Millener and H. Stöcker, Ann. Phys. NY 235 (1994) 35

Stable Multiply-strange systems in RMF (Model2, with Y-Y interaction) calculations of Schaffner et al. with ^{56}Ni and ^{180}Th core, Set-II of Balberg et al. and this work (CS2).

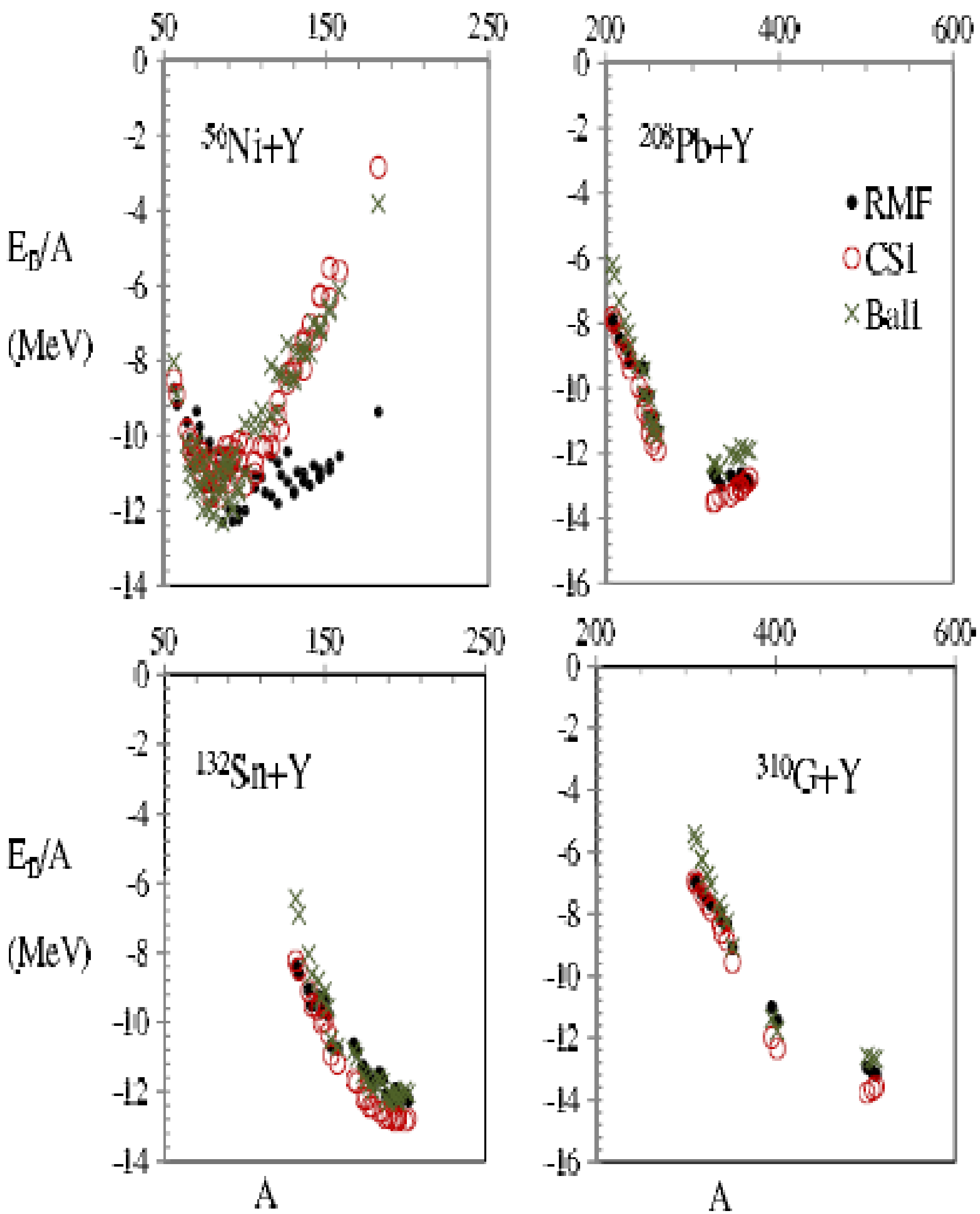


RMF: J. Schaffner, C.B. Dover, A. Gal, C. Greiner, D.J. Millener and H. Stöcker, Ann. Phys. NY 235 (1994) 35

Bal2: S. Balberg, A.Gal, J. Schaffner, Prog. Theo. Phys. Suppl. 117 (1994) 325

Multi-Strange Nuclei: Model-1 with NO Y-Y interaction

C.Samanta, JPG 37 (2010) 075104



(Current wisdom: Y-Y interaction is weak.)

Model-1=No Y-Y interaction
Model-2=Strong Y-Y interaction

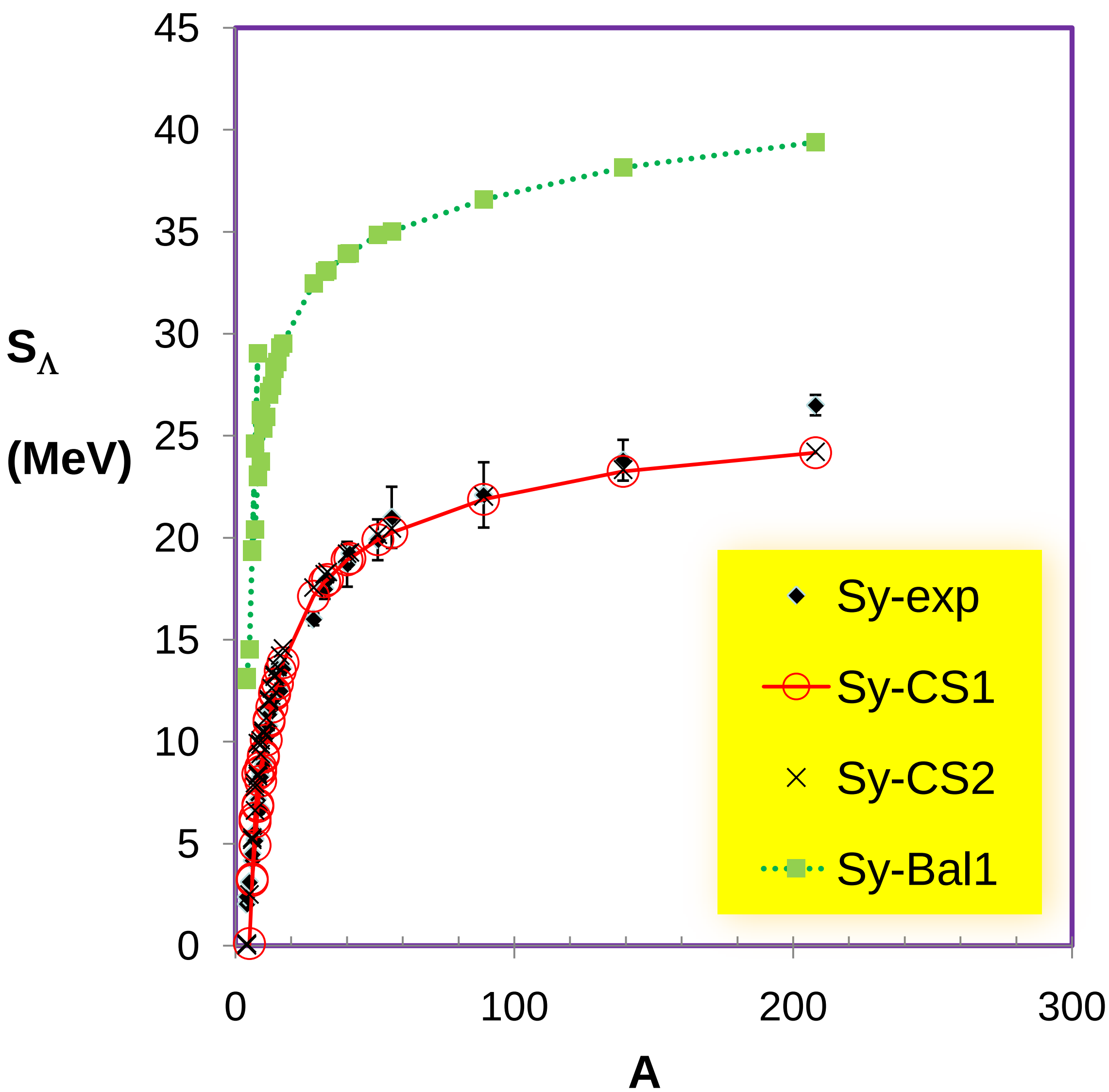
CS1(BE,Model1)=CS2(BE,Model2) -Cr

$$C_r = 12.0 A f_s (f_\Lambda + f_{\Xi^0} + f_{\Xi^-})$$
$$f_s = \sum_Y n_Y |S| / A.$$

Stable-multiply-strange systems in RMF calculations, based on ^{56}Ni, ^{132}Sn, ^{208}Pb and ^{310}G(Zc=126, n=184) core, by:

1. Model-1 of Schaffner et al.,
2. SET-I of Balberg et al.,
3. this work CS1.

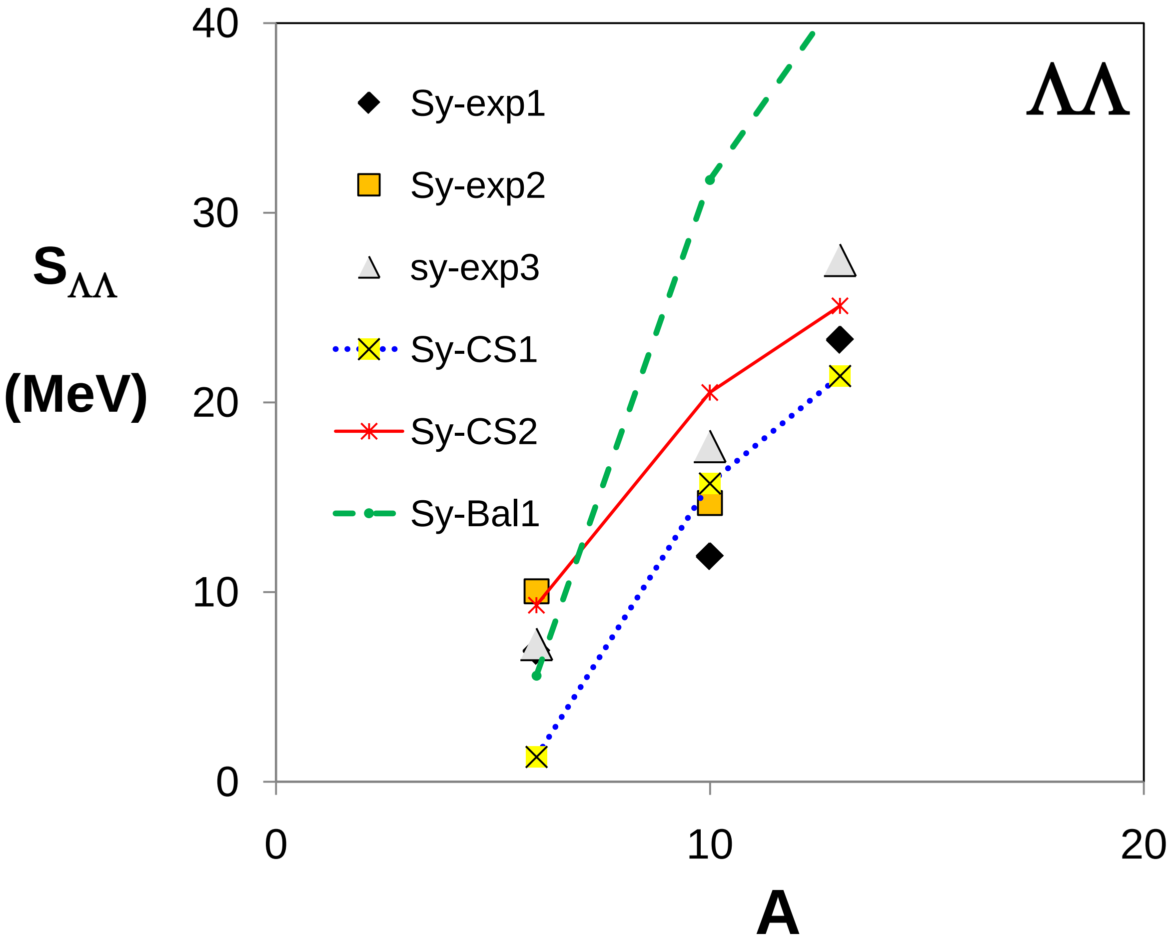
Λ-Separation Energies of Lambda Hypernuclei



Set-I (as well as Set-II) of Balberg et al. over predicts the experimental data.

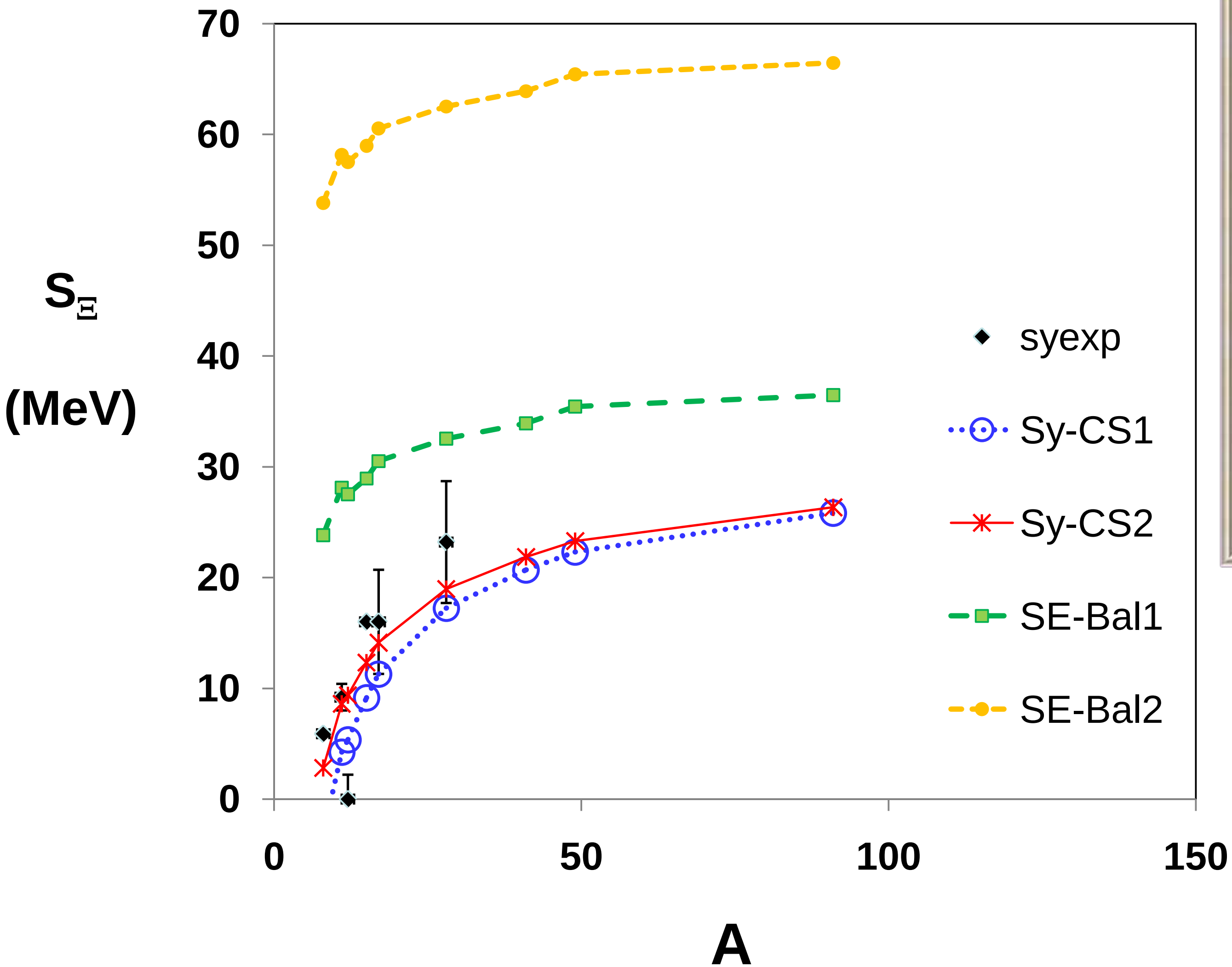
Sy-CS1 and Sy-CS2 give comparable results, except for very light elements where Sy-CS1 is lower..

ΛΛ-Separation Energies of Lambda-Lambda Hypernuclei



- Sy-CS2 (Model2) gives a reasonably good fit to the experimental data
- Sy-CS1 (Model1) differs more with Sy-CS2 at lower A values.
- Sy-Bal1 (Balberg et al, Set-I) predicts much larger values..
- Sy-Bal2, not shown here, gives even higher values of $S_{\Lambda\Lambda}$.

Ξ- Separation Energies from Cascade-Hypernuclei



Old emulsion data

(Ref: Bando et al.

IJMPA 5 (1990)4021).

Need more precise

experimental data for

Cascade hypernuclei.

Experimental data on Ξ hypernuclei are tabulated in: J. Phys. G: Nucl. Part. Phys. 32 (2006) 363

What do we learn from the Separation Energy Versus neutron number plot?

Lambda particle makes a nucleus more bound.

For example, ^{10}Li is known to be unbound, but **$^{10}{}_{\Lambda}Li$ is bound.**

Expt: $S_{\Lambda}(^{10}{}_{\Lambda}\mathbf{Li})$ ~10-12 MeV [P.K. Saha,PRL94(2005) 052502]

Th: CS1(10.2 MeV), CS2(11.4 MeV).

Does this mean that addition of Λ can make very neutron-rich hypernuclei – far beyond the normal drip line?

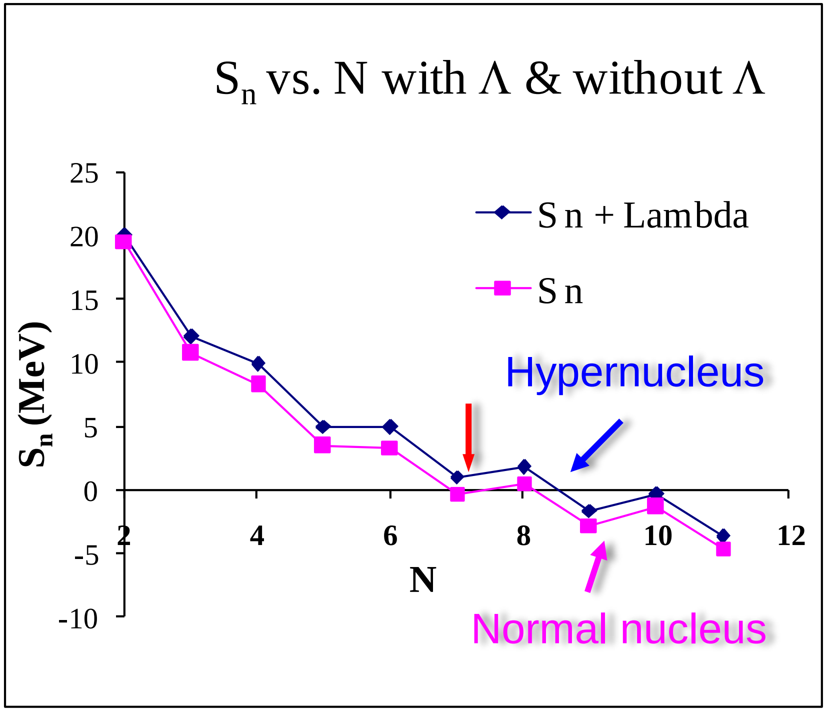
NO!

For $^{10}Li_{\Lambda}$: S_n ~ 1.06 MeV.

As neutrons are added one by one, S_n decreases in both hyper and normal nuclei, thus reaching the n-drip line at N=8.

Beyond N=8, the neutron-separation energy is negative, although S_{Λ} is positive.

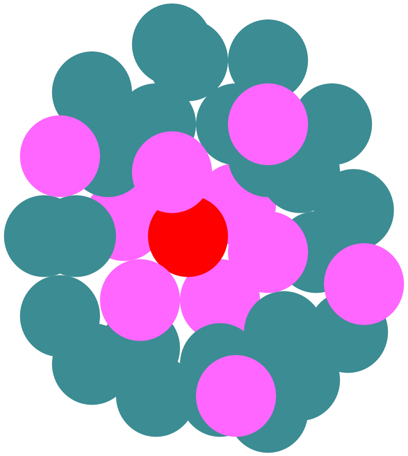
^{11}Li and $^{12}{}_{\Lambda}Li$ are dripline nuclei.



Nucleus $^{13}{}_{\Lambda}Li$ (Z=3, N=9, Λ=1) , if found, will be truly exotic & beyond n-dripline.

Lambda Hyperonic Effect on the Normal Drip lines

J. Phys. G: Nucl. Part. Phys. 35 (2008) 065101



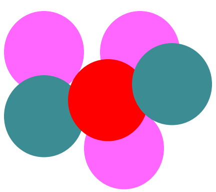
^{32}Ne

$^{35}_{\Lambda}$Ne

n-drip nucleus:
Ne (Z=10) N = 22(Normal)

Λ=1, N = 24 (Hyper)

Ca (Z=20) N = 44 (Normal)
N = 46 (Hyper)



^{5}Li

$^{5}_{\Lambda}$Li

p-drip nucleus:
Li (Z=3) N = 2 (Normal)

N = 1 (Hyper)

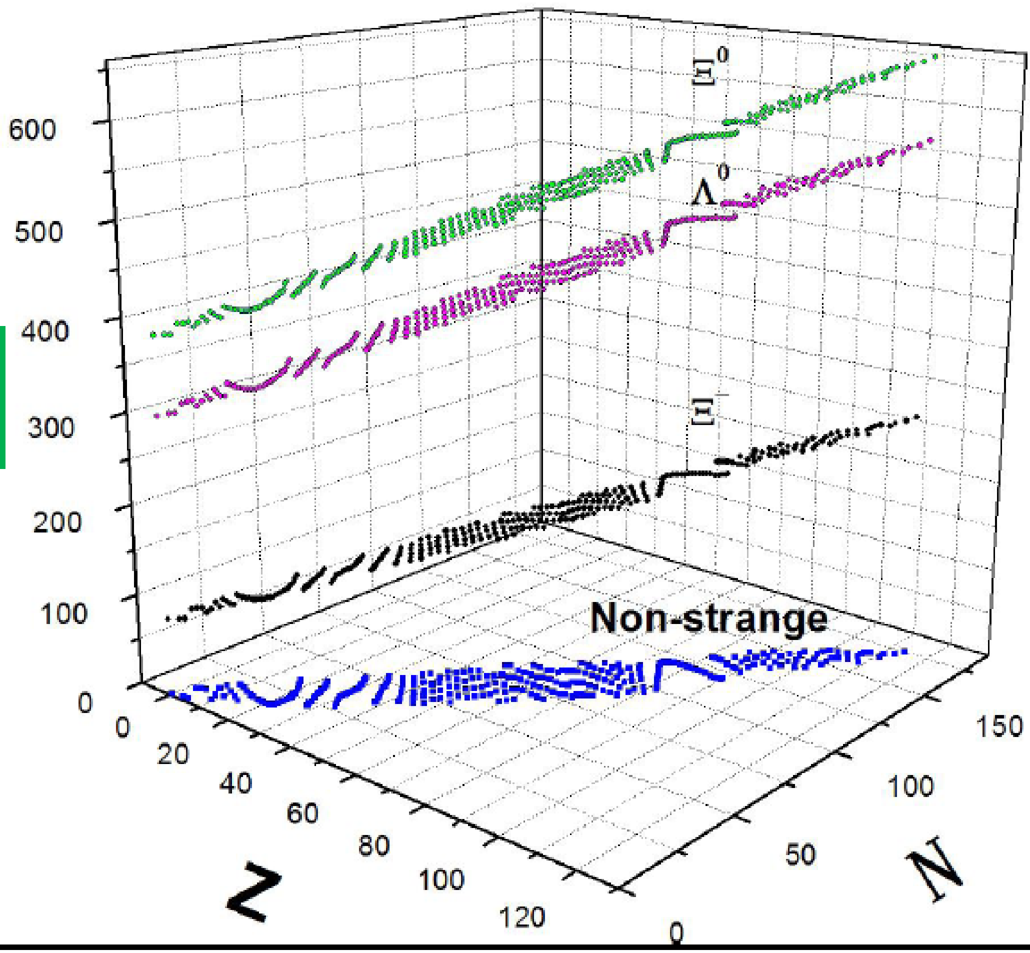
O (Z=8) N = 5 (Normal)
N = 4 (Hyper)

Neutron dripline moves out, proton dripline moves inside!

This effect however varies from nucleus to nucleus.

Is there an upper limit to the number of hyperons that could be bound?

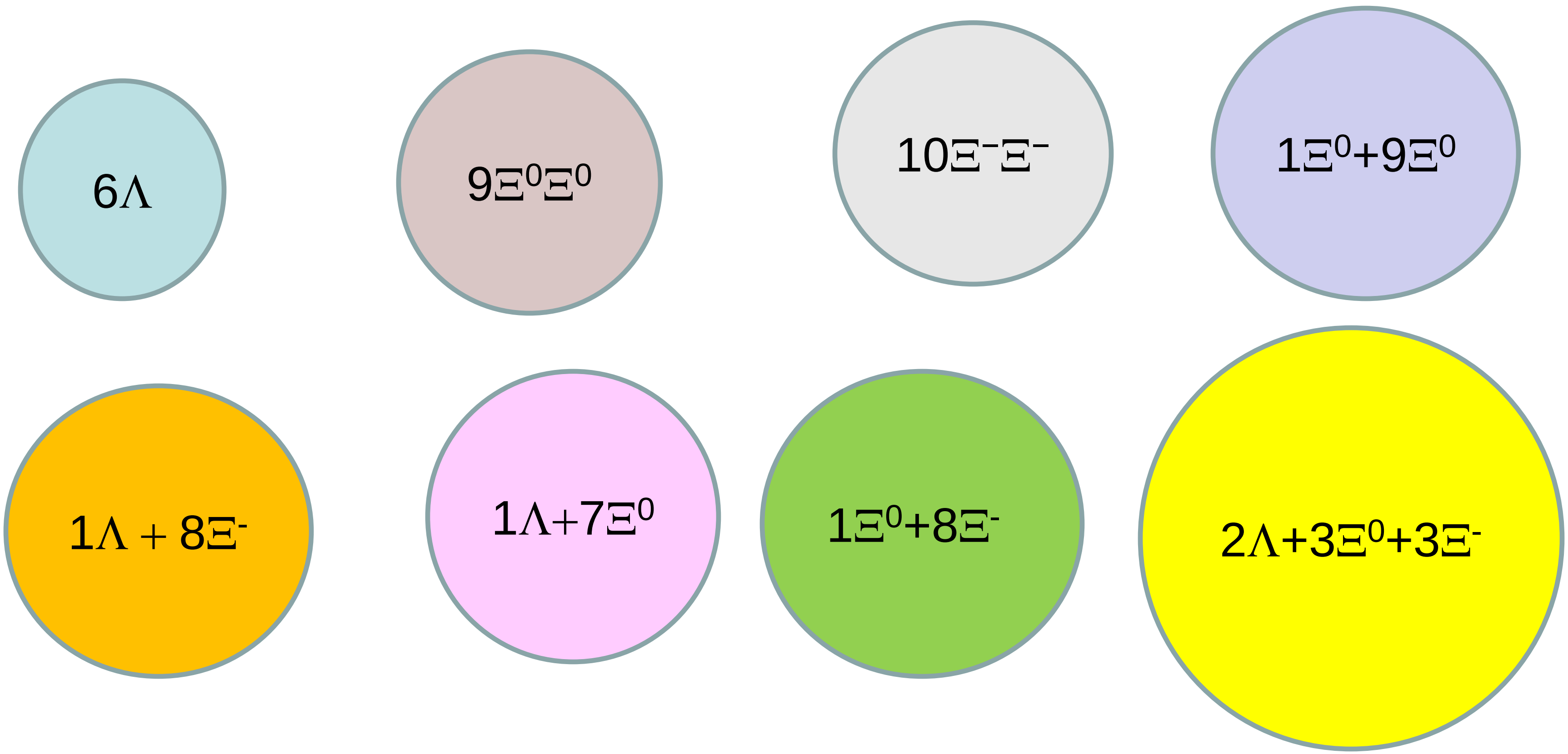
Maximum no. of hyperons



Hyper-drip points predicted by the mass formula CS2.

Lightest Bound Nuclei without any Neutron and Proton!

C. Samanta, Jour. Phys. G: Nucl. Part. Phys. 37 (2010)075104



PURE HYPERONIC SYSTEMS WHICH ARE BOUND (PREDICTED BY CS2)

No bound pure-hyperonic matter is possible by Model-1

Exotic bound nuclear systems

$_{2\Lambda}{}^{6}He = 2p + 2n + 2\Lambda$ **Is bound**

$S_{\Lambda\Lambda}$ (CS2) = 9.32

$^{A}{}_{Y}Z$	$S_{\Lambda\Lambda}$ (exp1)	$S_{\Lambda\Lambda}$ (exp2)
$^{6}{}_{\Lambda\Lambda}He$	6.91 +/- 0.16	10.06 +/- 1.72

Exotic nuclear systems like $_{2\Xi^0 2\Xi^- 2\Lambda}{}^{10}n$, $_{2\Xi^0 2\Lambda}{}^{8}He$ are also bound (CS2).

$_{2\Xi^0 2\Lambda}{}^{8}He = 2p + 2n + 2\Lambda + 2\Xi^0$

$_{2\Xi^0 2\Xi^- 2\Lambda}{}^{10}n = 2p + 2n + 2\Lambda + 2\Xi^0 + 2\Xi^-$

Need experimental data

Can this mass formula be used for hyperfragment yield calculation in heavy ion collisions?

Production of hypernuclei in multifragmentation

PHYSICAL REVIEW C **76**, 024909 (2007)

Production of hypernuclei in multifragmentation of nuclear spectator matter

A. S. Botvina[1] and J. Pochodzalla[2]
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[2]*Institut für Kernphysik, Johannes Gutenberg-University, Mainz, D-55099 Germany*
(Received 20 March 2007; published 23 August 2007)

We have found that there is a sensitivity of the fragment yields in multifragmentation to the mass formulas used for description of binding energy of hypernuclei. In Fig. 3 we compare SMM calculations performed with the liquid-drop hyperterm (3) in free energy of individual fragments, and with the Samanta term (2). There is a clear difference in the chemical potential ξ, and, as a result, the yields of hyperfragments are also different. As one can see from the bottom panels the liquid-drop formula predicts more strangeness in IMF's than the Samanta formula. The difference in the yields is particularly large for small double hyperfragments. In future, this observable may allow to test experimentally different mass formulas for hypernuclei in multifragmentation.

SMM= Statistical Multifragmentation model

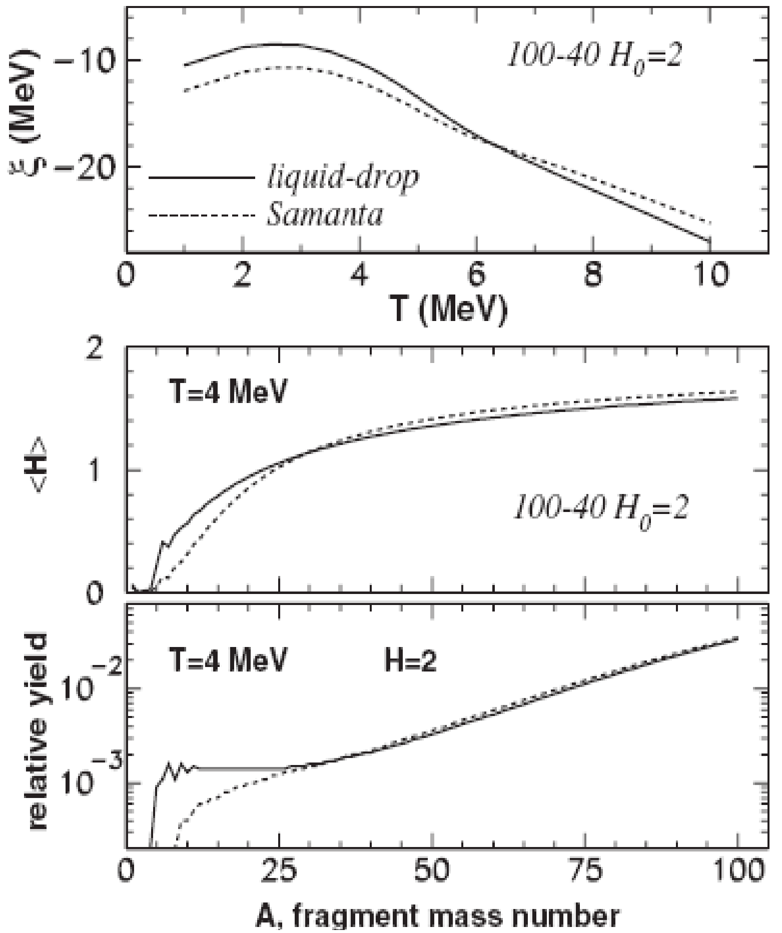

FIG. 3. Comparison of SMM calculations with the liquid-drop and Samanta descriptions of hyper terms in the mass formula, for the same sources as in Fig. 2. Top panel – the strangeness chemical potential ξ versus temperature T. Middle panel – average number of Λ hyperons in fragments, and bottom panel – yields of fragments with two Λ, at $T = 4$ MeV.

Summary

❖A generalised mass formula is formulated by extending the Bethe-Weizsäcker mass formula. It has no shell effect and it is not applicable for repulsive potential.

❖ It depends on the strangeness and mass of Hyperons.

❖Without changing any parameter, it can estimate binding energies of normal non-strange nuclei, singly-strange nuclei and multiply-strange nuclei.

❖This mass formula reproduces results of the relativistic mean field calculations of Schaffner et al., and the experimental data for Λ, $\Lambda\Lambda$, Ξ (and one Σ) hypernuclei as well.

➢CS1: Model1(no Y-Y interaction) and CS2: Model 2 (Y-Y interaction).

❖ Neutron and proton driplines are found to shift on addition of hyperons to normal nuclei. This shift is different for different nuclei.

❖For the first time possible hyperon-drip points for Λ, Ξ^0 and Ξ^- hypernuclei are predicted. These limiting values depend on the mass, strangeness as well as charge of the hyperon.

❖Mass formula (CS2) suggests existence of bound pure hyperonic matter and some exotic bound nuclear systems with a mixture of hyperons; (CS1 does not).

❖Due to its simplicity, this mass formula can be easily used for estimating the production yield of multi-strange nuclear systems. Certainly more experimental data are needed!

Thank you!