# A polynomial root solver that preserves the multiplicities of the roots

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The computation of the roots of a polynomial is a classical problem and considerable effort has been devoted to the development of robust algorithms. A persistent problem has, however, been the computation of multiple roots of polynomials. In particular, numerous computational experiments have shown that roundoff errors due to floating point arithmetic, without the addition of errors that represent data and experimental errors, are sufficient to cause multiple roots to break up into simple roots. An obvious and simple solution is cluster analysis on the computed roots, but this method is appropriate provided the computed roots form isolated and well-separated clusters, which cannot be guaranteed.

This talk discusses a new polynomial root solver for the computation of multiple roots of inexact polynomials. The theoretical motivation of the algorithm requires that the geometry of multiple roots be considered, but it is shown that there are several non-trivial computational issues, not related to the geometry, that must be addressed. In particular, the algorithm requires that several greatest common divisor and polynomial division computations be performed, but these are ill-posed computations for which robust algorithms must be used.

The algorithm for the polynomial root solver does not require that the level of the noise imposed on the polynomial coefficients be known. Furthermore, all the parameters in the algorithm are calculated from the data, such that user intervention is not required. The talk will include several non-trivial computational examples of the computation of the roots of inexact polynomials whose theoretically exact forms contain multiple roots of high degree.

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