Analyticity and crossing symmetry in the K-matrix formalism.

Simona Stoica

KVI, Groningen

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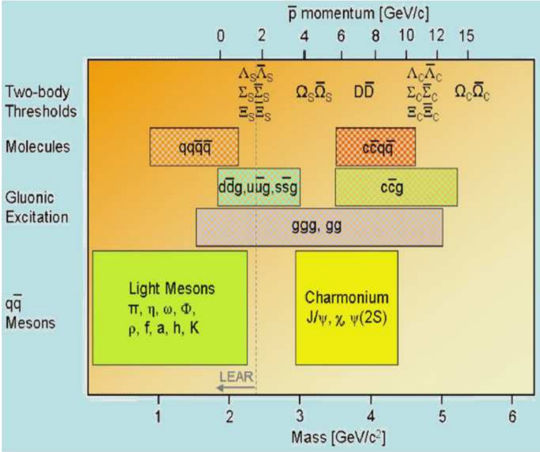
Overview

- Motivation
- Symmetries in scattering formalism
- K-matrix formalism
  - Simple K-matrix (K_S)
  - Analytic continued K-matrix (K_A)
- Pions and photons off the nucleon

PANDA physics



- Meson spectroscopy
  - charmonium states
  - open charm production
  - exotic matter
    - glueballs
    - hybrids
    - molecules
  - light mesons
- Baryon-Antibaryon production
- Charm in nuclei
- Hypernuclei physics
- Hadrons in nuclear medium

Symmetries

A scattering matrix should obey various 'symmetries':

1. Unitarity
   - $S^\dagger S = SS^\dagger = 1$; conservation of flux.
2. Crossing symmetry
   - Take all Feynman diagrams into account.
   - Low energy theorems.
3. Analyticity/causality
   - Should obey a dispersion relation.
4. Covariance
5. Gauge invariance
6. Chiral symmetry

→ *Are these symmetries obeyed in the K-matrix formalism?*

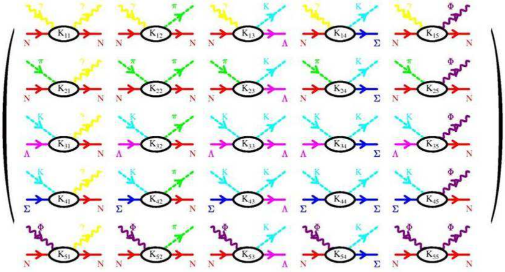
K-matrix formalism: the simple K-matrix

$$S_S = 1 + 2iT_S$$

$$T_S = \frac{K_S}{1 - iK_S}$$

K = sum of all tree level diagrams
Calculated per partial wave.

$K =$



- Algebraic $\longrightarrow$ Full coupled channels in large model space
- Unitary
- Gauge invariant, Chiral symmetry
- Covariant

$\left\{ \begin{array}{l} \textit{Causality} \\ \textit{Analitycity} \end{array} \right.$

Simple K-matrix: Analyticity

The scattering matrix should obey a dispersion relation:

$$\mathrm{Re}(T(s)) = (s - s_0)\frac{\mathcal{P}}{\pi}\int_{s_0}^{\Lambda} \frac{\mathrm{Im}(T(s'))}{(s'-s)(s'-s_0)}\mathrm{d}s'$$

- $\mathrm{Im}(T(s'))$ is calculated from the K-matrix.
- $\mathrm{Re}(T(s))$ calculated from the dispersion relation should have the same shape as $\mathrm{Re}(T(s))$ calculated from the K-matrix.

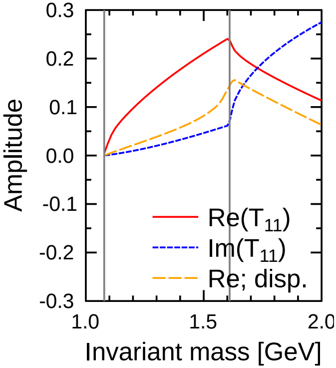
Simple K-matrix: Analyticity

- The dispersion relation is not obeyed.
- Simple K-matrix is not continuous along the threshold of channel 2:

$$K_S = \begin{pmatrix} K_{S11} & 0 \\ 0 & 0 \end{pmatrix} \xrightarrow{thr.2} \begin{pmatrix} K_{S11} & K_{S12} \\ K_{S21} & K_{S22} \end{pmatrix}$$

- S_S is not analytic $\rightarrow$ Problem!

Make an analytic continuation of K_S.

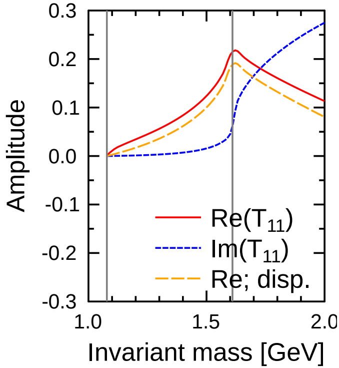


Analytic continued K-matrix: Analyticity

$$K_A = \begin{pmatrix} \rho_1 A_{11} & \sqrt{\rho_1\rho_2} A_{21} \\ \sqrt{\rho_1\rho_2} A_{12} & \rho_2 A_{22} \end{pmatrix}$$

Analytic continuation of the momenta of the particles below their thresholds:

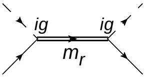
$\rho_i = k_i;\ k_i = \sqrt{\vec{k}_i^2}$



Be careful with the definition of ρ; it's double valued! ρ is double valued: $\rho = \pm\sqrt{\vec{k}^2}$;
if $\vec{k}^2 < 0$, $\rho = \pm i|k|$.

Analytic continued K-matrix: Analyticity

Explicitly add a pole and consider the decay $\propto e^{-\Gamma t}$:

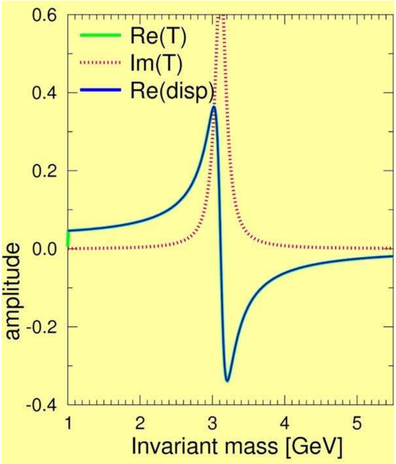


$$A = \frac{-g^2}{s - m_r^2}$$

gives

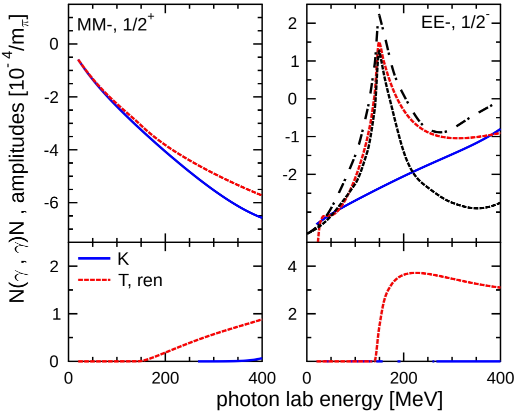
$$S_A = \frac{s - m_r^2 - ig^2\rho}{s - m_r^2 + ig^2\rho}$$

Compare with results from dispersion integral



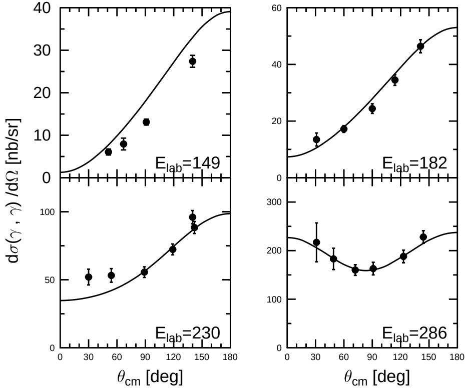
Compton Scattering

Consistently re-summed u & t channel contributions added as contact terms to the kernel



Renormalize at threshold.
Cusp at pion production threshold agrees with data analysis.
Data from:
Pfeil et al, NPB73, 166
Bergstrom et al, PRC48, 1508

Compton Scattering, Cross sections



Renormalization:

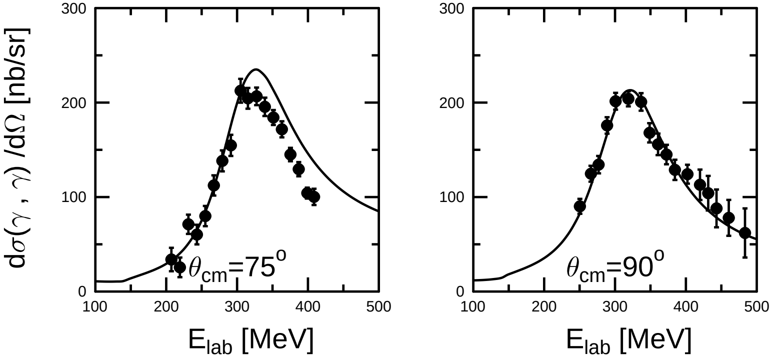
$$K_A = \begin{pmatrix} \rho_1 A_{11} & \sqrt{\rho_1\rho_2}A_{21} \\ \sqrt{\rho_1\rho_2}A_{12} & \rho_2 A_{22} \end{pmatrix}$$

Gives:

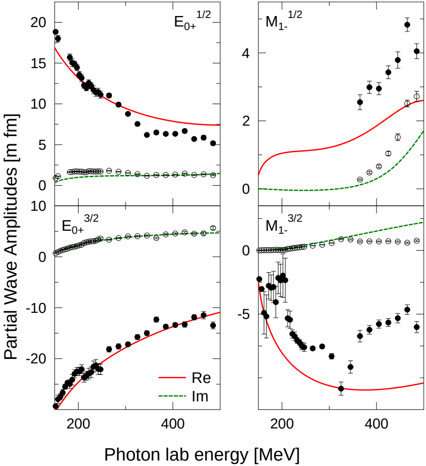
$$T_{11} = \rho_1 \frac{A_{1,1} + Z}{1 - i\rho_1(A_{1,1} + Z)}$$

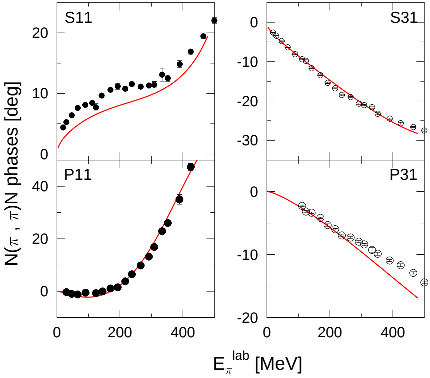
$$Z = i\frac{\rho_2 A_{1,2} A_{2,1}}{1 - i\gamma}$$

Compton Scattering, Cross sections



Pion scattering and photoproduction





η, ρ meson production not yet included

Conclusion

- Symmetries in the *simple* K-matrix formalism
  - Unitarity: by construction
  - Analyticity/Causality: not fulfilled
- Symmetries in the *analytic continued* K-matrix formalism
  - Unitarity: by construction
  - Analyticity/Causality: fulfilled
- Renormalization of off-diagonal matrix elements should solve the crossing symmetry problem partially
- Pion Photoproduction
- Renormalization gives promising results