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Combinatorial and Arithmetical Properties of Infinite Words Associated with Quadratic Non-simple Parry Numbers

Abstract

We study some arithmetical and combinatorial properties of β-integers for β being the larger root of the equation $x^2 = mx - n, m, n \in \mathbb{N}, m \geq n+2 \geq 3$. We determine with the accuracy of ± 1 the maximal number of β-fractional positions, which may arise as a result of addition of two β-integers. For the infinite word u_β coding distances between the consecutive β-integers, we determine precisely also the balance. The word u_β is the only fixed point of the morphism $A \to A^{m-1}B$ and $B \to A^{m-n-1}B$. In the case $n = 1$, the corresponding infinite word u_β is sturmian, and, therefore, 1-balanced. On the simplest non-sturmian example with $n \geq 2$, we illustrate how closely the balance and the arithmetical properties of β-integers are related.