
Courtesy
H. Todt

Clumping in Hot-Star Winds

Achim Feldmeier
Univ. Potsdam

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W-R. Hamann

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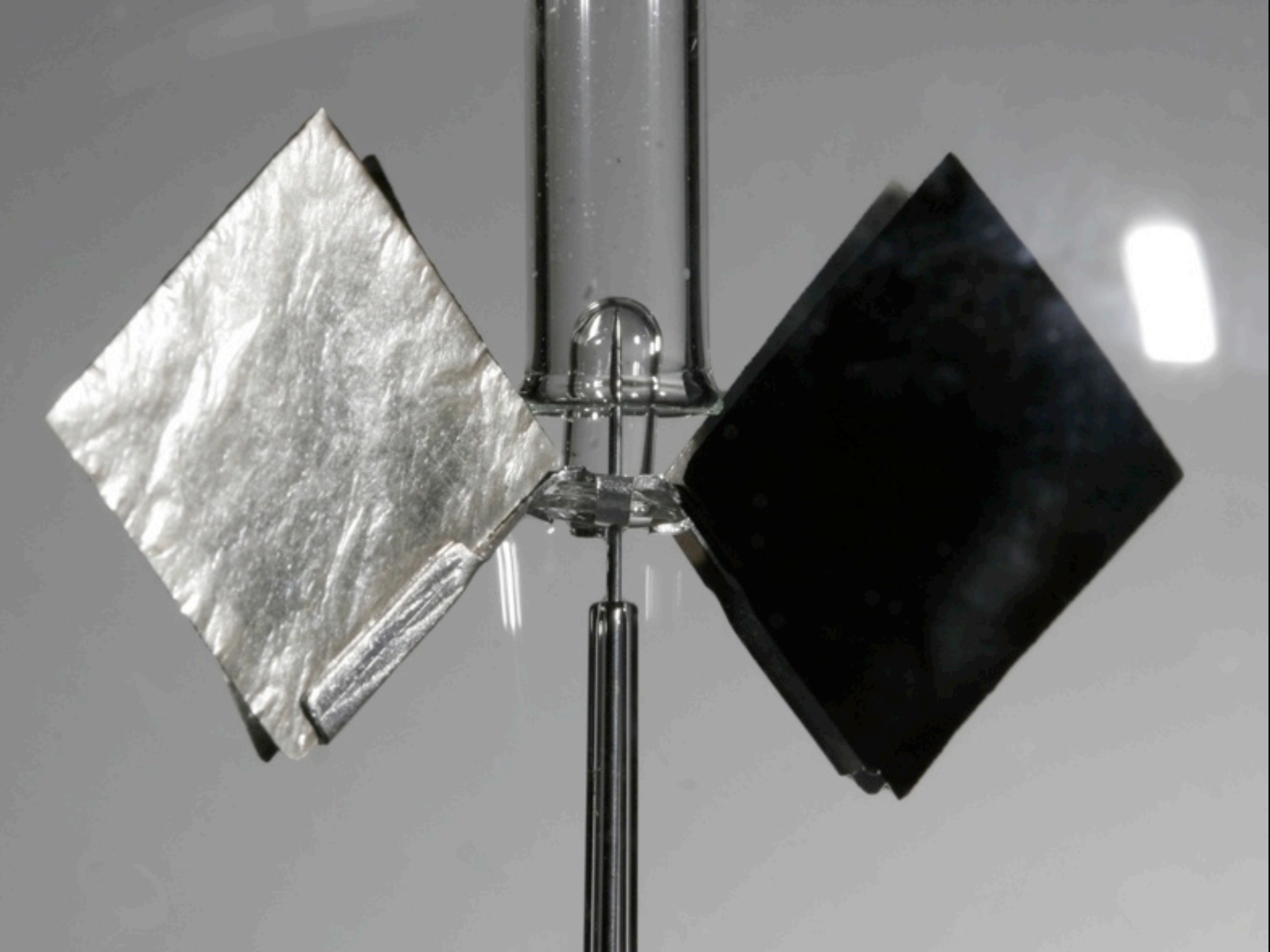
J. Krticka
Brno



What is line-radiation driving?

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Momentum transfer

from photons to metals

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Momentum transfer

from photons to metals

via scattering

in spectral lines

What is line-radiation driving?

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plus

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Momentum transfer | from photons to metals

via scattering | in spectral lines

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Momentum transfer | from metal ions to H/He

What is line-radiation driving?

Momentum transfer	from photons to metals
via scattering	in spectral lines

plus

Momentum transfer	from metal ions to H/He
via elastic	Coulomb collisions

Why line-driven winds?

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O &WR stars lose 1/2 their mass through wind

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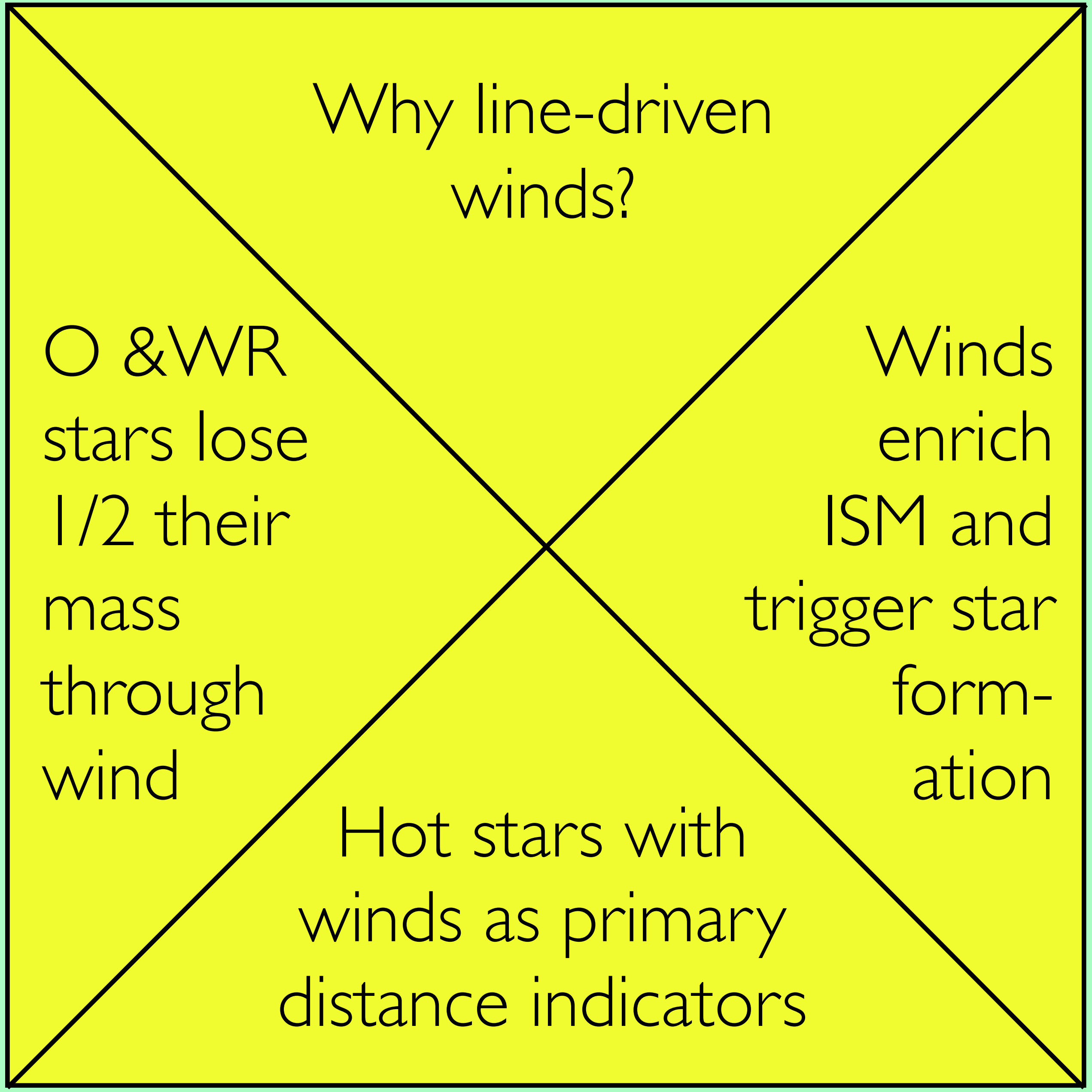
O &WR stars lose 1/2 their mass through wind

Hot stars with winds as primary distance indicators


Why line-driven winds?
O &WR stars lose 1/2 their mass through wind
Winds enrich ISM and trigger star form-ation
Hot stars with winds as primary distance indicators

History of radiation-driven stellar winds

Wolf & Rayet	1867	stars with very broad spectral lines

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Milne	1926	new plasma instability in solar chromosphere

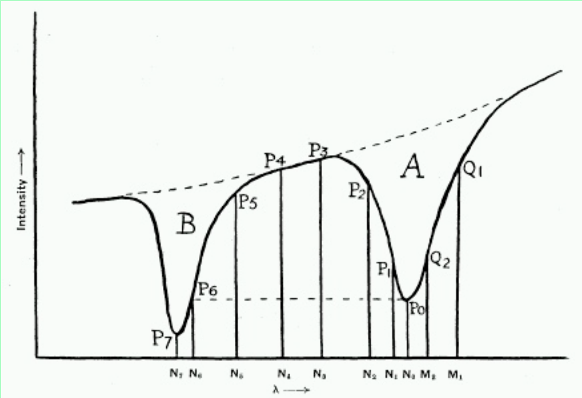
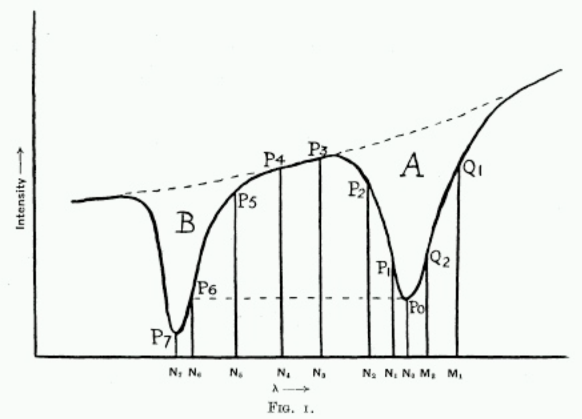
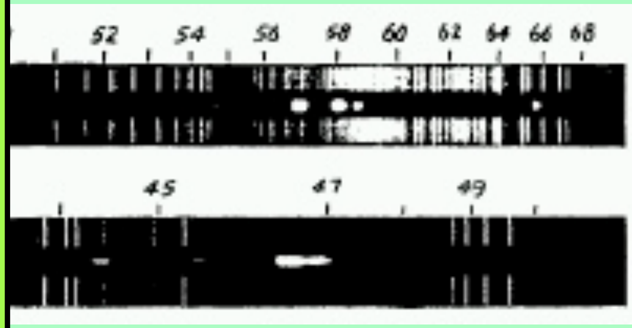


Fig. 1.

History of radiation-driven stellar winds

Wolf & Rayet	1867	stars with very broad spectral lines
Milne	1926	new plasma instability in solar chromosphere
Beals Chandrasekhar	1929 1934	broad emission lines from W-R stars form in continuous outflow = wind





Parker	1958	Transonic (!) hydrodynamic (!) outflow from sun

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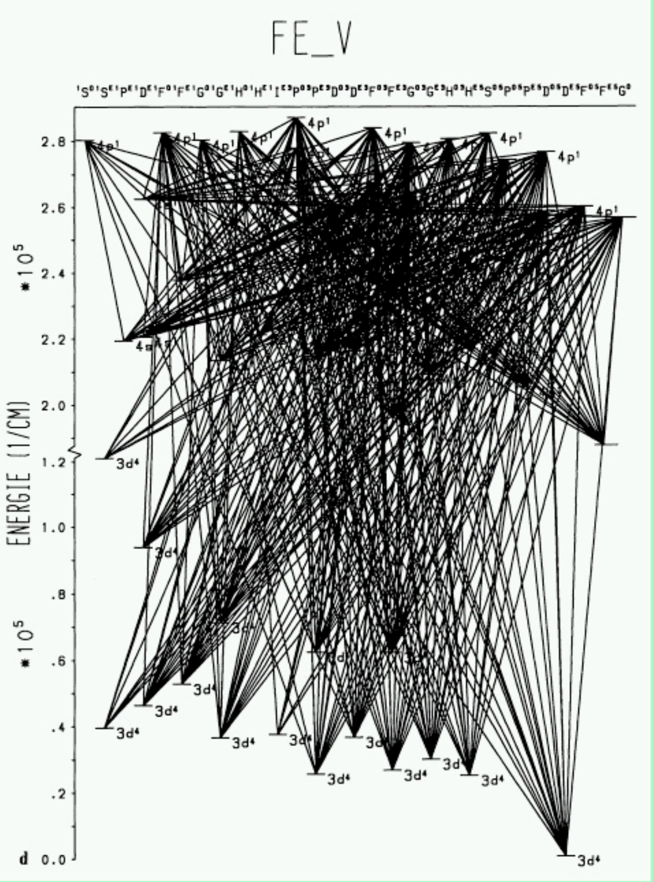
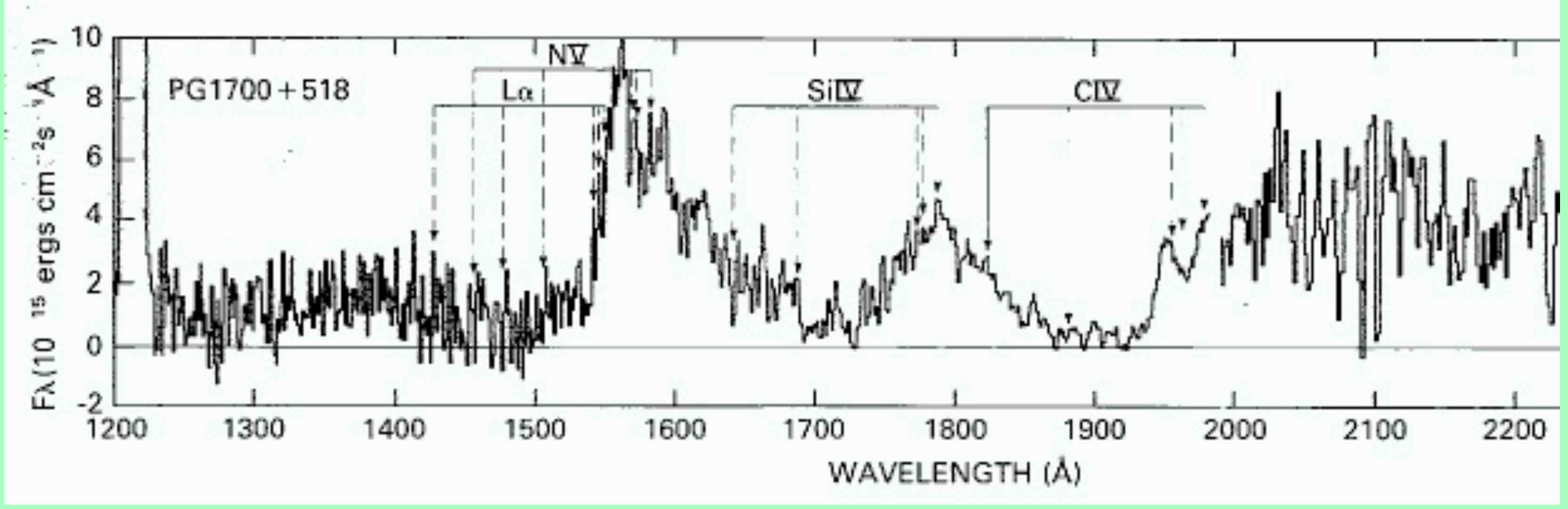
Lucy CAK	1970 1975	stationary theory of line-radiation driven winds
Cannon Scharmer Hamann Werner	1973 1981 1985 1985	Accelerated Lambda Iteration = ALI for radiative transfer
Hamann Hillier Pauldrach Puls	1986-	Full radiative transfer + nonLTE wind models with line blanketing etc



Begelmann
Turnshek
Weymann
Norman
Shlosman
Proga

1980-

Line-driven winds from accretion disks in quasars, cataclysmic variables, and young stellar obj



WIND HYDRODYNAMICS: TOPICS

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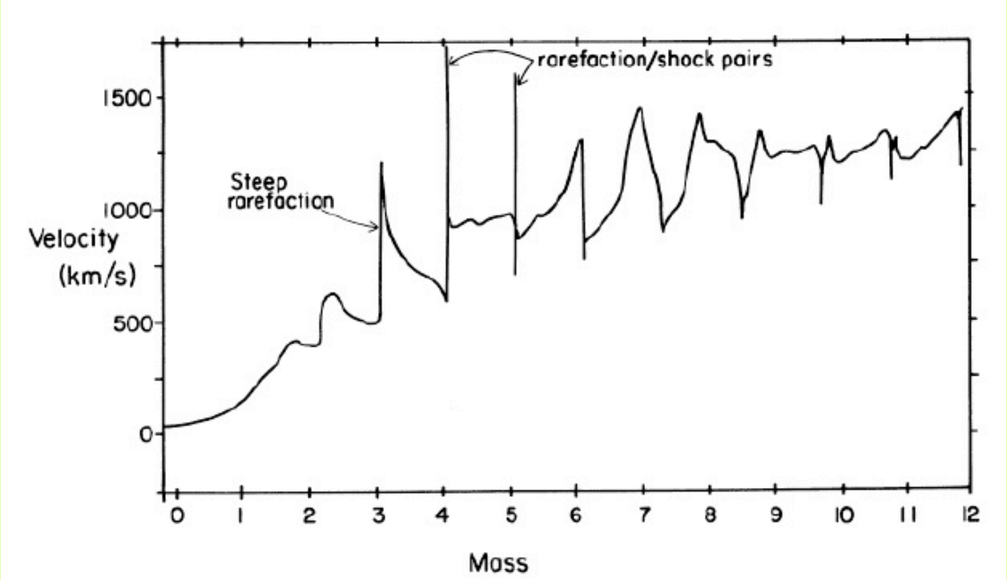
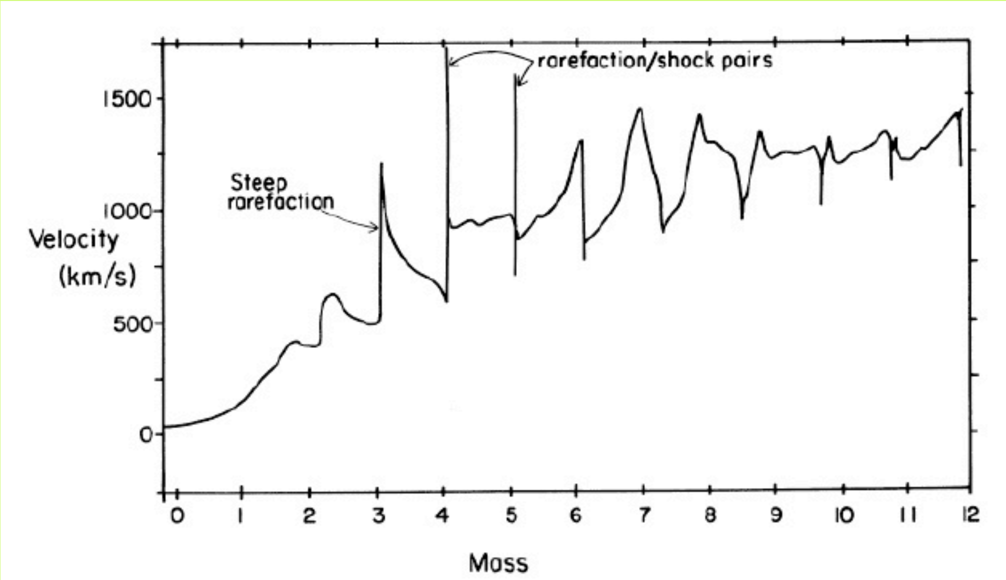
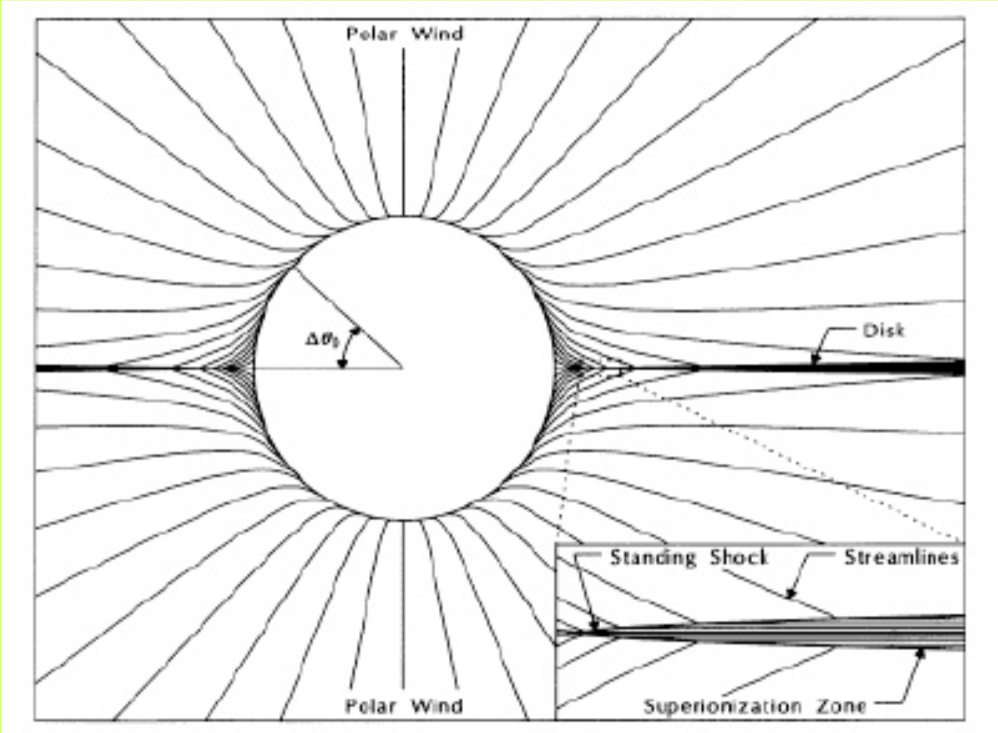


Shocks from line-driven instability

Owocki et al. 1988, ApJ

WIND HYDRODYNAMICS: TOPICS



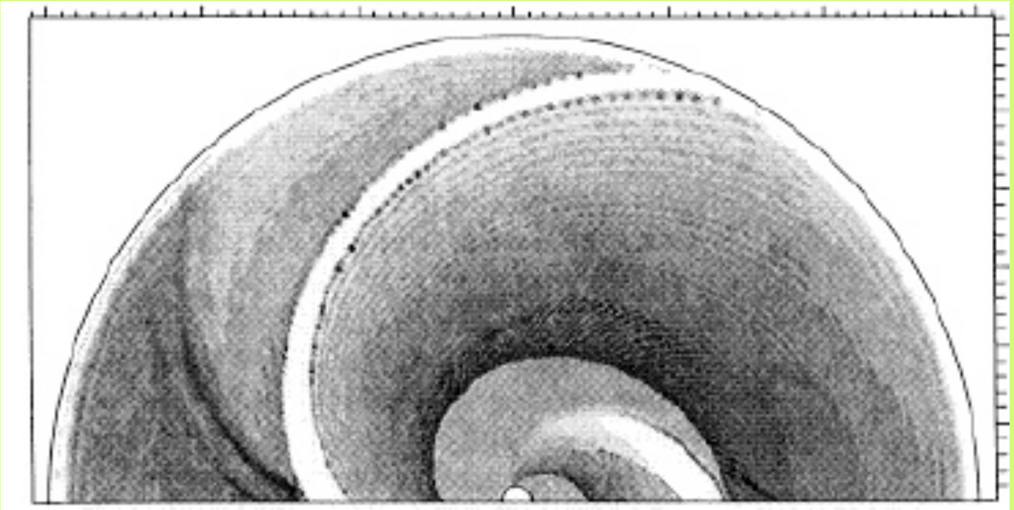


Shocks from line-driven instability

Owocki et al. 1988, ApJ

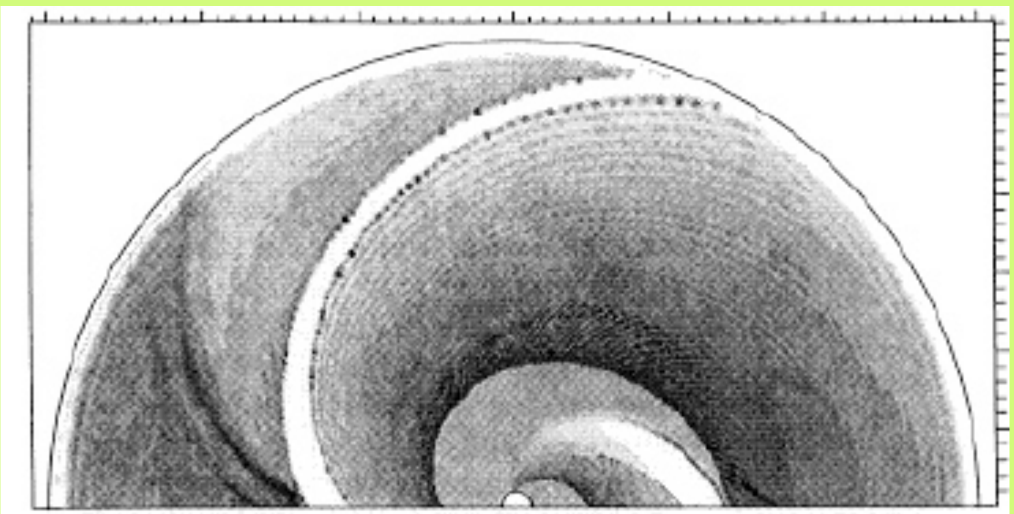
Wind-compressed disks

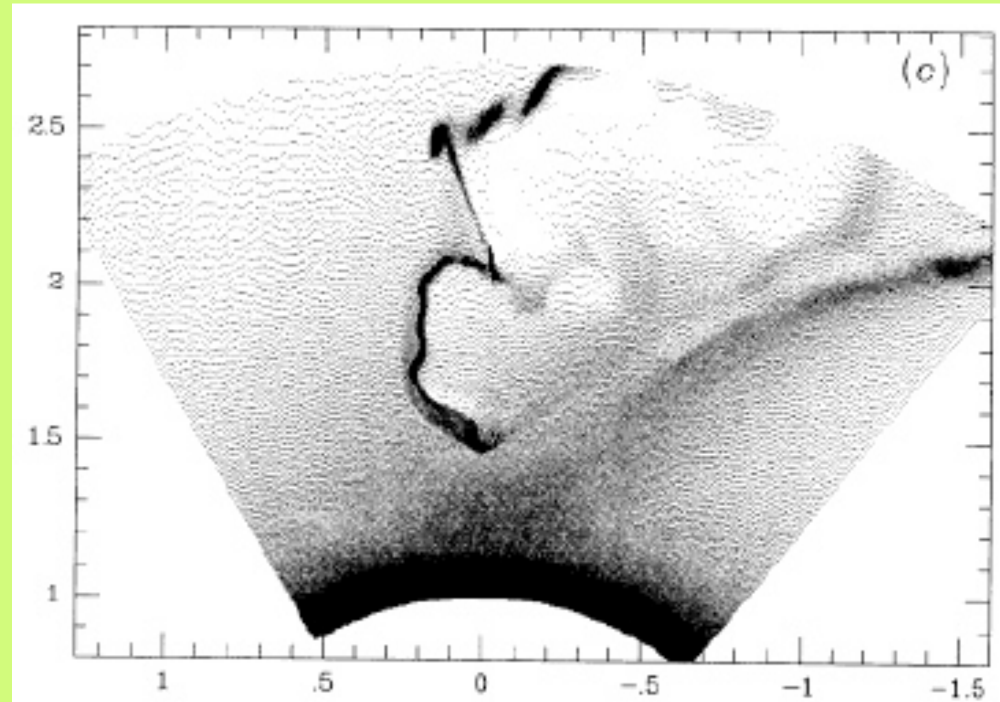
Bjorkman & Cassinelli 1993, ApJ



Corotating interaction regions

Cranmer & Owocki 1996, ApJ



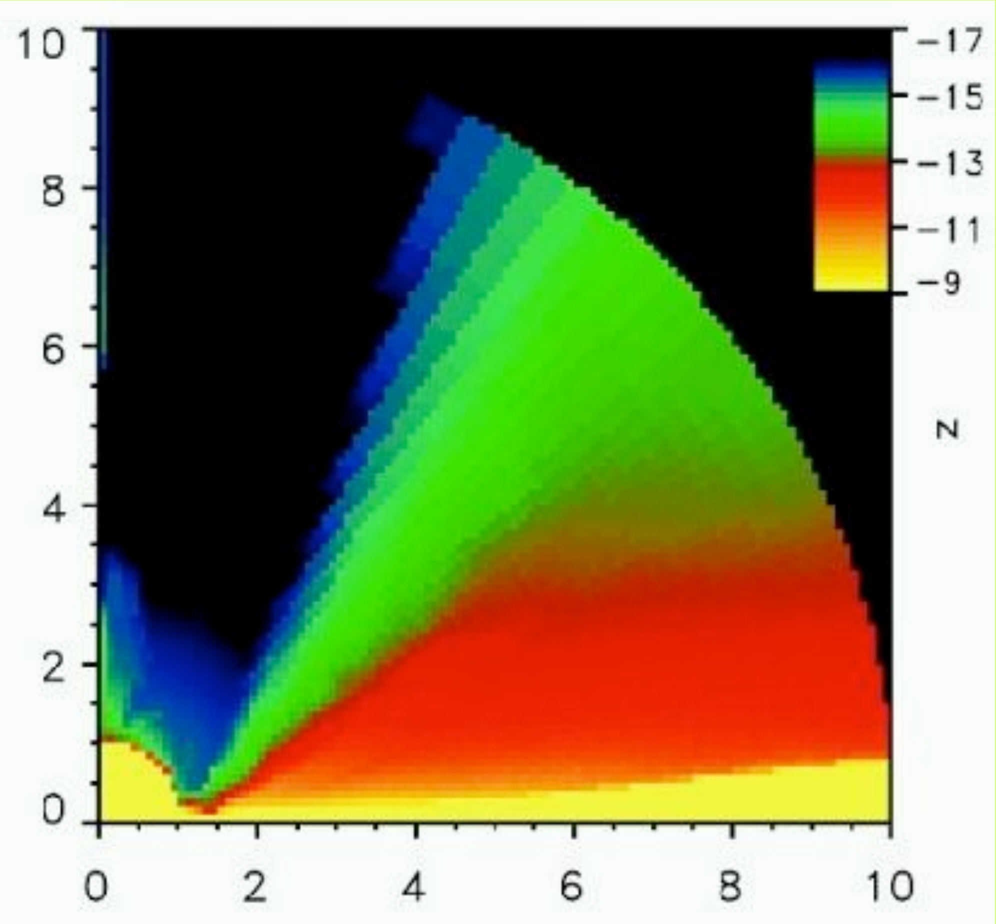


Corotating interaction regions

Cranmer & Owocki 1996, ApJ

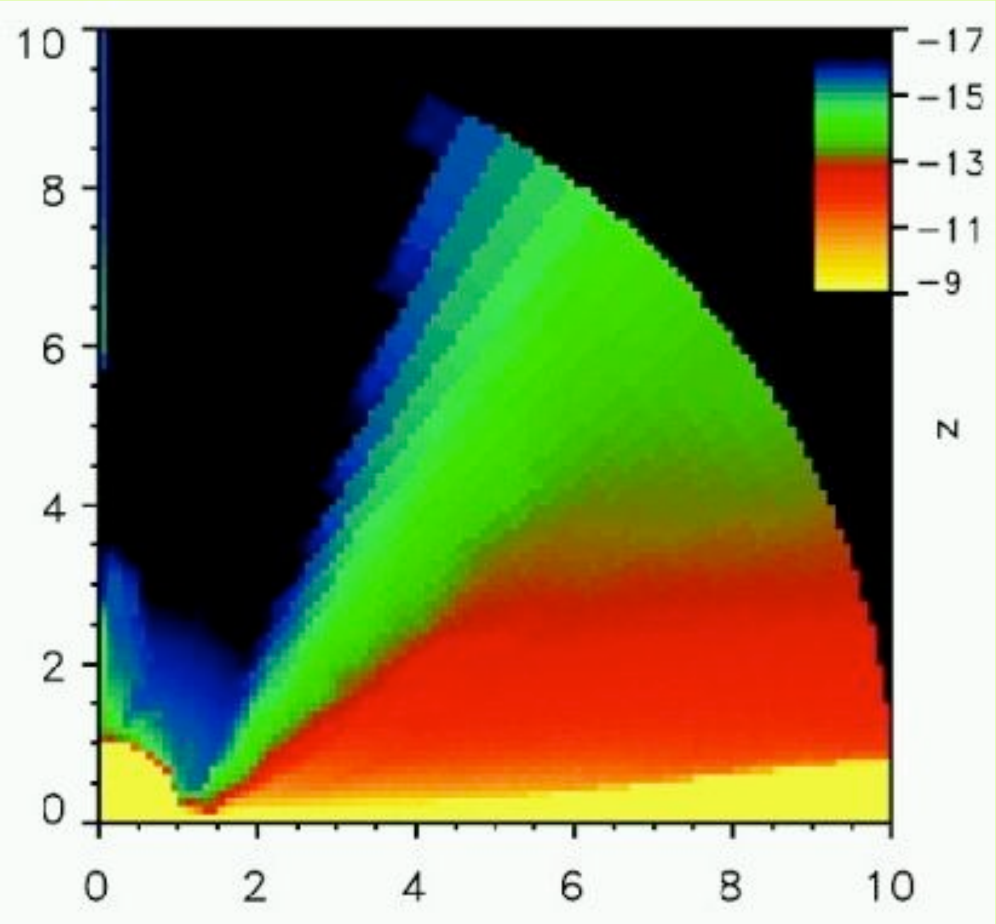
High-mass X-ray binaries

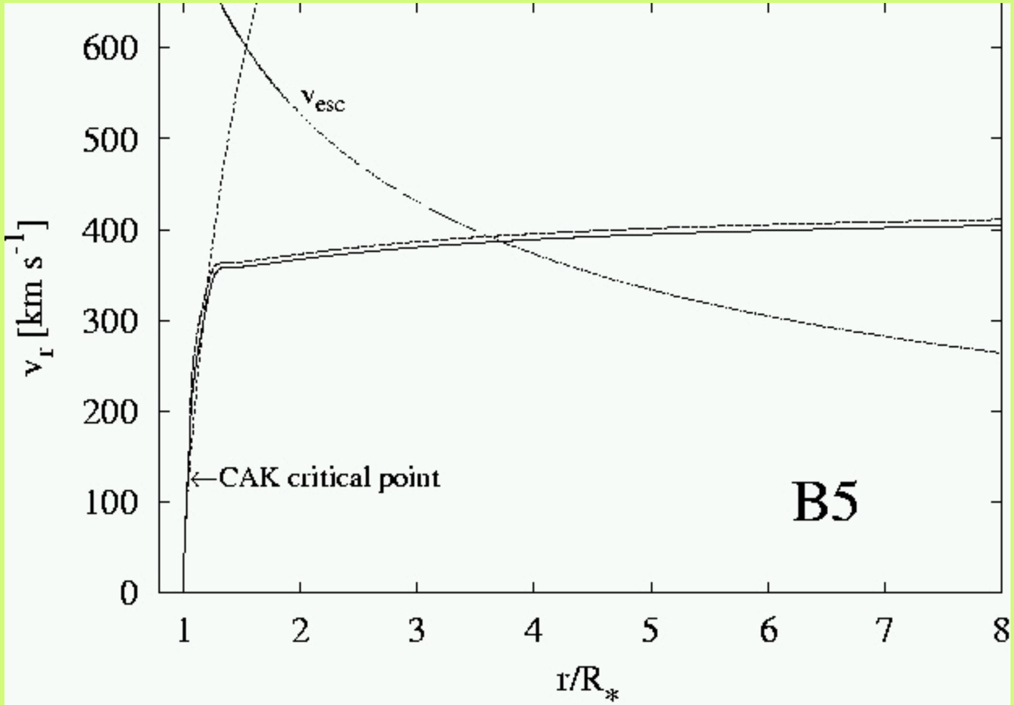
Blondin et al. 1990, ApJ



Quasar winds

Proga 2004, ApJ





Quasar winds

Proga 2004, ApJ

Thin winds with
ion decoupling

Krticka & Kubat 2000, A&A

The minimum formalism

1. Planar, 1-D wind
2. Zero sound speed
3. Sobolev approximation
4. CAK line distribution

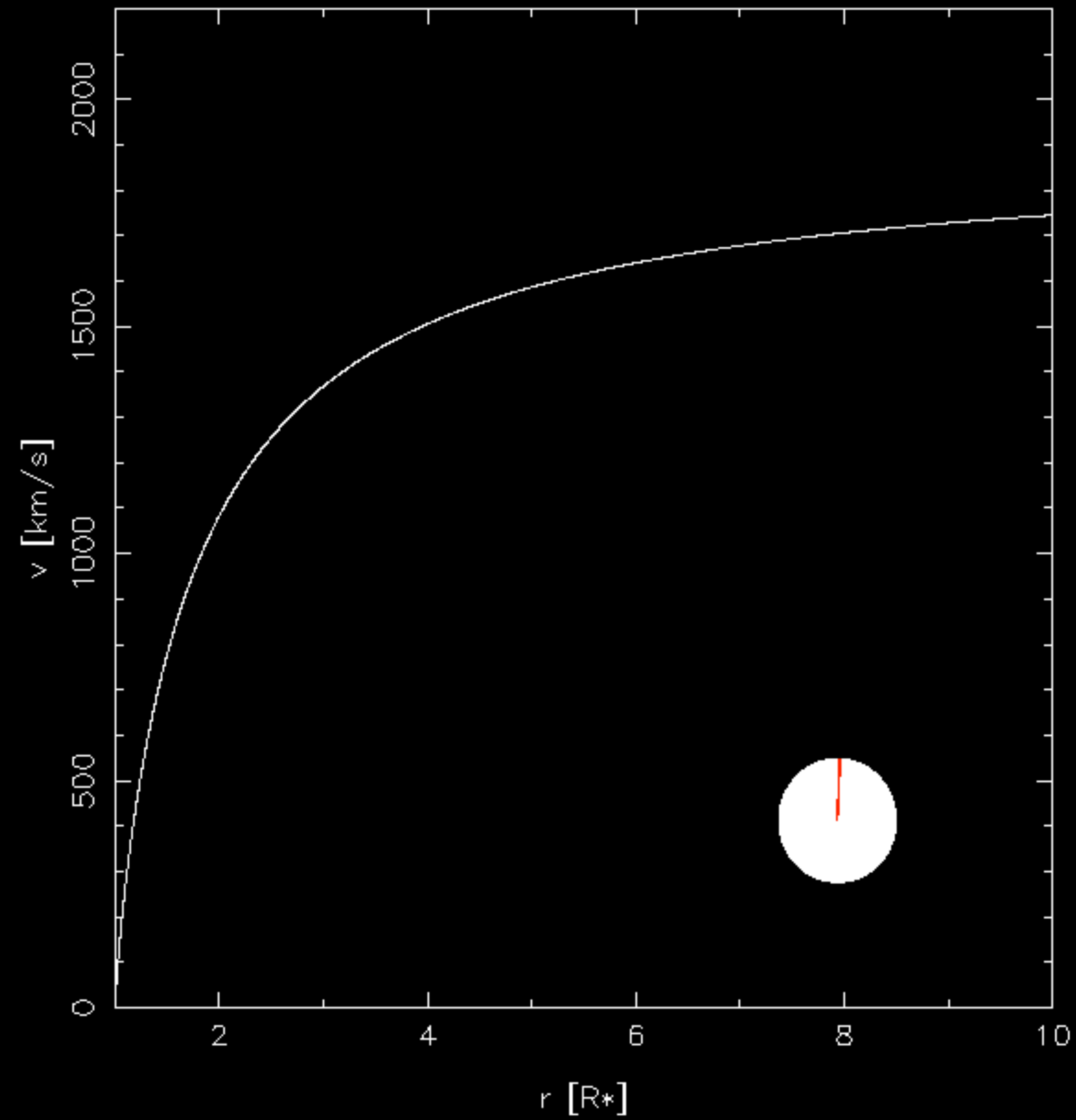
$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial z} + \rho\frac{\partial v}{\partial z} = 0,$$

$$E \equiv \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} + g(z) - CF(z)\left(\frac{\partial v/\partial z}{\rho}\right)^{\alpha} = 0.$$

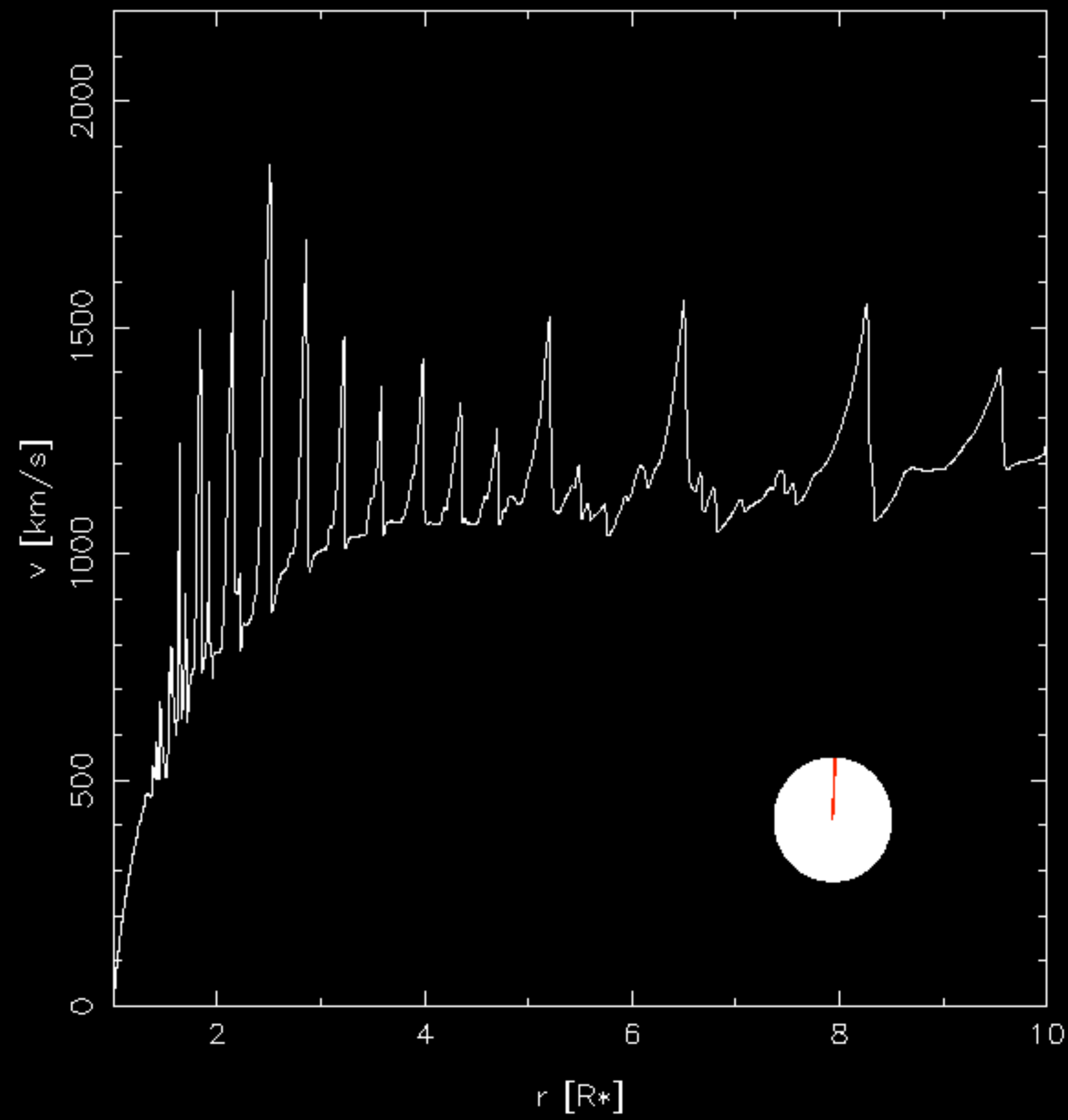
INSTA

BILITY

Line-driven instability

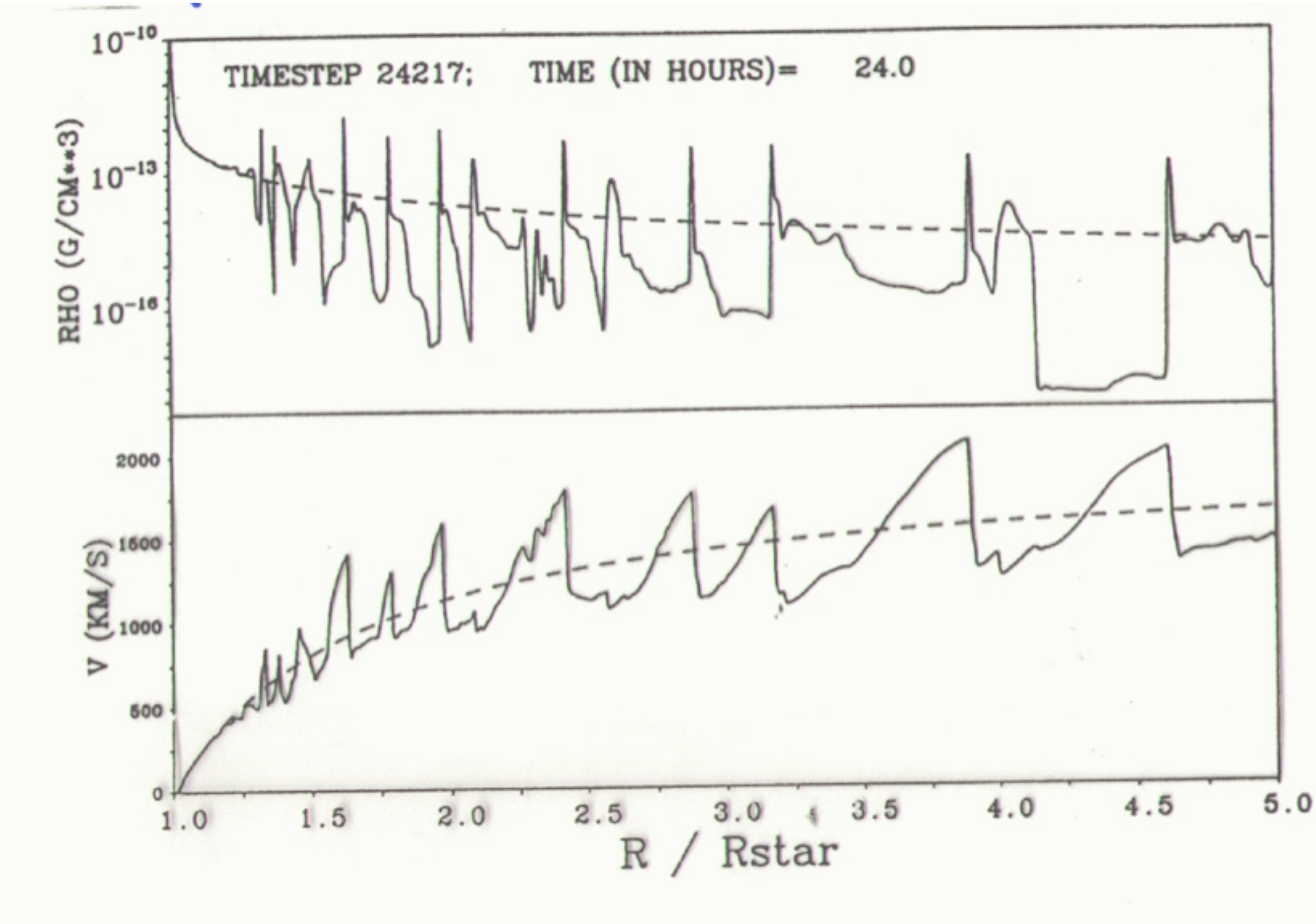


Line-driven instability

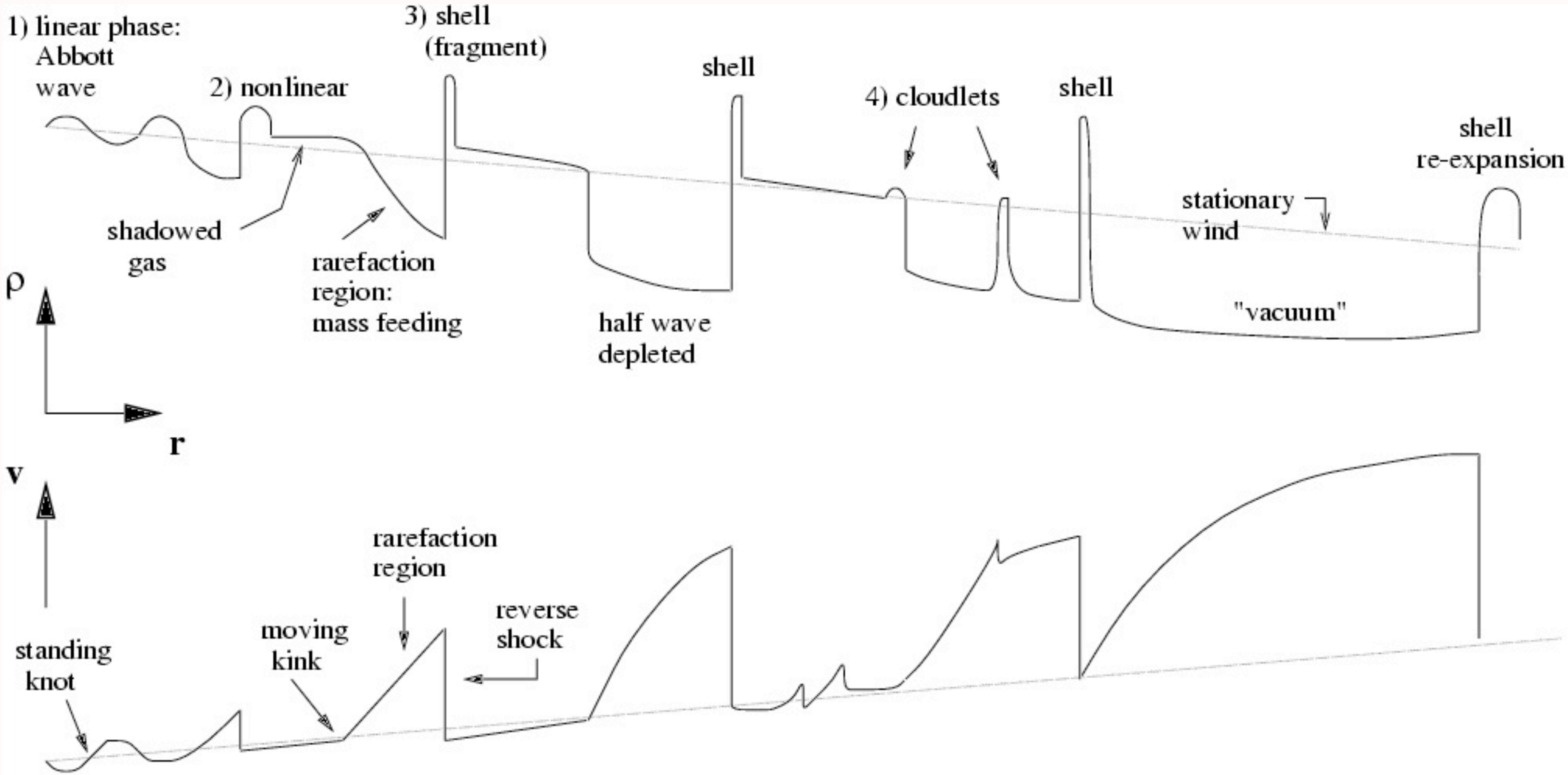


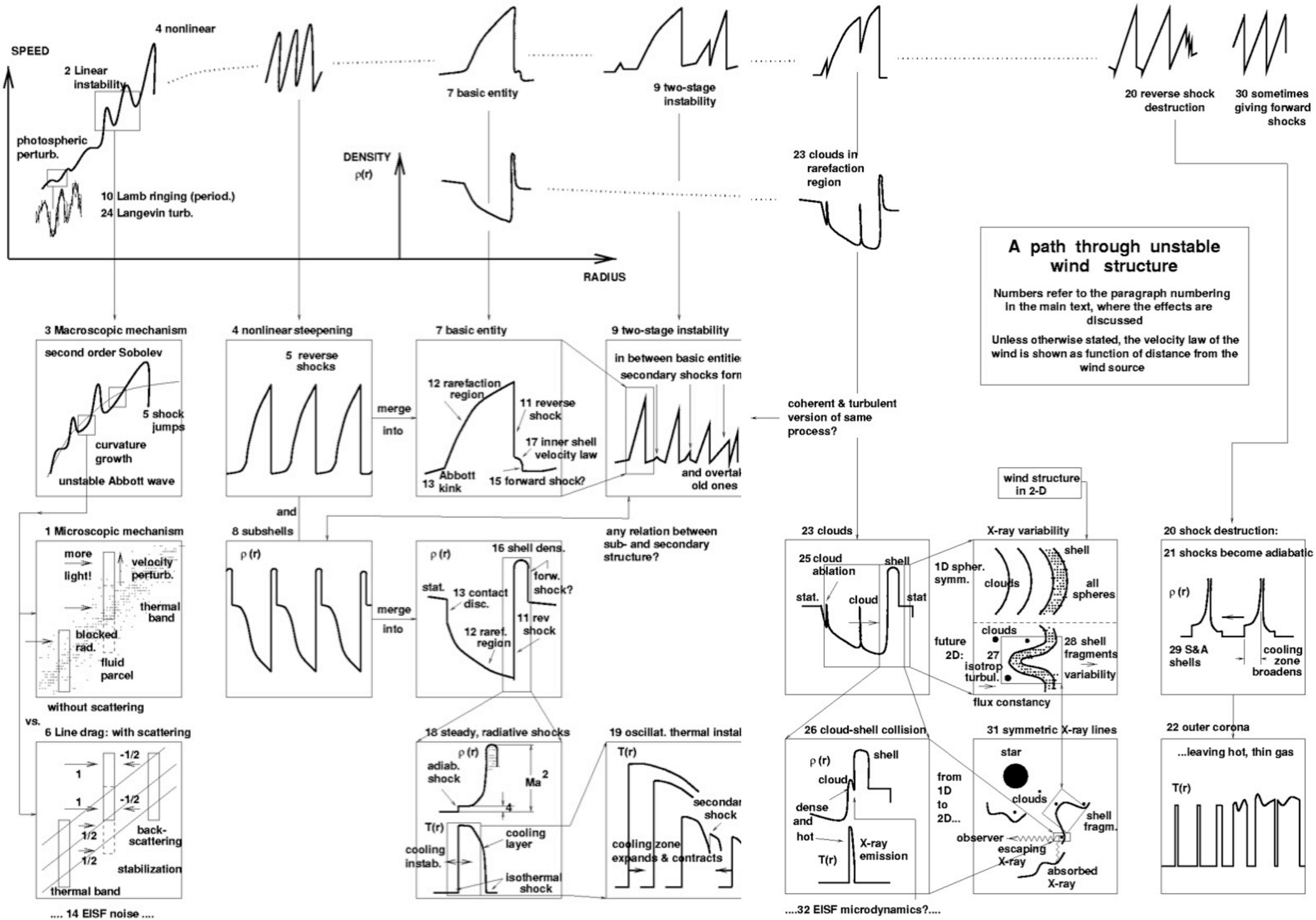
Numerical wind structure

Owocki et al.	1988	ApJ	: Absorption
Owocki	1991		: Scattering
Feldmeier	1995	A&A	: Energy eq.



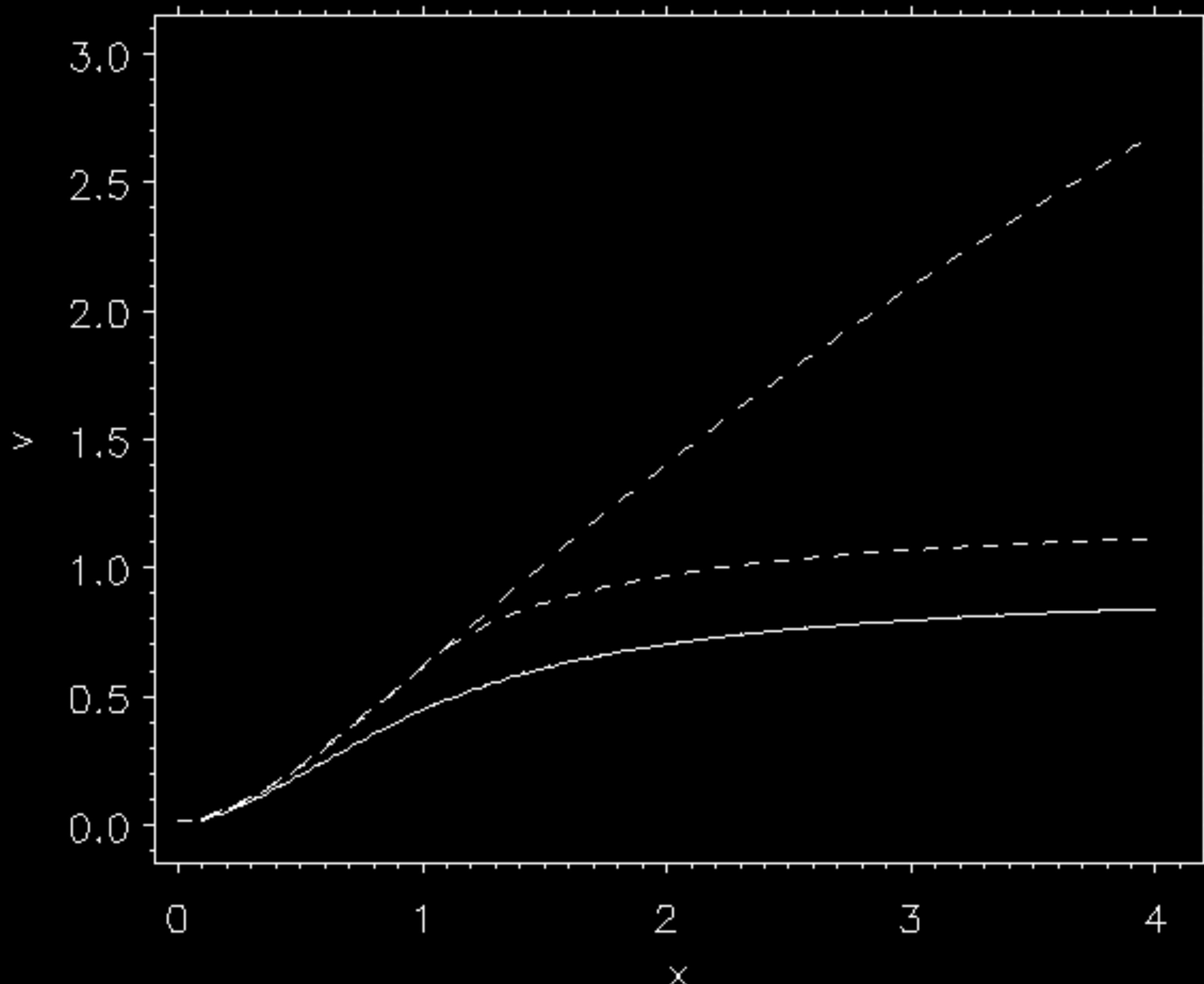
Generic wind structure

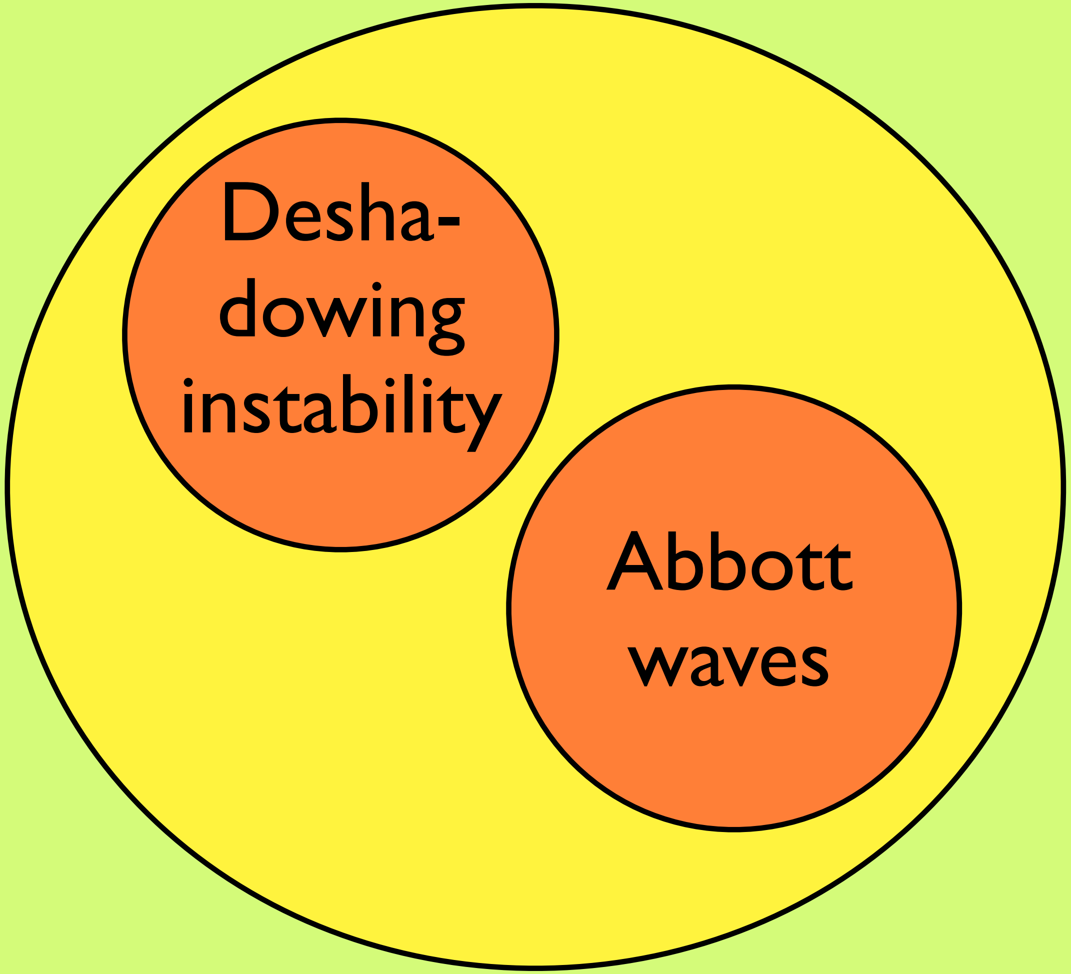


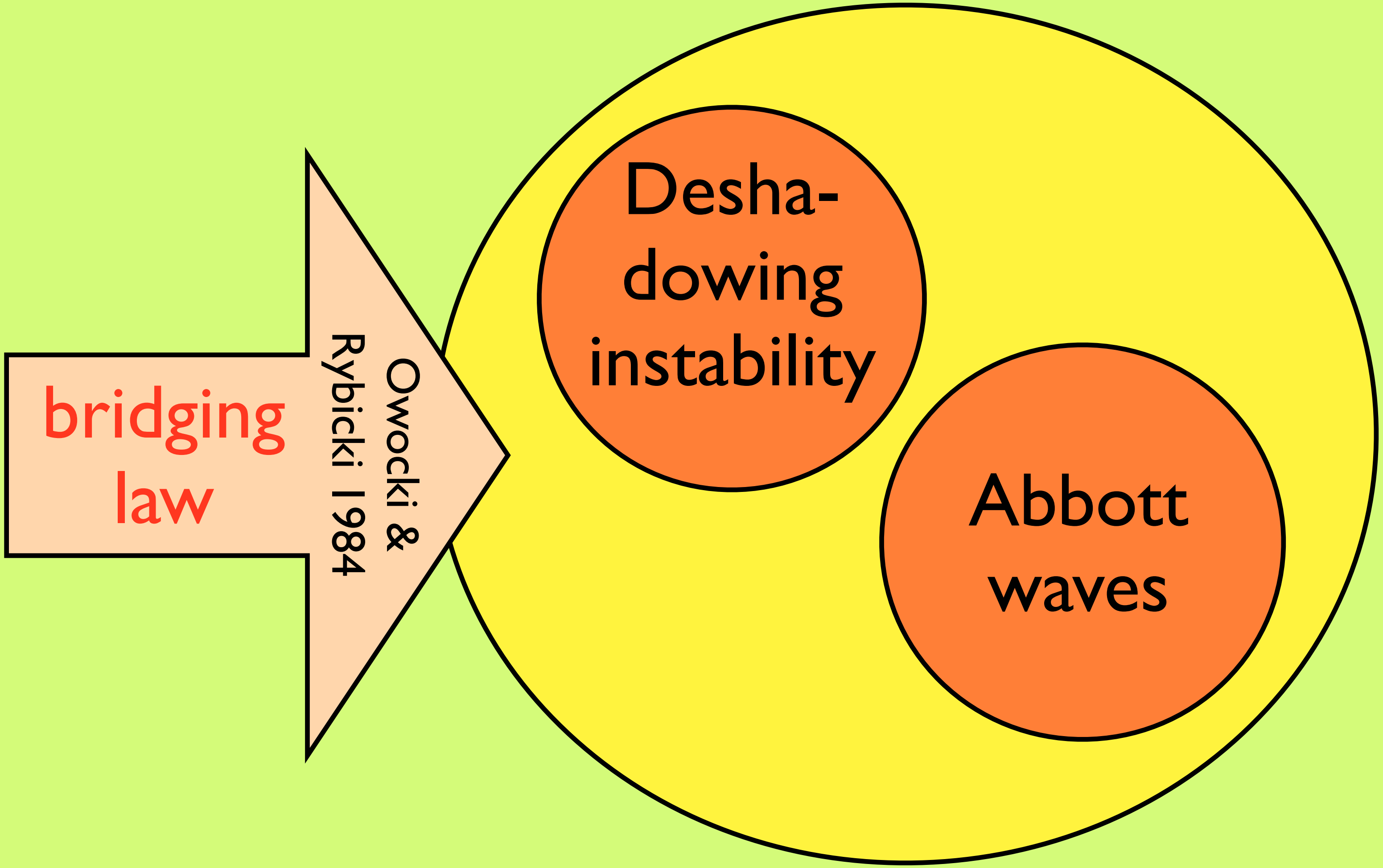


SPEED
2 Linear instability
4 nonlinear
photospheric perturb.
10 Lamb ringing (period.)
24 Langevin turb.
DENSITY ρ(r)
RADIUS
7 basic entity
9 two-stage instability
20 reverse shock destruction
30 sometimes giving forward shocks
23 clouds in rarefaction region
A path through unstable wind structure
Numbers refer to the paragraph numbering in the main text, where the effects are discussed
Unless otherwise stated, the velocity law of the wind is shown as function of distance from the wind source
3 Macroscopic mechanism
second order Sobolev
5 shock jumps
curvature growth
unstable Abbott wave
4 nonlinear steepening
5 reverse shocks
merge into
7 basic entity
12 rarefaction region
11 reverse shock
17 inner shell velocity law
13 Abbott kink
15 forward shock?
9 two-stage instability
in between basic entities secondary shocks form
and overtake old ones
coherent & turbulent version of same process?
and
8 subshells
any relation between sub- and secondary structure?
1 Microscopic mechanism
more light!
velocity perturb.
thermal band
blocked rad.
fluid parcel
without scattering
vs.
6 Line drag: with scattering
1
-1/2
1/2
back-scattering
stabilization
thermal band
.... 14 EISF noise ....
ρ (r)
16 shell dens.
stat.
13 contact disc.
forw. shock?
11 rev shock
12 raref. region
18 steady, radiative shocks
adiab. shock
Ma 2
4
T(r)
cooling layer
cooling instab.
isothermal shock
19 oscillat. thermal instal
secondary shock
cooling zone expands & contracts
wind structure in 2-D
23 clouds
25 cloud ablation
shell
cloud
stat
X-ray variability
1D spher. symm.
clouds
all spheres
future 2D:
27 isotrop turbul.
28 shell fragments
variability
flux constancy
20 shock destruction:
21 shocks become adiabatic
29 S&A shells
cooling zone broadens
26 cloud-shell collision
dense and hot
X-ray emission
from 1D to 2D...
31 symmetric X-ray lines
star
shell fragm.
observer
escaping X-ray
absorbed X-ray
22 outer corona
...leaving hot, thin gas
....32 EISF microdynamics?....

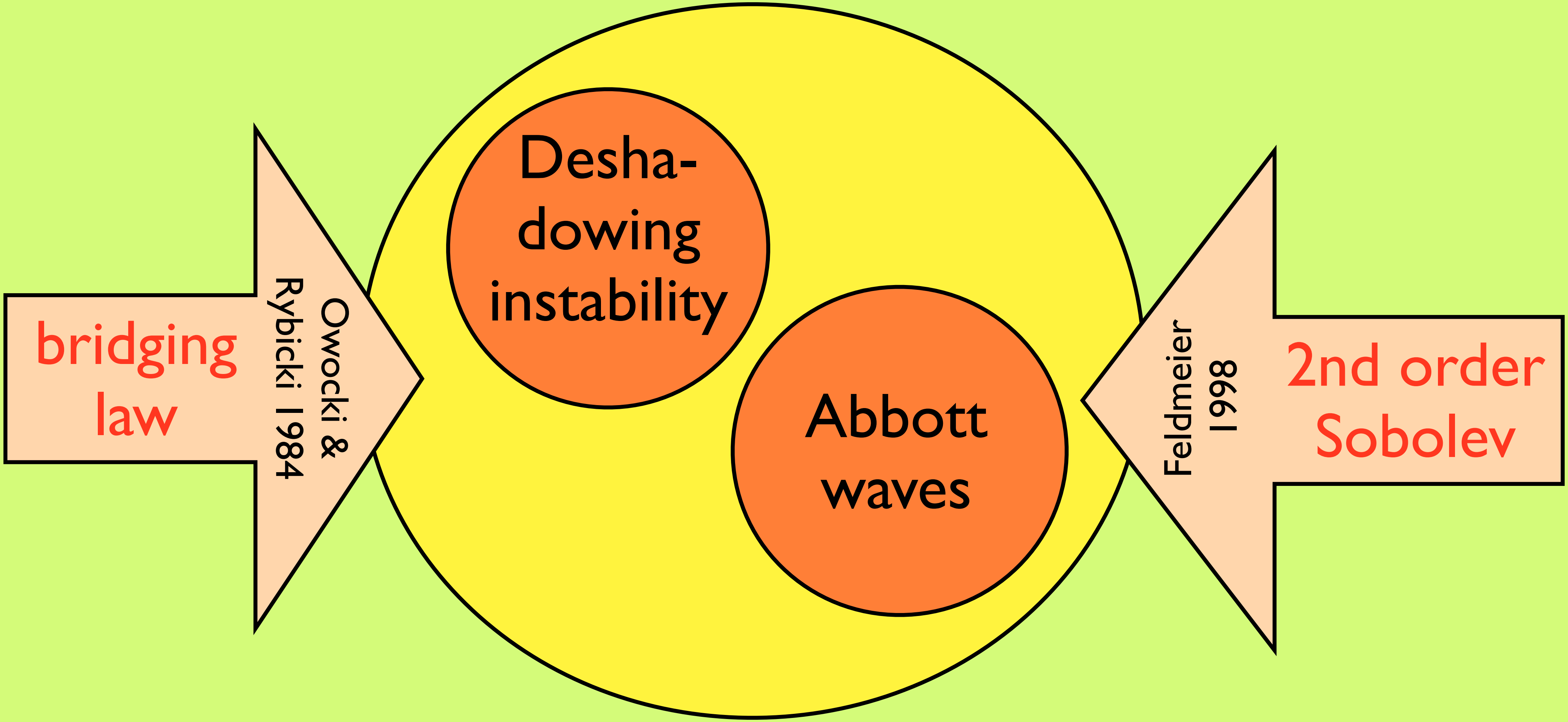
ABBOTT WAVES




Desha-
dowing
instability
Abbott
waves

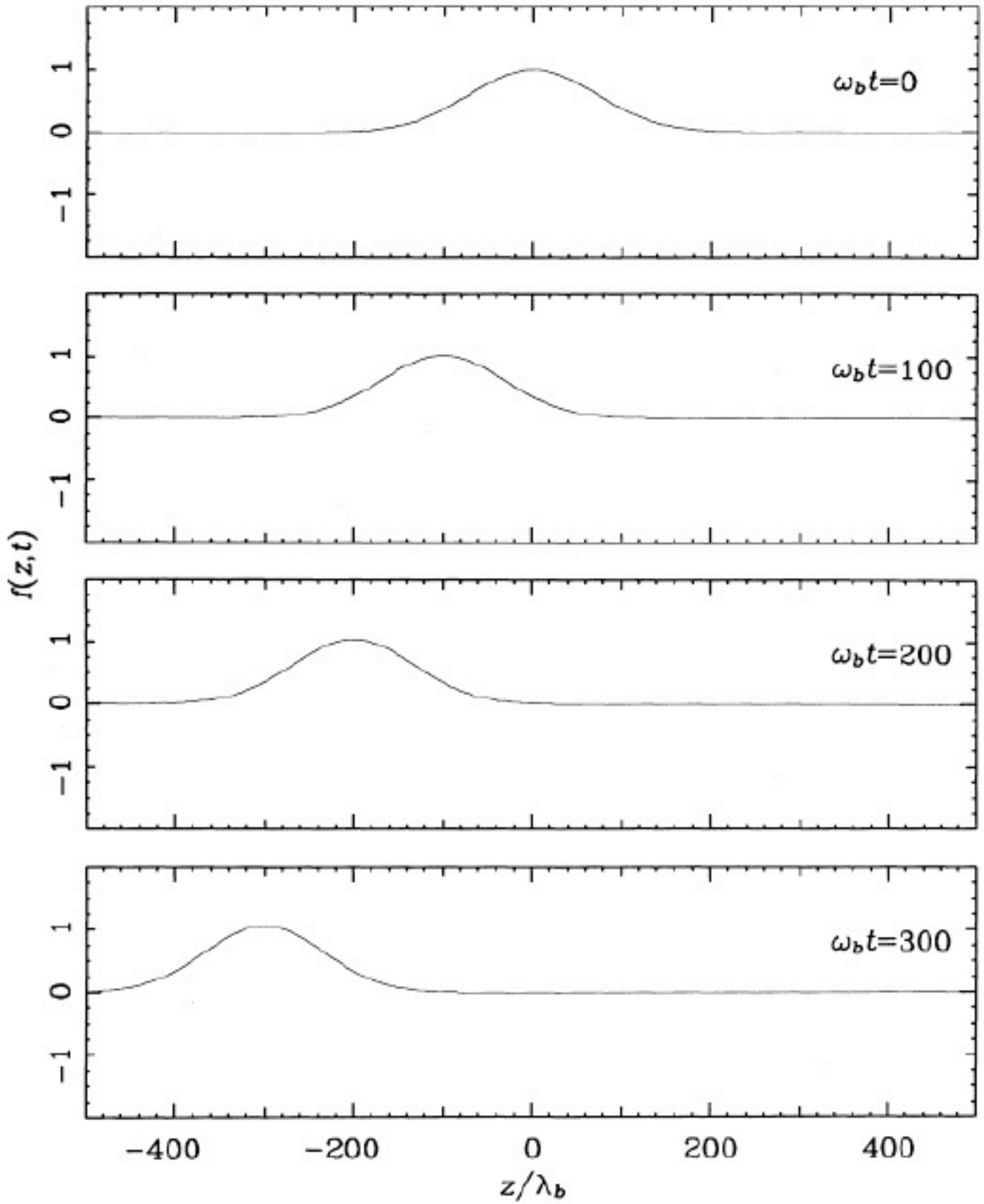


bridging law
Owocki & Rybicki 1984
Desha-dowing instability
Abbott waves


bridging law
Owocki & Rybicki 1984
Desha-dowing instability
Abbott waves
Feldmeier 1998
2nd order Sobolev

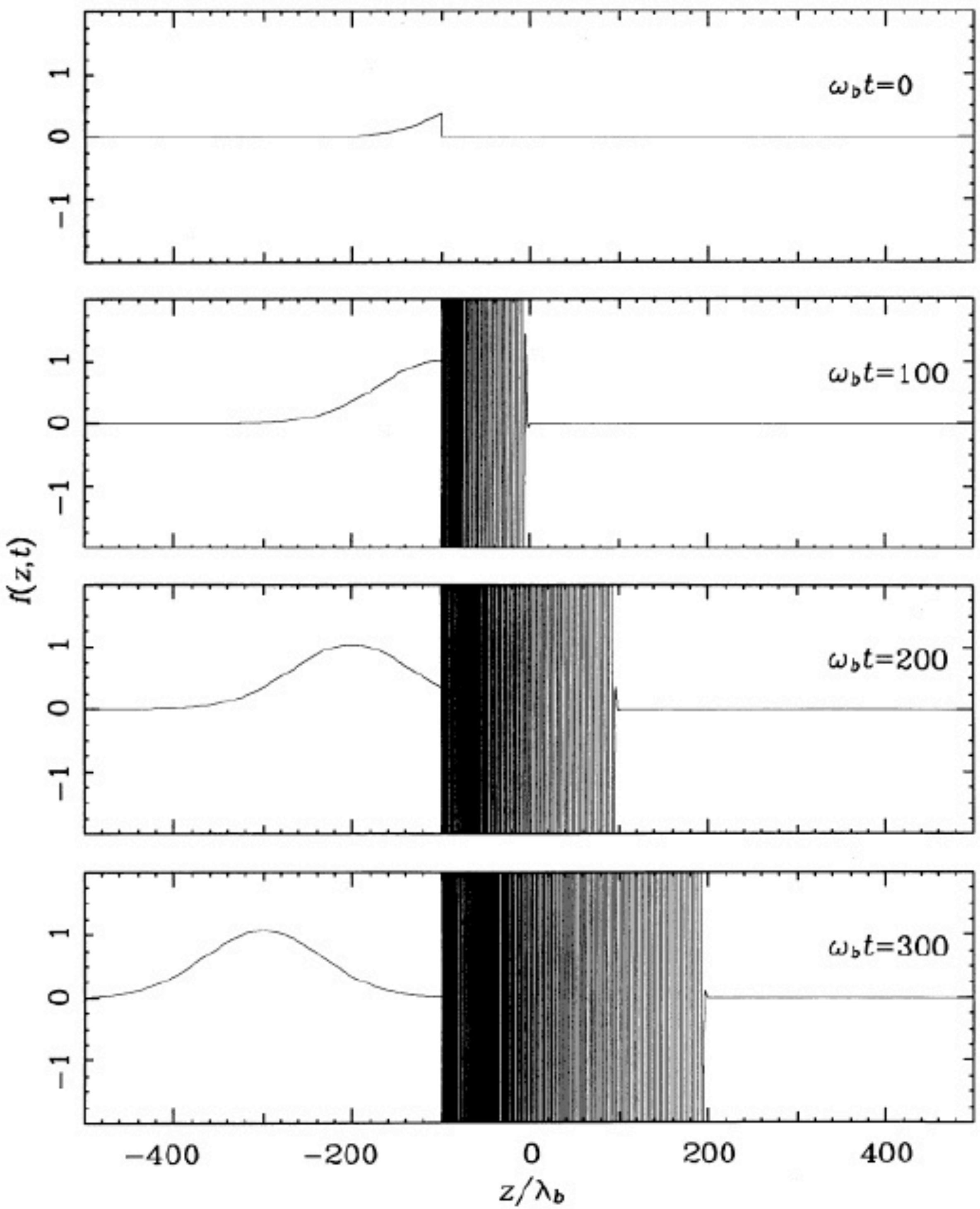
GREENS

FUNCTION



Propagation of smooth Gaussian pulse as Abbott wave

Owocki & Rybicki 1986, ApJ



Reconstruction & propagation of FULL Gaussian from TRUNCATED one

Run-

away

I.

IONS

Chandrasekhar

1943

Dynamical friction: momentum exchange due to gravity

DYNAMICAL FRICTION

I. GENERAL CONSIDERATIONS: THE COEFFICIENT OF DYNAMICAL FRICTION

S. CHANDRASEKHAR
Yerkes Observatory
Received January 7, 1943

ABSTRACT

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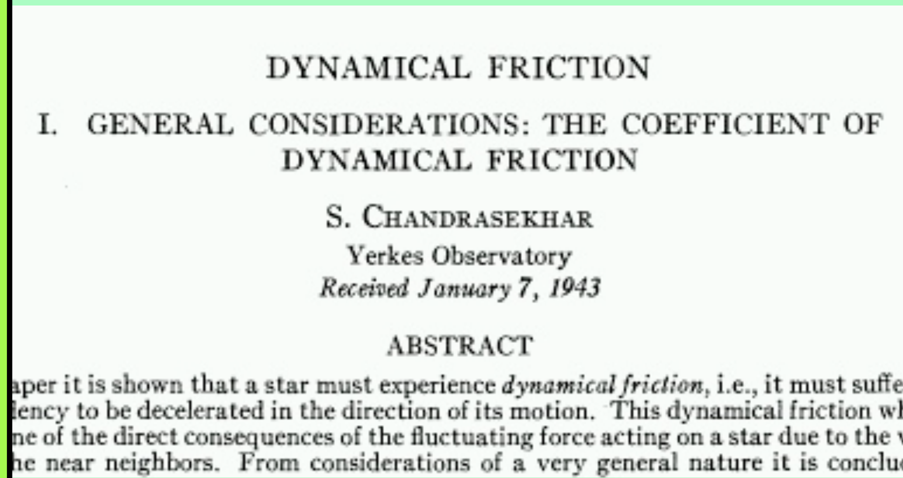
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Castor, Abbott, Klein

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Coulomb interactions between rad-acc ions and protons

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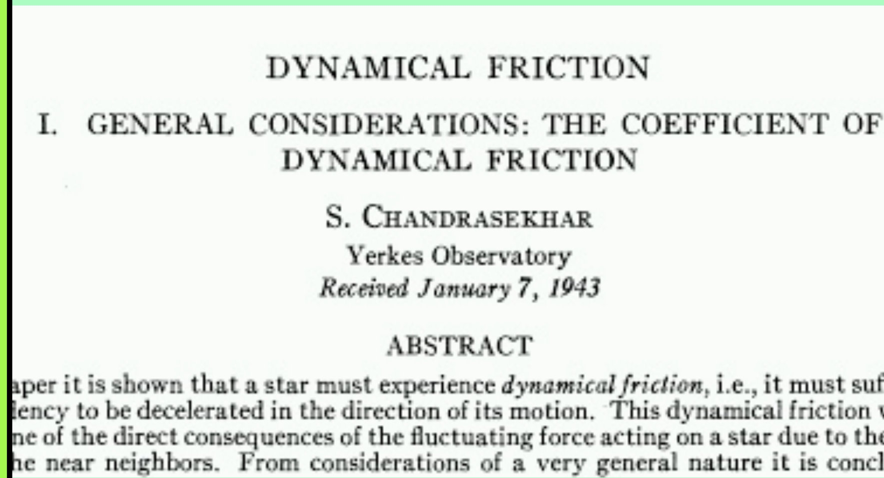
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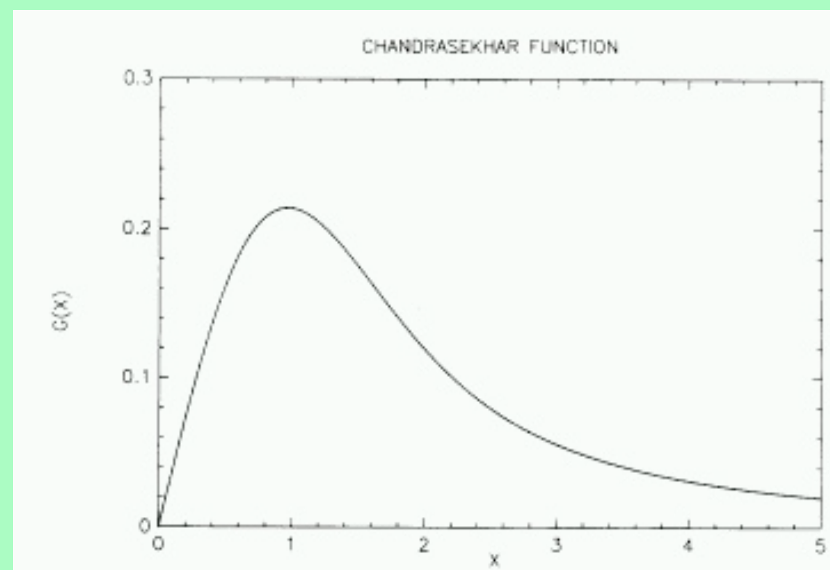
1976

Coulomb interactions between rad-acc ions and protons

Springmann & Pauldrach

1992

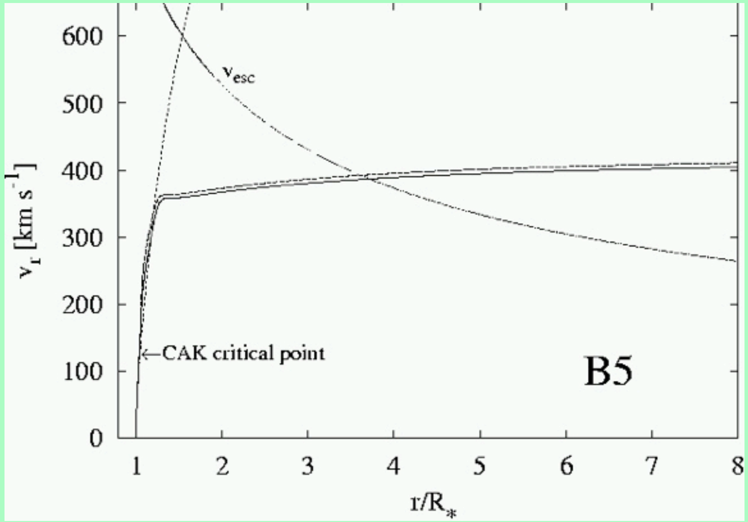
Ion runaway after decoupling (see plasma literature of 1950ies)

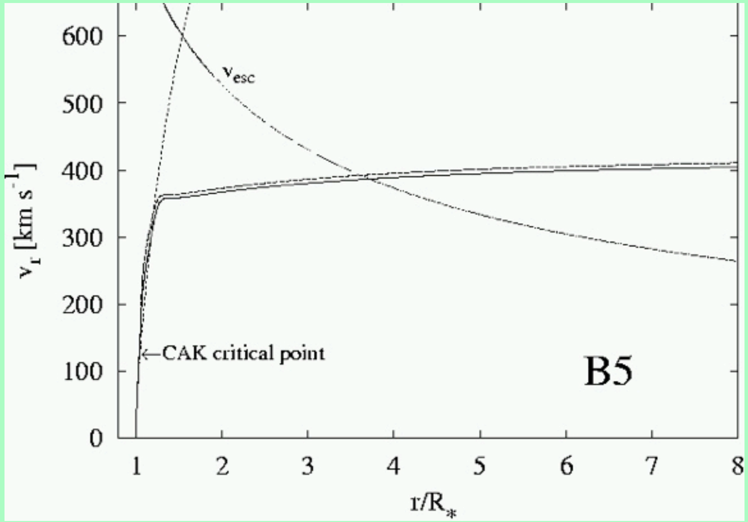
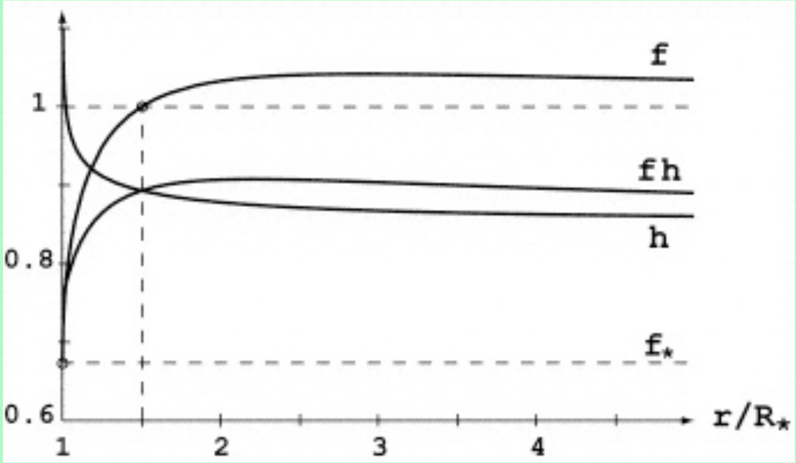


Krticka & Kubat

2000

Ions and protons switch together (single fluid!) to slow solution branch



Krticka & Kubat	2000	Ions and protons switch together (single fluid!) to slow solution branch	
Owocki & Puls	2002	This is a breeze solution. "Unstable"?	

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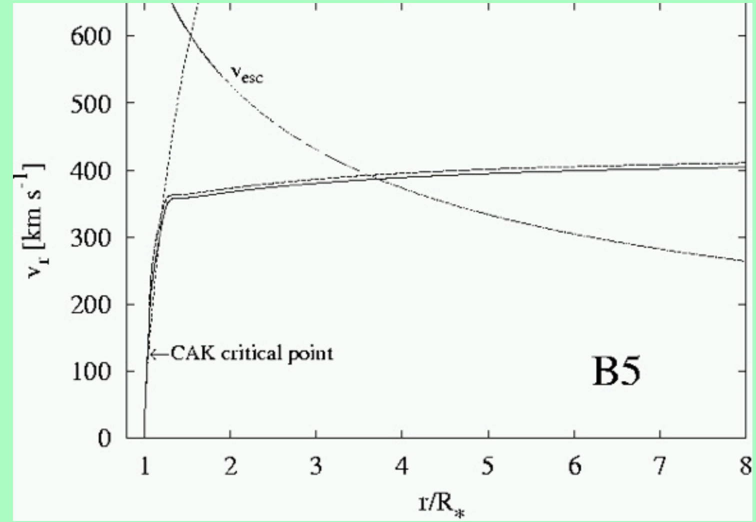

Owocki & Puls

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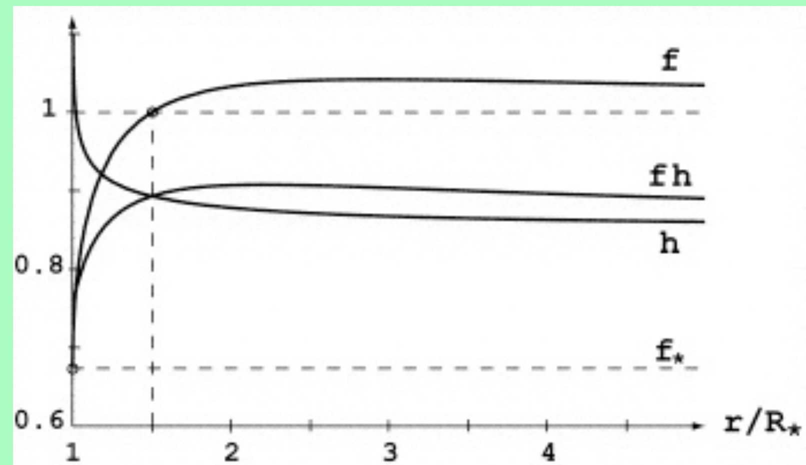
This is a breeze solution.

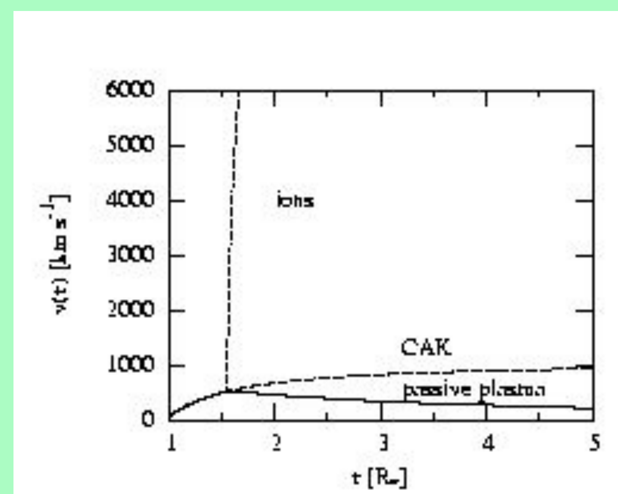
"Unstable"?



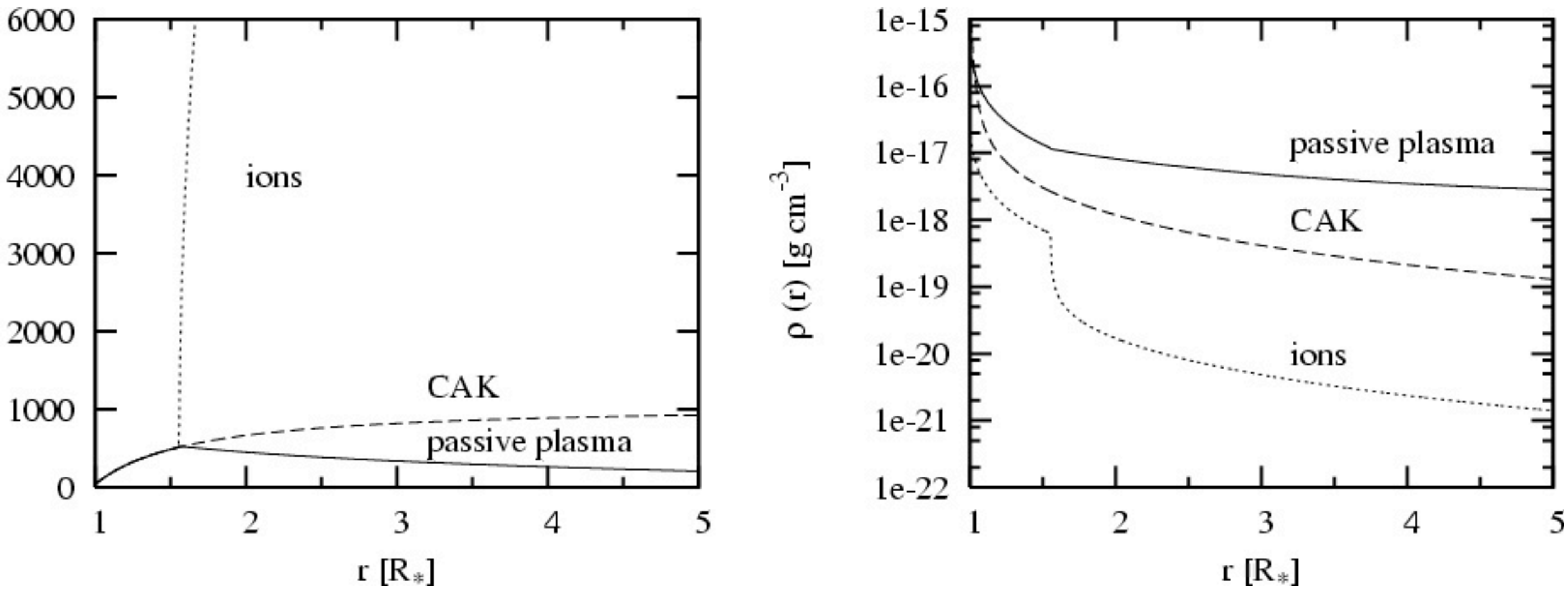
Votruba, Feldmeier, Kubat

2007

First time-dependent hydrodynamic simulation of thin wind



ION DECOUPLING IN THIN WINDS



Votruba, Feldmeier, Kubat 2007, A&A

II.

WAVES

Start-up Question:

why does wind prefer unique critical solution over infinite variety of breeze solutions?

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Answer, I. part (Abbott 1980, ApJ)

CAK critical point is to Abbott waves,
what sonic point is to sound waves: information barrier

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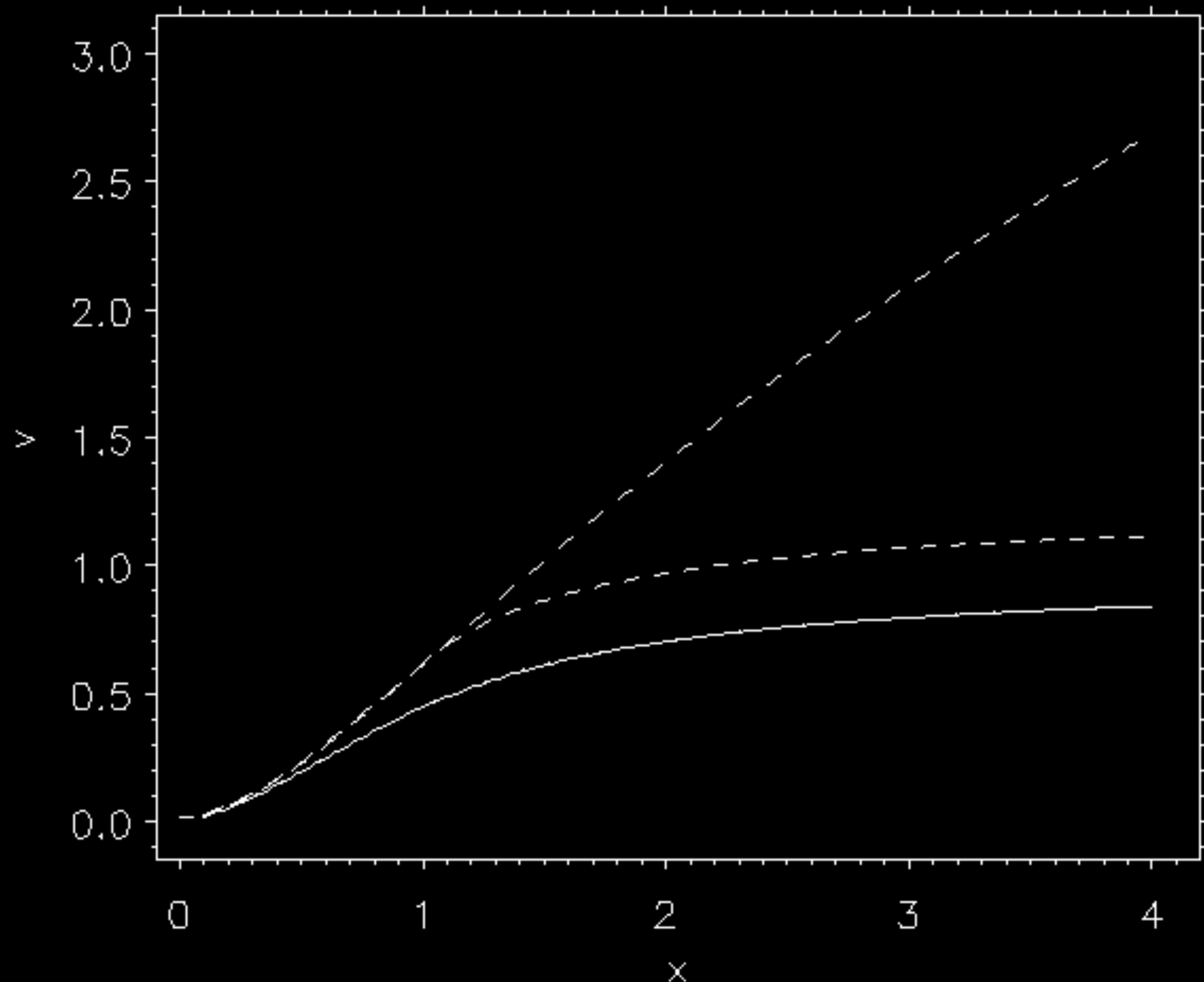
why does wind prefer unique critical solution
over infinite variety of breeze solutions?

Answer, I. part (Abbott 1980, ApJ)

CAK critical point is to Abbott waves,
what sonic point is to sound waves: information barrier

Answer, II. part (Feldmeier & Shlosman 2000 & 2002, ApJ)

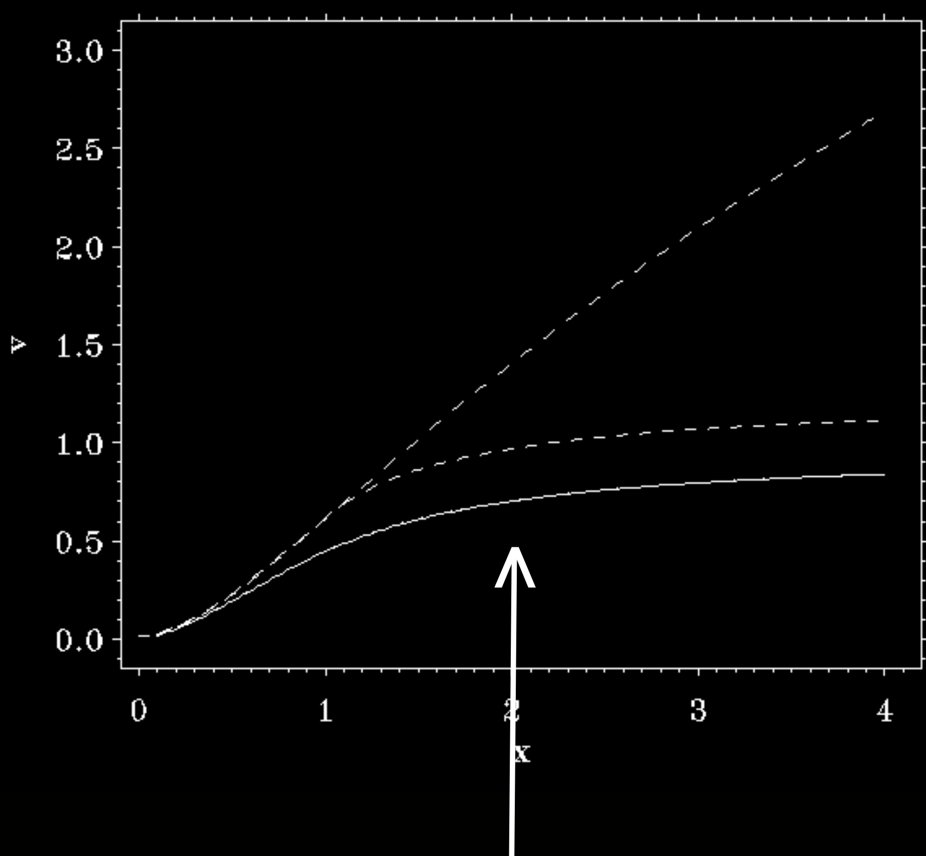
Strange dispersion of Abbott waves accelerates wind
till CAK solution is reached

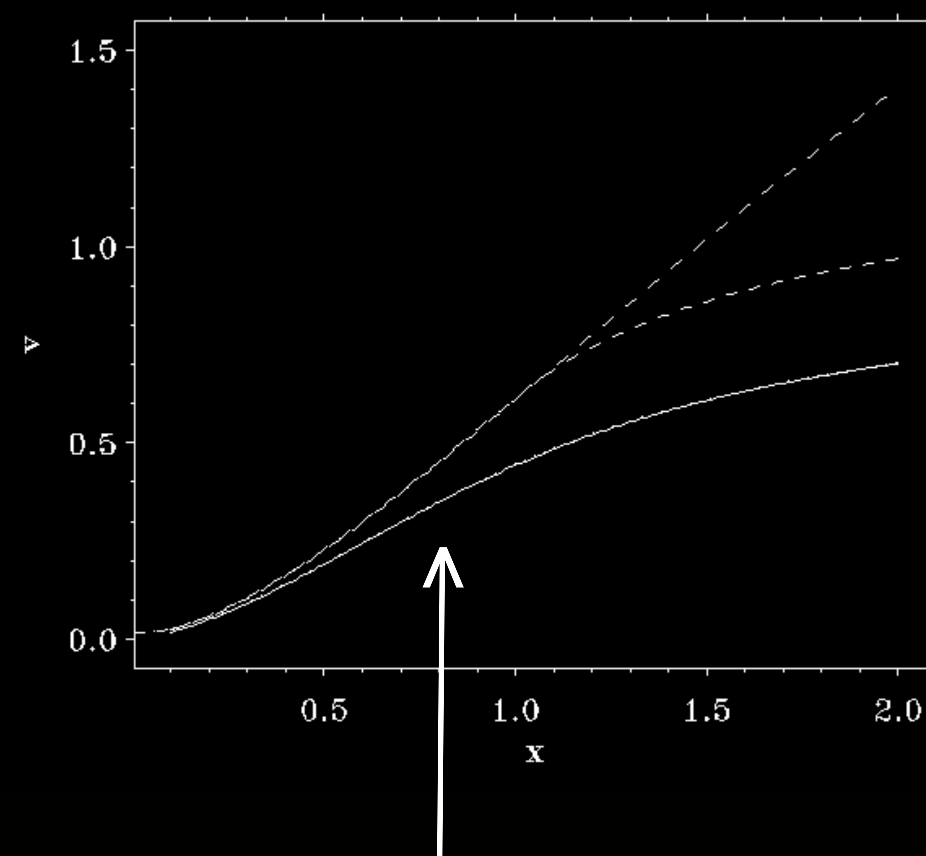


3.0
2.5
2.0
1.5
1.0
0.5
0.0
v
0
1
2
3
4
x

Study physical origin of runaway

...including new solution types



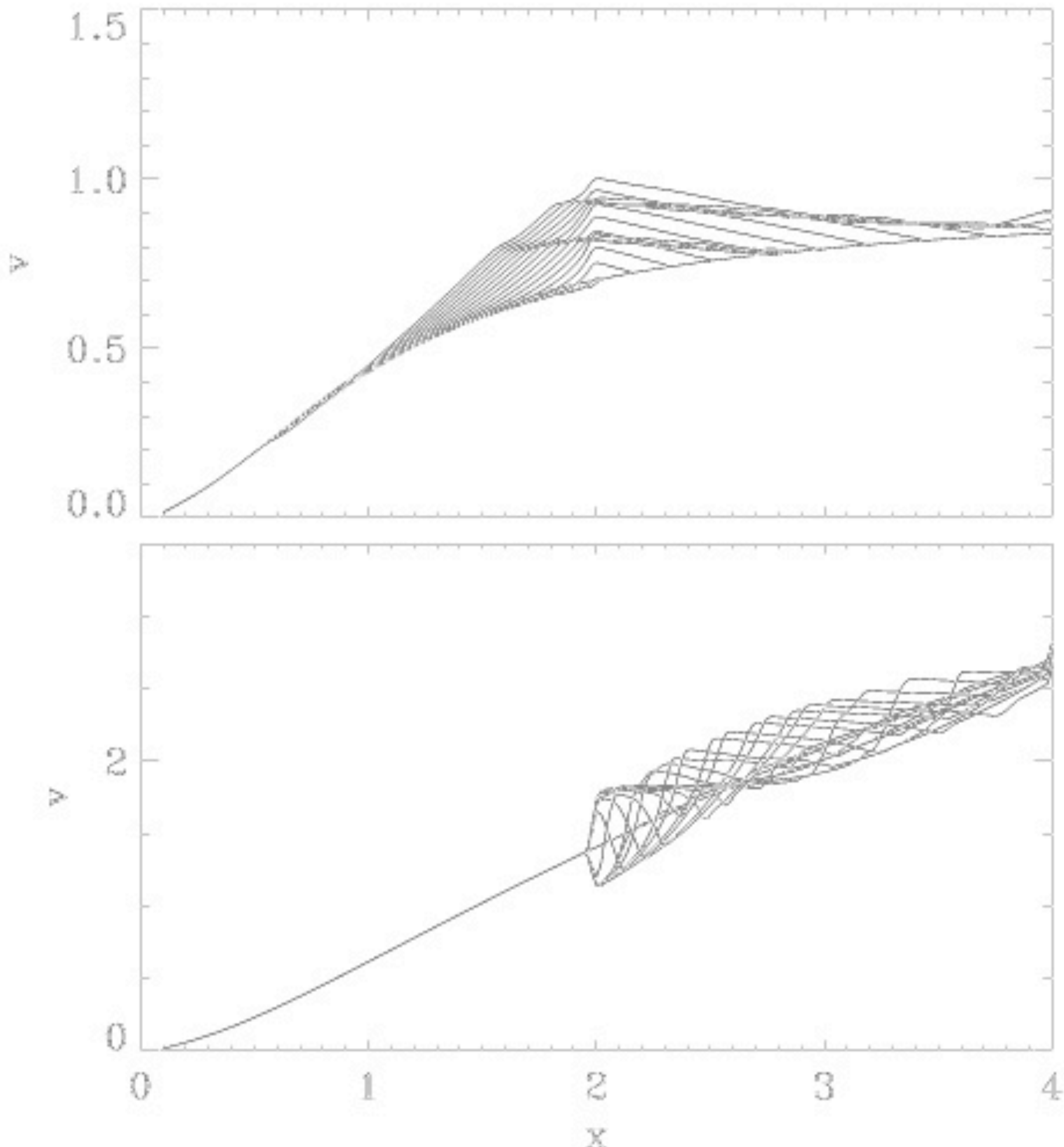


If perturbation is strong enough to cause v’<0: Abbott wave runaway

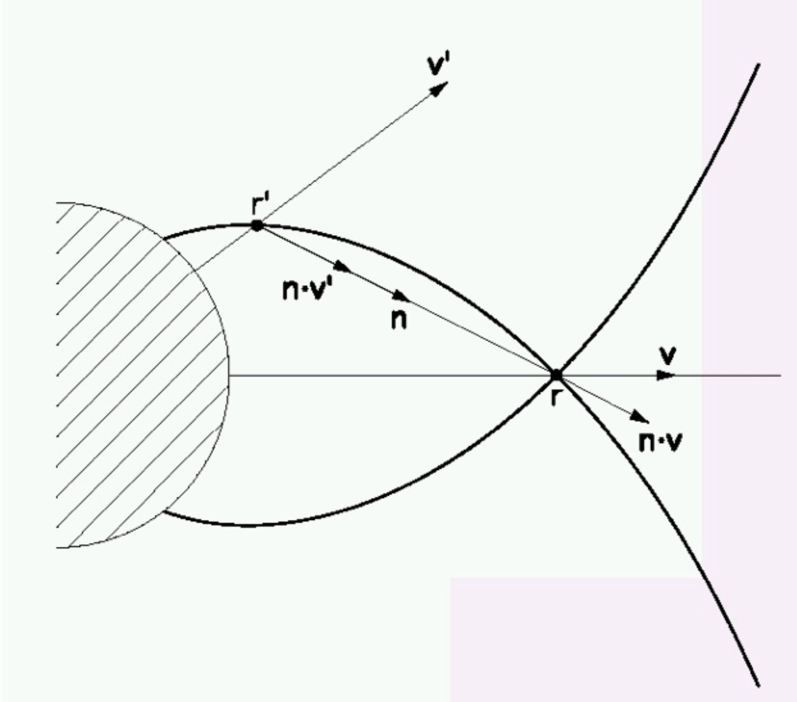
If perturbation is located sufficiently far in: overloading

Wave runaway of breeze wind

Critical CAK wind is stable



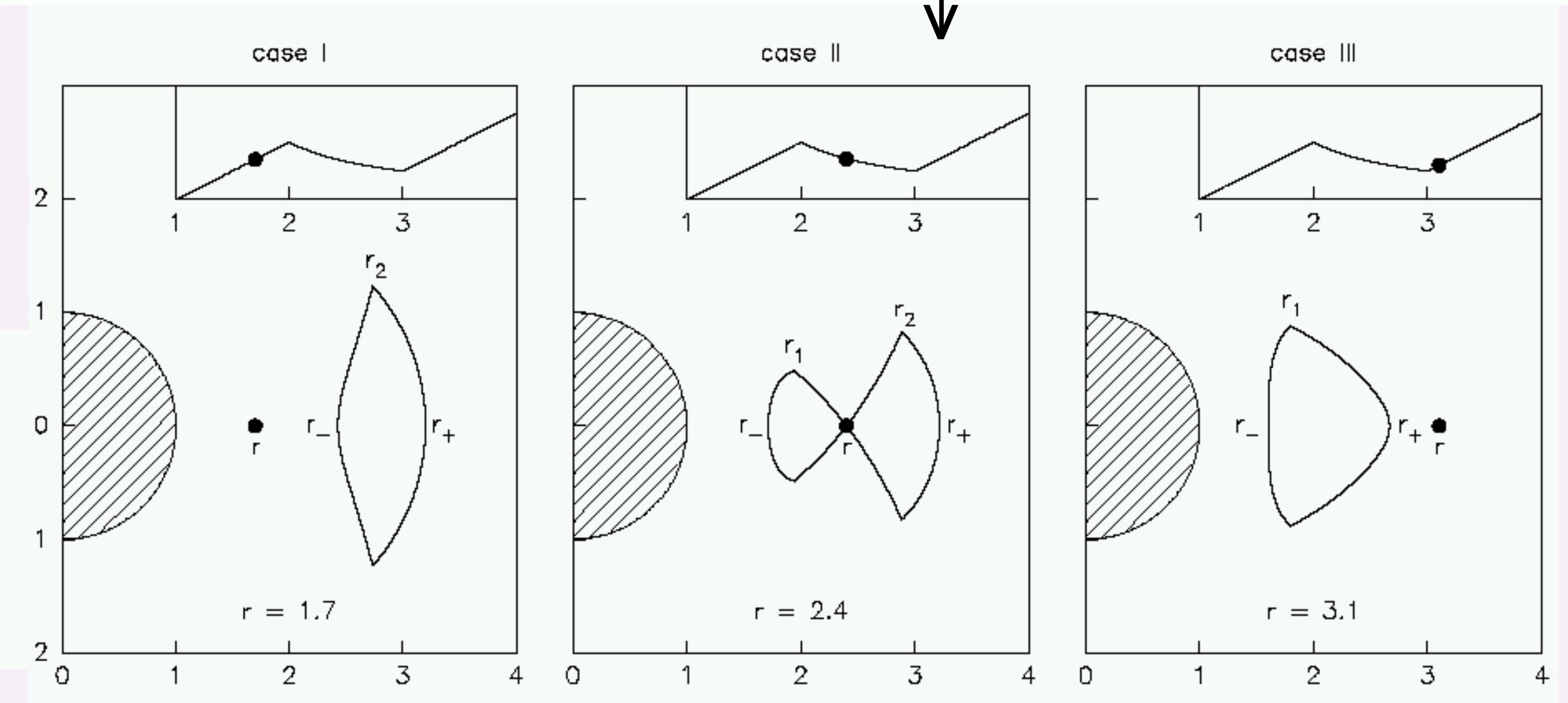
Multiple Resonance



← Rybicki & Hummer 1978, ApJ

Shape of resonance region of one point

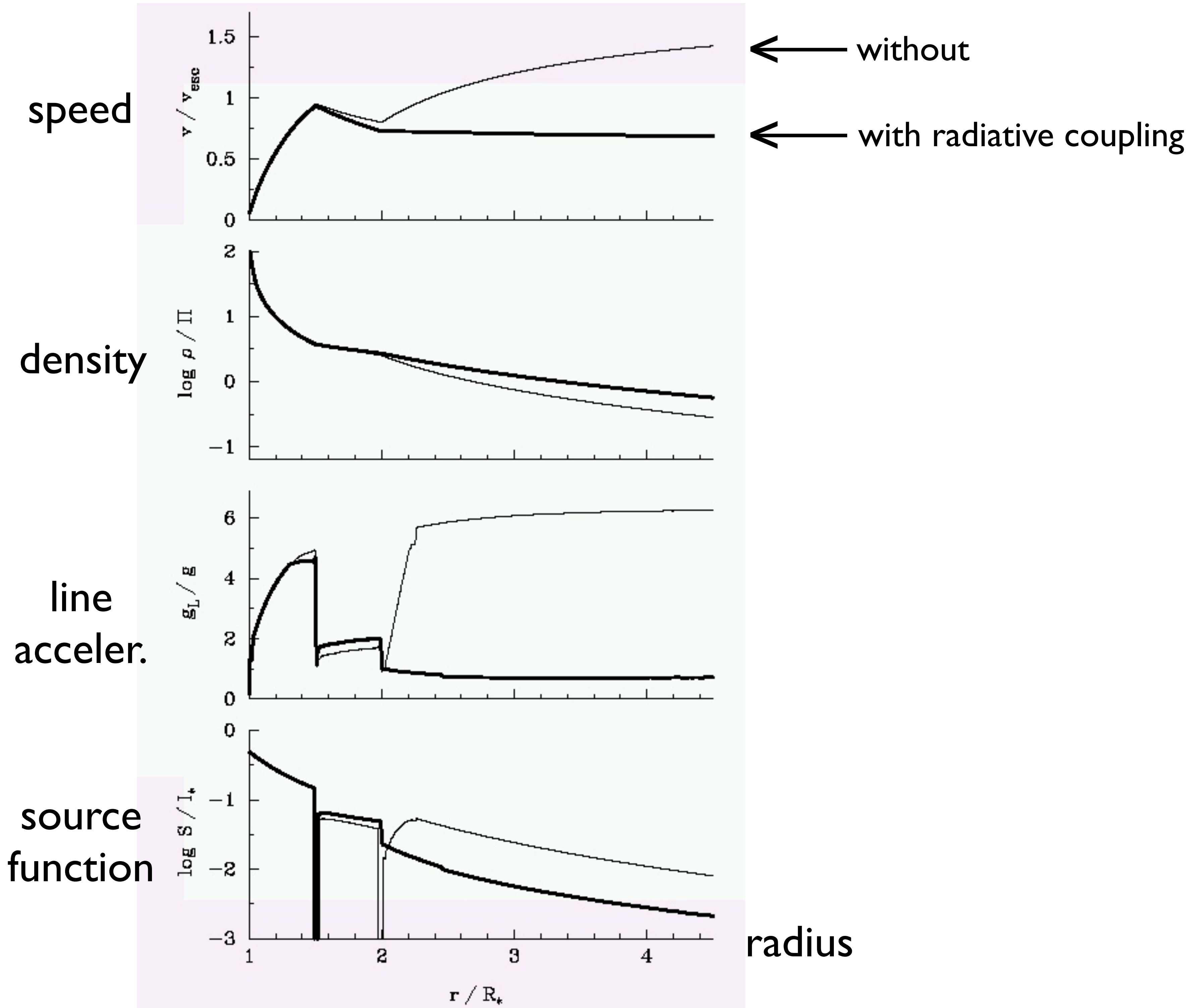
Feldmeier & Nikutta 2006, A&A



Integral equation for source function

$$\left\{ \begin{array}{l} \dfrac{g_{\mathrm{L}}(r)}{g_* \ \Xi} - \tilde{\gamma}_{\mathrm{Lc}}(r) \\ \dfrac{S_{\mathrm{e}}(r)\tilde{\beta}_{\mathrm{L}}(r)}{I_*} - \tilde{\beta}_{\mathrm{Lc}}(r) \end{array} \right\} = \frac{1}{2}\kappa_0 v_{\mathrm{th}} \frac{R\Pi}{V} \int_{r_-}^{r_+} \mathrm{d}r' r'^2 \rho' \left\{ \begin{array}{l} \Upsilon_1 \\ \Upsilon_2 \end{array} \right\} \frac{S_{\mathrm{e}}(r')}{I_*}$$

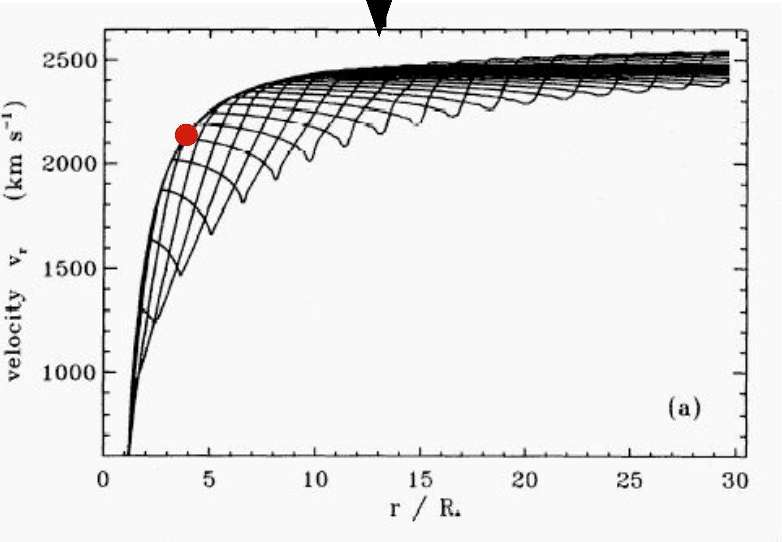
$$\times \frac{1}{\tau_0 \tau_0'} \left\{ \left(\tau_0 + \tau_0''\right)^{1-\alpha} + \left(\tau_0' + \tau_0''\right)^{1-\alpha} - \left(\tau_0 + \tau_0' + \tau_0''\right)^{1-\alpha} - \tau_0''^{\,1-\alpha} \right\}. \qquad (28)$$



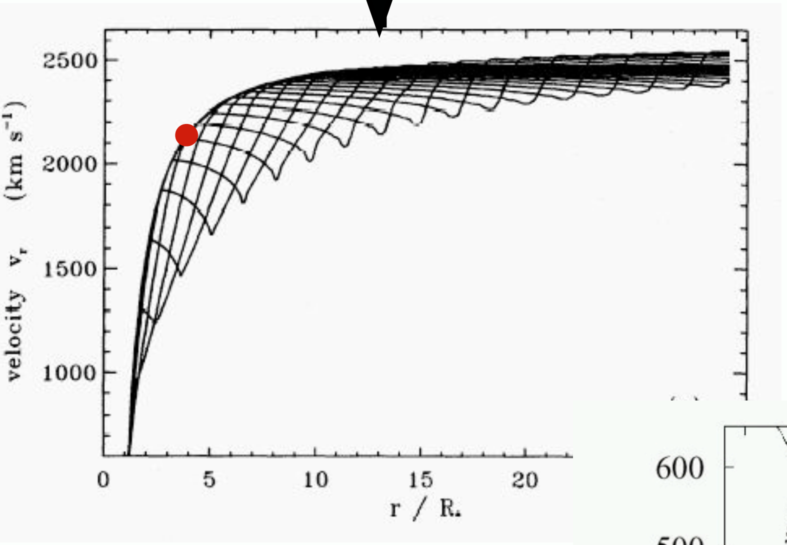
speed
density
line acceler.
source function
without
with radiative coupling
radius
v / v_esc
log ρ / Π
g_L / g
log S / I_*
r / R_*

kinks

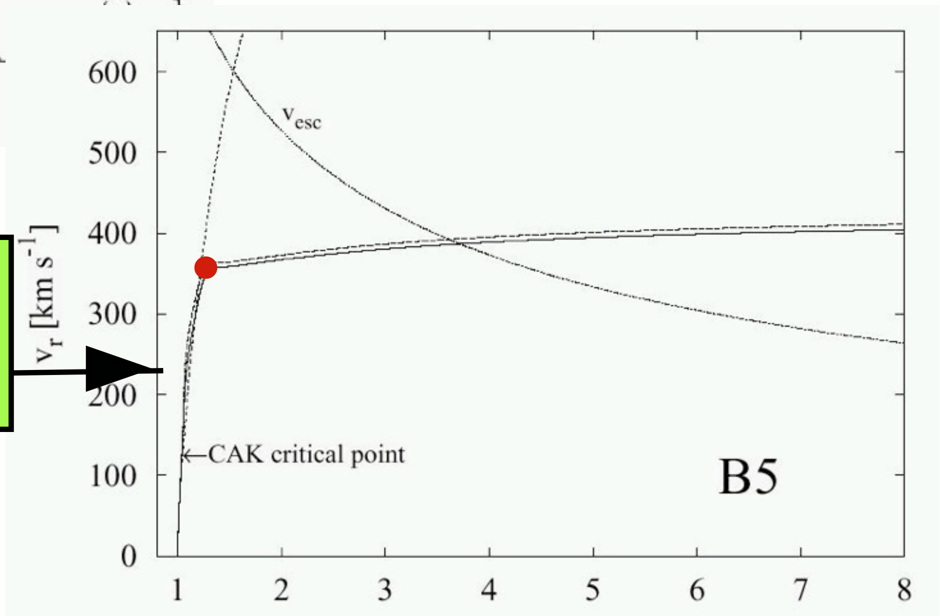
Cranmer & Owocki 1996



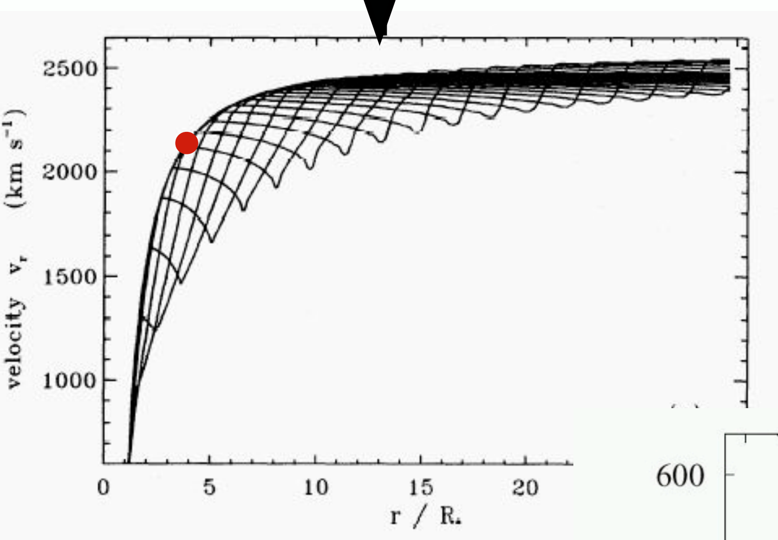
Cranmer & Owocki 1996



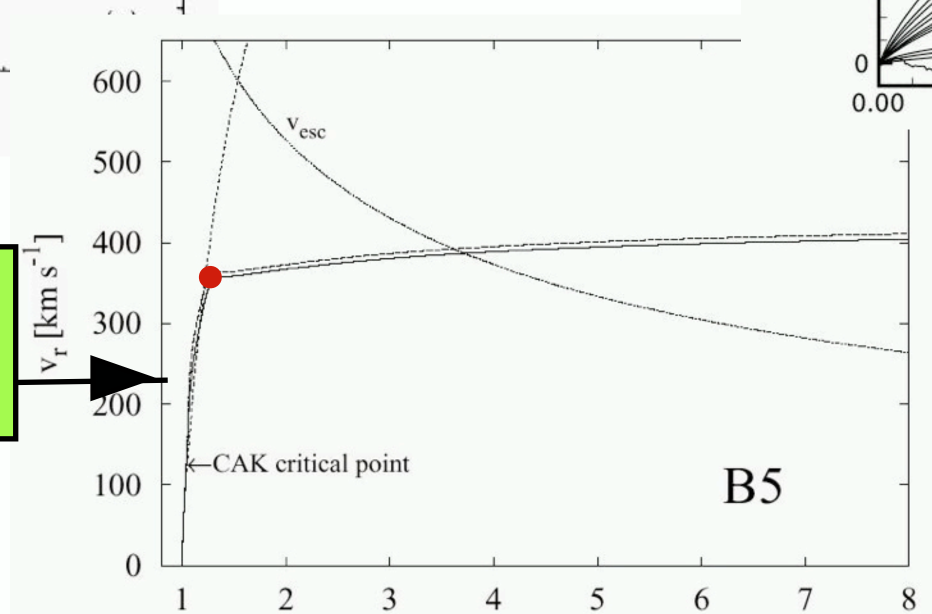
Krticka & Kubat 2000



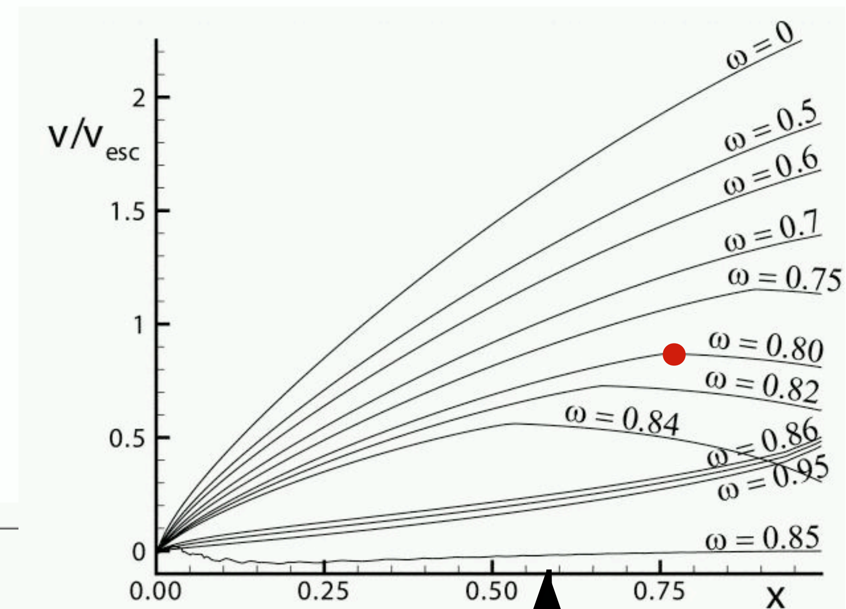
Cranmer & Owocki 1996

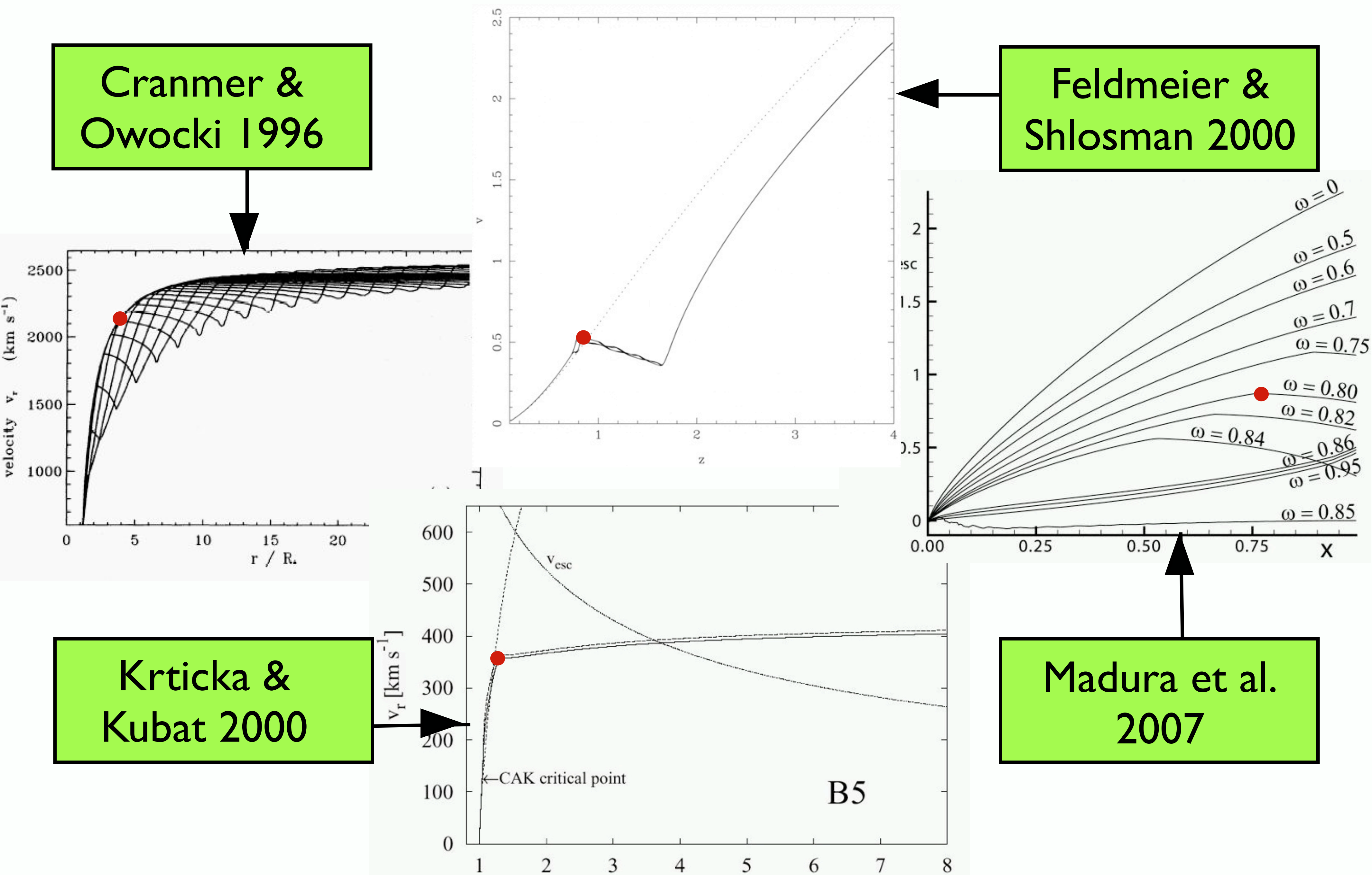


Krticka & Kubat 2000



Madura et al. 2007





Cranmer & Owocki 1996
Feldmeier & Shlosman 2000
Krticka & Kubat 2000
Madura et al. 2007
velocity v_r (km s^{-1})
r / R*
v
z
v_r [km s^{-1}]
v_{esc}
←CAK critical point
B5
ω = 0
ω = 0.5
ω = 0.6
ω = 0.7
ω = 0.75
ω = 0.80
ω = 0.82
ω = 0.84
ω = 0.86
ω = 0.95
ω = 0.85
x

$$[\dot{v}] = -U[v']$$

$$q(x_2,t)-q(x_1,t)=\frac{d}{dt}\int_{x_2}^{s(t)}\rho(x,t)\,dx+\frac{d}{dt}\int_{s(t)}^{x_1}\rho(x,t)\,dx$$

$$=\rho(s^-,t)\dot{s}-\rho(s^+,t)\dot{s}+\int_{x_2}^{s(t)}\rho_t(x,t)\,dx+\int_{s(t)}^{x_1}\rho_t(x,t)\,dx,$$

$$[\dot{v}]=-U[v']$$

$$[\dot{v}] = -U[v']$$

$$[\dot{v}] = -U\,[v']$$

1 where $[f] = f_{right} - f_{left}$ for any function $f(x,t)$

$$[\dot{v}] = -U\,[v']$$

1. where $[f] = f_{right} - f_{left}$ for any function $f(x,t)$
2. $v = v(x,t)$ is wind velocity

$$[\dot{v}] = -U\,[v']$$

1. where $[f] = f_{right} - f_{left}$ for any function $f(x,t)$
2. $v = v(x,t)$ is wind velocity
3. U is kink speed

Kink Speed

Feldmeier, Raetzel, & Owocki 2008, ApJ

Kink Speed

Feldmeier, Raetzel, & Owocki 2008, ApJ

Euler eq at kink

$$[\dot{v}] + v[v'] = K\left[\sqrt{v'/\varrho}\,\right]$$

Kink Speed

Feldmeier, Raetzel, & Owocki 2008, ApJ

Euler eq at kink	$[\dot{v}] + v[v'] = K\left[\sqrt{v'/\varrho}\,\right]$
Whitham relation	$[\dot{v}] = -U[v']$

Kink Speed

Feldmeier, Raetzel, & Owocki 2008, ApJ

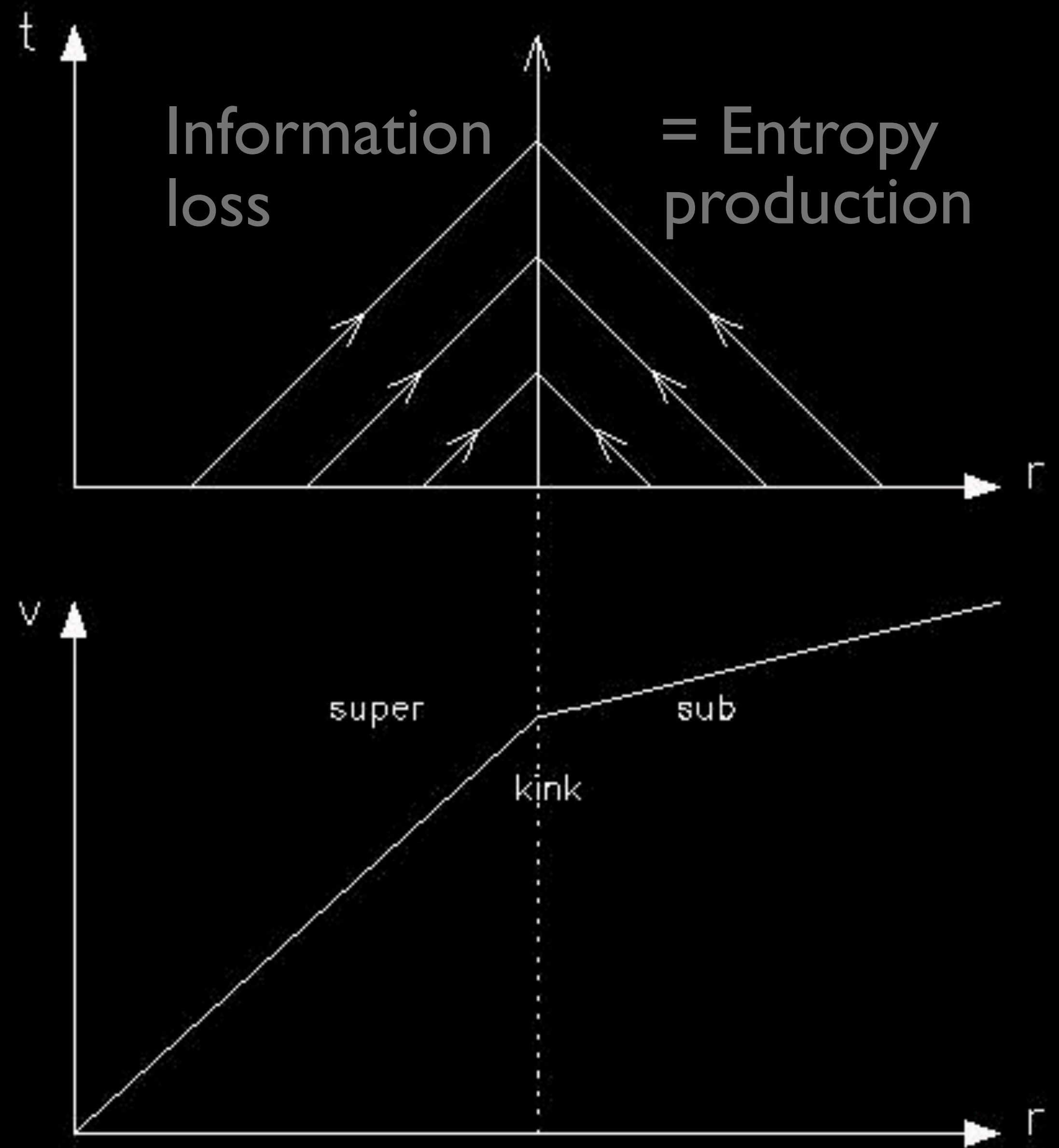
Euler eq at kink	$[\dot{v}] + v[v'] = K\left[\sqrt{v'/\varrho}\,\right]$
Whitham relation	$[\dot{v}] = -U[v']$
Kink speed	$U = v - \dfrac{K}{\sqrt{\varrho}} \dfrac{[\sqrt{v'}]}{[v']} \overset{v_r'=0}{=} v - \dfrac{K}{\sqrt{\varrho v_l'}}$

Kink Speed

Feldmeier, Raetzel, & Owocki 2008, ApJ

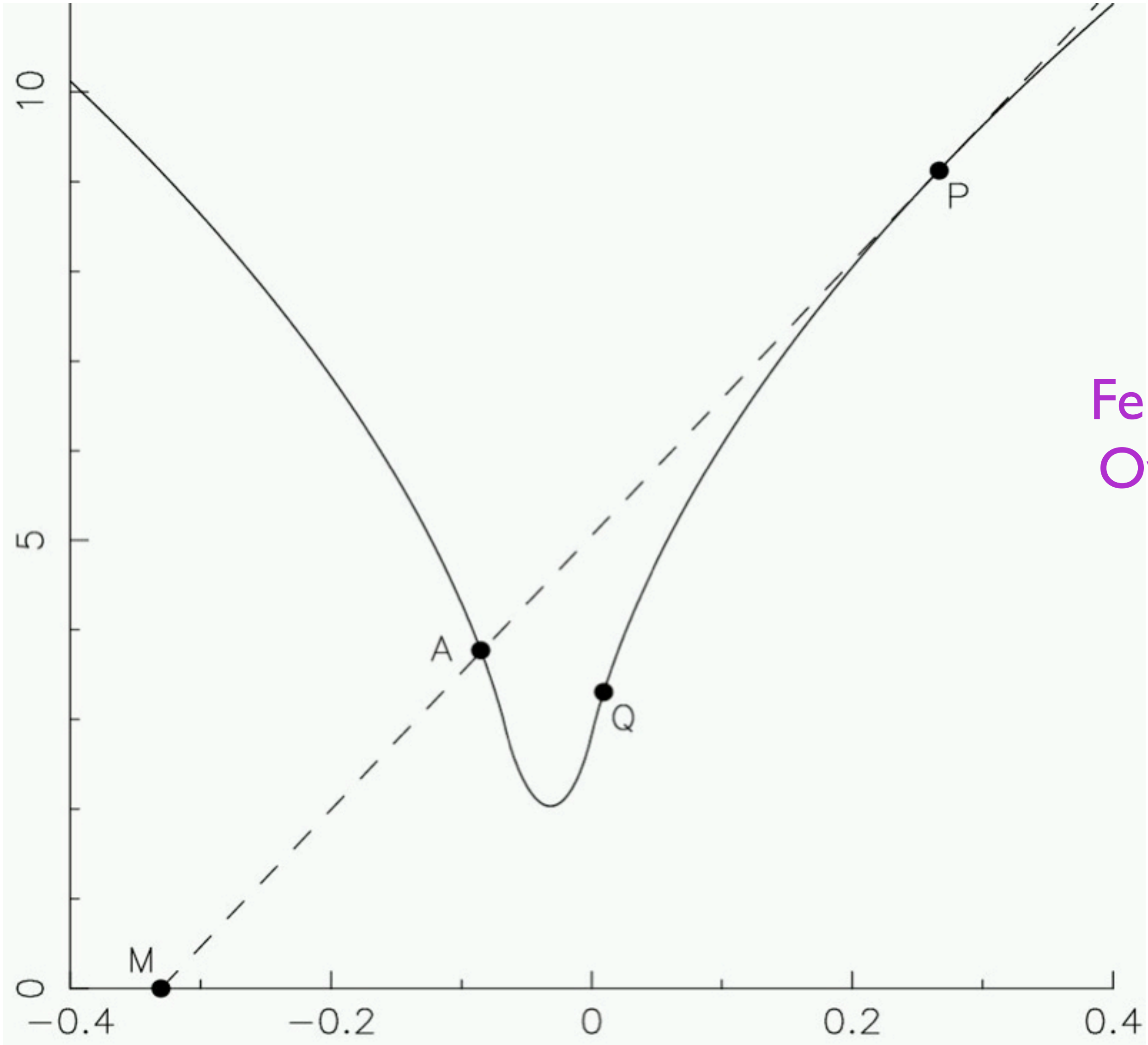
Euler eq at kink	$[\dot{v}] + v[v'] = K\left[\sqrt{v'/\varrho}\,\right]$
Whitham relation	$[\dot{v}] = -U[v']$
Kink speed	$U = v - \frac{K}{\sqrt{\varrho}} \frac{[\sqrt{v'}]}{[v']} \overset{v'_r=0}{=} v - \frac{K}{\sqrt{\varrho v'_l}}$
Abbott wave speed	$A = v - \frac{\partial(K\sqrt{v'/\varrho})}{\partial v'} = v - \frac{K}{2\sqrt{\varrho v'}}$

Kinks are new type of radiative(-acoustic) shock



A shock polytrope for kinks

$\frac{g_{line}}{g}$ (instead of p)



$\frac{r\ dv}{v\ dr}$ (instead of r)

Feldmeier, Raetzel, Owocki 2008, ApJ

G A S

G A S

Parker

1958

G A S

Parker

1958

v=a is saddle point in r-v plane

G	A	S	L	D	W

Parker

1958

$v=a$ is saddle point in r-v plane

GAS	LDW
Parker	Bjorkman
1958	1995

$v = a$ is saddle point in r-v plane

GAS	LDW
Parker	Bjorkman
1958	1995
$v=a$ is saddle point in r-v plane	$v=A$ is saddle point in r-v' plane

G A S

Parker

1958

v=a is saddle point in r-v plane

^

WAVE

weak discont.

L D W

Bjorkman

1995

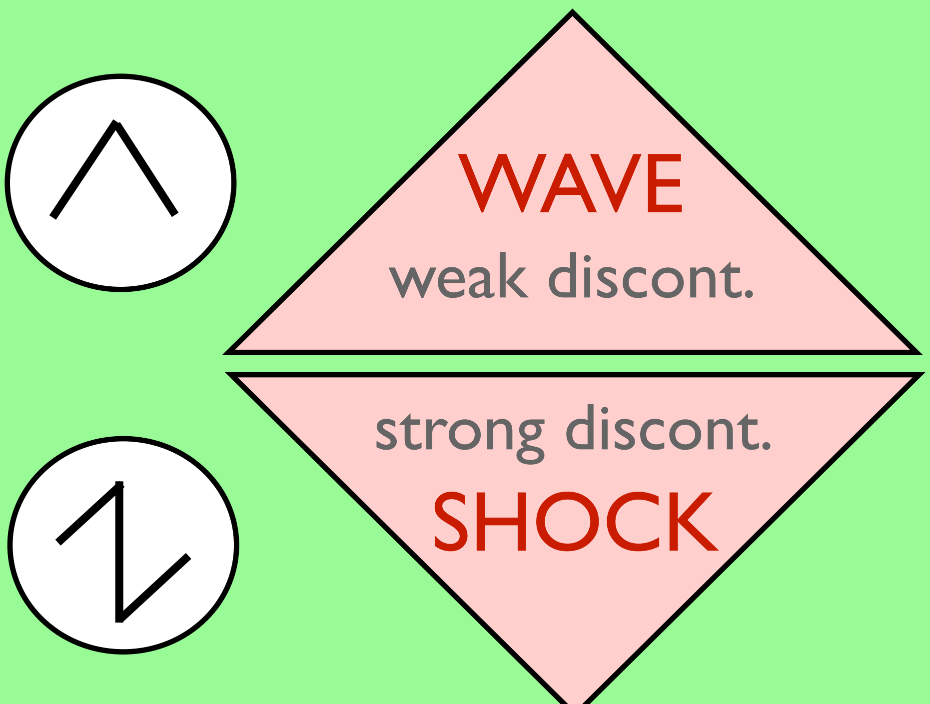
v=A is saddle point in r-v′ plane

G A S

Parker

1958

$v=a$ is saddle point in r-v plane



L D W

Bjorkman

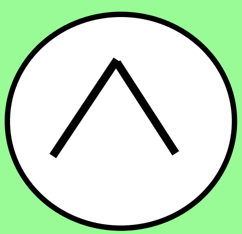
1995

$v=A$ is saddle point in r-v' plane

G A S

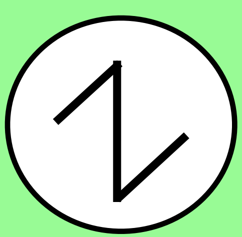
Parker | 1958

$v=a$ is saddle point in r-v plane



WAVE

weak discont.



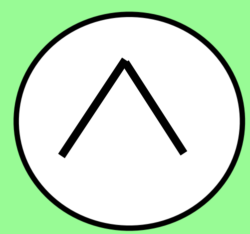
strong discont.

SHOCK

L D W

Bjorkman | 1995

$v=A$ is saddle point in r-v' plane



SHOCK

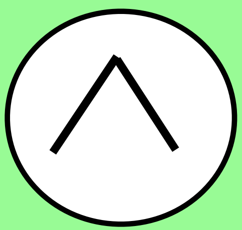
weak discont.

G A S

Parker

1958

$v=a$ is saddle point in r-v plane

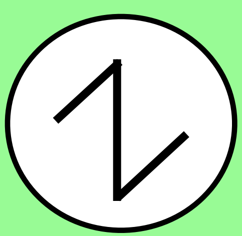


WAVE

weak discont.

strong discont.

SHOCK

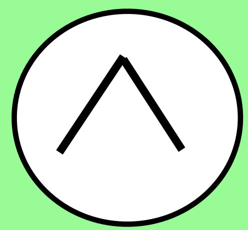


L D W

Bjorkman

1995

$v=A$ is saddle point in r-v' plane

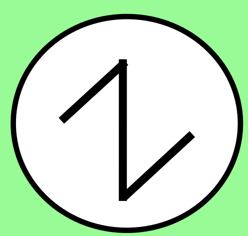


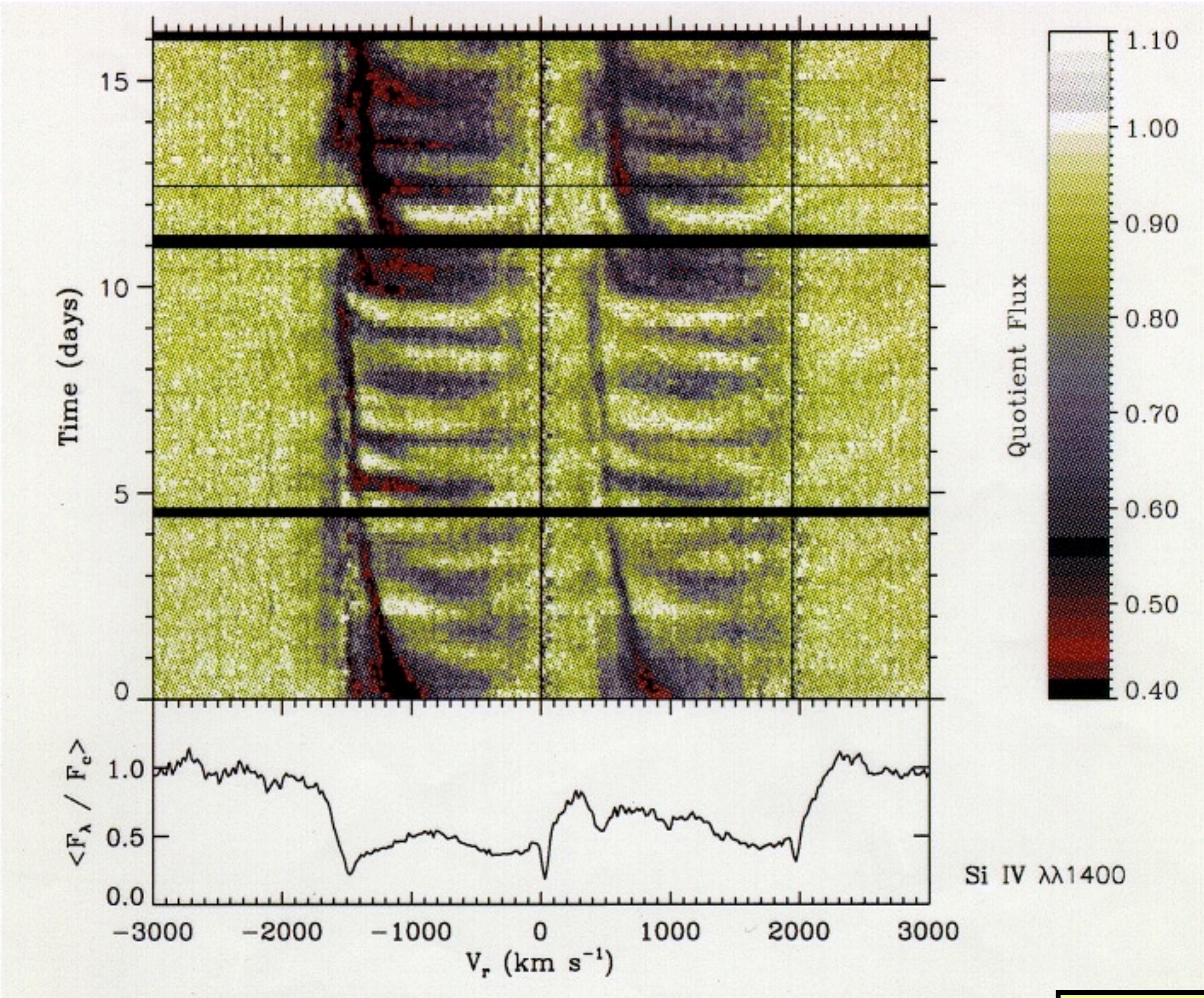
SHOCK

weak discont.

strong discont.

SHOCK

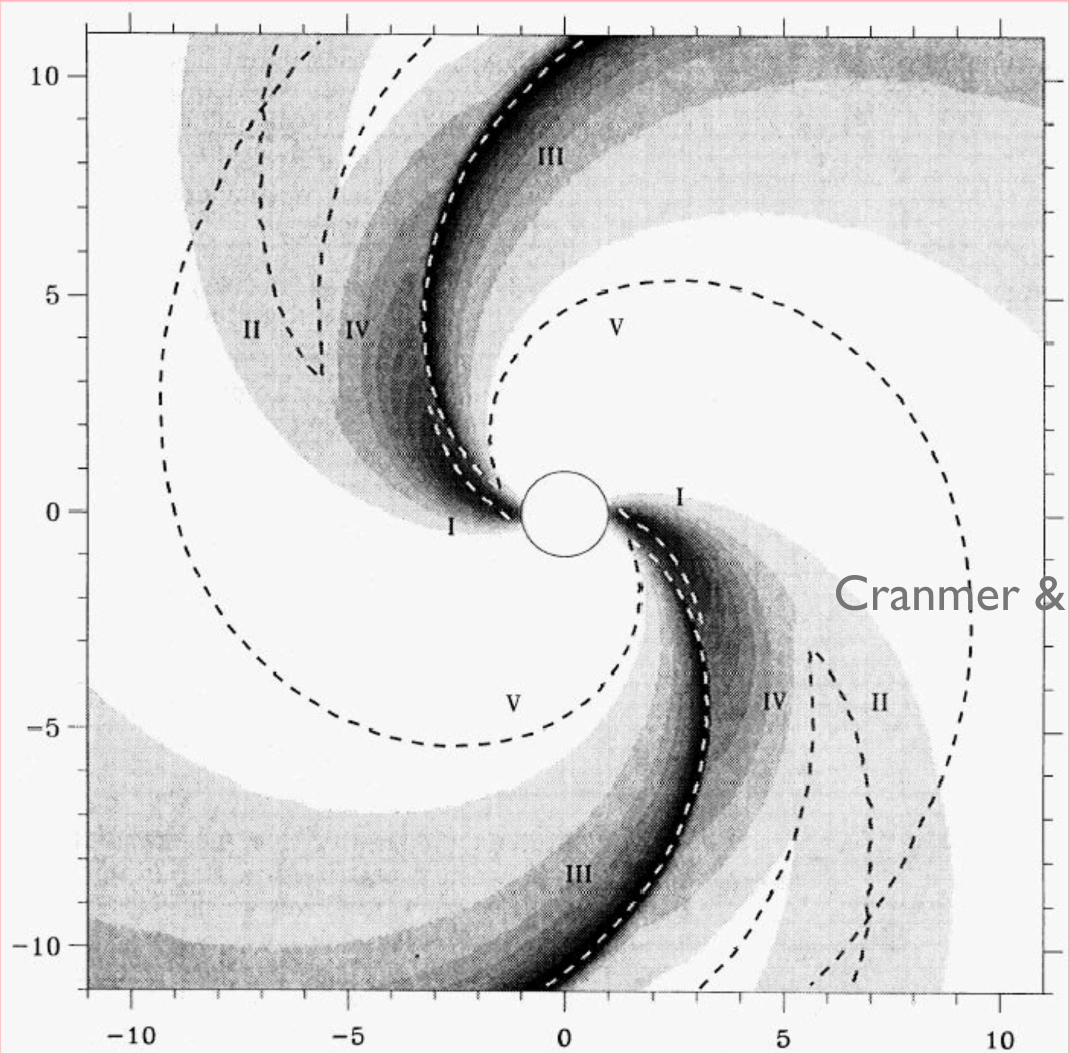


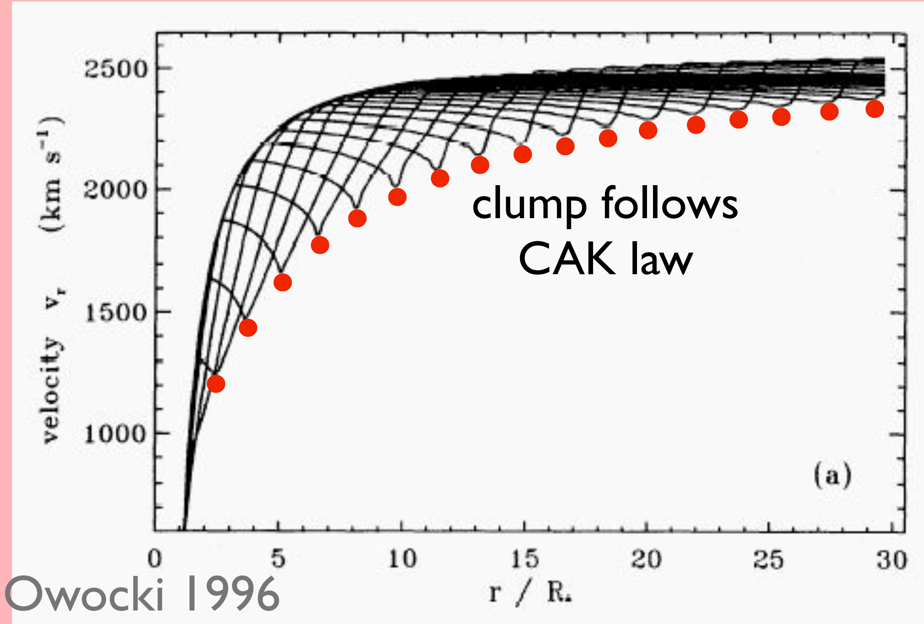


IUE MEGA campaign

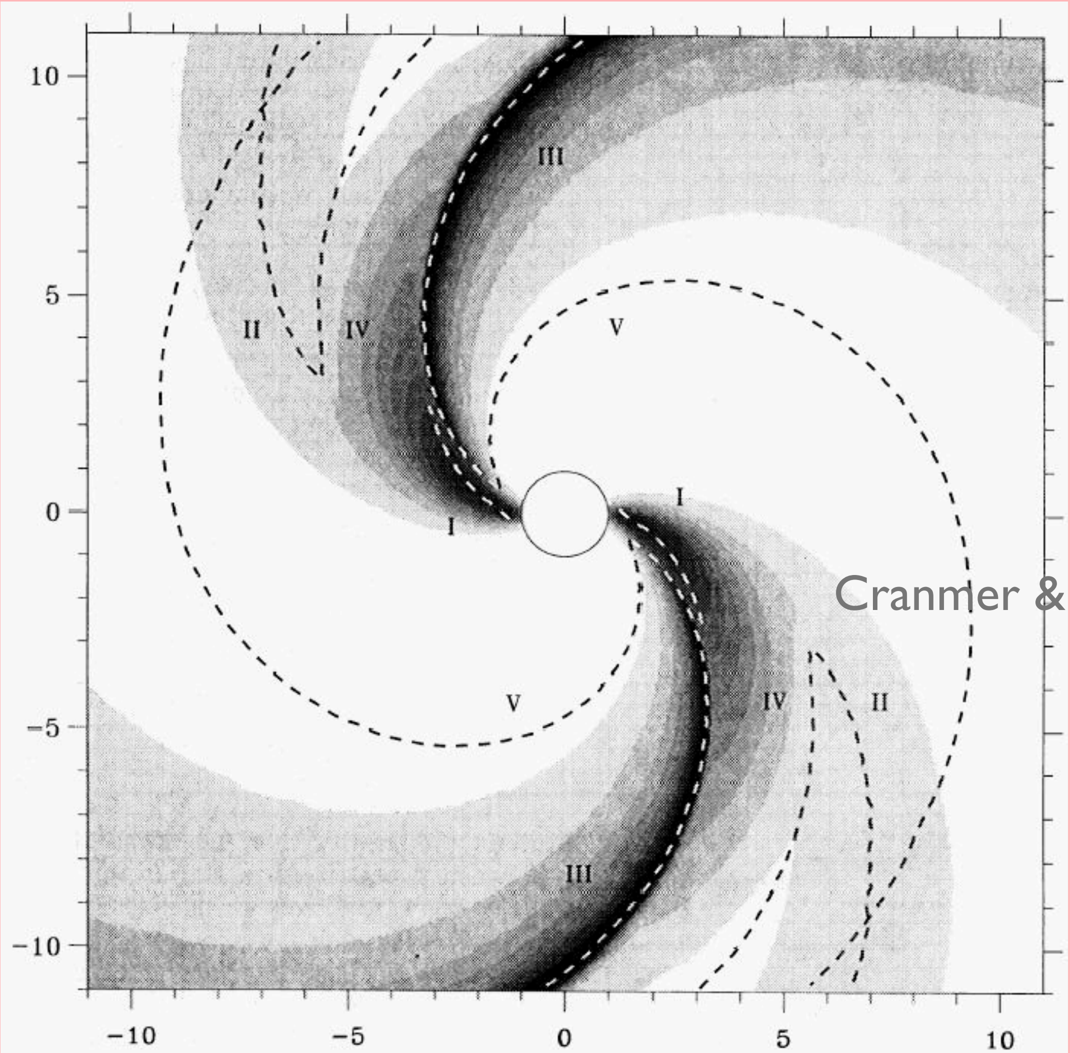
Observations of DACs:

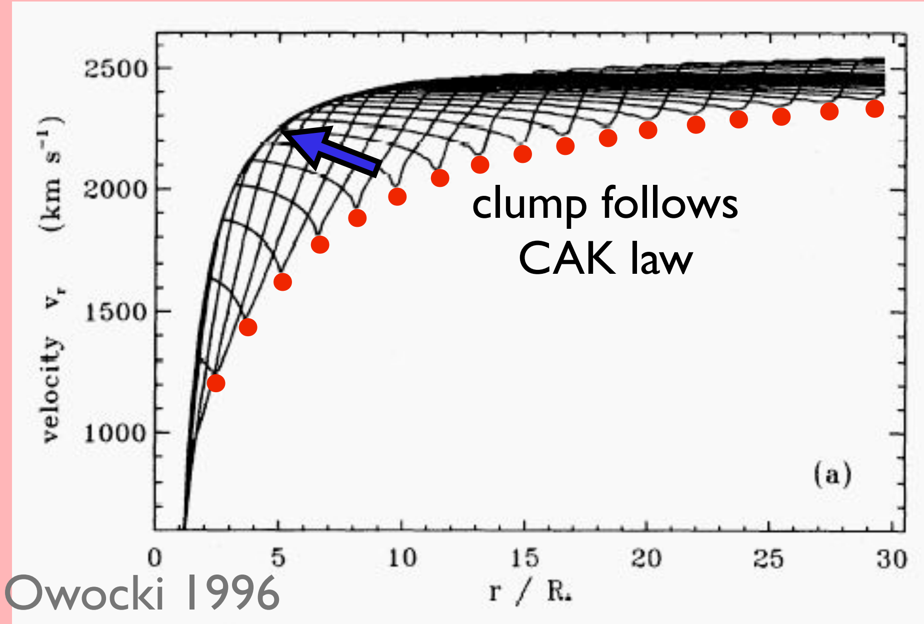
Prinja & Howarth
Fullerton & Massa
Henrichs & Kaper



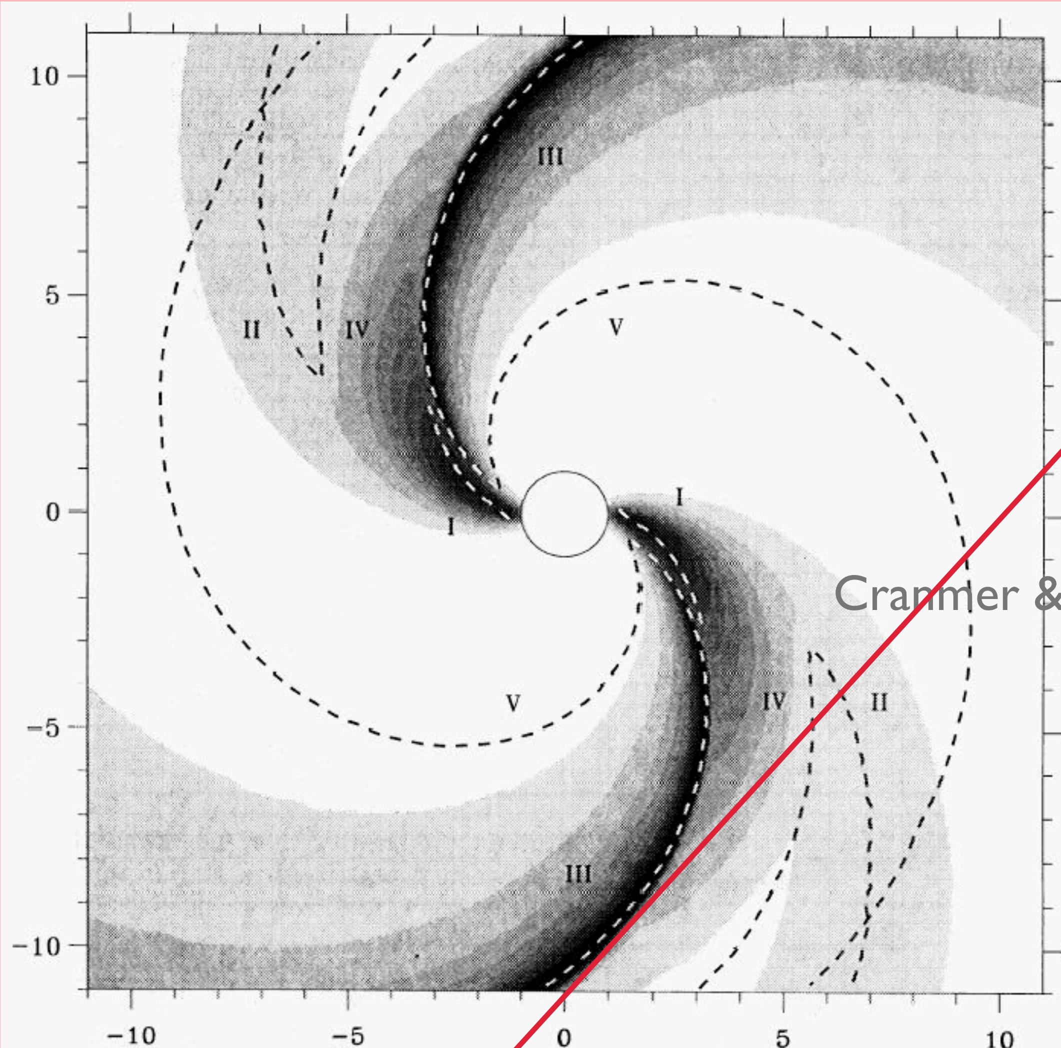


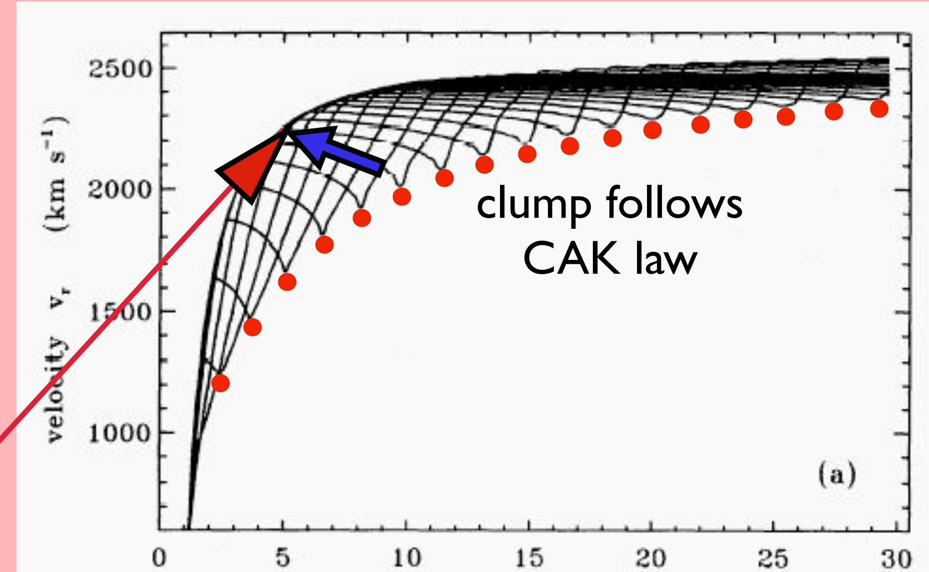
Cranmer & Owocki 1996





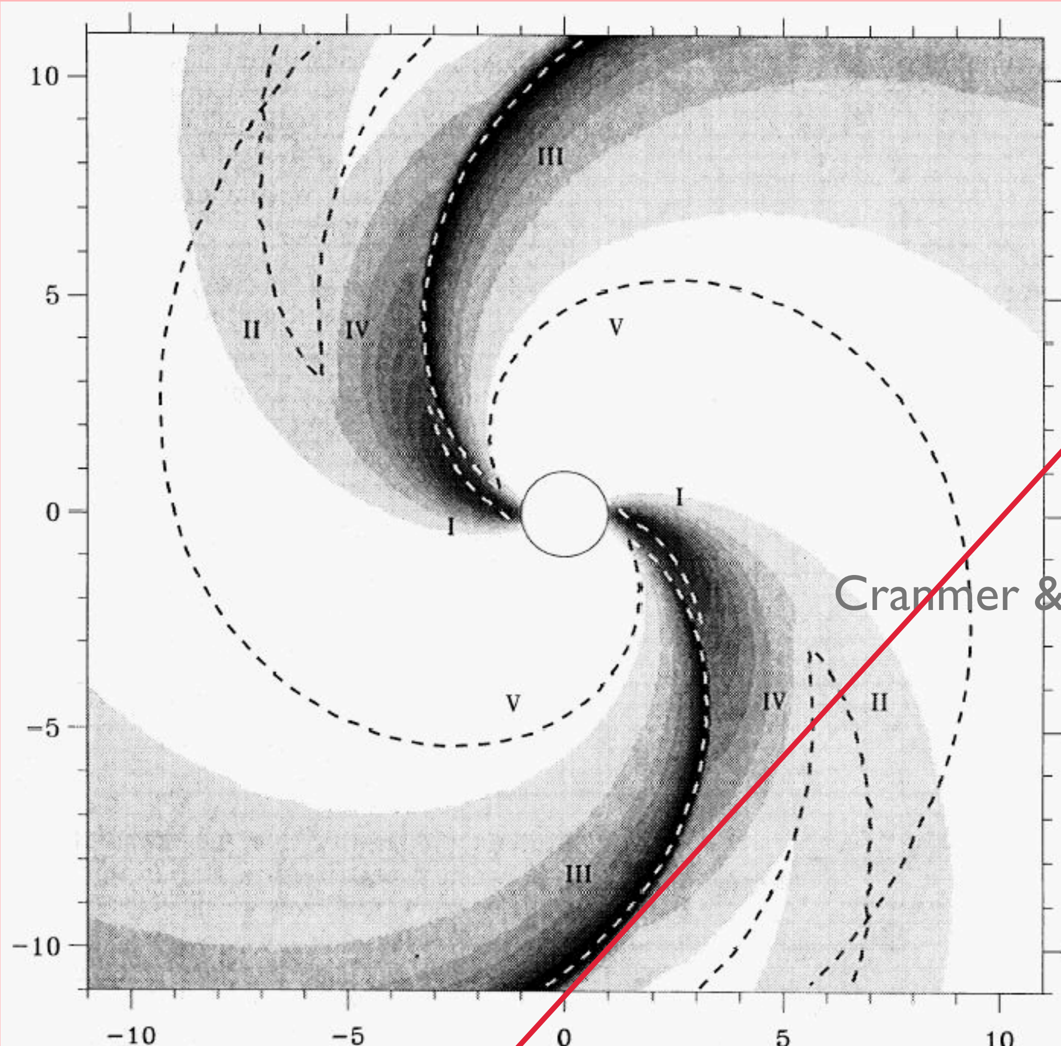
Cranmer & Owocki 1996

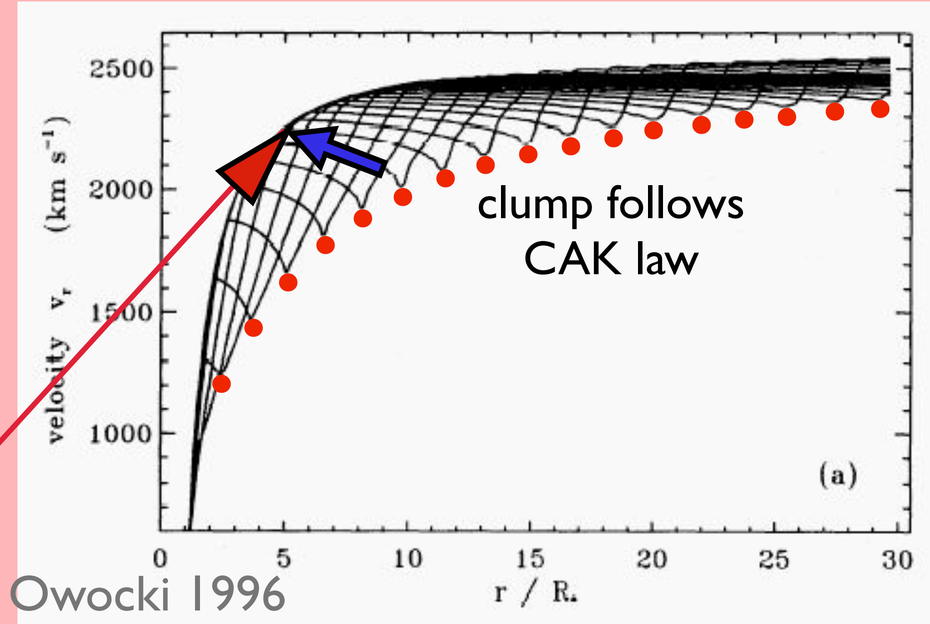




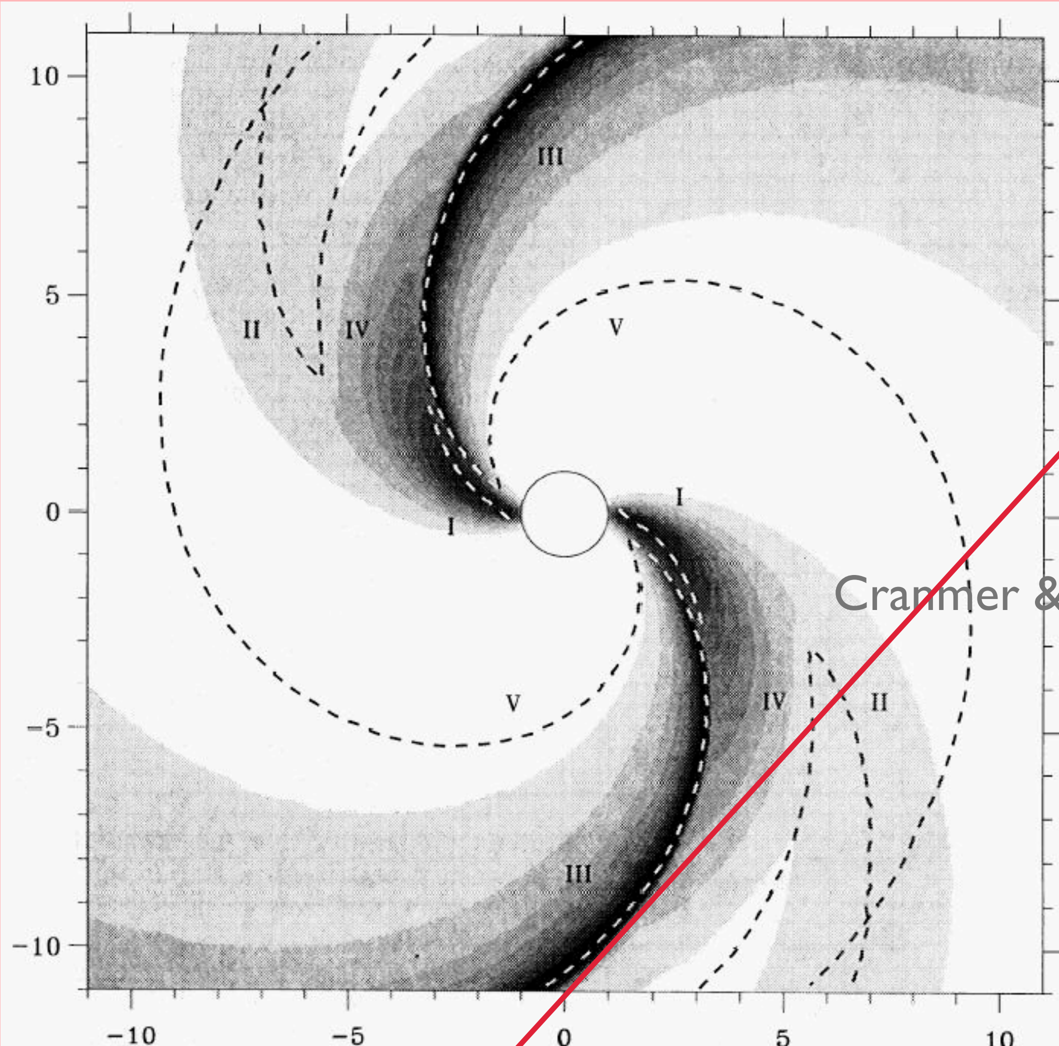
Cranmer & Owocki 1996

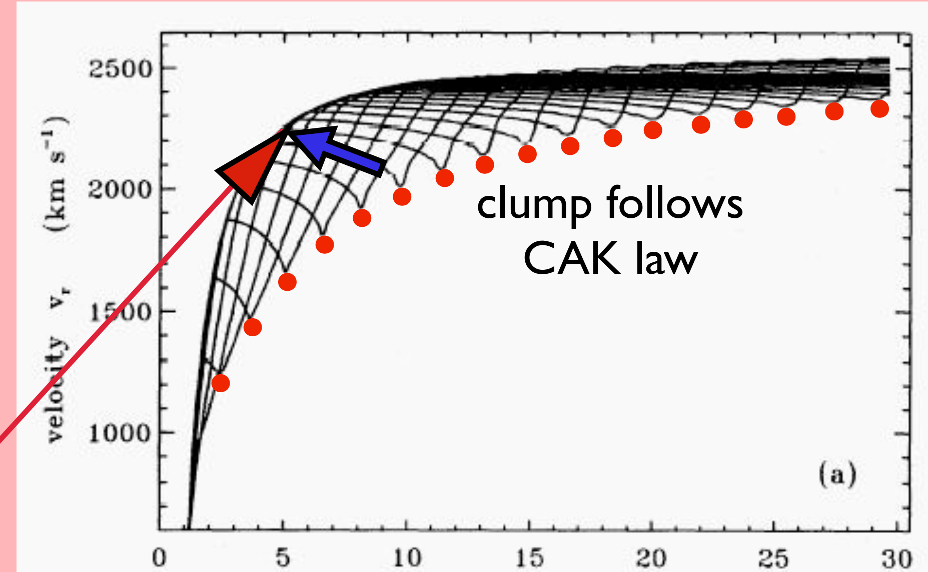
clump sends Abbott wave upstream



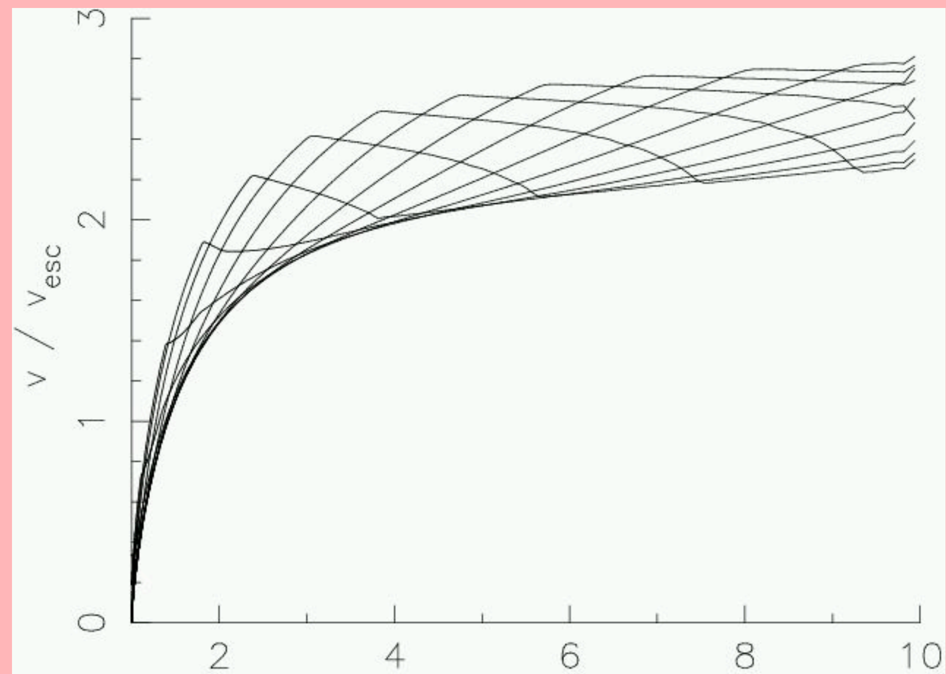


- clump sends Abbott wave upstream
- creates precursor kink





Cranmer & Owocki 1996

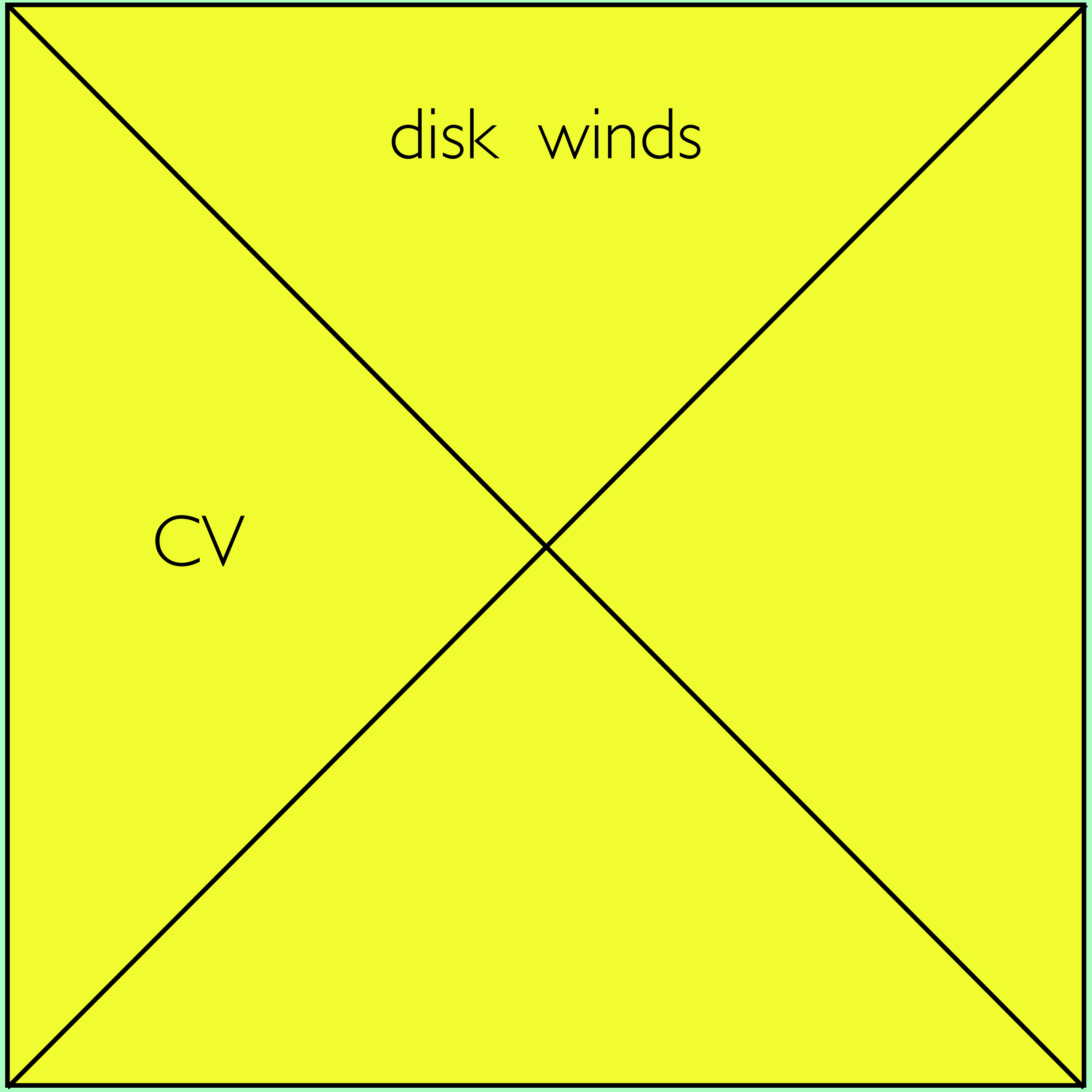


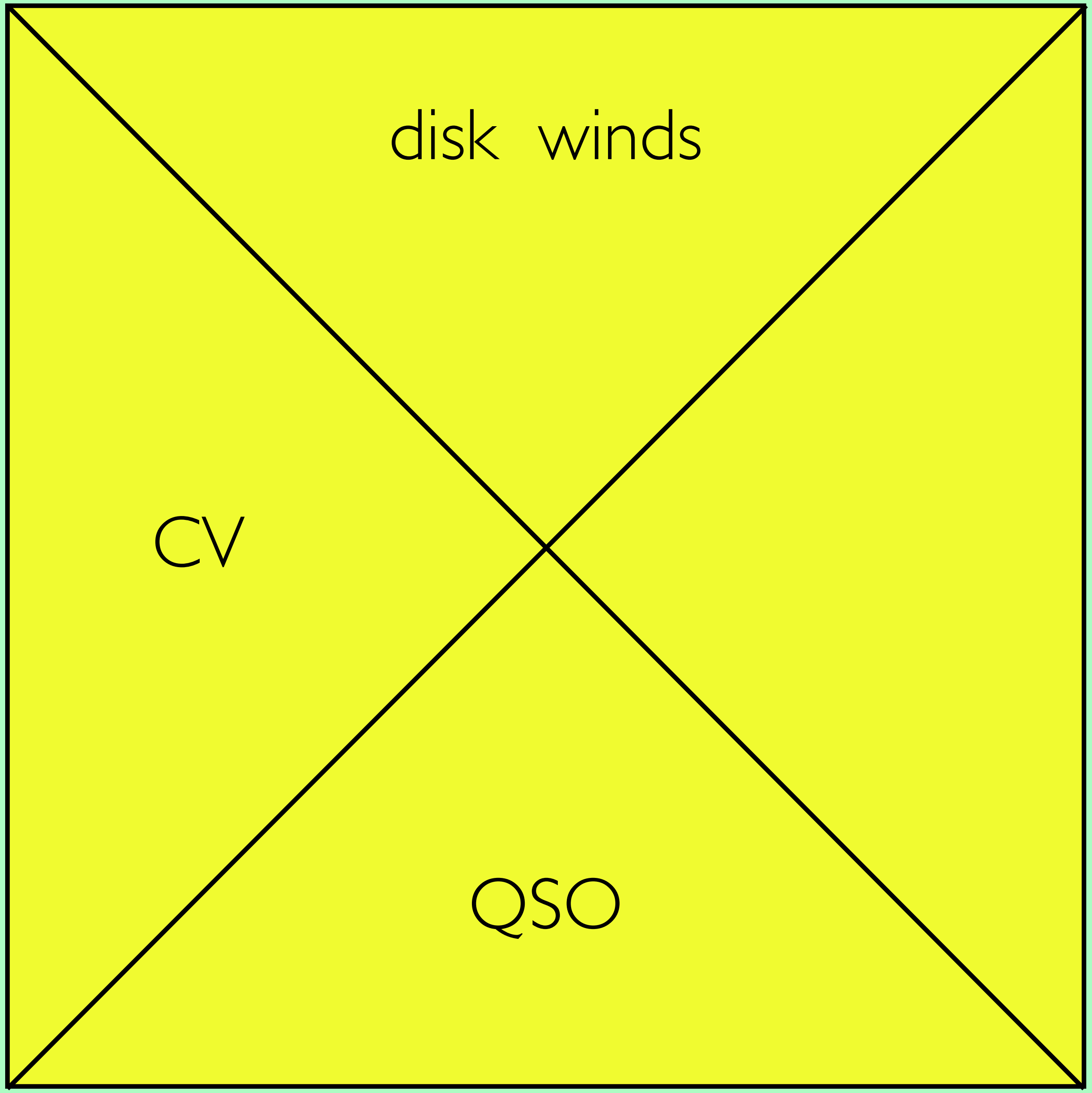
we, from 1D calculation

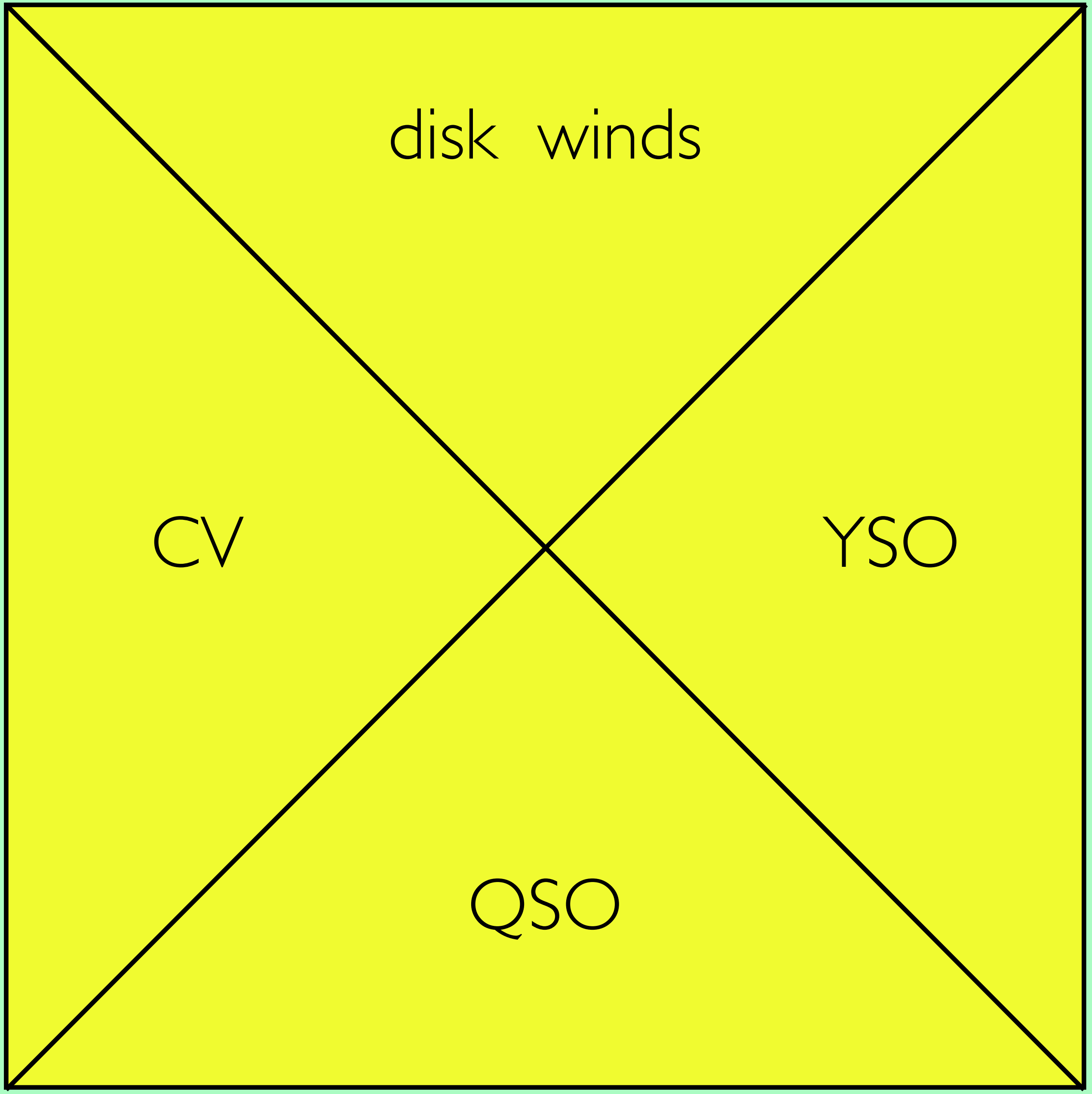
- clump sends Abbott wave upstream
- creates precursor kink

disk
wind

disk winds


disk winds
CV

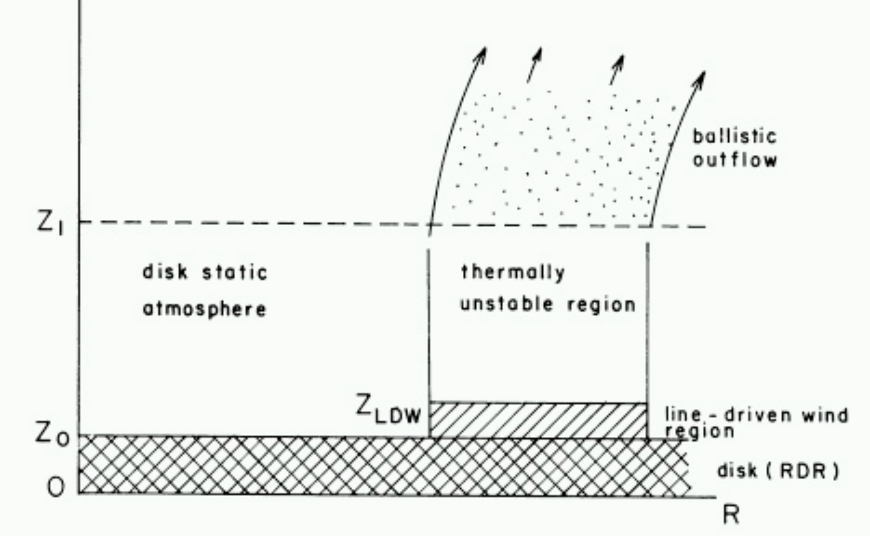

disk winds
CV
QSO

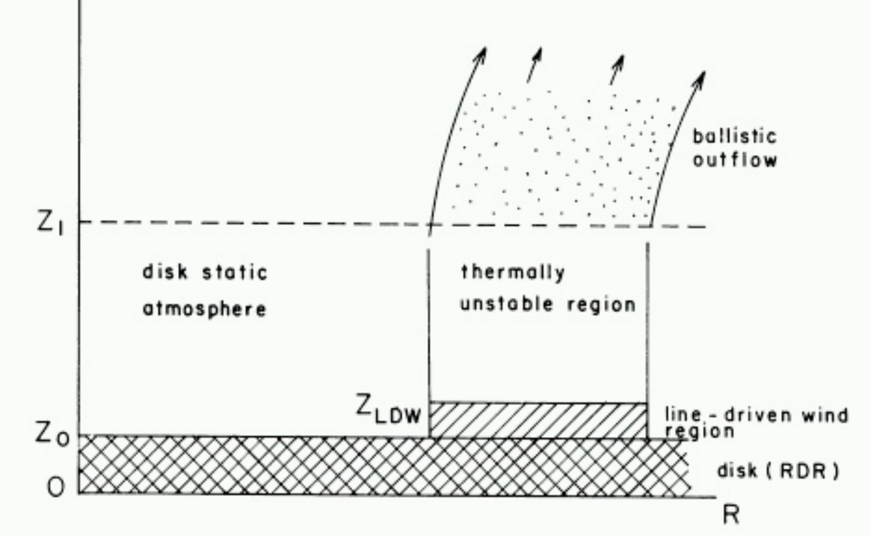
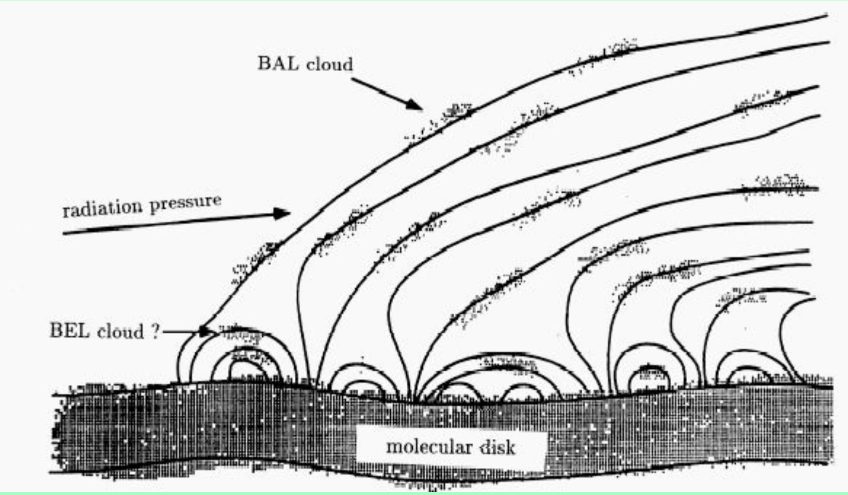

disk winds
CV
YSO
QSO

Shlosman

1985

Radiation from central QSO region shielded by disk atmosphere

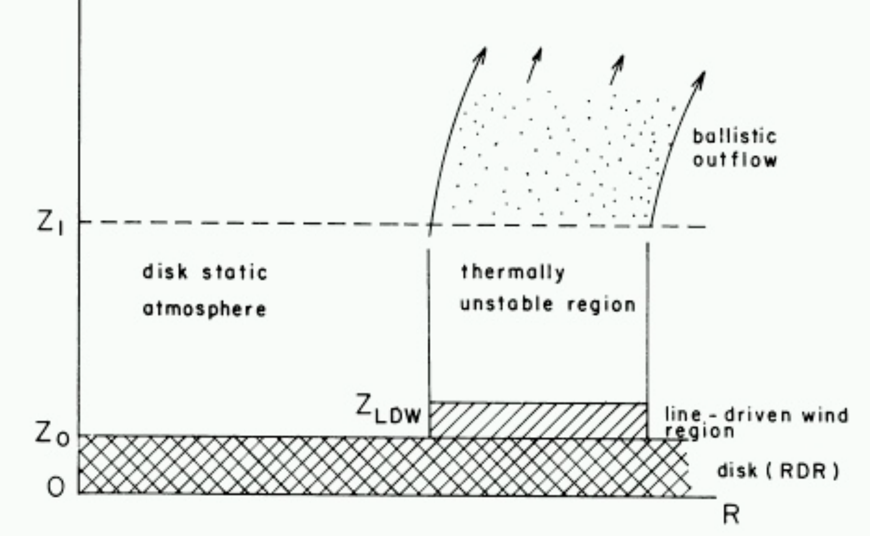


Shlosman	1985	Radiation from central QSO region shielded by disk atmosphere	
de Kool & Begelman	1994	Similarity solution for magnetic wind with scaled-up continuum driving (no line force)	

Shlosman

1985

Radiation from central QSO region shielded by disk atmosphere



de Kool & Begelman

1994

Similarity solution for magnetic wind with scaled-up continuum driving
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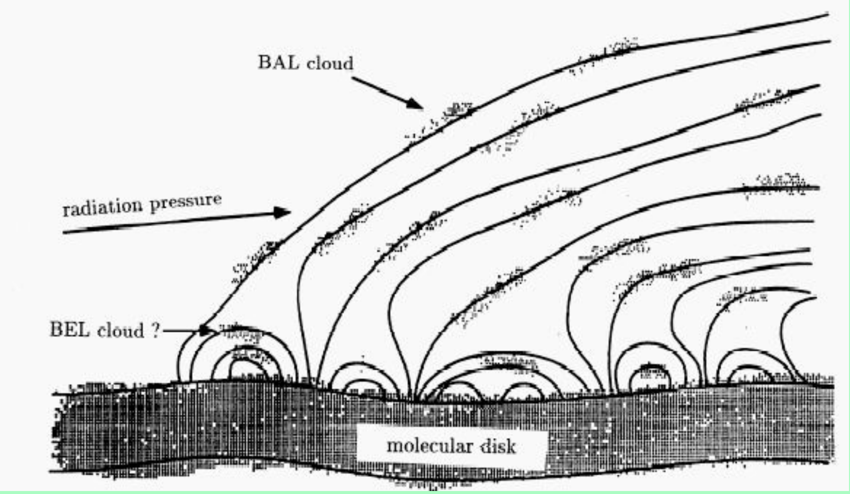
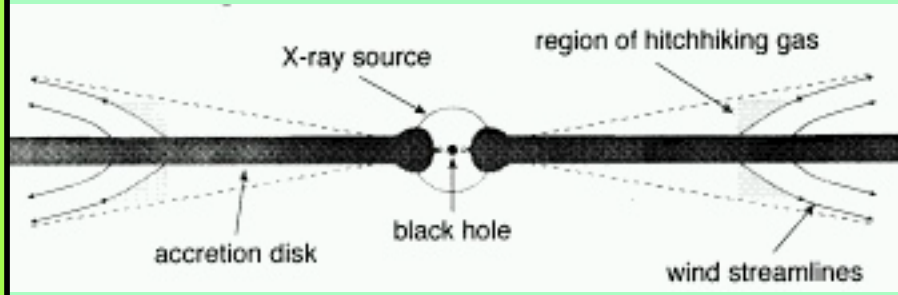


Murray et al.

1995

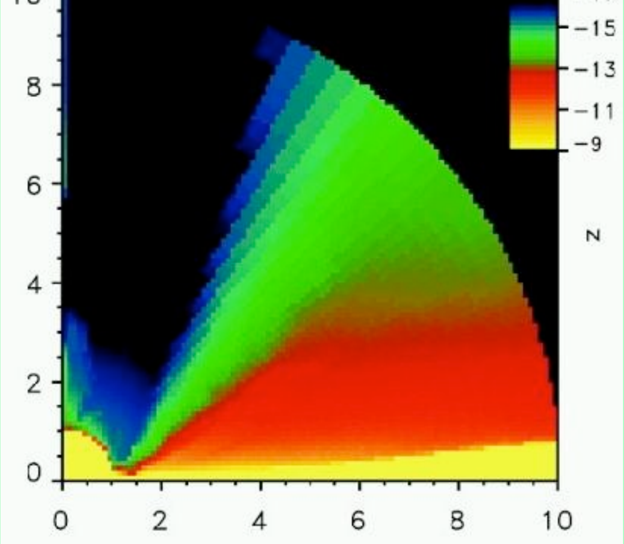
"Hitchhiking gas": Inner wind is overionized and shields outer wind from central QSO radiation



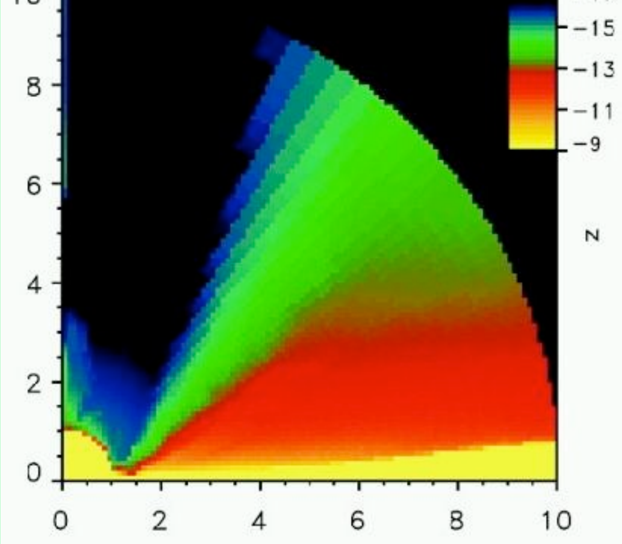
Proga, Stone, Drew

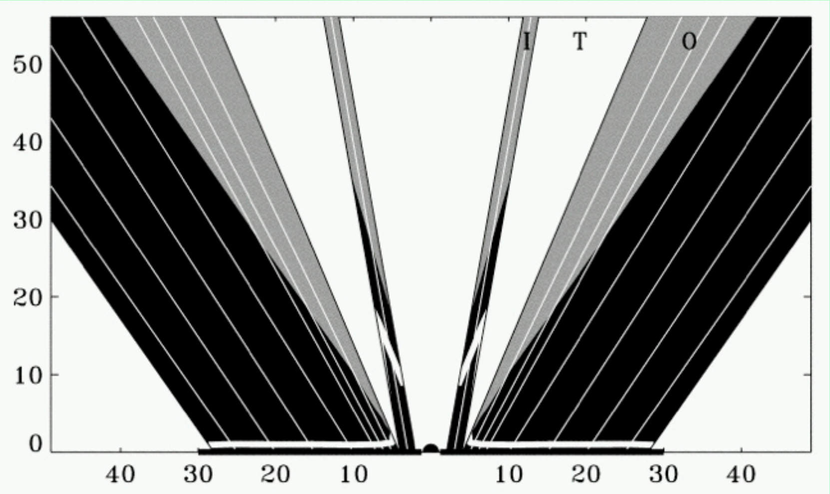
1998

First time-dependent wind models for CVs using Zeus



Proga, Stone, Drew	1998	First time-dependent wind models for CVs using Zeus
Feldmeier & Shlosman	1999	Semi-analytic CAK model: 2-D eigenvalue problem





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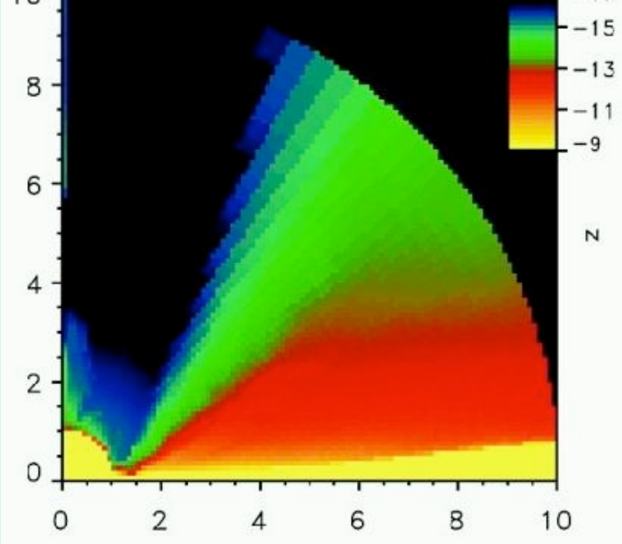
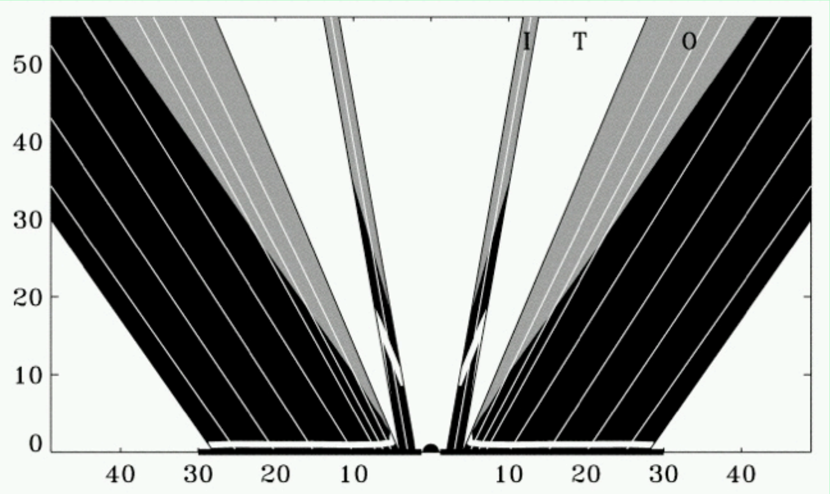


Feldmeier & Shlosman

1999

Semi-analytic CAK model: 2-D eigenvalue problem



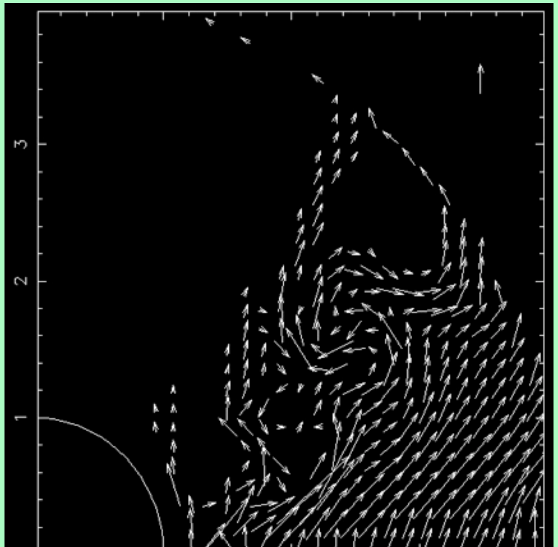
Proga

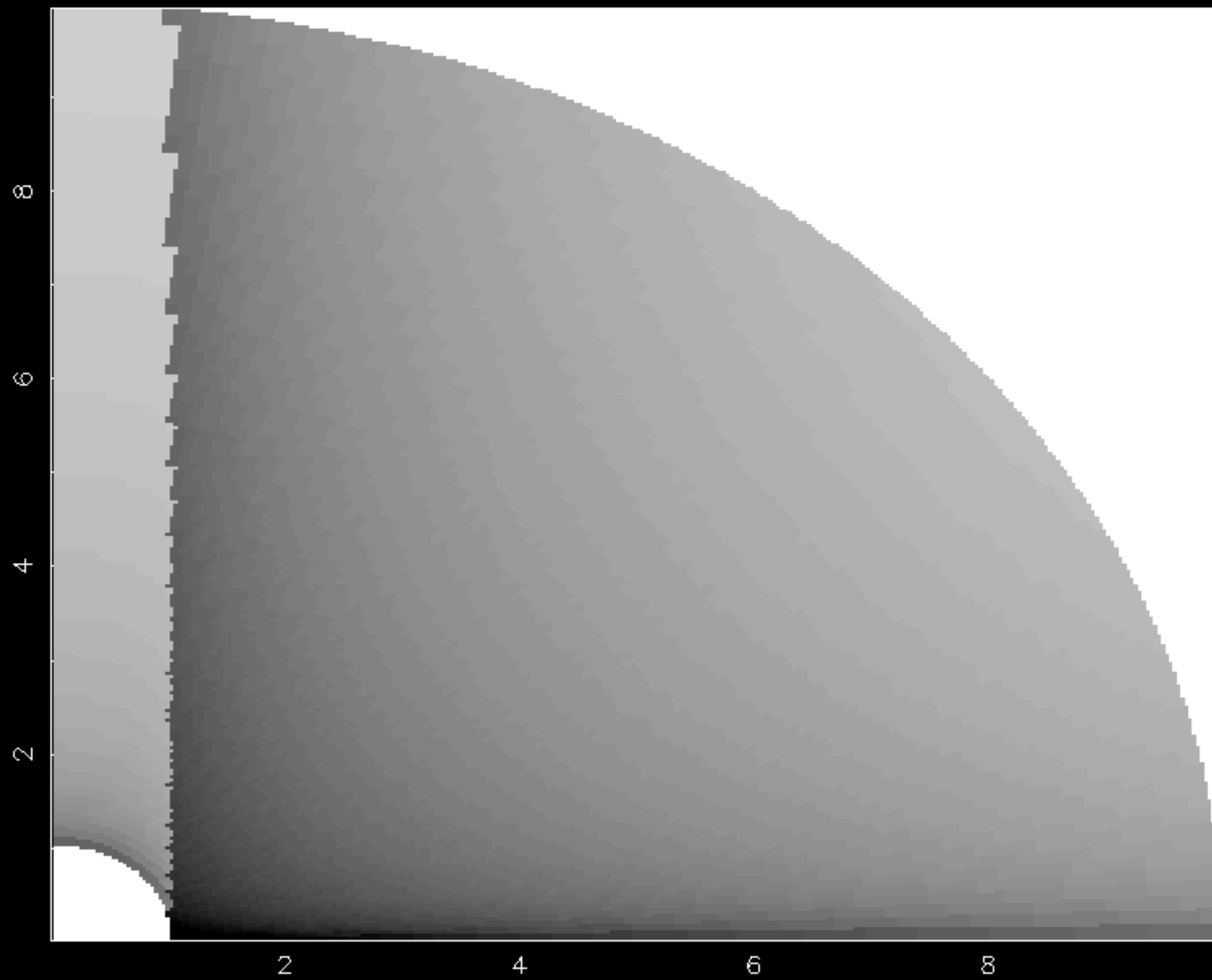
2004

Feldmeier

2001

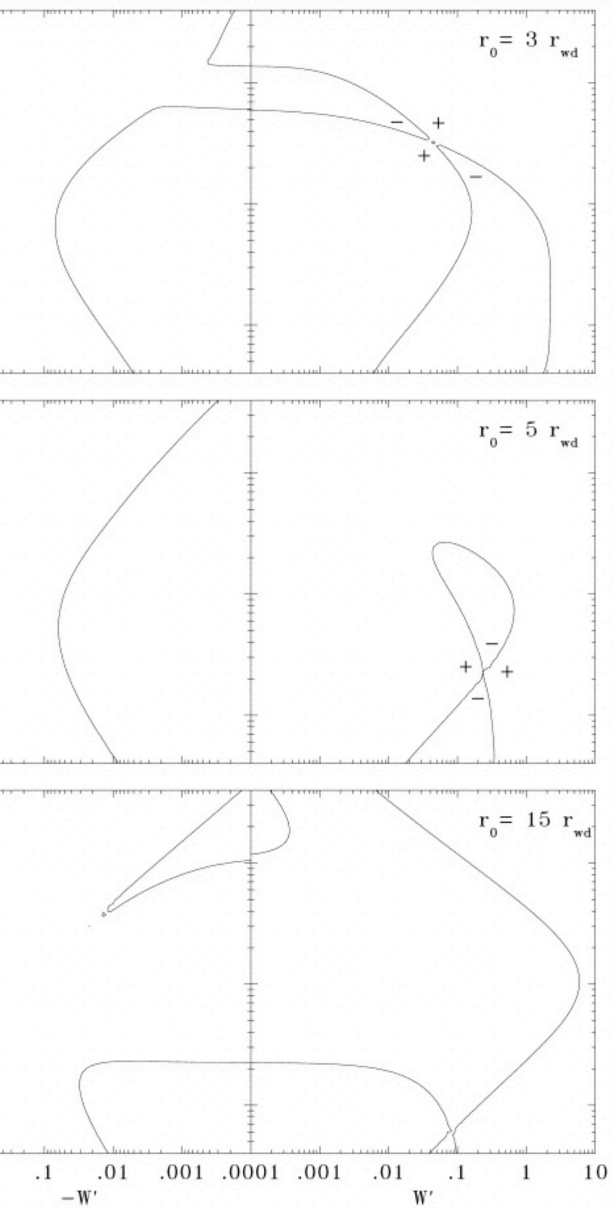
Time-dependent simulations of magnetized, line-driven disk winds

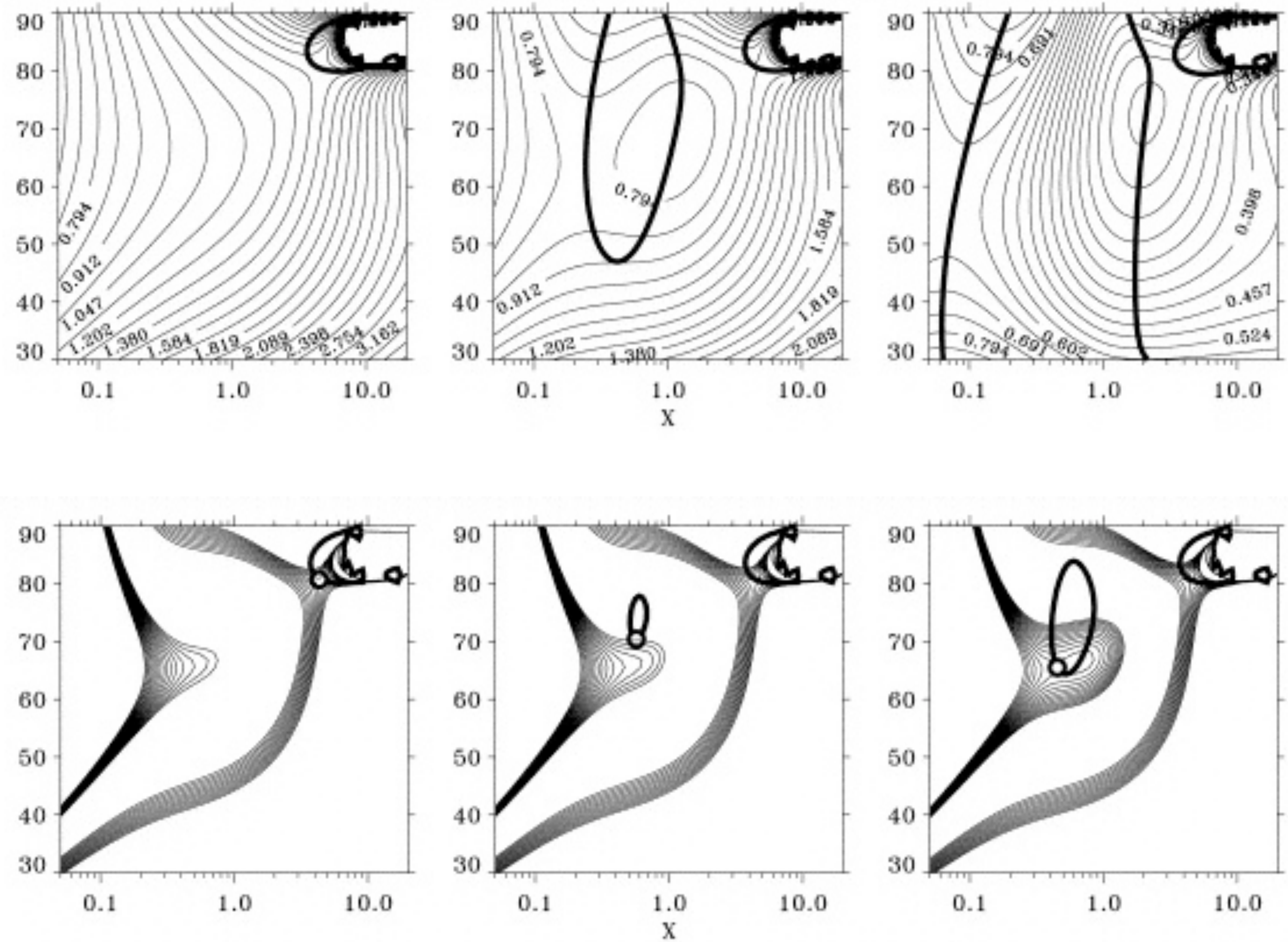



2
4
6
8
2
4
6
8

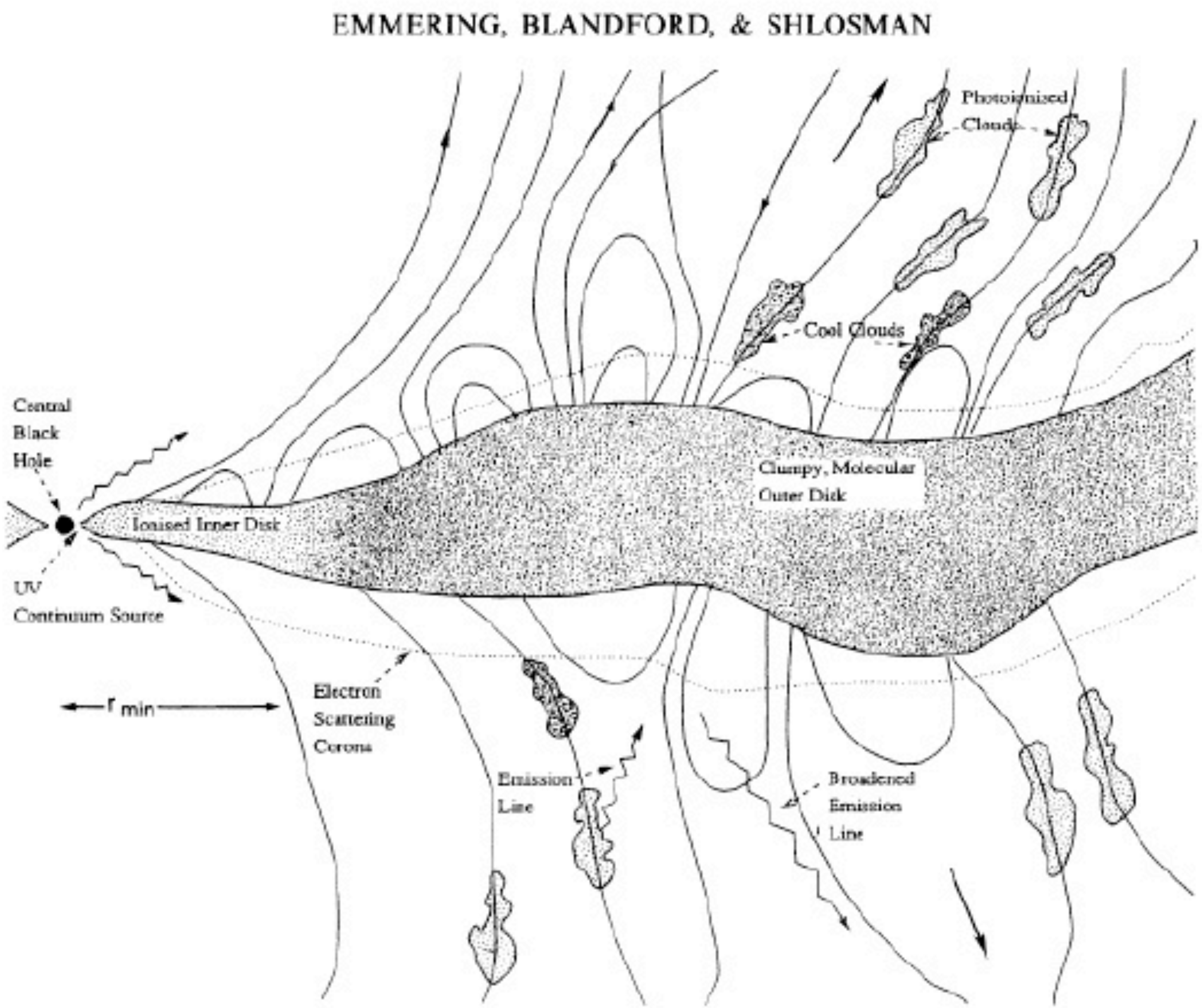
Semi-analytic wind model

Feldmeier & Shlosman 1999, ApJ
Feldmeier et al. 1999, ApJ

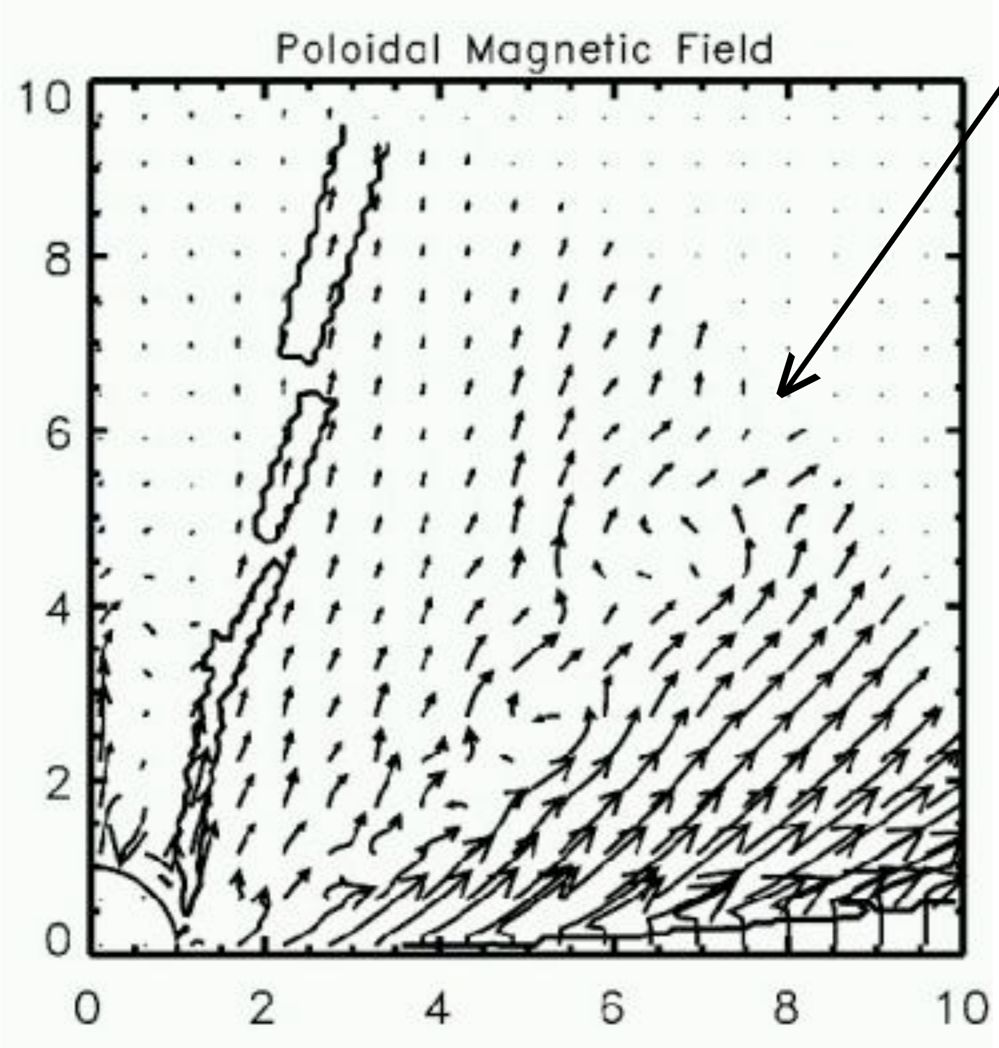




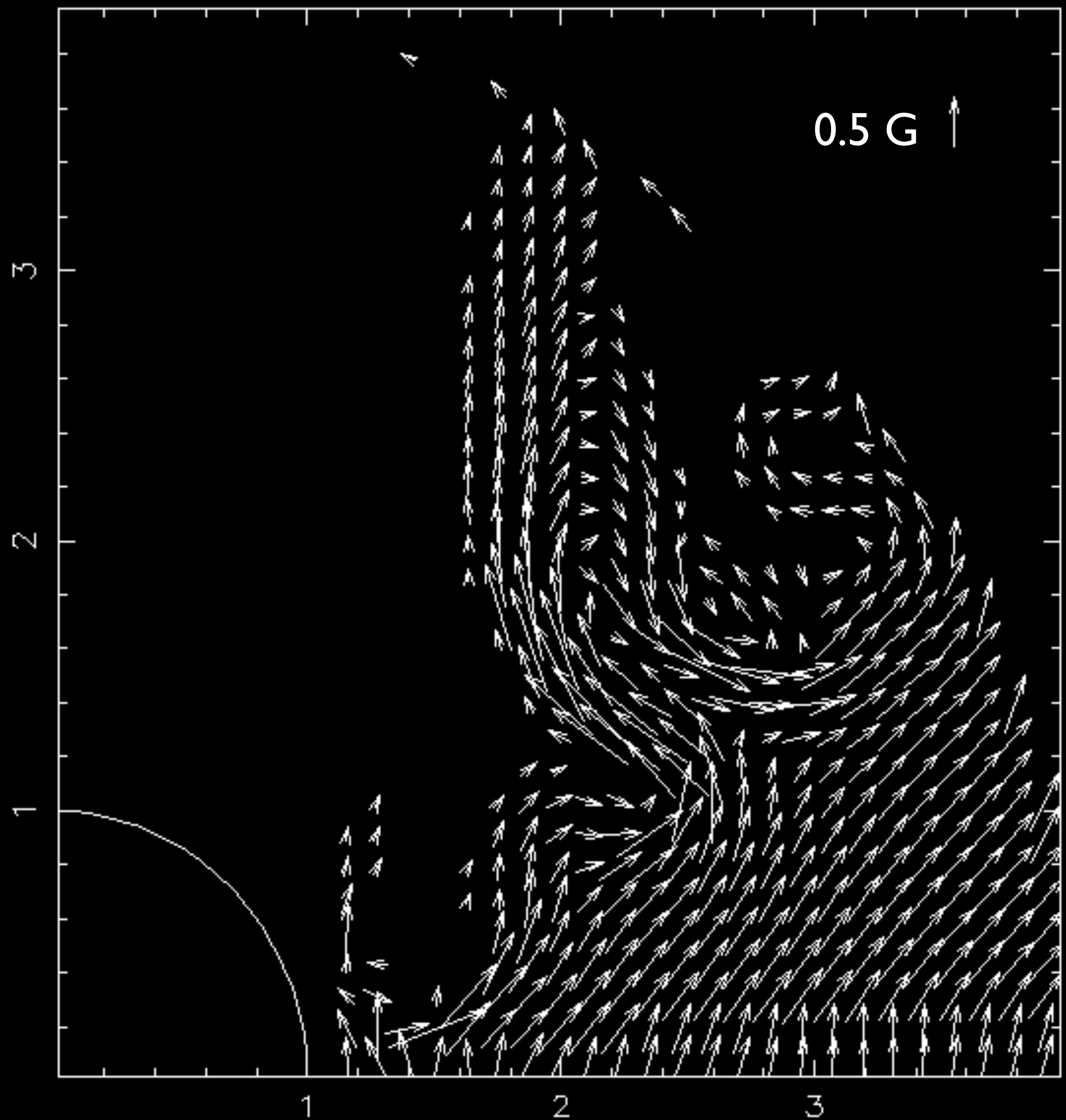
Magnetic winds: Blandford & Payne (1982)



Vortex sheet in B_{pol}



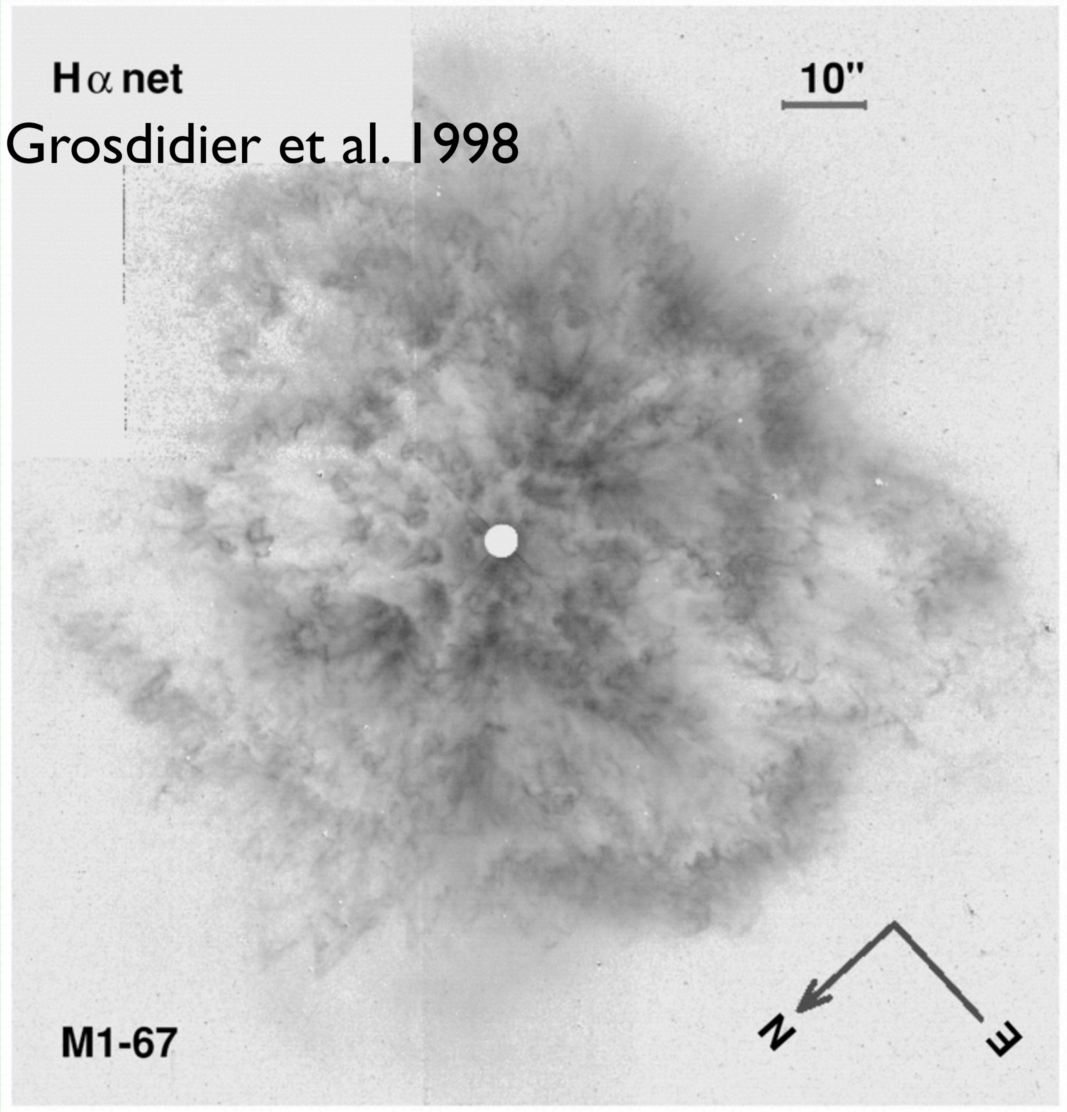
Proga 2004, ApJ



Feldmeier 2001
Habilitation

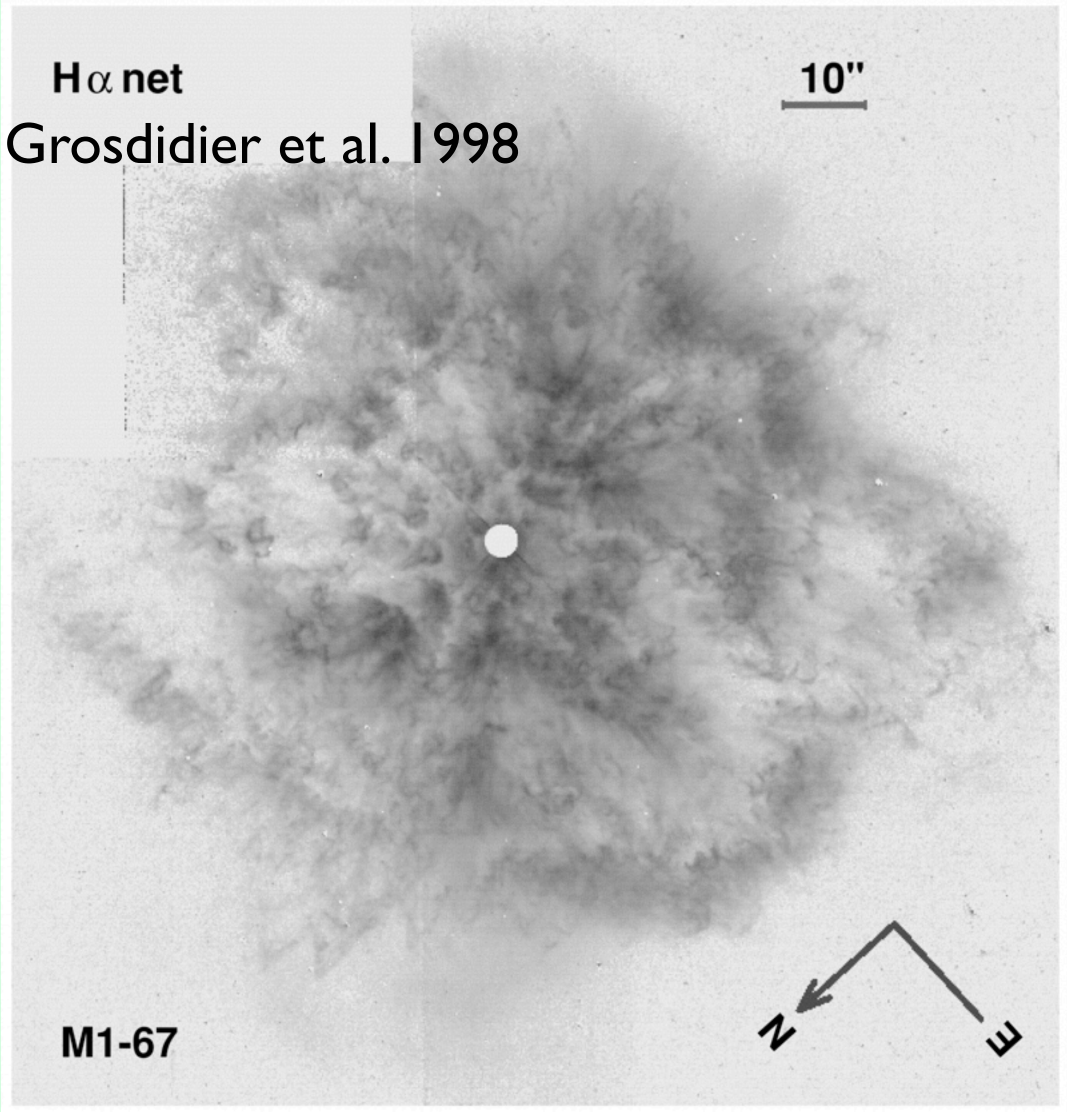
STOCHASTIC TRANSFER

Stochastic Transfer



Grosdidier et al. 1998

Stochastic Transfer



Grosdidier et al. 1998

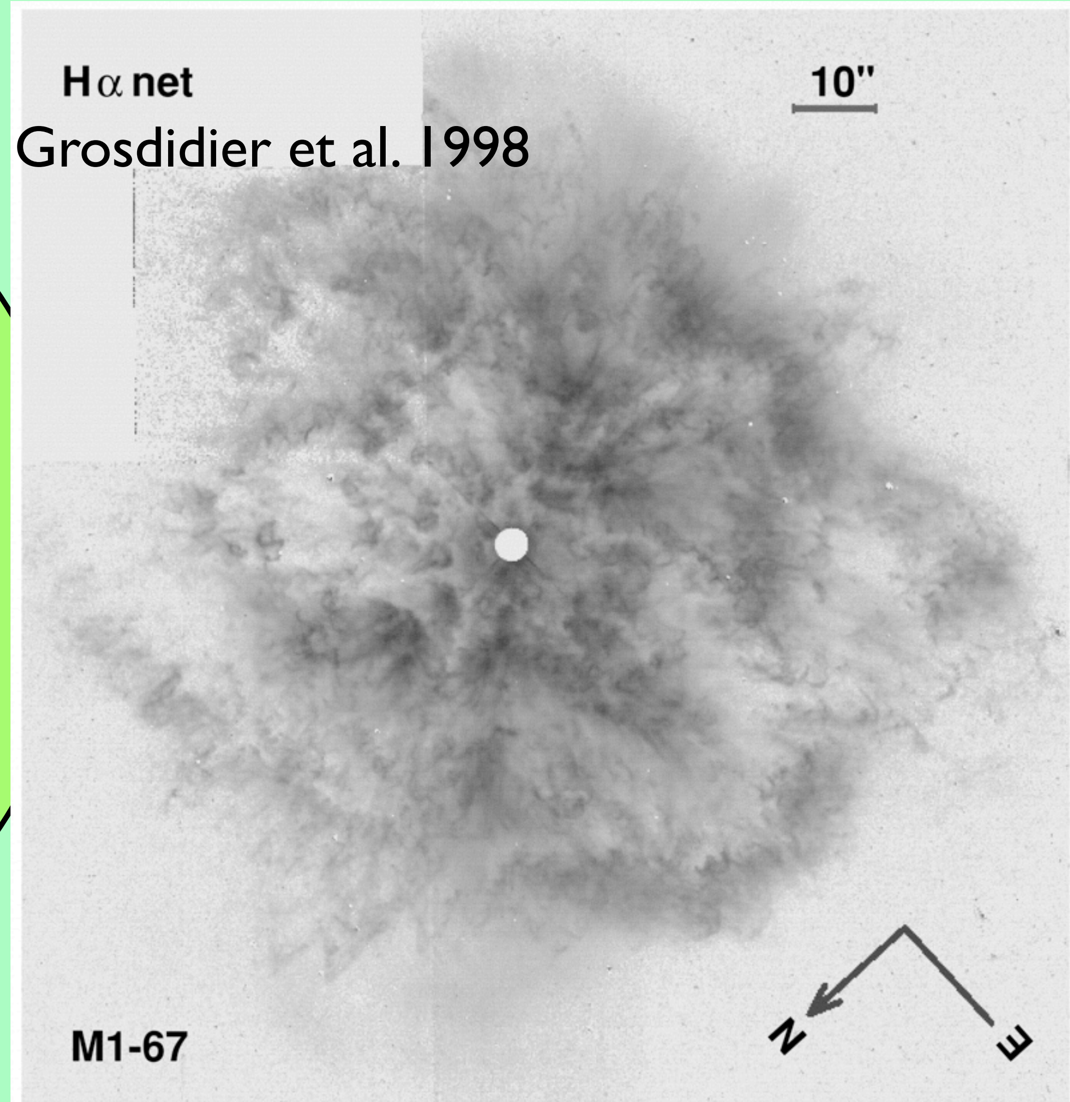
Radiative transfer in clumped wind

.NE.

RT in smooth wind

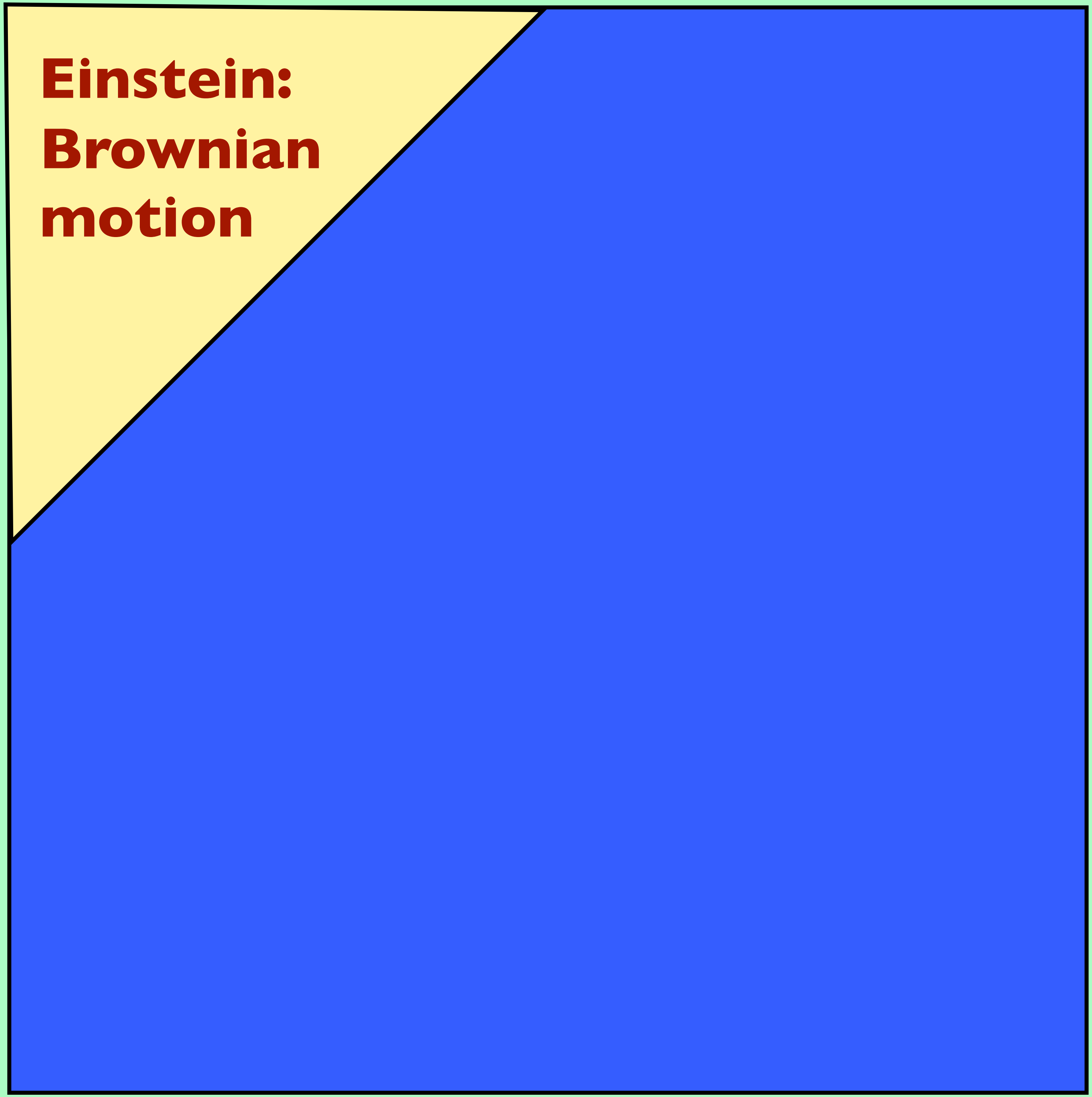
Stochastic Transfer

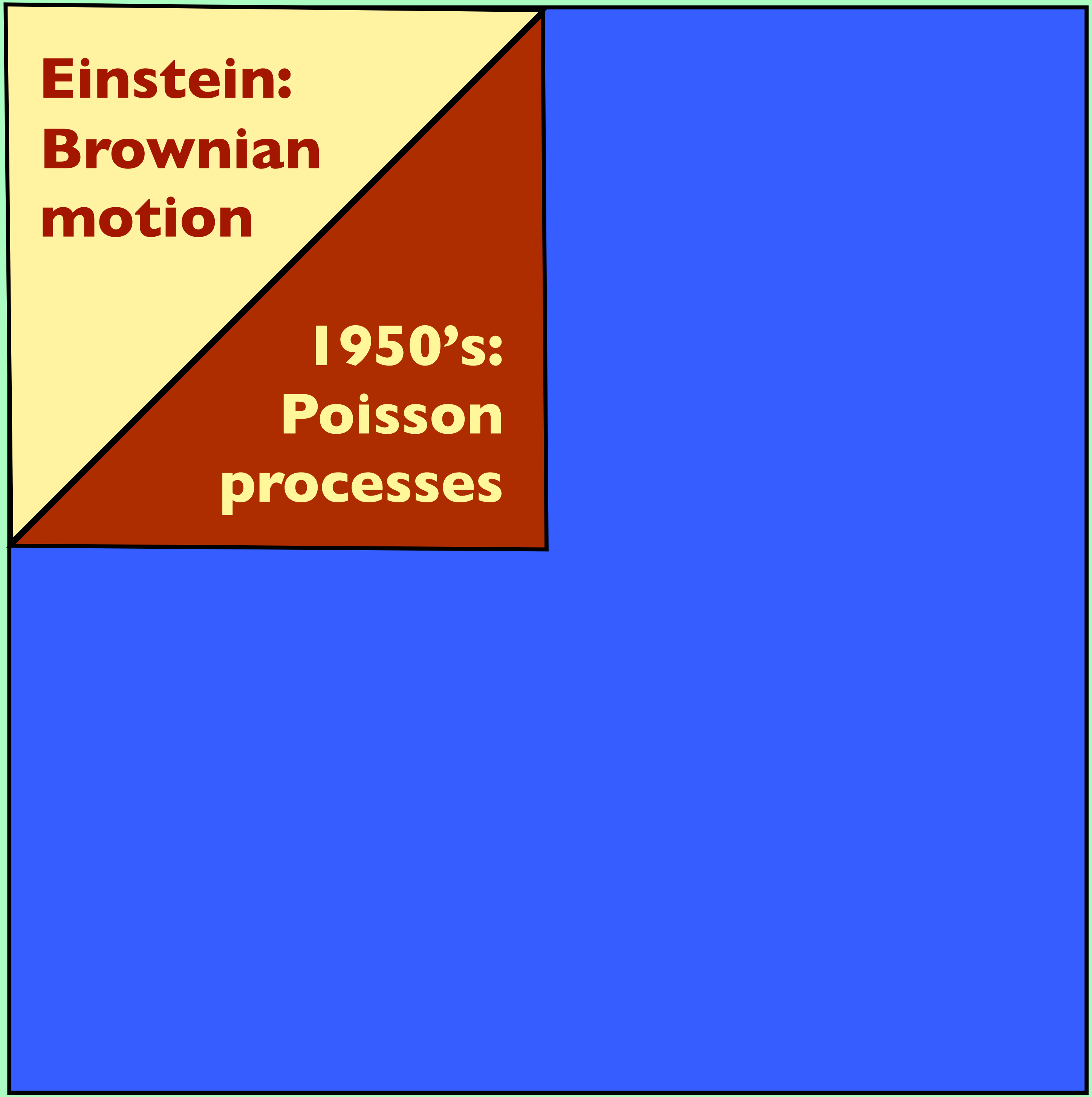
since 1960's
stochastic neutron transport
in nuclear reactor walls



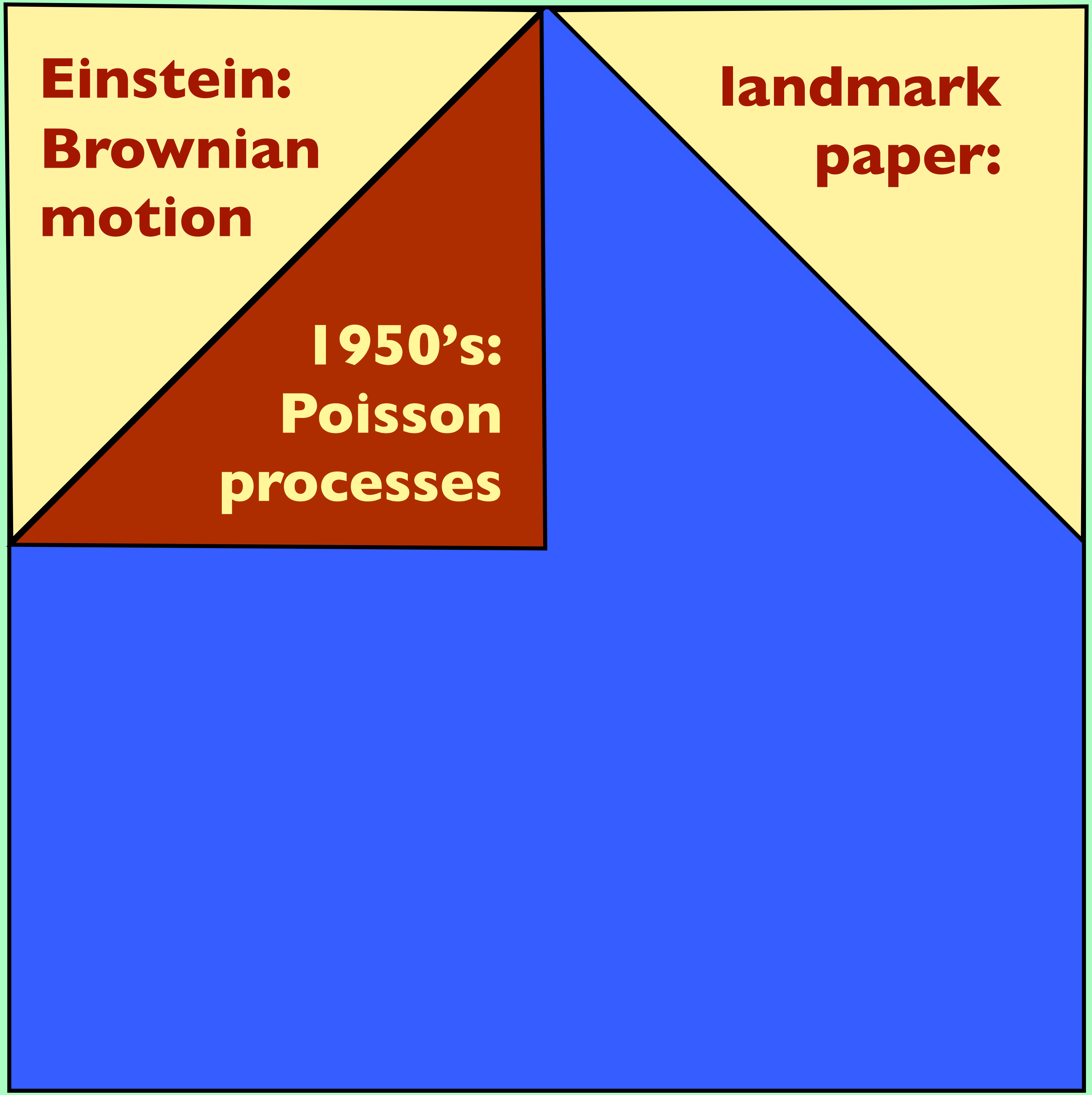
Grosdidier et al. 1998

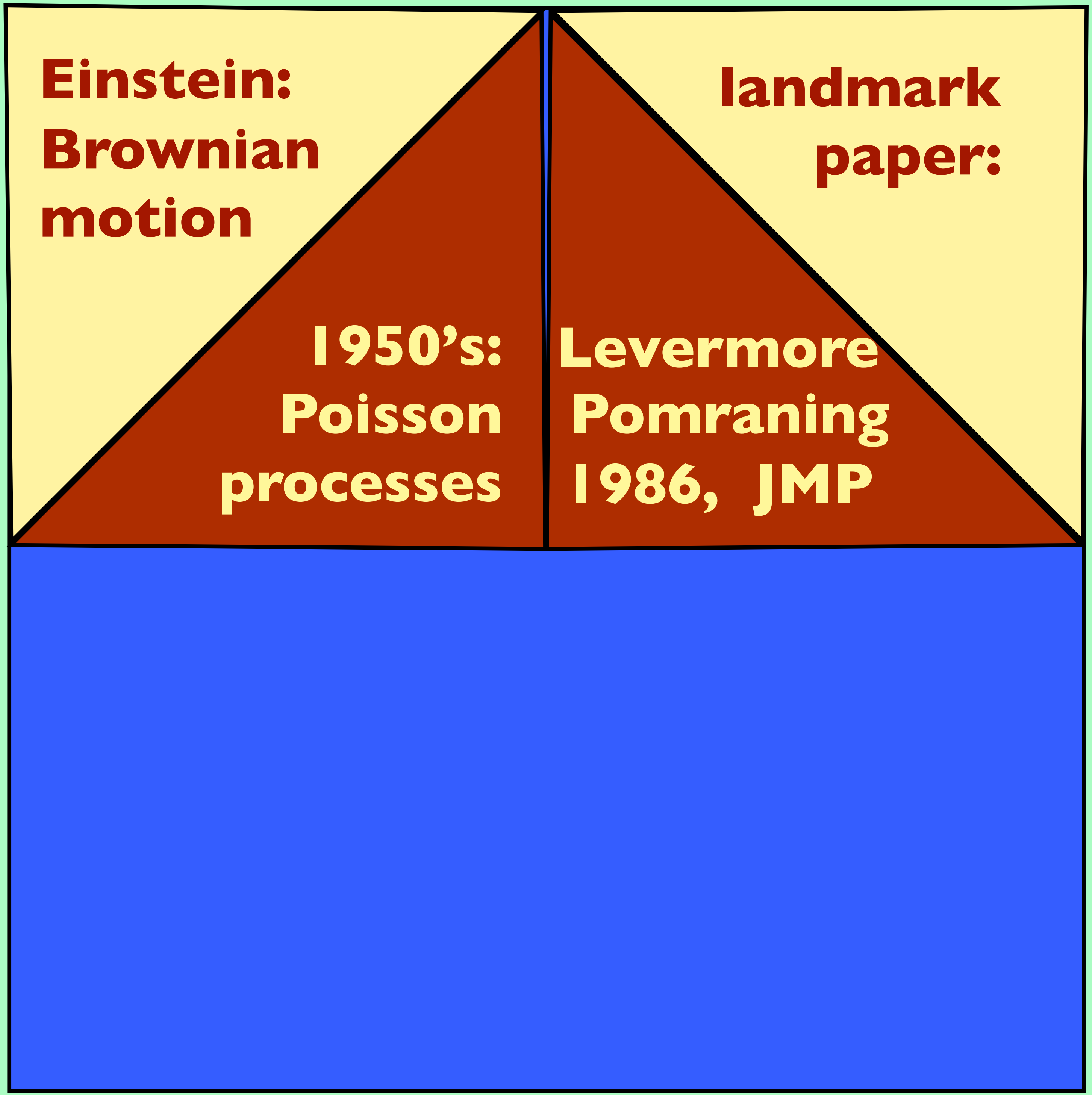
Radiative transfer in clumped wind
.NE.
RT in smooth wind

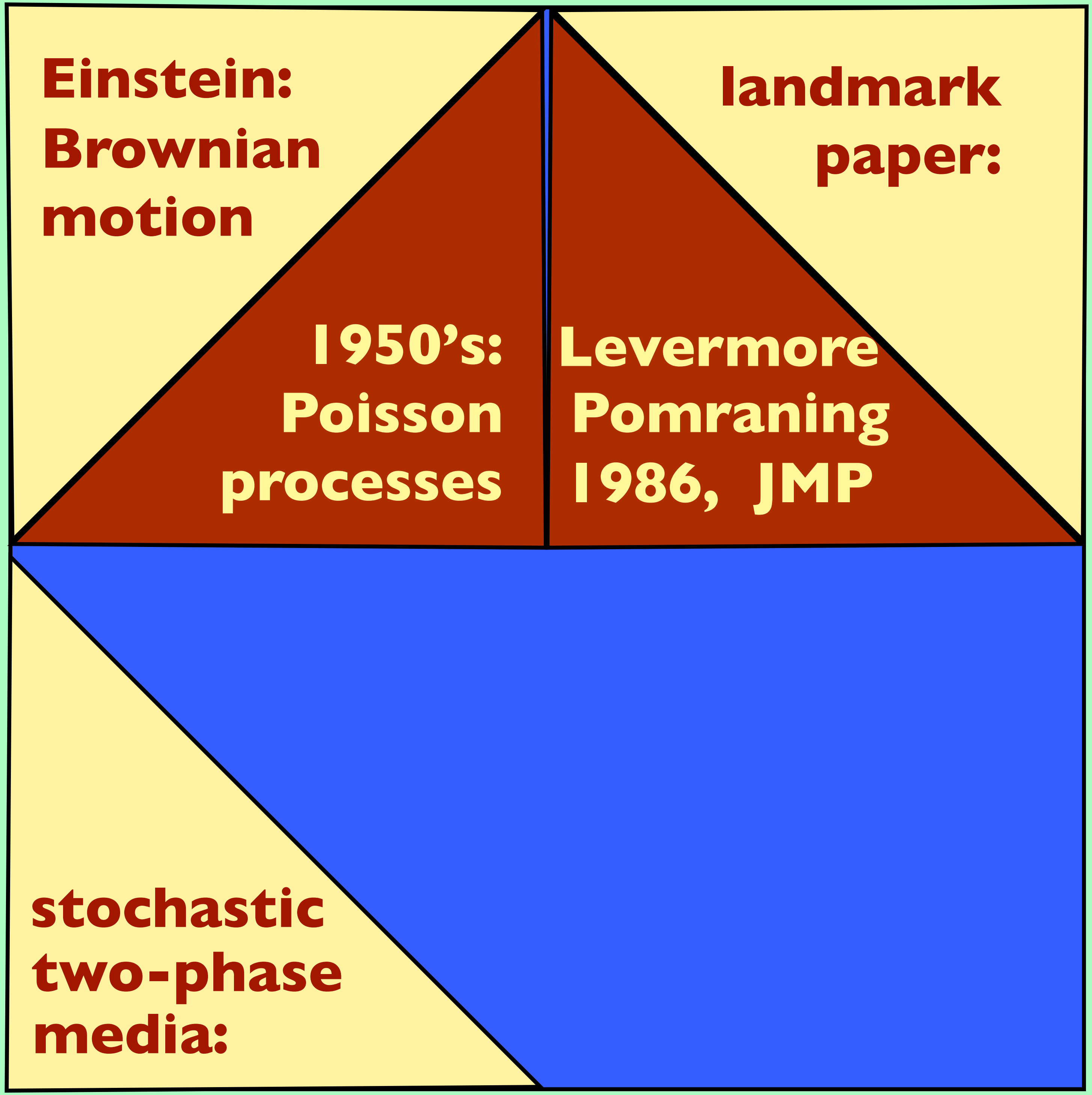

Einstein:
Brownian
motion

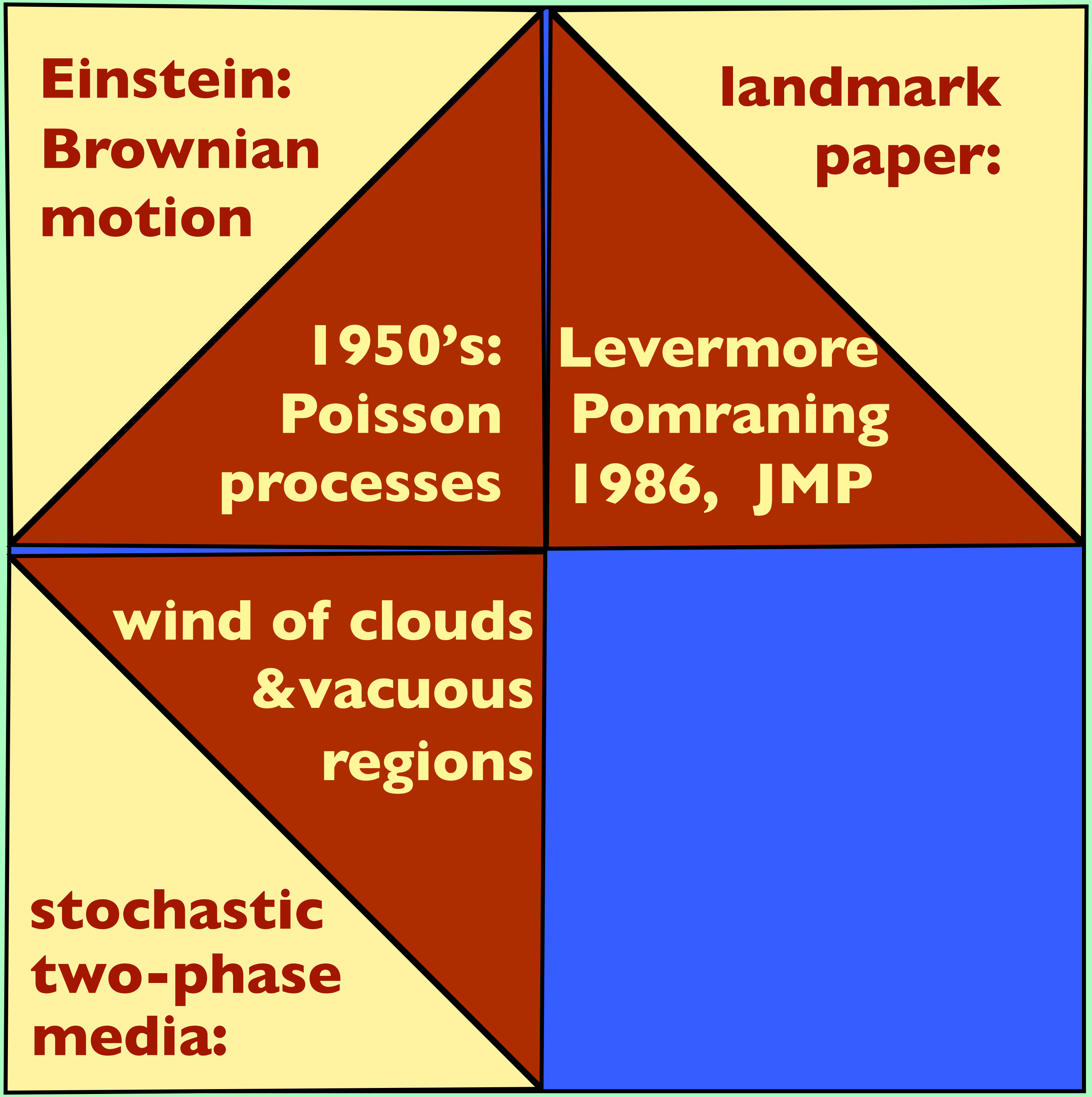


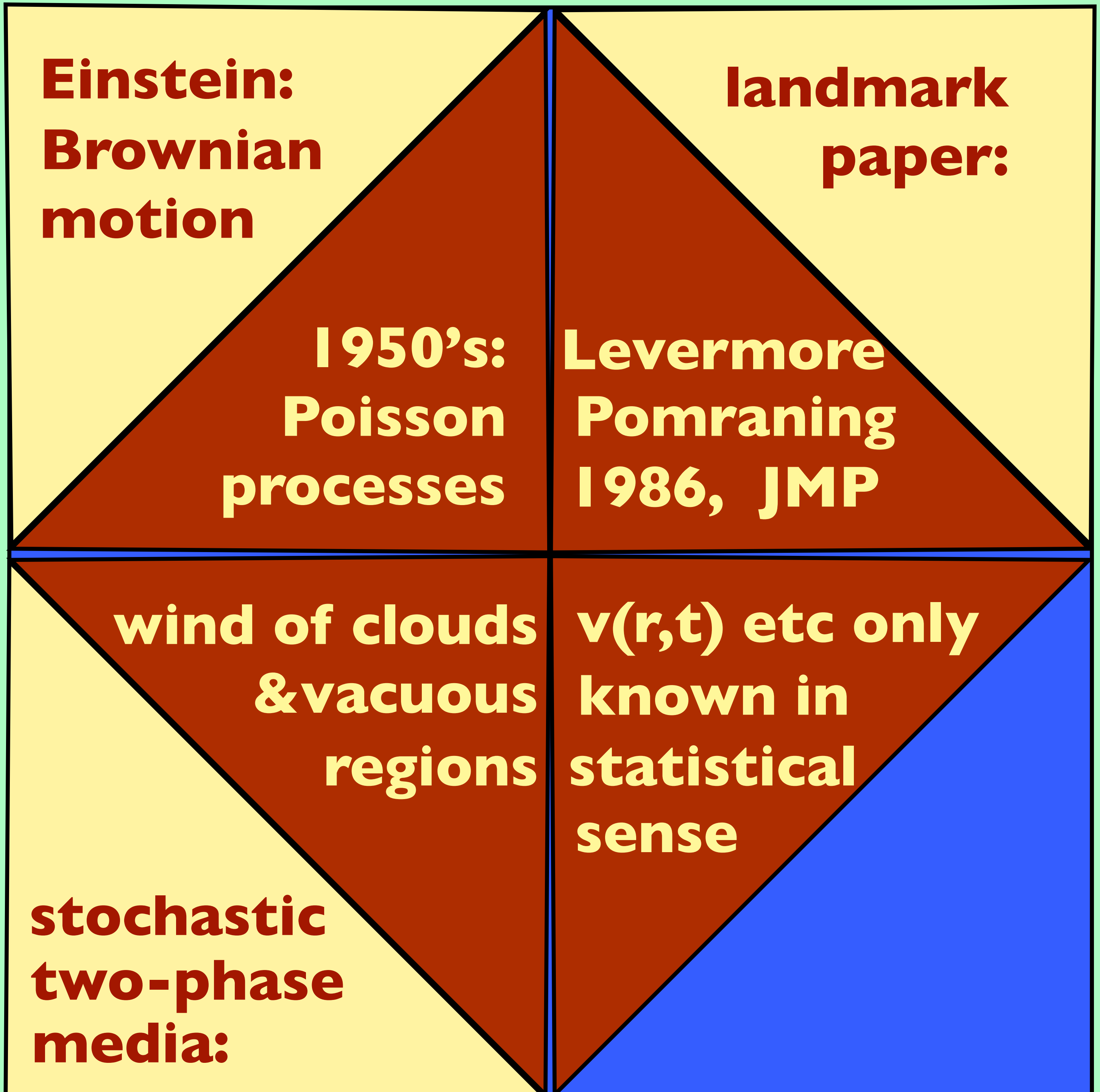
Einstein: Brownian motion
1950's: Poisson processes


Einstein:
Brownian
motion
1950's:
Poisson
processes
landmark
paper:


Einstein:
Brownian
motion
landmark
paper:
1950's:
Poisson
processes
Levermore
Pomraning
1986, JMP


Einstein:
Brownian
motion
landmark
paper:
1950's:
Poisson
processes
Levermore
Pomraning
1986, JMP
stochastic
two-phase
media:


Einstein:
Brownian
motion
landmark
paper:
1950's:
Poisson
processes
Levermore
Pomraning
1986, JMP
wind of clouds
&vacuous
regions
stochastic
two-phase
media:


Einstein:
Brownian
motion
landmark
paper:
1950's:
Poisson
processes
Levermore
Pomraning
1986, JMP
wind of clouds
&vacuous
regions
v(r,t) etc only
known in
statistical
sense
stochastic
two-phase
media:

Einstein:
Brownian
motion

landmark
paper:

1950's:
Poisson
processes

Levermore
Pomraning
1986, JMP

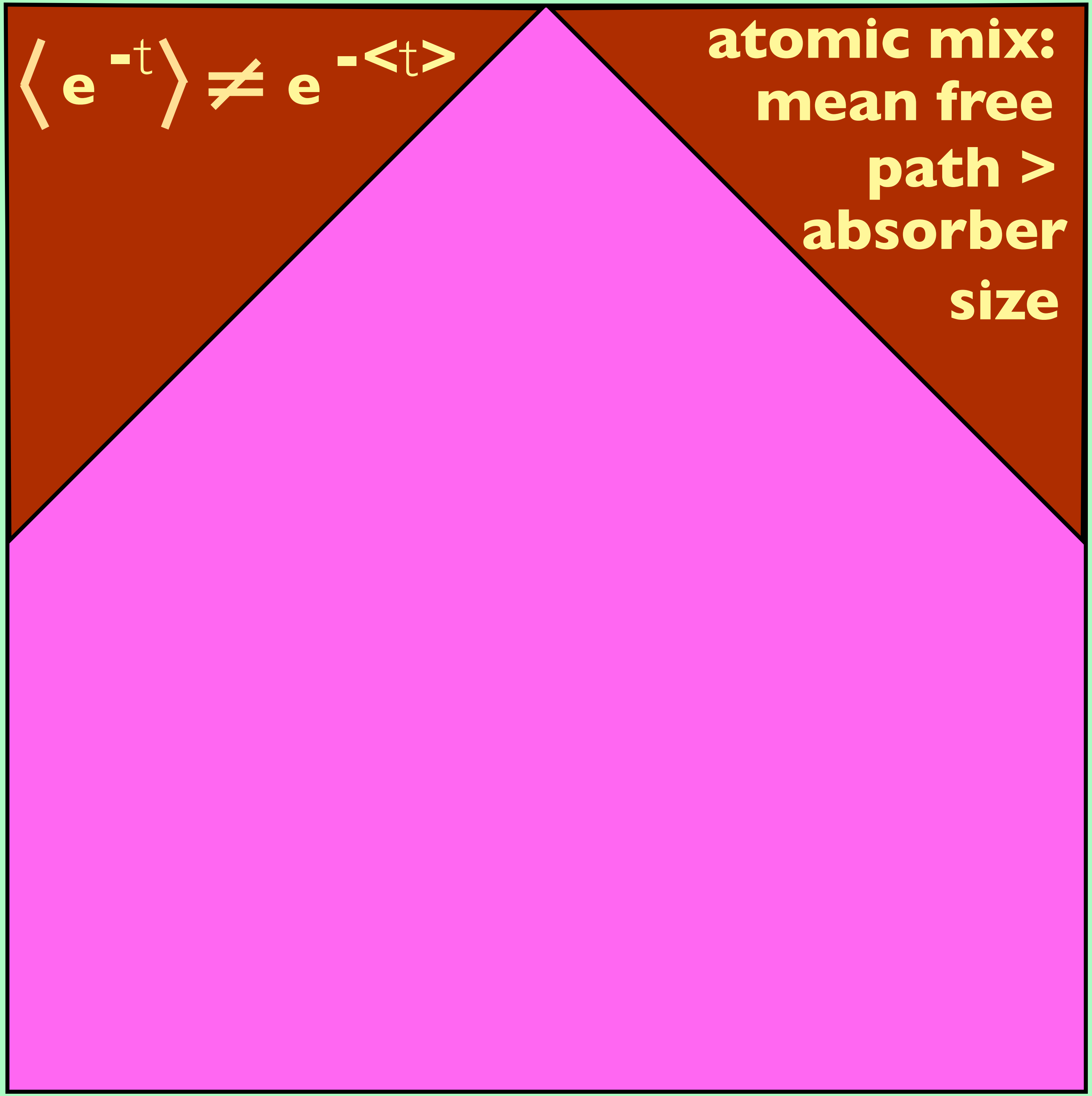
wind of clouds
&vacuous
regions

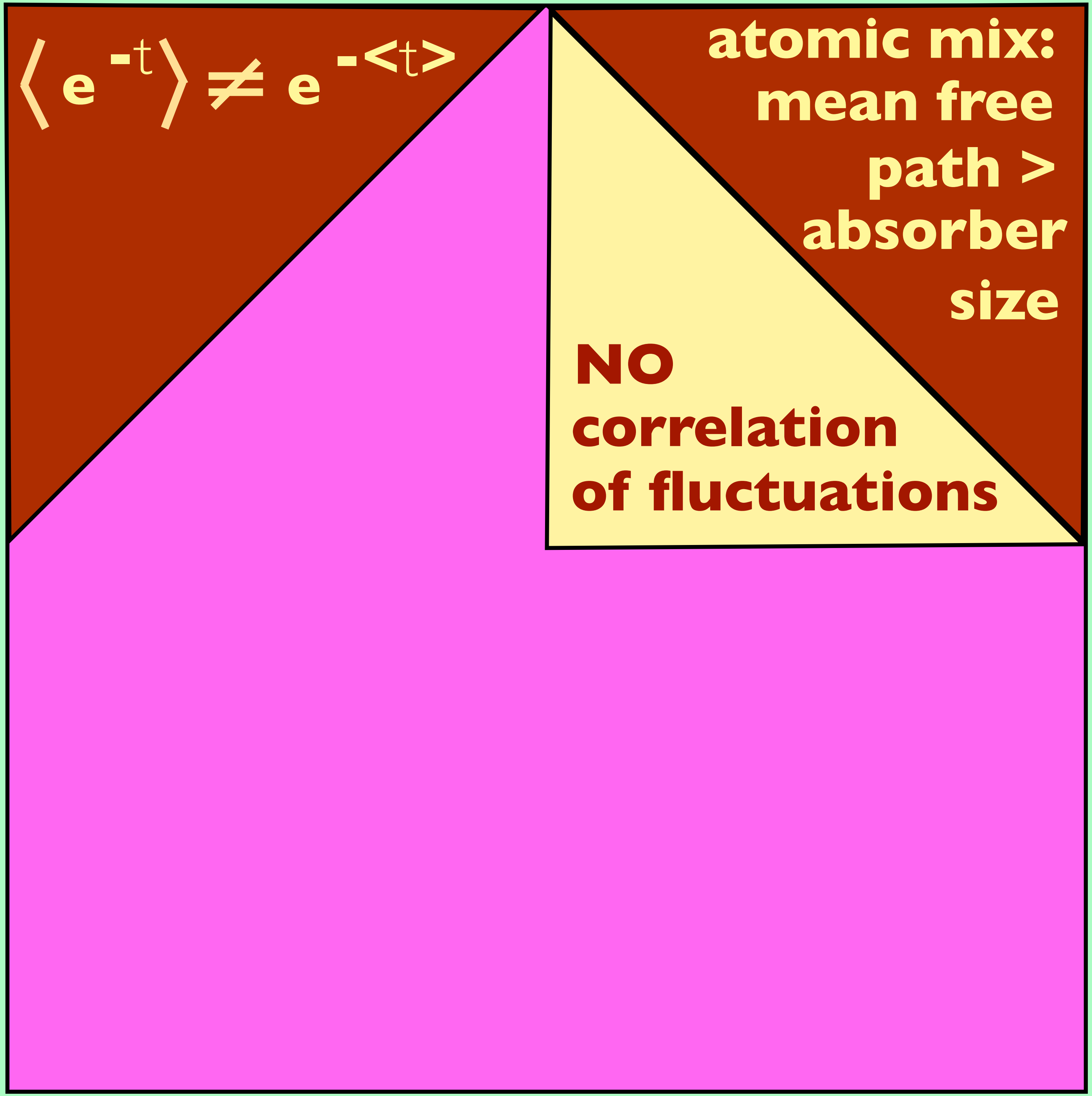
v(r,t) etc only
known in
statistical
sense

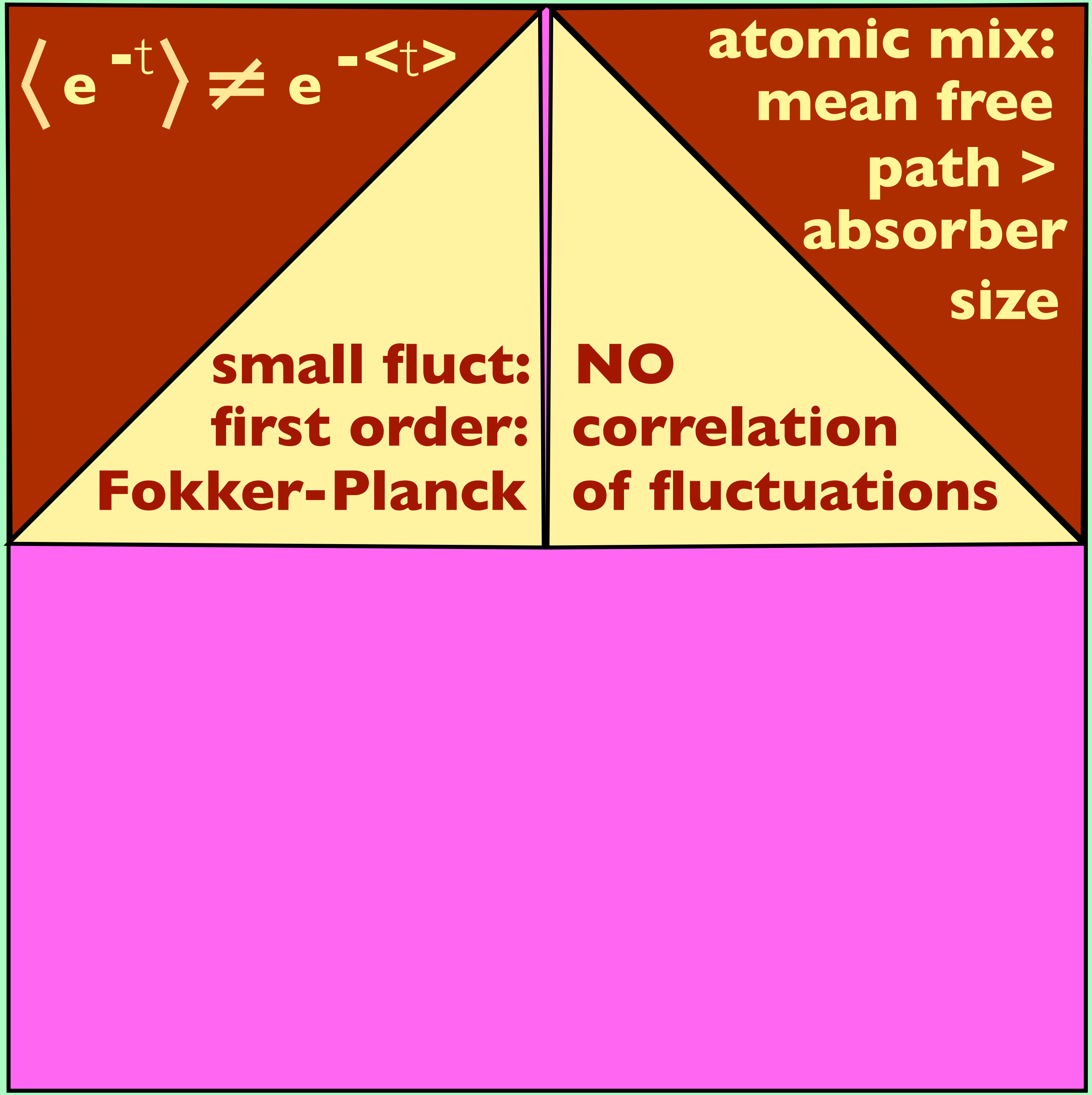
stochastic
two-phase
media:

ensemble
average of
rad. flux

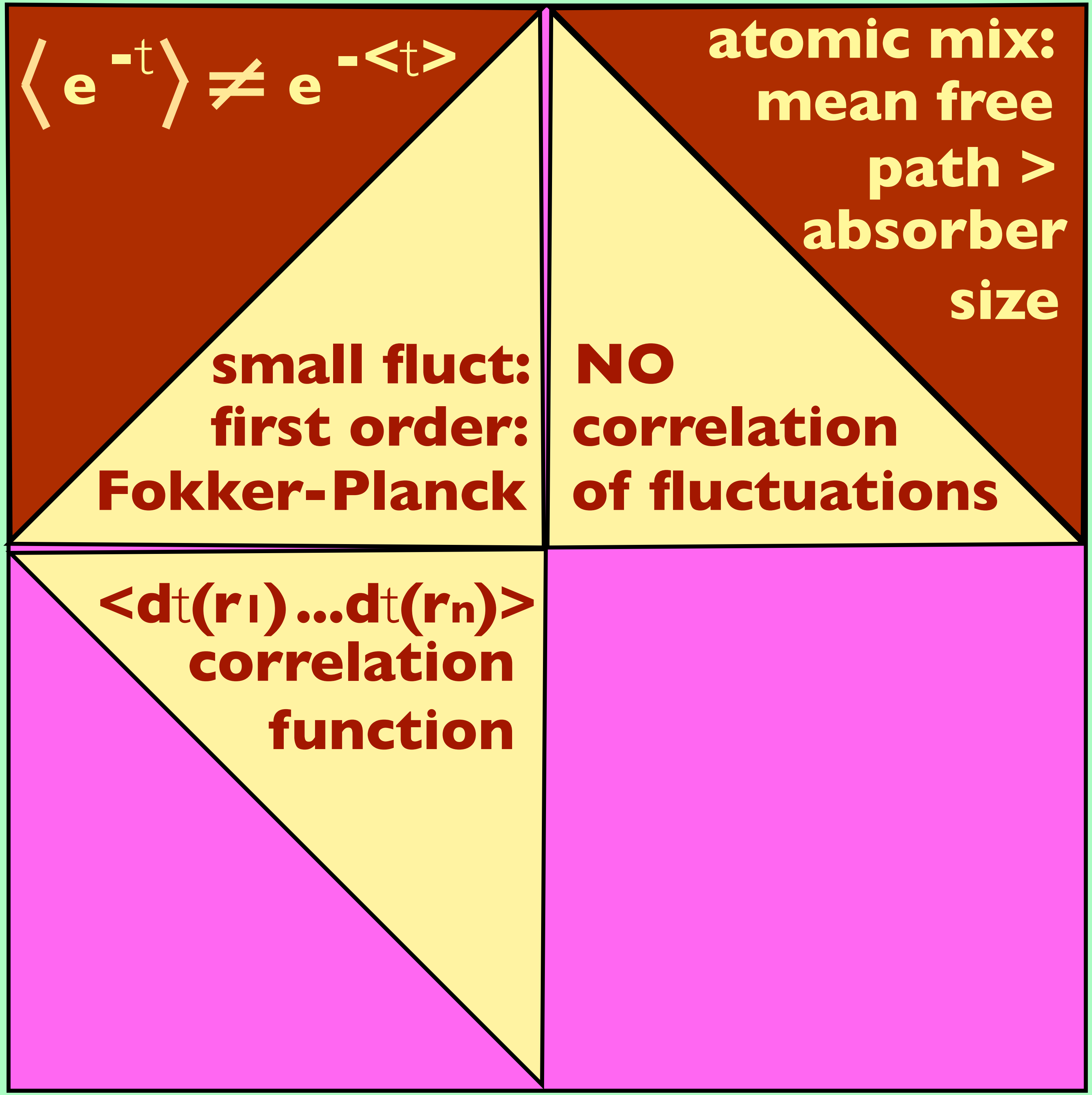
$$\langle e^{-t} \rangle \neq e^{-\langle t \rangle}$$


$\langle e^{-t} \rangle \neq e^{-\langle t \rangle}$
atomic mix: mean free path > absorber size

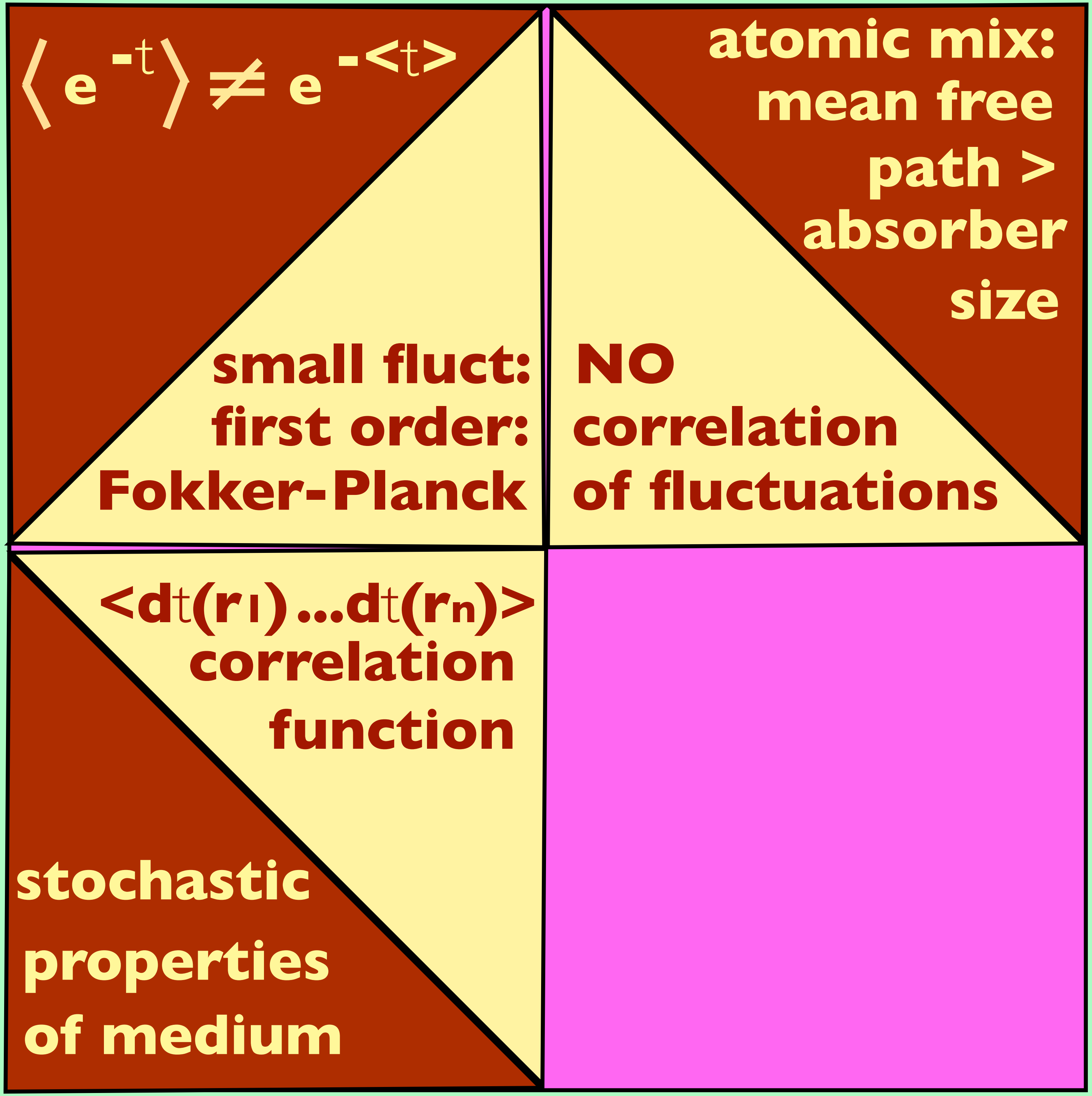

$\langle e^{-t} \rangle \neq e^{-\langle t \rangle}$
atomic mix: mean free path > absorber size
NO correlation of fluctuations

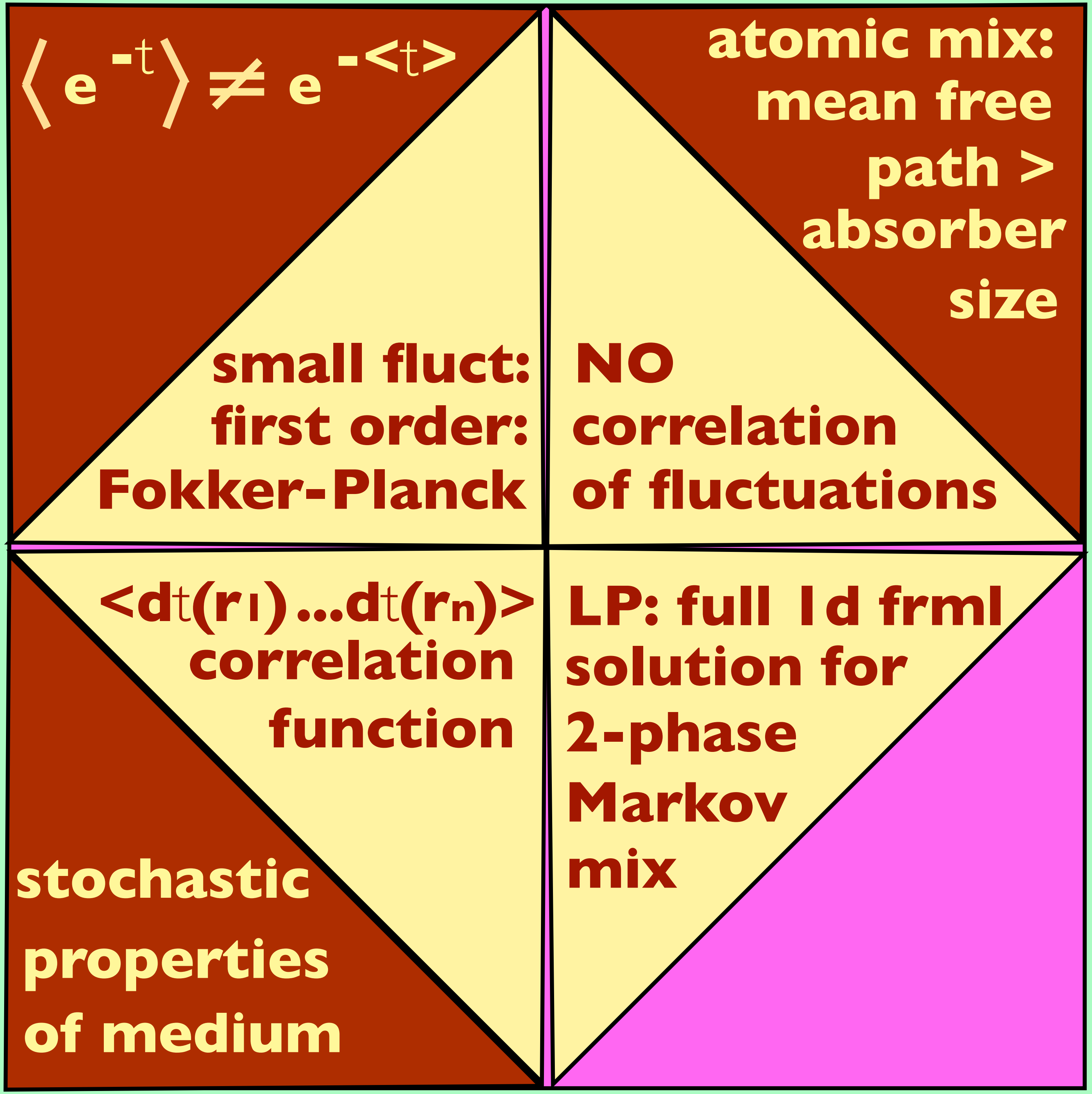


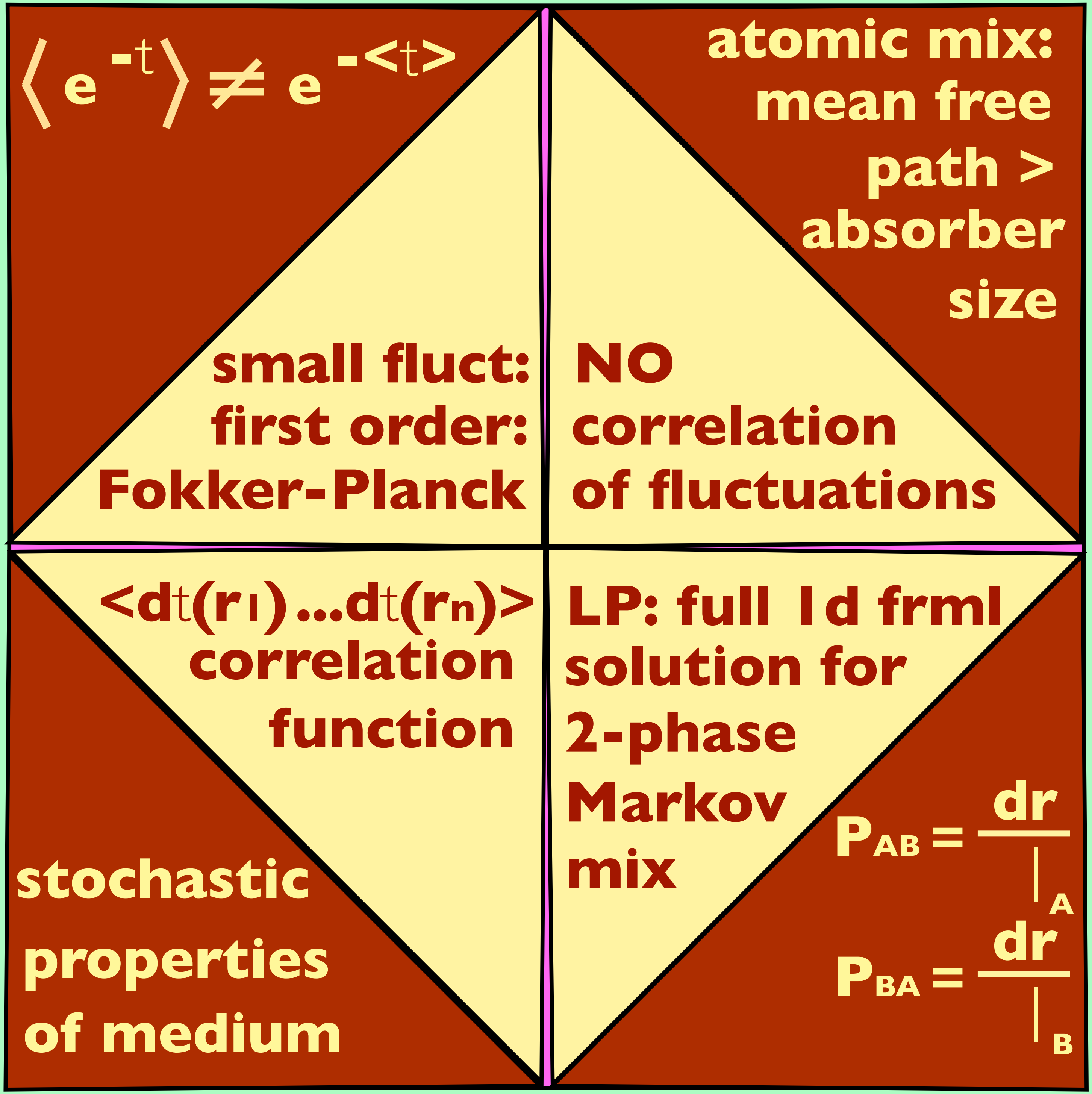
$\langle e^{-t} \rangle \neq e^{-\langle t \rangle}$
atomic mix: mean free path > absorber size
small fluct: first order: Fokker-Planck
NO correlation of fluctuations



⟨e -t⟩ ≠ e -<t>
atomic mix: mean free path > absorber size
small fluct: first order: Fokker-Planck
NO correlation of fluctuations
<dt(r1)...dt(rn)> correlation function


$\langle e^{-t} \rangle \neq e^{-\langle t \rangle}$
atomic mix: mean free path > absorber size
small fluct: first order: Fokker-Planck
NO correlation of fluctuations
$\langle dt(r_1) \ldots dt(r_n) \rangle$ correlation function
stochastic properties of medium


$\langle e^{-t} \rangle \neq e^{-\langle t \rangle}$
atomic mix: mean free path > absorber size
small fluct: first order: Fokker-Planck
NO correlation of fluctuations
$\langle dt(r_1) \ldots dt(r_n) \rangle$ correlation function
LP: full 1d frml solution for 2-phase Markov mix
stochastic properties of medium


⟨e^-t⟩ ≠ e^-<t>
atomic mix: mean free path > absorber size
small fluct: first order: Fokker-Planck
NO correlation of fluctuations
<dt(r1)...dt(rn)> correlation function
LP: full 1d frml solution for 2-phase Markov mix
stochastic properties of medium
PAB = dr / lA
PBA = dr / lB

From their definitions according to Eqs. (19) and (20), one can easily deduce recurrence relationships for $\widehat{T}_n$ and $\widetilde{T}_n$ given by

$$\widehat{T}_n = \langle (L^{-1}M)^{n-1}L^{-1}q\rangle - \sum_{i=2}^{n-2} \langle (L^{-1}M)^i\rangle \widehat{T}_{n-i}, \quad n\geqslant 3, \tag{22}$$

$$\widetilde{T}_n = \langle (L^{-1}M)^{n}\rangle\langle\Psi\rangle - \sum_{i=2}^{n-2} \langle (L^{-1}M)^i\rangle \widetilde{T}_{n-i}, \quad n\geqslant 3. \tag{23}$$

These recurrence relationships are initiated by the explicit $n=2$ expressions

$$\widehat{T}_2 = \langle L^{-1}ML^{-1}q\rangle, \quad \widetilde{T}_2 = \langle L^{-1}ML^{-1}M\rangle\langle\Psi\rangle. \tag{24}$$

From these recurrence relationships one can prove, by induction, that $\widehat{T}_n$ and $\widetilde{T}_n$ can be written in explicit form as

$$\widehat{T}_n = \sum_i a_i \langle (L^{-1}M)^{p_1}\rangle\langle (L^{-1}M)^{p_2}\rangle \cdots \langle (L^{-1}M)^{p_i}L^{-1}q\rangle, \quad n\geqslant 2, \tag{25}$$

$$\widetilde{T}_n = \sum_i a_i \langle (L^{-1}M)^{p_1}\rangle\langle (L^{-1}M)^{p_2}\rangle \cdots \langle (L^{-1}M)^{p_i}\rangle\langle\Psi\rangle, \quad n\geqslant 2. \tag{26}$$

The powers p_k can assume any non-negative integer values subject to the constraint

$$p_1 + p_2 + \cdots + p_i = m, \tag{27}$$

where $m = n-1$ for Eq. (25) and $m = n$ for Eq. (26). The sum over i in Eqs. (25) and (26) is over all possible combinations for the powers p_k, and $a_i = +1$ for an odd number of terms in the product involving ensemble averaged operators, and $a_i = -1$ for an even number of terms. As an explicit example, we have

$$\widehat{T}_5 = \langle (L^{-1}M)^4L^{-1}q\rangle - \langle (L^{-1}M)^2\rangle\langle (L^{-1}M)^2L^{-1}q\rangle - \langle (L^{-1}M)^3\rangle\langle L^{-1}ML^{-1}q\rangle, \tag{28}$$

$$\widetilde{T}_5 = \langle (L^{-1}M)^5\rangle\langle\Psi\rangle - \langle (L^{-1}M)^2\rangle\langle (L^{-1}M)^3\rangle\langle\Psi\rangle - \langle (L^{-1}M)^3\rangle\langle (L^{-1}M)^2\rangle\langle\Psi\rangle. \tag{29}$$

To proceed, we define the nth-order spatial correlations according to

$$\widehat{N}_n(s_1,\ldots,s_n) = \langle M(s_1)M(s_2)\cdots M(s_{n-1})q(s_n)\rangle, \tag{30}$$

and

$$\widetilde{N}_n(s_1,\ldots,s_n) = \langle M(s_1)M(s_2)\cdots M(s_{n-1})M(s_n)\rangle, \tag{31}$$

and, in analogy to Eq. (4), we define τ_n as the optical depth corresponding to a distance s_n, i.e.,

$$\tau_n = \int_0^{s_n} ds'\sigma(s'). \tag{32}$$

In terms of these definitions, we can write, using Eq. (15) for L^{-1},

$$L\widehat{T}_n = \int_0^s ds_1 \int_0^{s_1} ds_2 \cdots \int_0^{s_{n-2}} ds_{n-1} \times \exp[-(\langle\tau\rangle - \langle\tau_{n-1}\rangle)] \times \sum_i a_i [\widetilde{N}_{p_1}\widetilde{N}_{p_2}\cdots\widetilde{N}_{p_{i-1}}\widehat{N}_{p_i}], \tag{33}$$

and x. The probability of exactly n transitions from state A in a distance $s-x$ is given by

$$P = G_n(s-x) - G_{n+1}(s-x). \tag{95}$$

Thus we may express the distribution for $\beta(s)$ as

$$P[\beta(s)<x] = \sum_{n=0}^{\infty} H_n(x)[G_n(s-x) - G_{n+1}(s-x)]. \tag{96}$$

In a similar fashion, we can deduce the distribution for $\alpha(s)$ as

$$P[\alpha(s)<x] = \sum_{n=0}^{\infty} G_n(x)[H_n(s-x) - H_{n+1}(s-x)]. \tag{97}$$

Now, it is known[12] that the sum of n identically distributed exponential random variables with parameter $1/\lambda$ is given by a gamma distribution with parameters n and $1/\lambda$. Thus we have in our case

$$G_n(x) = \int_0^x dx' \frac{(x'/\lambda_A)^{n-1}e^{-x'/\lambda_A}}{\lambda_A(n-1)!}, \quad n\geqslant 1, \tag{98}$$

$$H_n(y) = \int_0^y dy' \frac{(y'/\lambda_B)^{n-1}e^{-y'/\lambda_B}}{\lambda_B(n-1)!}, \quad n\geqslant 1. \tag{99}$$

From Eqs. (88), (98), and (99) we deduce

$$G_n(s-x) - G_{n+1}(s-x) = \frac{1}{n!}\left(\frac{s-x}{\lambda_A}\right)^n e^{-(s-x)/\lambda_A}, \quad n\geqslant 0,$$

$$H_n(s-x) - H_{n+1}(s-x) = \frac{1}{n!}\left(\frac{s-x}{\lambda_B}\right)^n e^{-(s-x)/\lambda_B}, \quad n\geqslant 0. \tag{100}$$

Thus $P[\beta(s)<x]$, given by Eq. (96), becomes

$$P[\beta(s)<x] = e^{-(s-x)/\lambda_A}\left\{1 + \left(\frac{s-x}{\lambda_A\lambda_B}\right)^{1/2} \times \int_0^x dy \frac{e^{-y/\lambda_B}}{y^{1/2}} I_1\left[2\left(\frac{(s-x)y}{\lambda_A\lambda_B}\right)^{1/2}\right]\right\}, \tag{101}$$

where we have recognized the Taylor series expansion for the modified Bessel function as

$$I_1(z) = \sum_{r=0}^{\infty} \frac{(z/2)^{2r+1}}{r!(r+1)!}. \tag{102}$$

A similar result is found for $P[\alpha(s)<x]$. Using the fact that $P[\beta(s)>x] = 1 - P[\beta(s)<x]$ and inserting these results into Eq. (93) gives the cumulative distribution function in the optical depths range $\sigma_B s<\tau<\sigma_A s$ as

$$F(\tau,s) = p_A\left\{1 - e^{-u}\left[1 + 2\int_0^{(uv)^{1/2}} dx\, I_1(2x)e^{-x^2/u}\right]\right\} + p_B\, e^{-v}\left[1 + 2\int_0^{(uv)^{1/2}} dx\, I_1(2x)e^{-x^2/v}\right], \quad \sigma_B s<\tau<\sigma_A s, \tag{103}$$

where we have defined

$$u = \frac{1}{\lambda_A}\left(\frac{\tau - \sigma_B s}{\sigma_A - \sigma_B}\right); \quad v = \frac{1}{\lambda_B}\left(\frac{\sigma_A s - \tau}{\sigma_A - \sigma_B}\right). \tag{104}$$

Clouds in winds:

Clouds in winds:

$s_{\mathbf{vac}} \rightarrow \mathbf{0}, \; s_{\mathbf{cloud}} \rightarrow \infty$

Clouds in winds:

$s_{vac} \rightarrow 0, \quad s_{cloud} \rightarrow \infty$

$$\frac{<I(z)>}{I_0} = \exp(-z/l_{vac})$$

Clouds in winds:

$s_{vac} \rightarrow 0, \quad s_{cloud} \rightarrow \infty$

$$\frac{\langle I(z) \rangle}{I_0} = \exp(-z/\underbrace{l_{vac}}_{\text{mean free path}})$$

Clouds in winds:

$s_{vac} \rightarrow 0, \; s_{cloud} \rightarrow \infty$

$$\frac{<I(z)>}{I_0} = \exp(-z/\overbrace{l_{vac}}^{\text{mean free path}})$$

cloud opacity (∞) does not enter!

Clouds in winds:

in general:

$$\langle \exp(-t) \rangle \gg \exp(-\langle t \rangle)$$

$$s_{vac} \rightarrow 0, \quad s_{cloud} \rightarrow \infty$$

mean free path

$$\frac{\langle I(z) \rangle}{I_0} = \exp(-z/l_{vac})$$

cloud opacity (∞) does not enter!

X-

rays

NUMERICAL PROBLEM

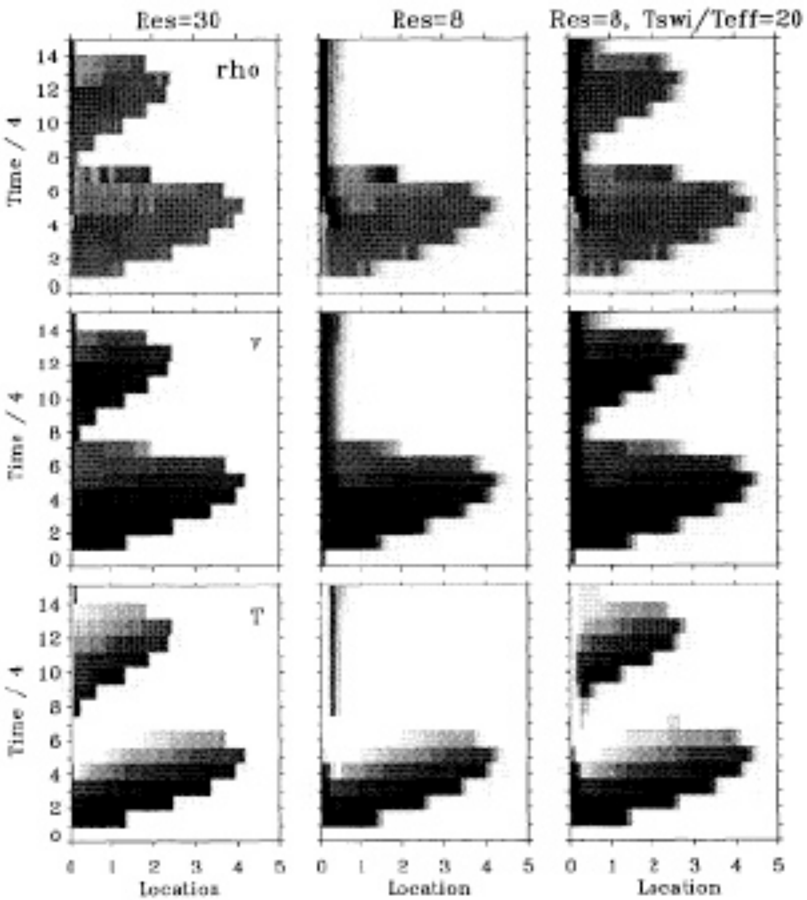


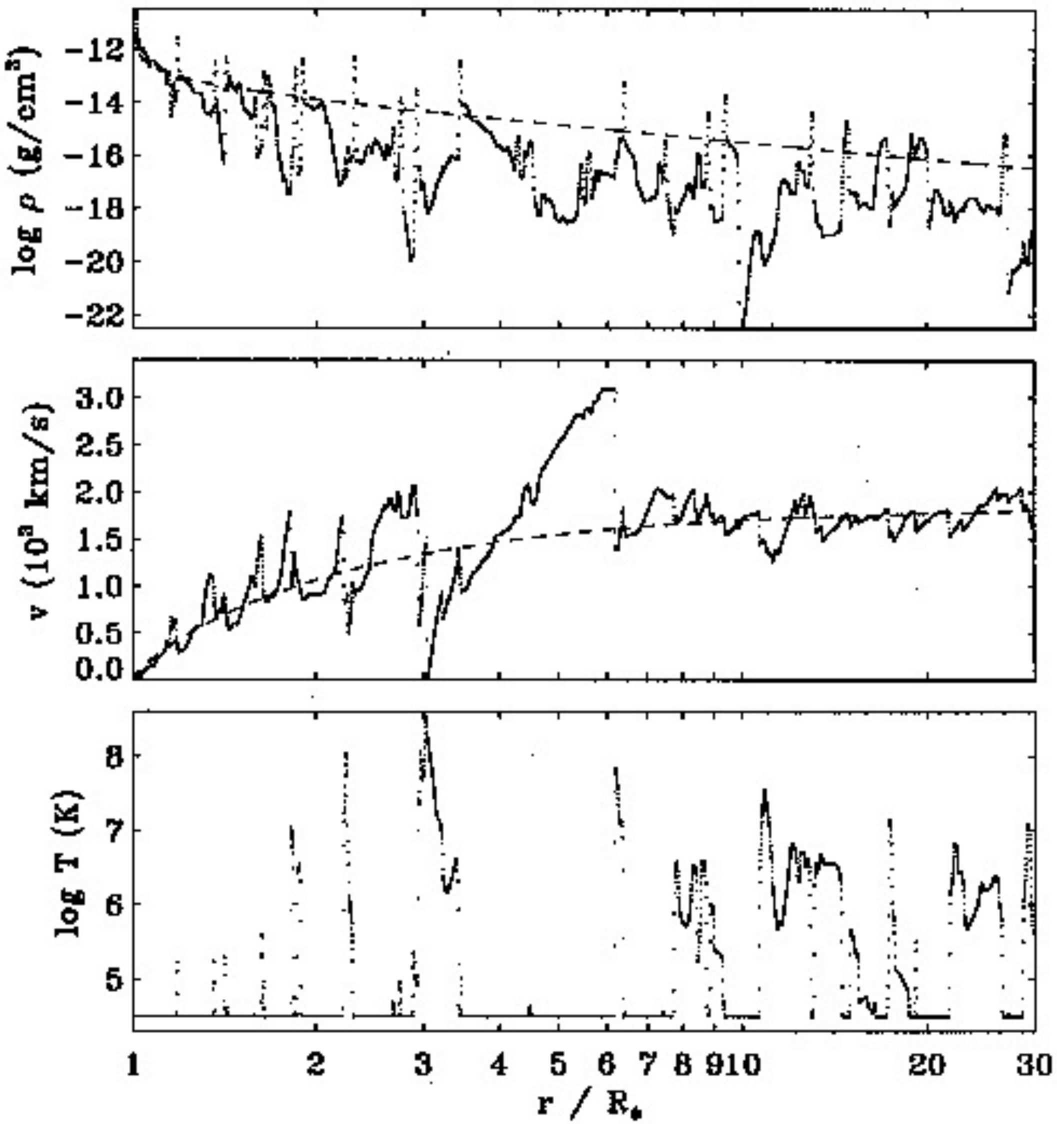
Fig. 6. Global thermal instability oscillations for $\alpha = -1/2$ at grid resolutions Res = 30 (resolved; left panel) and Res = 8 (collapsed; mid panel). The snapshots are separated by the cooling time $t_c = 4$. The right panel is for the same resolution as the mid panel, but now using $T_{swi}/T_{eff} = 20$, resp. $T_{post}/T_{swi} = 5$. This hinders the collapse of the cooling zone

irreversible collapse of radiative cooling zone

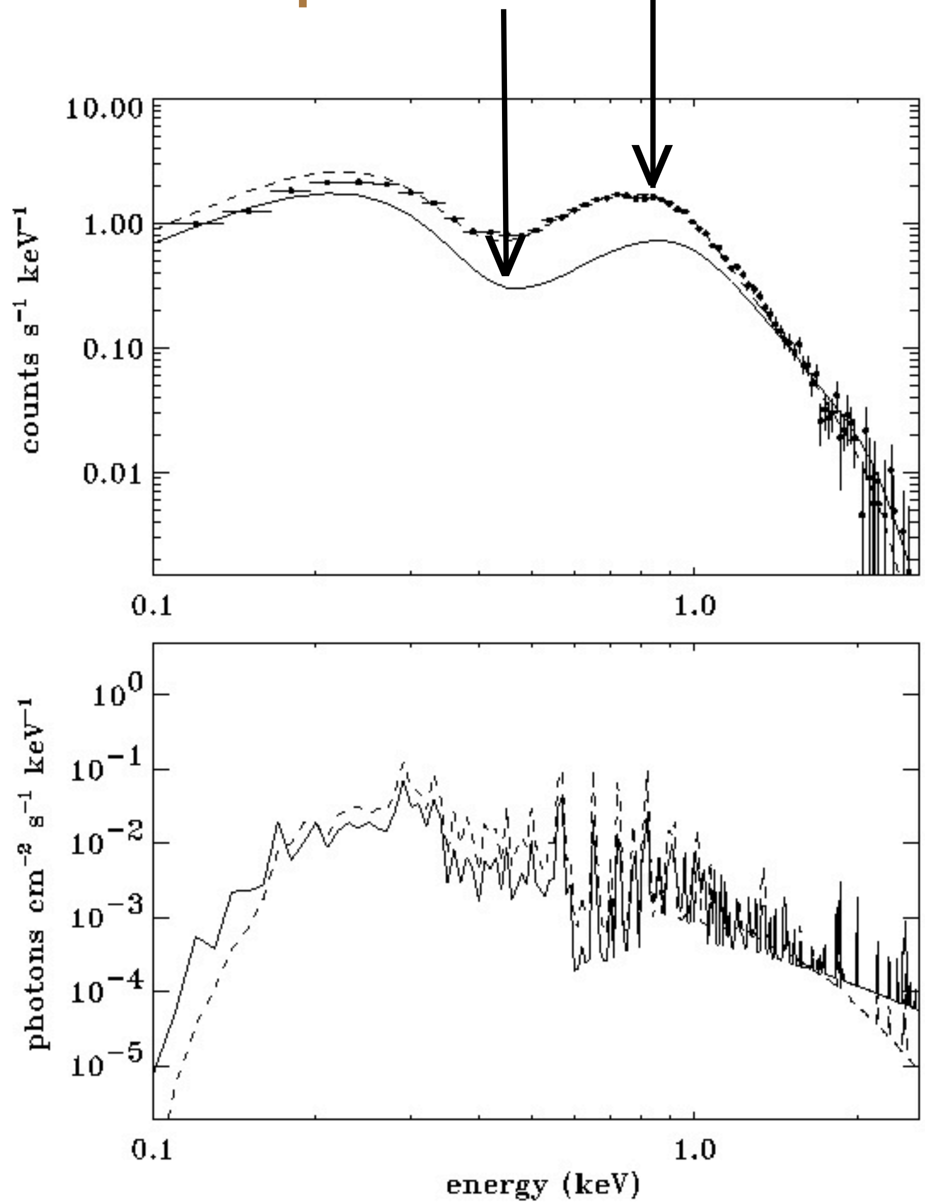
due to oscillatory thermal instability

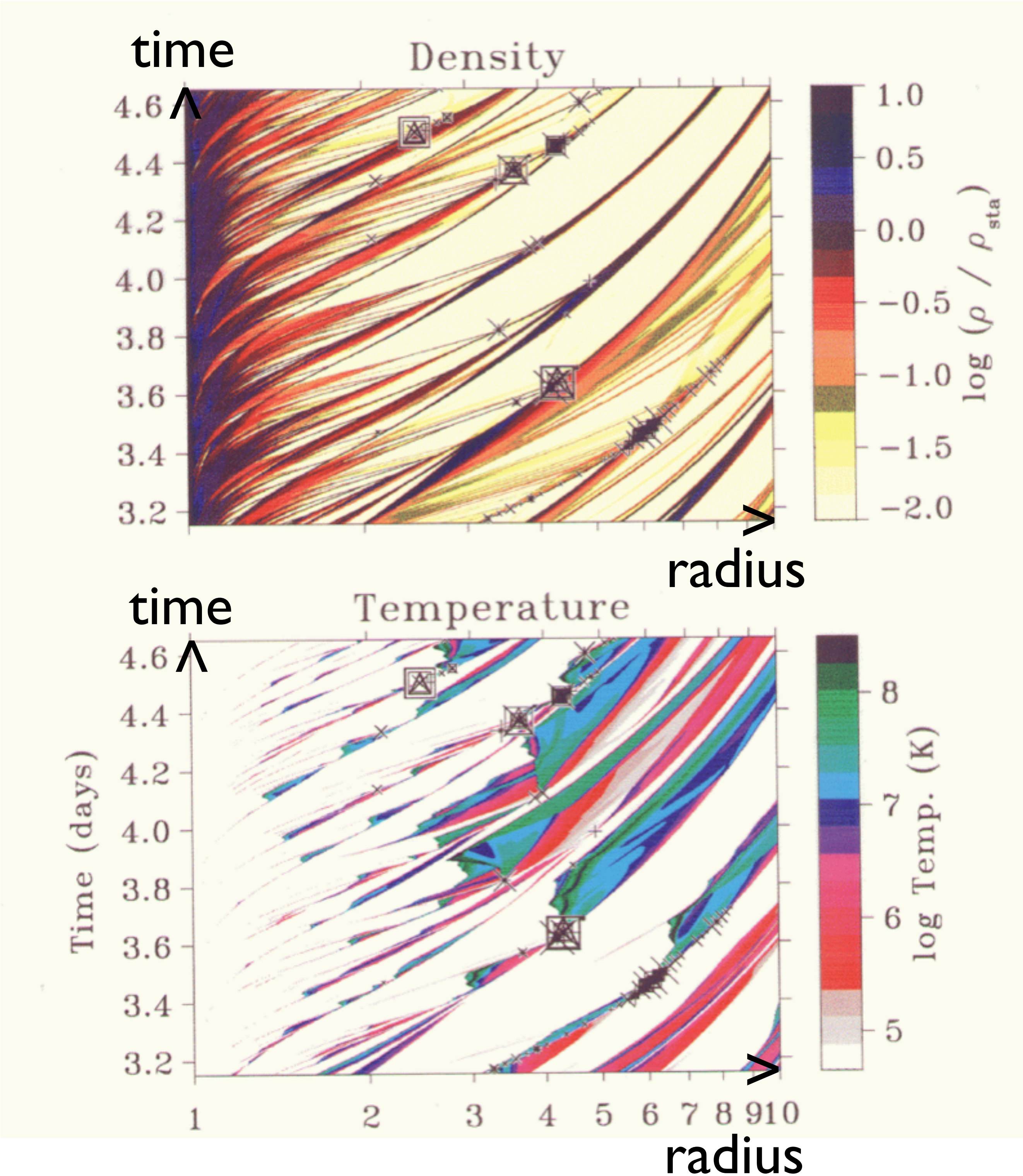
solution: modify cooling function artificially at low temperature (Feldmeier 1995)

Wind-shock model for zeta Ori

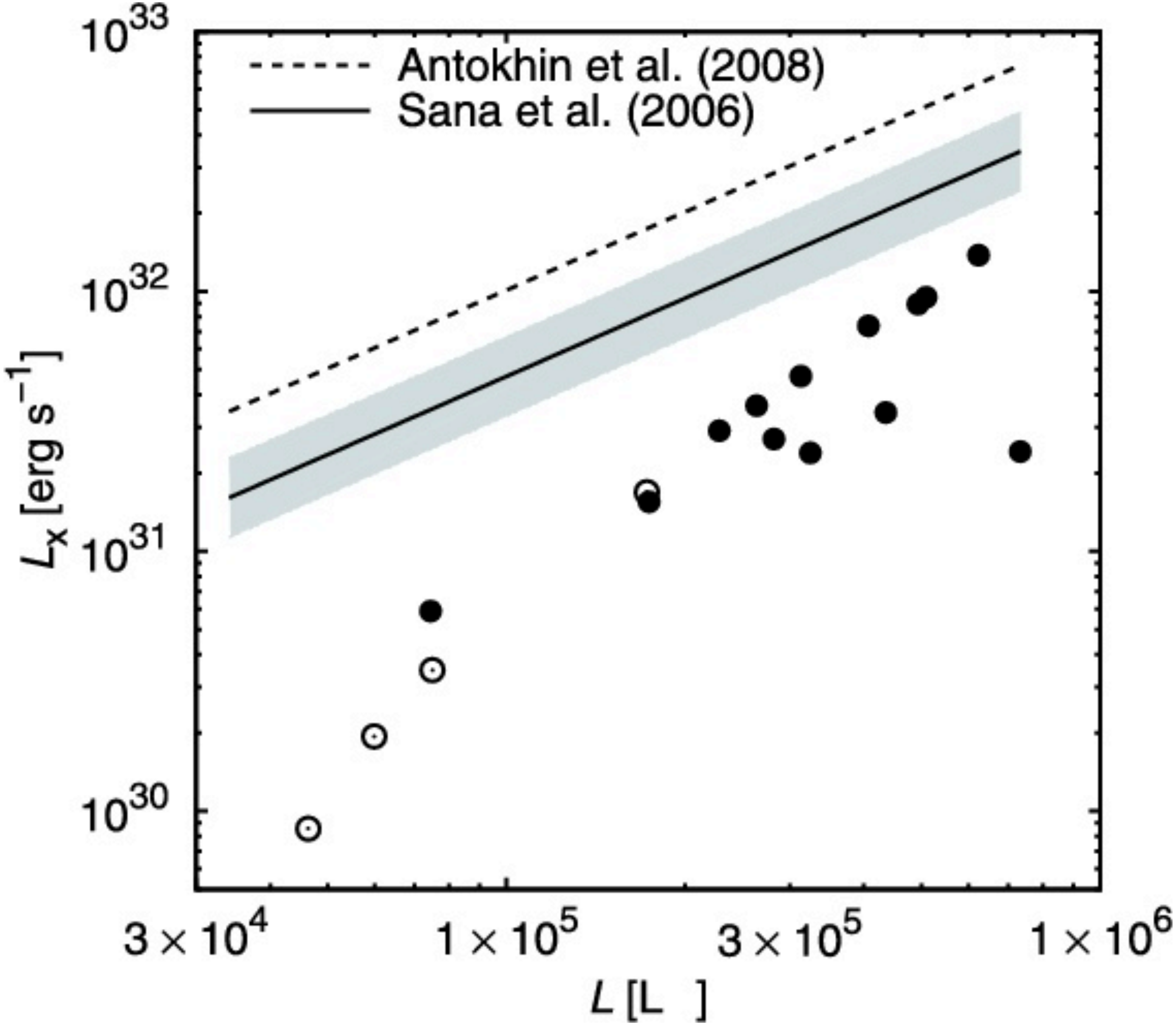


Rosat data and model prediction



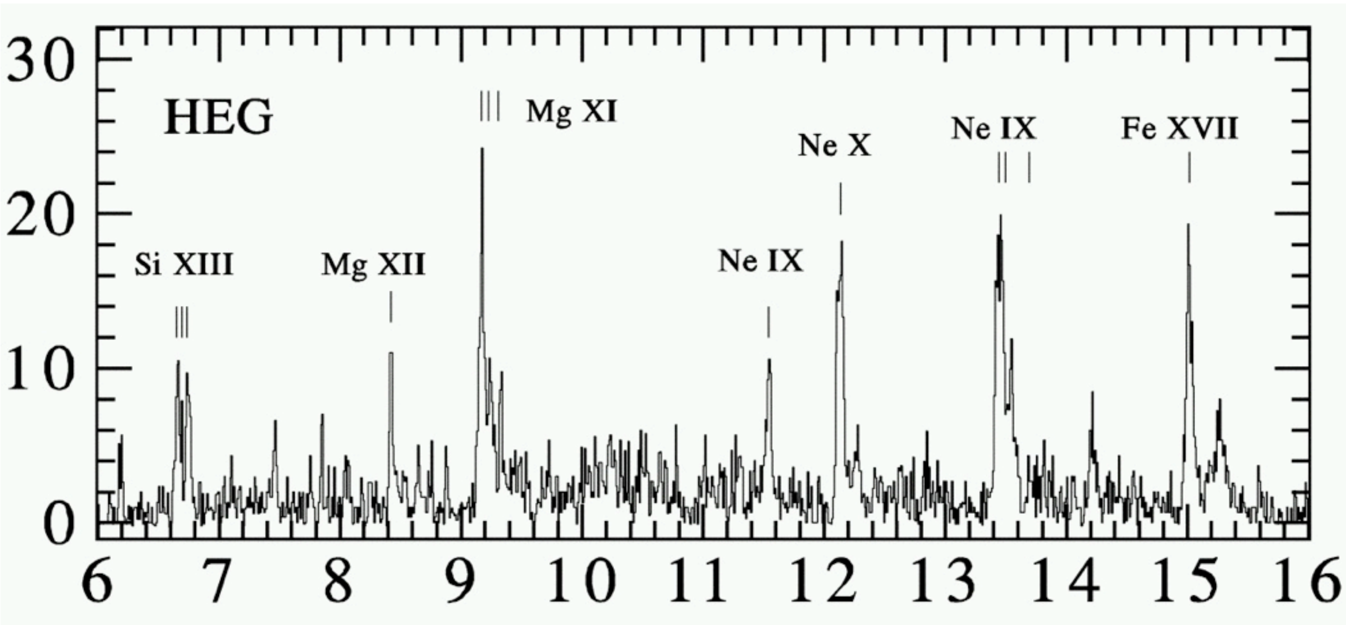


Cloud-shell collisions as origin of X-ray emission

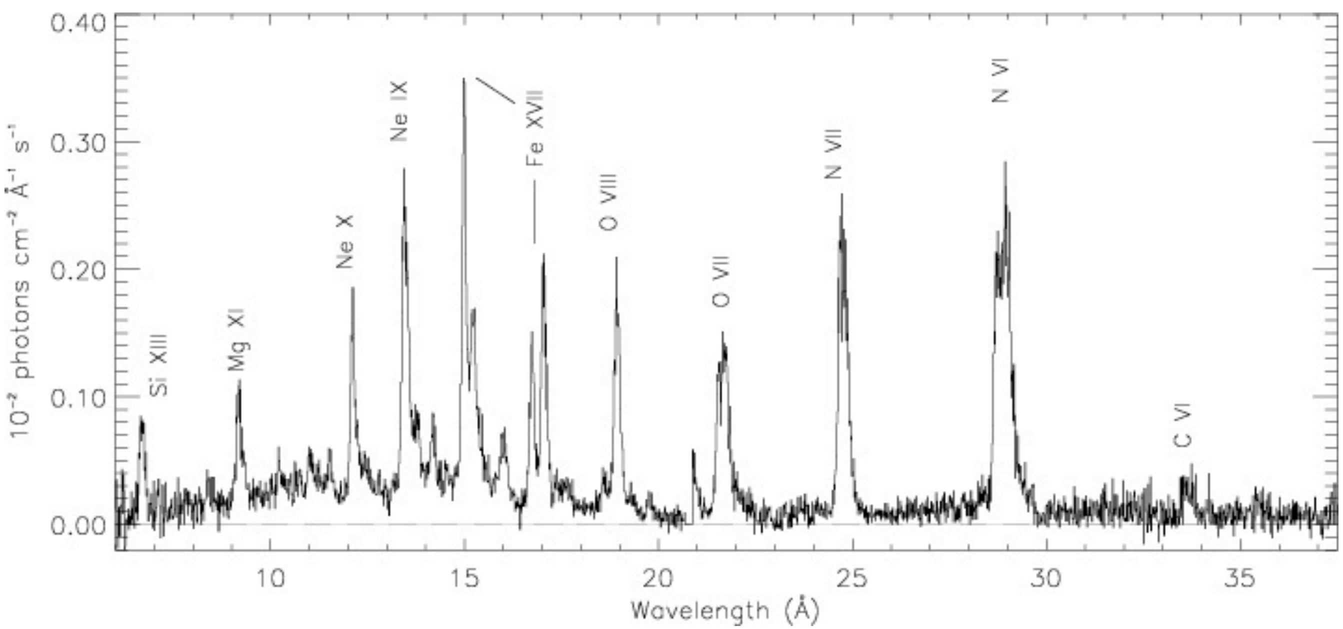


Krticka, Feldmeier, Kubat et al. 2009, A&A

X-ray lines

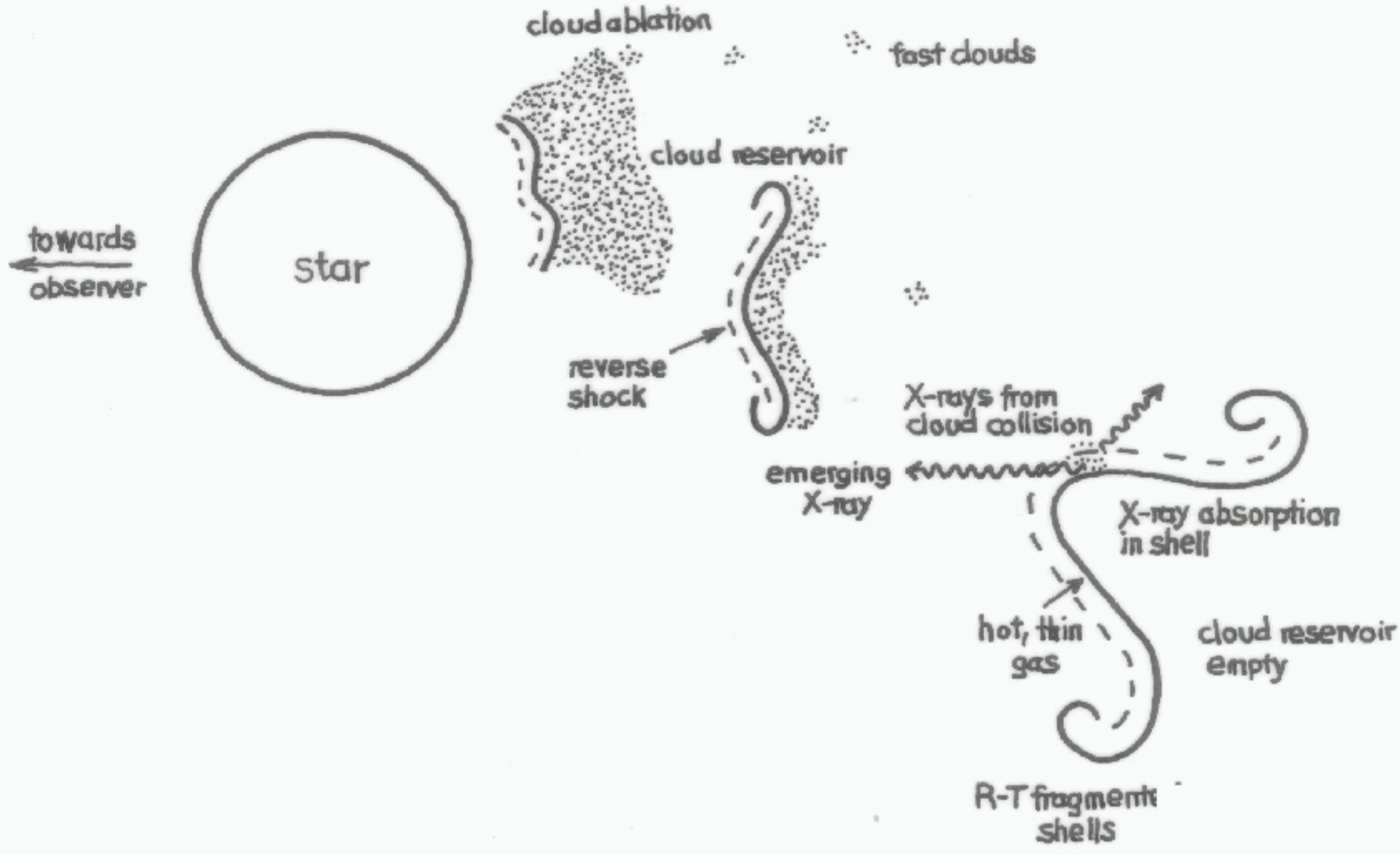


Chandra
zeta Ori
Waldron & Cassinelli 2001, ApJ

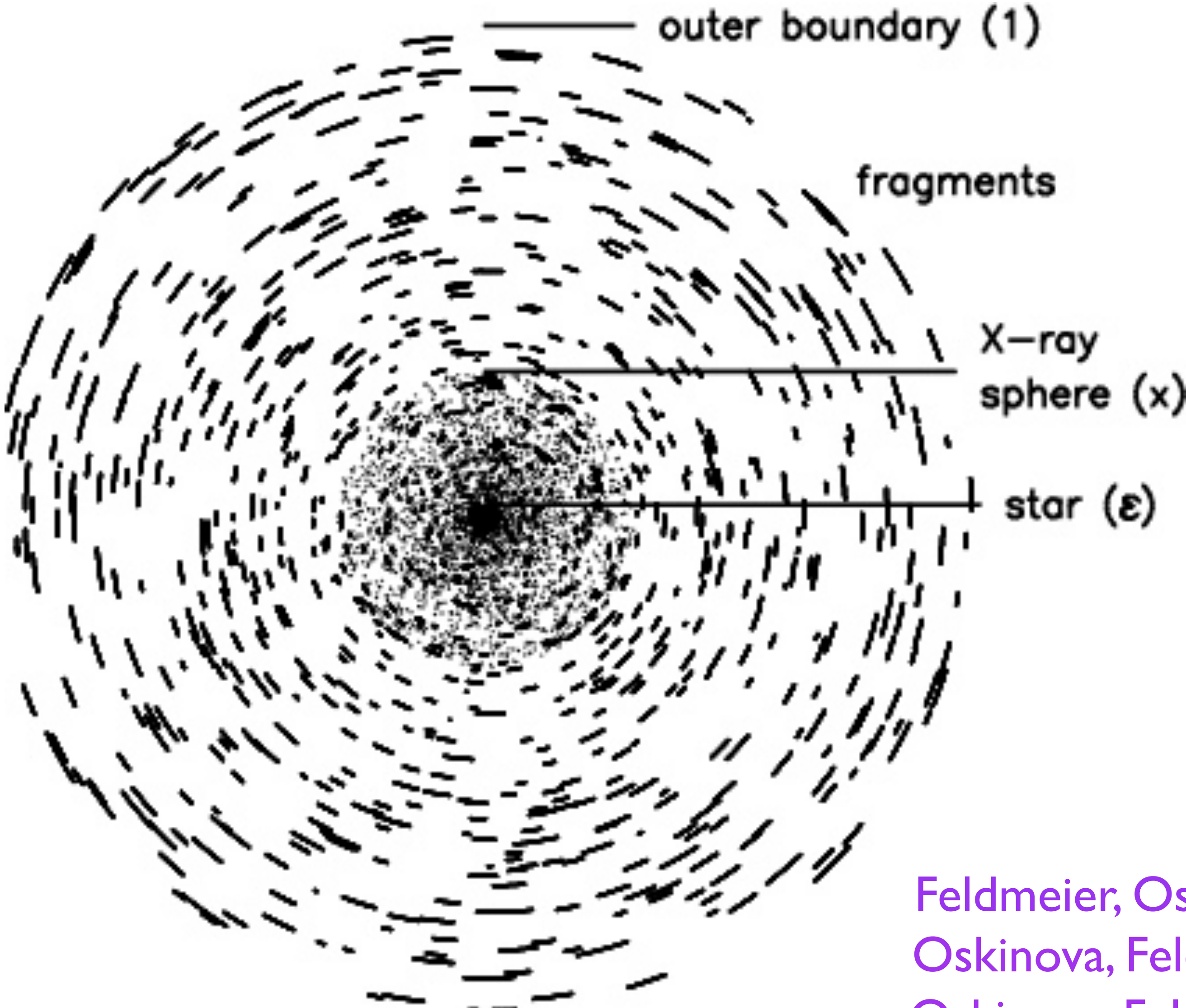


XMM
zeta Pup
Kahn et al. 2001, A&A

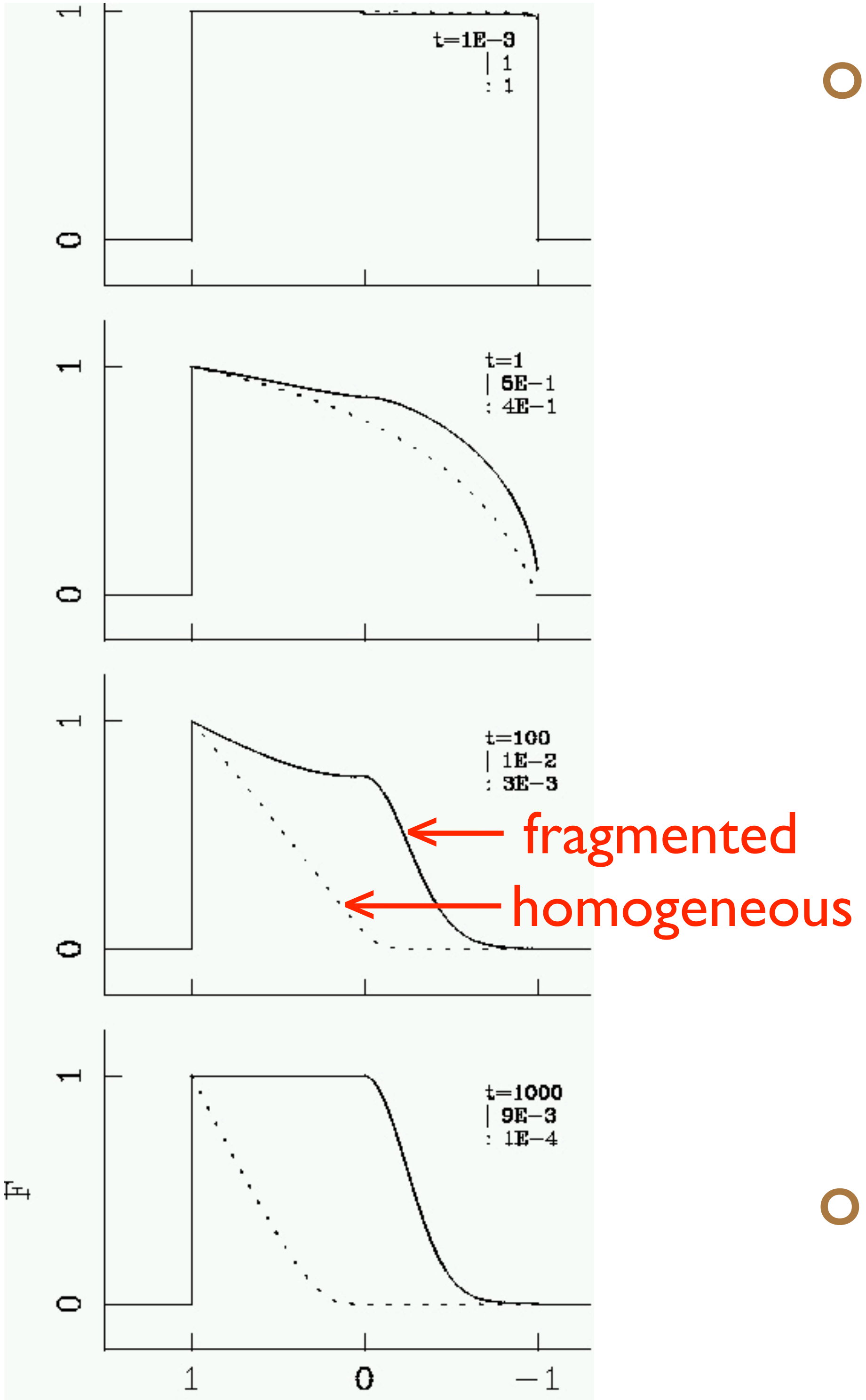
Wind Structure



... and its model realization



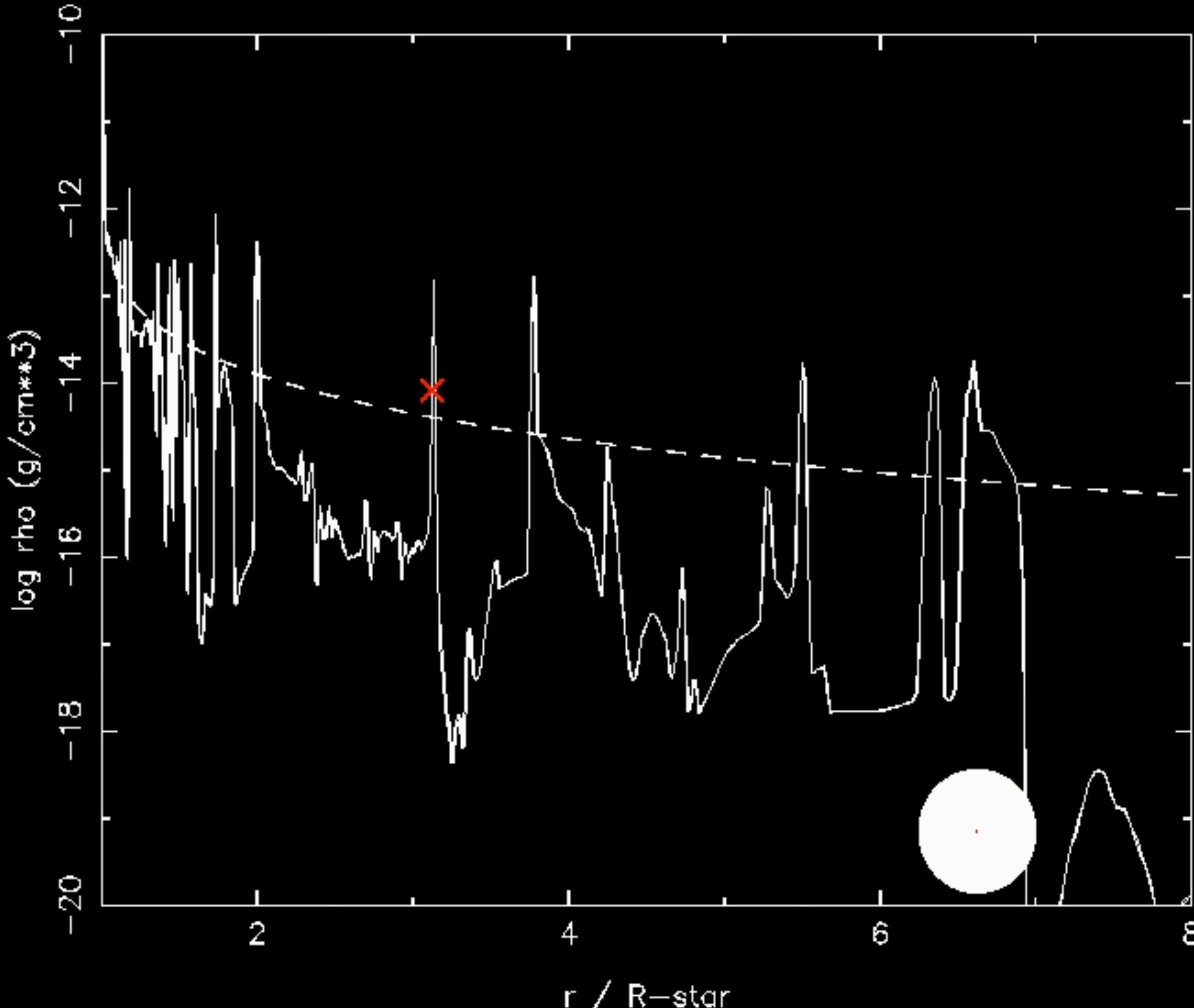
Feldmeier, Oskinova, Hamann 2003, A&A
Oskinova, Feldmeier, Hamann 2004, A&A
Oskinova, Feldmeier, Hamann 2006, MNRAS
Oskinova, Hamann, Feldmeier 2007, A&A



optically thin

optically thick

CALCULATED X-RAY EMISSION LINE PROFILES



-10
-12
-14
-16
-18
-20
log rho (g/cm**3)
2
4
6
8
r / R-star