Mass and angular momentum loss via decretion disks

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Stellar evolution: initial parameters

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* chemical composition

Stellar evolution: initial parameters

* the most important parameter: mass
* chemical composition
* rotation may also play a role

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* additional mixing: due to instabilities caused by differential rotation
* change of the stellar shape

Influence of the rotation on the stellar shape

- Roche model
  - gravitation force: point source approximation
  - rigid body rotation

Influence of the rotation on the stellar shape

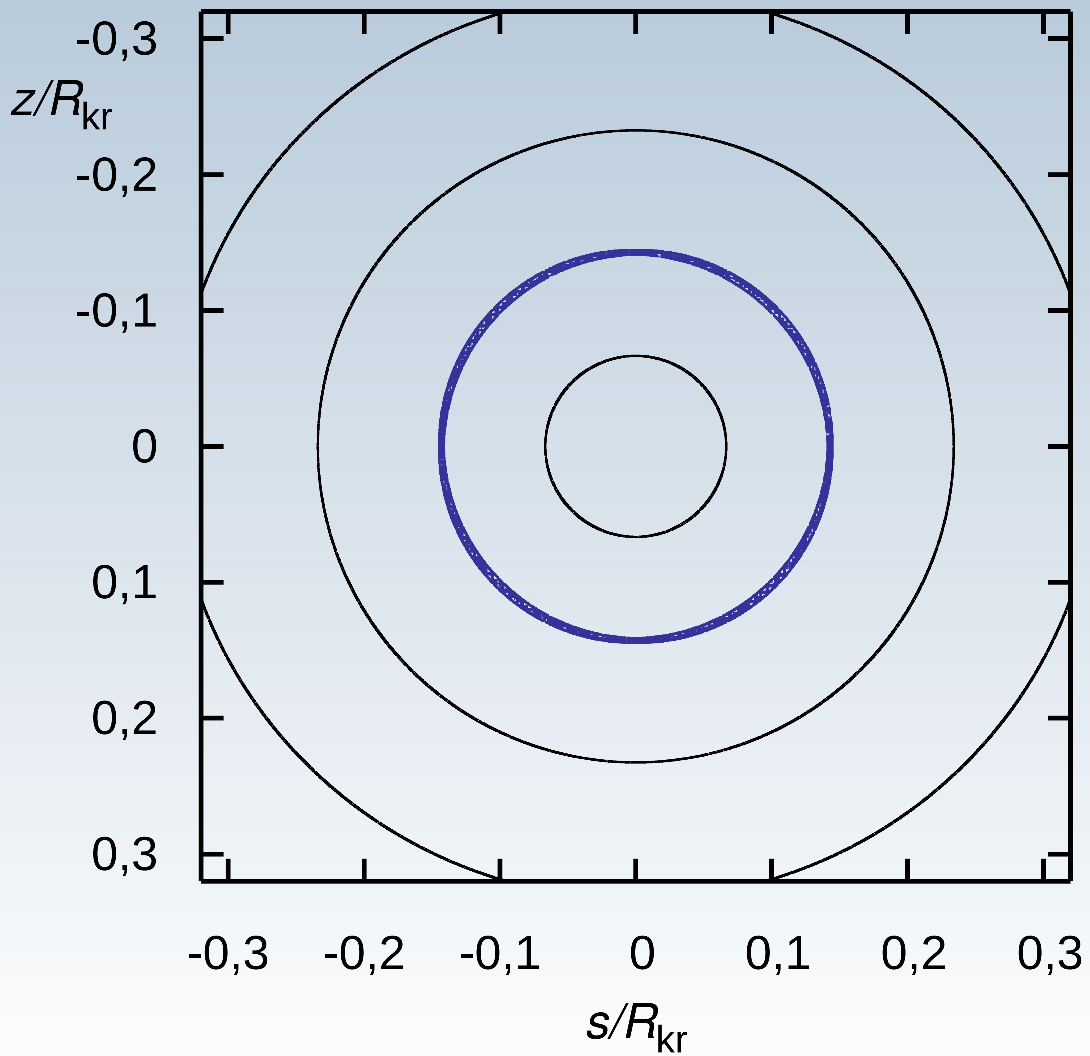
- Roche model
  - gravitation force: point source approximation
  - rigid body rotation
- potential

$$\Phi = -\frac{GM}{r} - \frac{1}{2}s^2\Omega^2$$

  - M is the stellar mass
  - Ω is the rotational frequency
  - s is the distance from the rotational axis

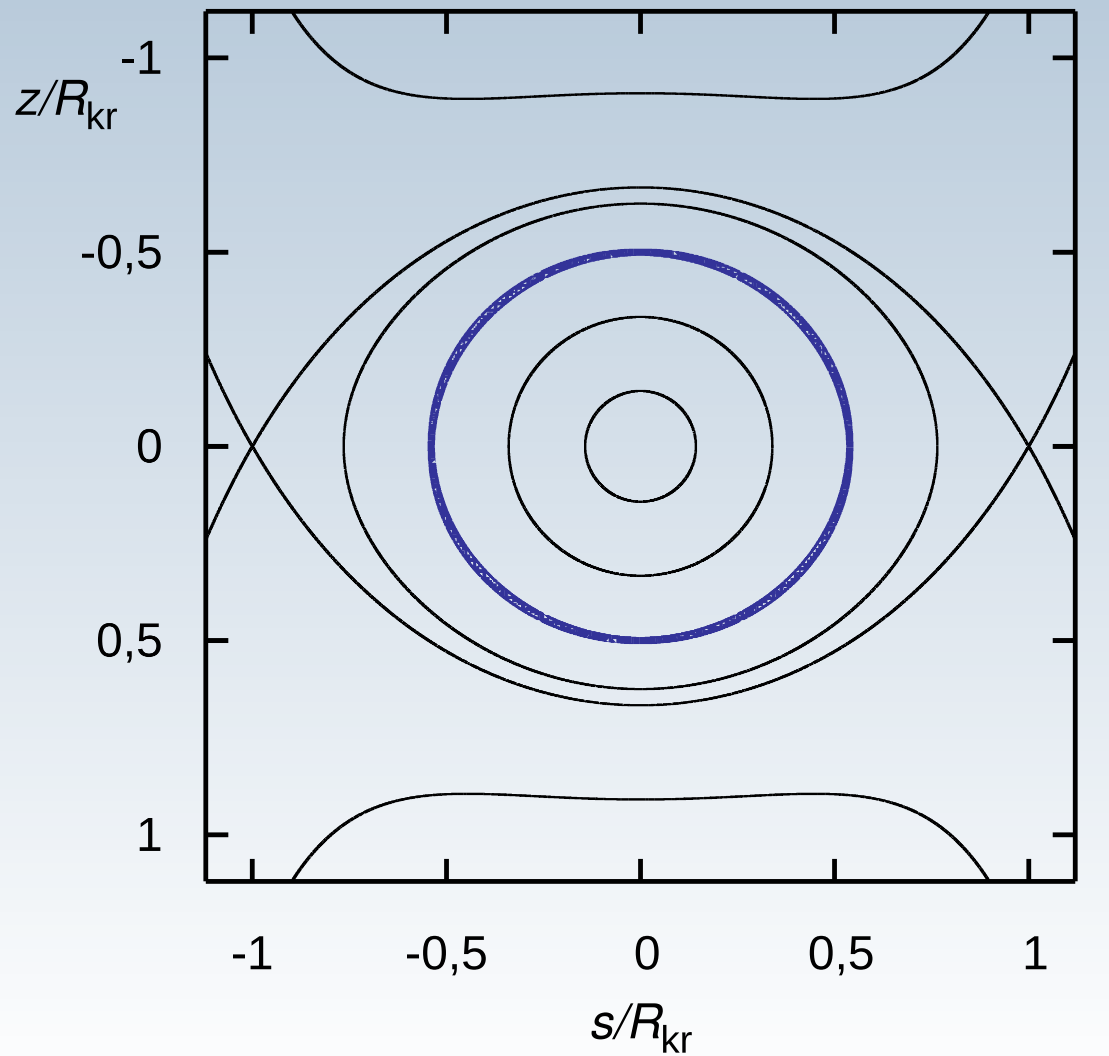
Influence of the rotation on the stellar shape

* slow rotation, $R_{\rm eq} \ll R_{\rm cr}$, $R_{\rm cr} = \left(\frac{GM}{\Omega^2}\right)^{1/3}$



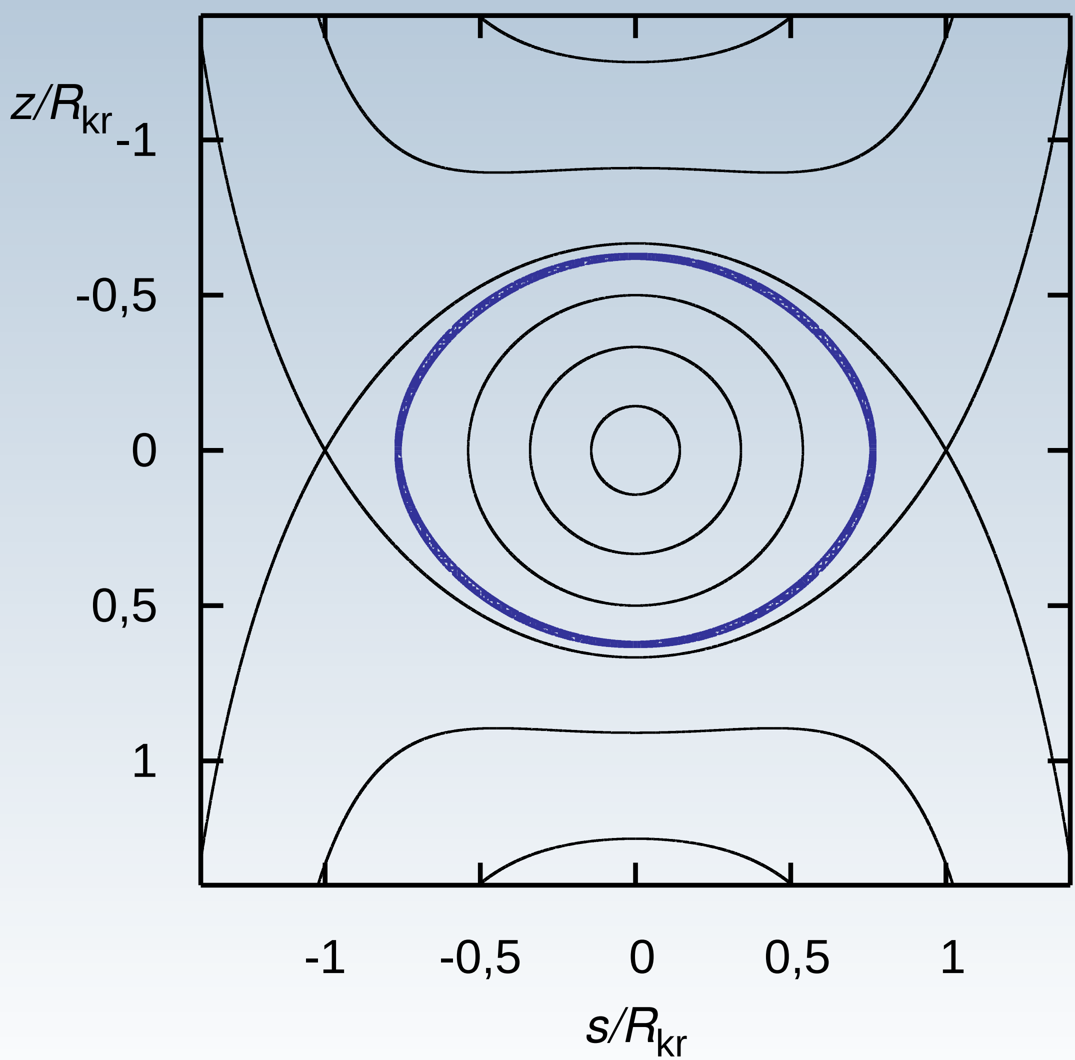
Influence of the rotation on the stellar shape

* faster rotation, $R_{eq} < R_{cr}$, $R_{cr} = \left(\frac{GM}{\Omega^2}\right)^{1/3}$



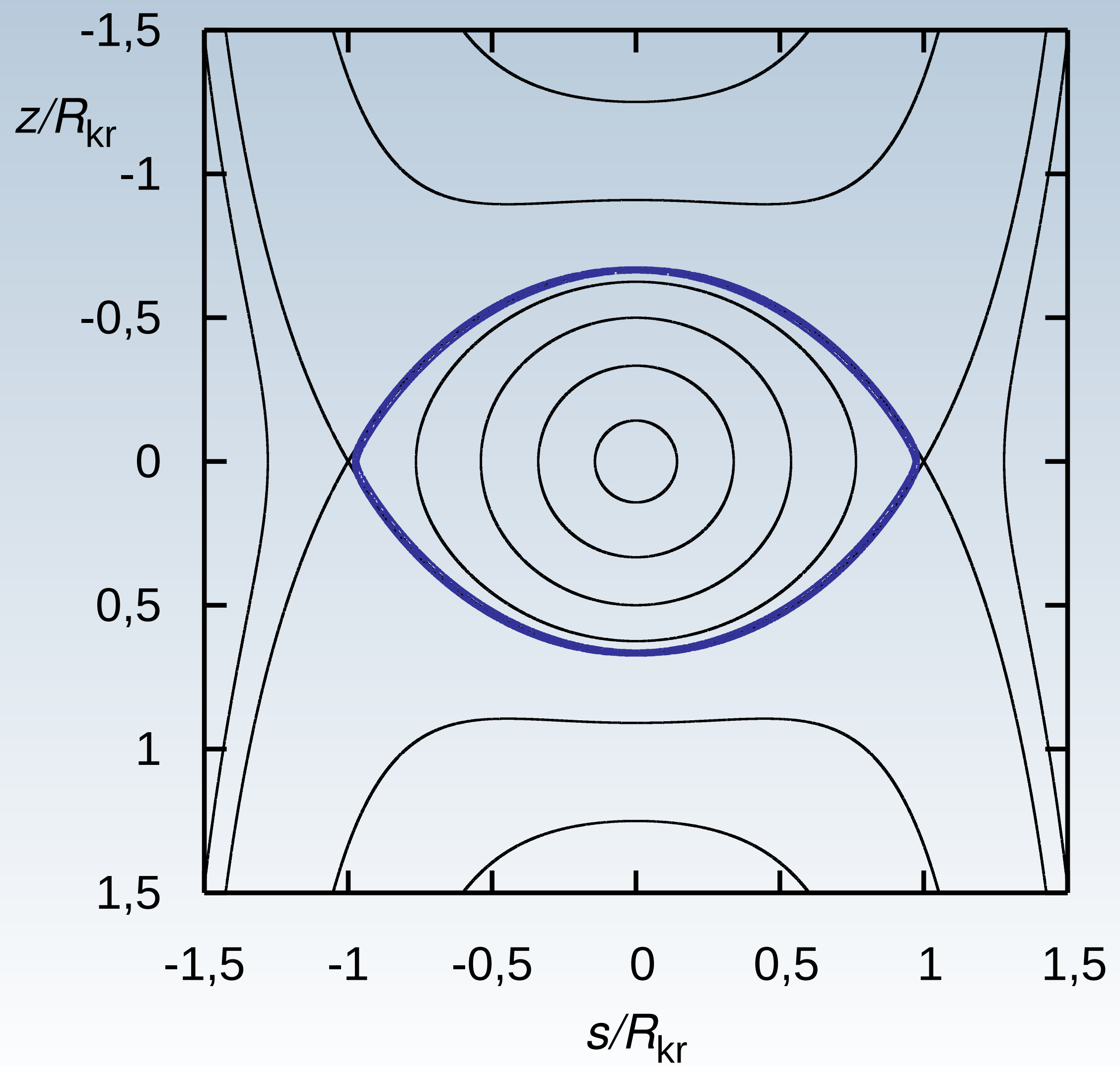
Influence of the rotation on the stellar shape

* near-critical rotation, $R_{eq} \approx R_{cr}$



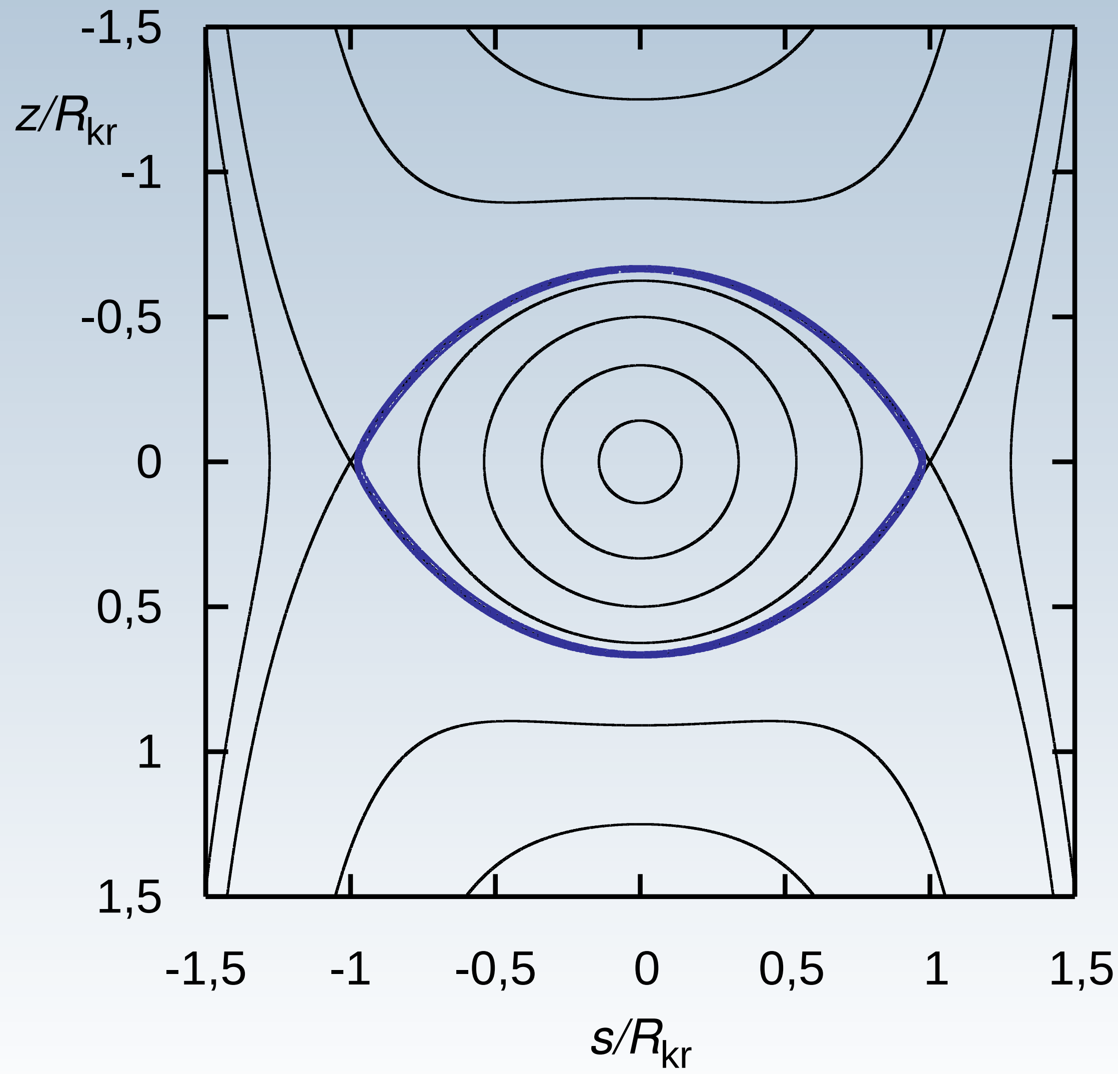
Influence of the rotation on the stellar shape

- critical rotation, $R_{\rm eq} = R_{\rm cr}$, $R_{\rm cr} = \left(\frac{GM}{\Omega^2}\right)^{1/3}$



Influence of the rotation on the stellar shape

* supercritical rotation?



Influence of the rotation on the stellar shape

* gravitational force in the equatorial plane balanced by the centrifugal force

$$R_{\rm eq}\Omega_{\rm crit}^2 = \frac{GM}{R_{\rm eq}^2}$$

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$$V_{\rm crit} = R_{\rm eq}\Omega_{\rm crit} = \sqrt{\frac{GM}{R_{\rm eq}}}$$

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- ∗ material in the equatorial plane rotates with the critical speed

$$V_{\mathrm{crit}} = R_{\mathrm{eq}}\Omega_{\mathrm{crit}} = \sqrt{\frac{GM}{R_{\mathrm{eq}}}}$$

$\Rightarrow$ rotational velocity in the equatorial plane lower than the escape velocity

$$V_{\mathrm{crit}} < V_{\mathrm{esc}} = \sqrt{\frac{2GM}{R_{\mathrm{eq}}^2}}$$

Influence of the rotation on the stellar shape

- ∗ gravitational force in the equatorial plane balanced by the centrifugal force
- ∗ material in the equatorial plane rotates with the critical speed

⇒ rotational velocity in the equatorial plane lower than the escape velocity

⇒ material cannot immediately escape to the infinity

Influence of the rotation on the stellar shape

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∗ material in the equatorial plane rotates with the critical speed

⇒ rotational velocity in the equatorial plane lower than the escape velocity

⇒ material cannot immediately escape to the infinity

∗ star does not rotate as a rigid body anymore

⇒ creation of *circumstellar disk* in the equatorial plane (due to a non-zero viscosity)

Stellar angular momentum

* the norm of the stellar angular momentum

$$J = I\Omega$$

  - I is the stellar moment of inertia
  - Ω is the rotation angular frequency

Stellar angular momentum

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$$J = I\Omega$$

* angular momentum change

$$\dot{J} = \dot{I}\Omega + I\dot{\Omega}$$

  * $\dot{J}$ is the angular momentum loss (e.g., in HD 37776 due to the wind, Mikulášek et al. 2008)

Stellar angular momentum

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  * $\dot{J}$ negligible, decline of I ($\dot{I} < 0$)
    $\Rightarrow$ spin up of the star

$$\frac{\dot{\Omega}}{\Omega} = -\frac{\dot{I}}{I}$$

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  - once the star reaches the critical rotation frequency ($\Omega = \Omega_{\text{crit}}$) $\Rightarrow$ spin up ends, angular momentum loss

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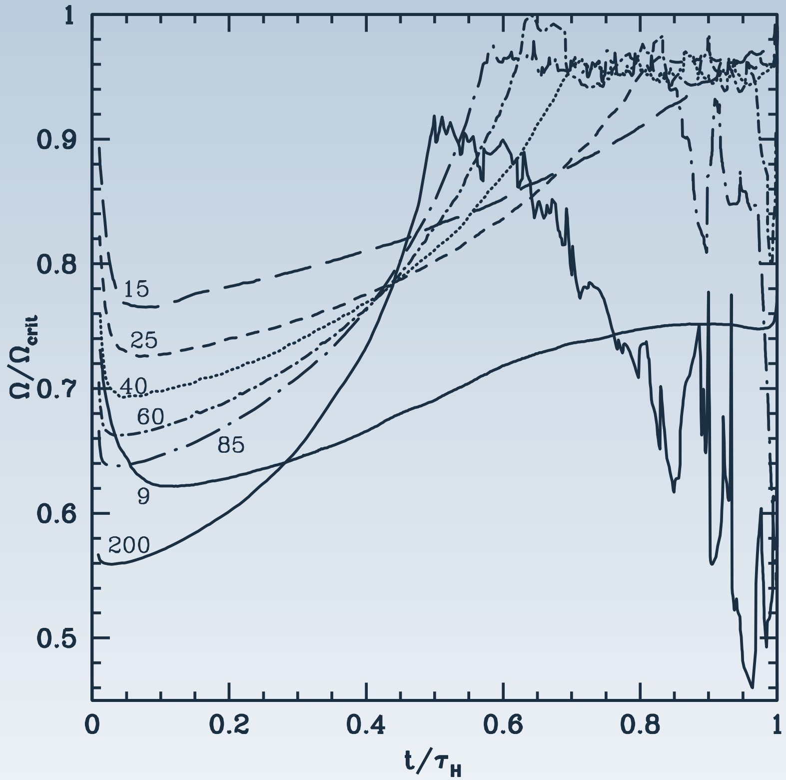
$$\dot{J} = \dot{I}\Omega_{\text{crit}}$$

  * $\dot{I}$ given by evolution $\Rightarrow$ also $\dot{J}$

Can stars reach the critical rotation?

* fast rotating stars may reach the critical rotation (Meynet et al. 2007)



* Ω/Ω_{crit} change during the main-sequence evolution ($Z = 0$) (Ekström et al. 2008)

Angular momentum loss due to a Keplerian disk

* material in the disk on Keplerian orbits
* orbital velocity

$$v_{\mathrm{K}}(r) = \sqrt{\frac{GM}{r}}$$

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* angular momentum loss per unit of time

$$\dot{J} \equiv \dot{J}_{\rm K}(R_{\rm out}) = \dot{M} v_{\rm K}(R_{\rm out}) R_{\rm out} \sim R_{\rm out}^{1/2}$$

* $R_{\rm out}$ is the outer disk radius

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* $\dot{J}$ given by the evolution $\Rightarrow$ to keep the critical rotation the star has to shed the angular momentum $\Rightarrow$ required mass-loss rate

$$\dot{M} = \frac{\dot{J}}{v_{\rm K}(R_{\rm out}) R_{\rm out}} \sim R_{\rm out}^{-1/2}$$

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* $\dot{J}$ given by the evolution $\Rightarrow$ to keep the critical rotation the star has to shed the angular momentum $\Rightarrow$ required mass-loss rate
* $\dot{M} \sim R_{\mathrm{out}}^{-1/2}$ $\Rightarrow$ lower mass loss for larger disks

Viscous decretion disks

* angular momentum of the material in the disk

$$j \sim r v_{\mathrm{K}}(r) \sim r^{1/2}$$

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* analogy with accretion disks: *artificial viscosity* (Shakura & Sunyaev 1973)
* artificial viscosity likely due to magnetorotational instability (Balbus & Hawley 1991)

Viscous decretion disk models

* disk described by hydrodynamic equations in cylindrical coordinates
* introduction of artificial viscosity
* axial symmetry
* stationarity

(Lightman 1974, Umin et al. 1991, Okazaki 2001, Jones et al. 2008)

Viscous decretion disk models

* continuity equation

$$\frac{1}{r}\frac{\mathrm{d}\left(r\Sigma v_r\right)}{\mathrm{d}r} = 0$$

* where
  * integrated disk density $\Sigma = \int_{-\infty}^{\infty} \rho \, \mathrm{d}z$
  * v_r is the radial disk velocity

Viscous decretion disk models

- continuity equation

$$\frac{1}{r}\frac{\mathrm{d}\,(r\Sigma v_r)}{\mathrm{d}r} = 0$$

- r component of the momentum equation

$$v_r\frac{\mathrm{d}v_r}{\mathrm{d}r} = \frac{v_\phi^2}{r} + g - \frac{1}{\Sigma}\frac{\mathrm{d}(a^2\Sigma)}{\mathrm{d}r} + \frac{3}{2}\frac{a^2}{r}$$

- where
  - the gravity acceleration is $g = -GM/r^2$
  - a is the sound speed, $a^2 = kT/(\mu m_{\mathrm{H}})$
  - μm_{H} is the mean molecular weight
  - temperature distribution $T = T_0\left(R_{\mathrm{eq}}/r\right)^p$

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- ϕ component of the momentum equation

$$\frac{v_r}{r}\frac{\mathrm{d}\left(r v_\phi\right)}{\mathrm{d}r} + \frac{\alpha}{r^2\Sigma}\frac{\mathrm{d}}{\mathrm{d}r}\left(a^2 r^2\Sigma\right) = 0$$

  - ⋆ artificial viscosity parameterized via α (Shakura & Sunyaev 1973)

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* θ component of the momentum equation

$$\rho = \rho_0 \exp\left(-\frac{1}{2}\frac{z^2}{H^2}\right), \qquad H = \frac{a}{v_\mathrm{K}} r$$

Viscous decretion disk models

- boundary conditions
  - sonic point $v_r = a$ at radius R_{crit}

$$\frac{v_\phi^2}{R_{\mathrm{crit}}} - \frac{GM}{R_{\mathrm{crit}}^2} + \frac{5}{2}\frac{a^2}{R_{\mathrm{crit}}} - \left.\frac{\mathrm{d}a^2}{\mathrm{d}r}\right|_{R_{\mathrm{crit}}} = 0$$

  - v_ϕ, and Σ specified at the stellar surface

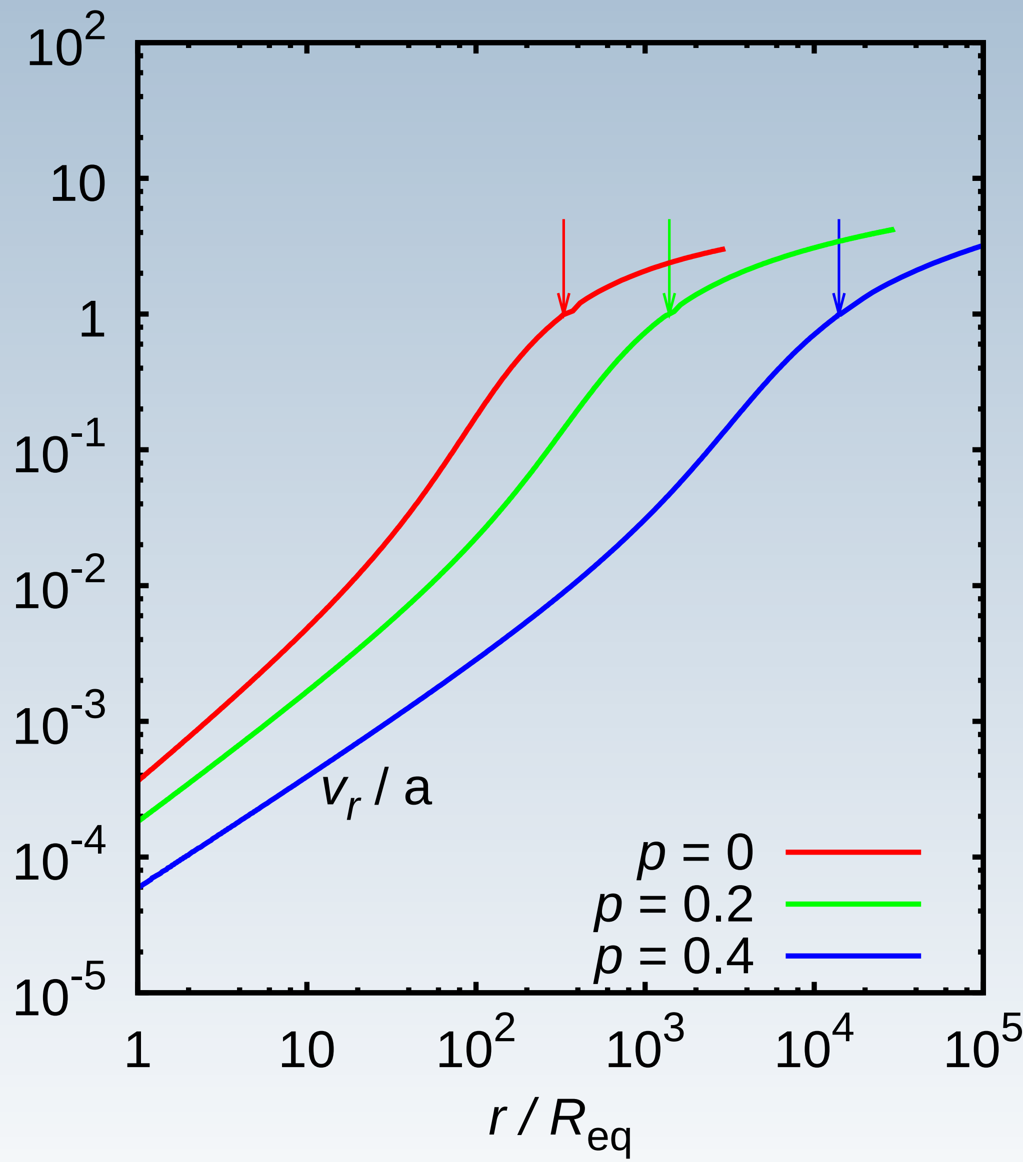
Viscous decretion disk models

* boundary conditions
  * sonic point $v_r = a$ at radius $R_{\rm crit}$

$$\frac{v_\phi^2}{R_{\rm crit}} - \frac{GM}{R_{\rm crit}^2} + \frac{5}{2}\frac{a^2}{R_{\rm crit}} - \left.\frac{{\rm d}a^2}{{\rm d}r}\right|_{R_{\rm crit}} = 0$$

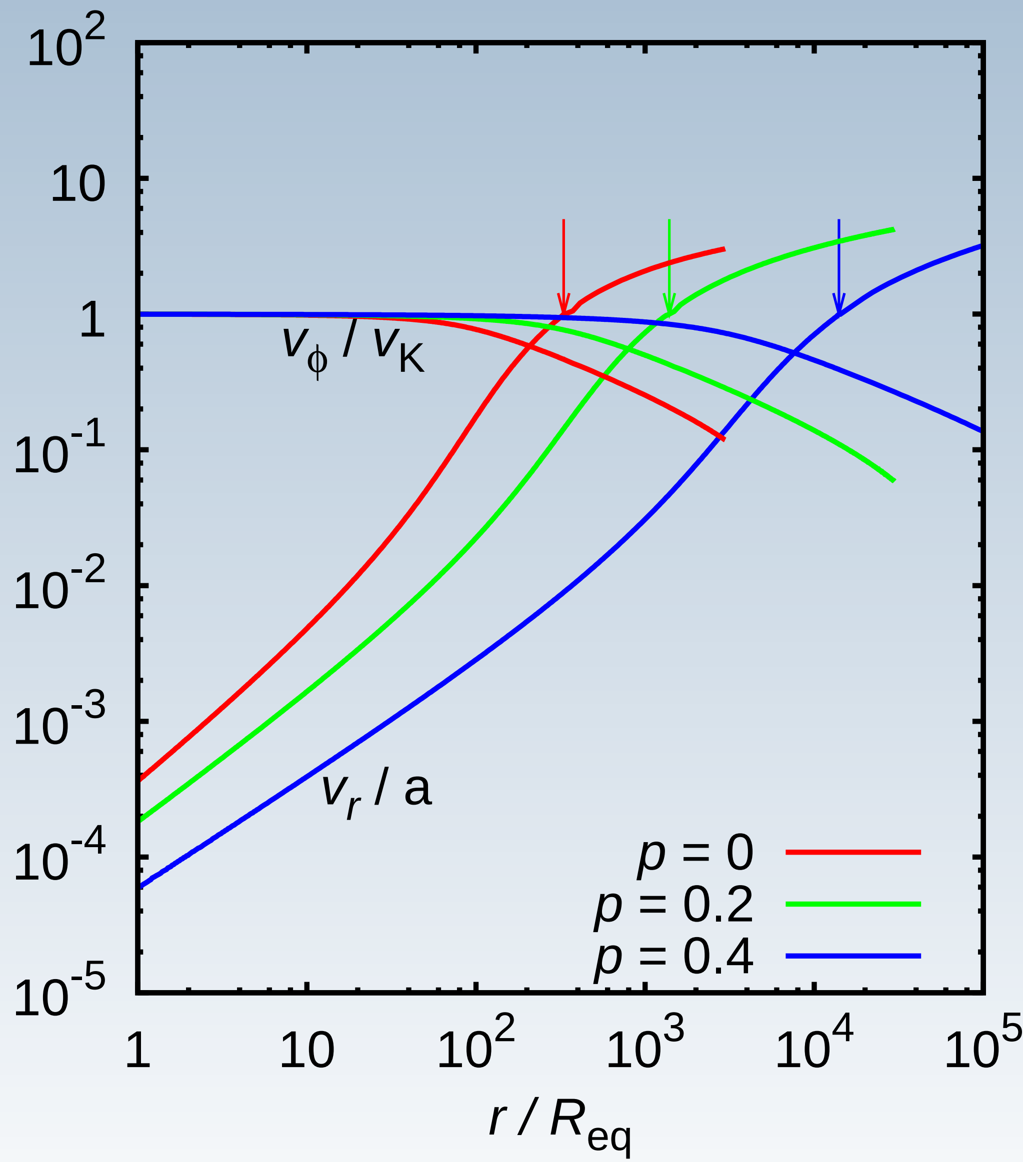
  * v_ϕ, and Σ specified at the stellar surface
* numerical solution using the Newton-Raphson method

Calculated disk models



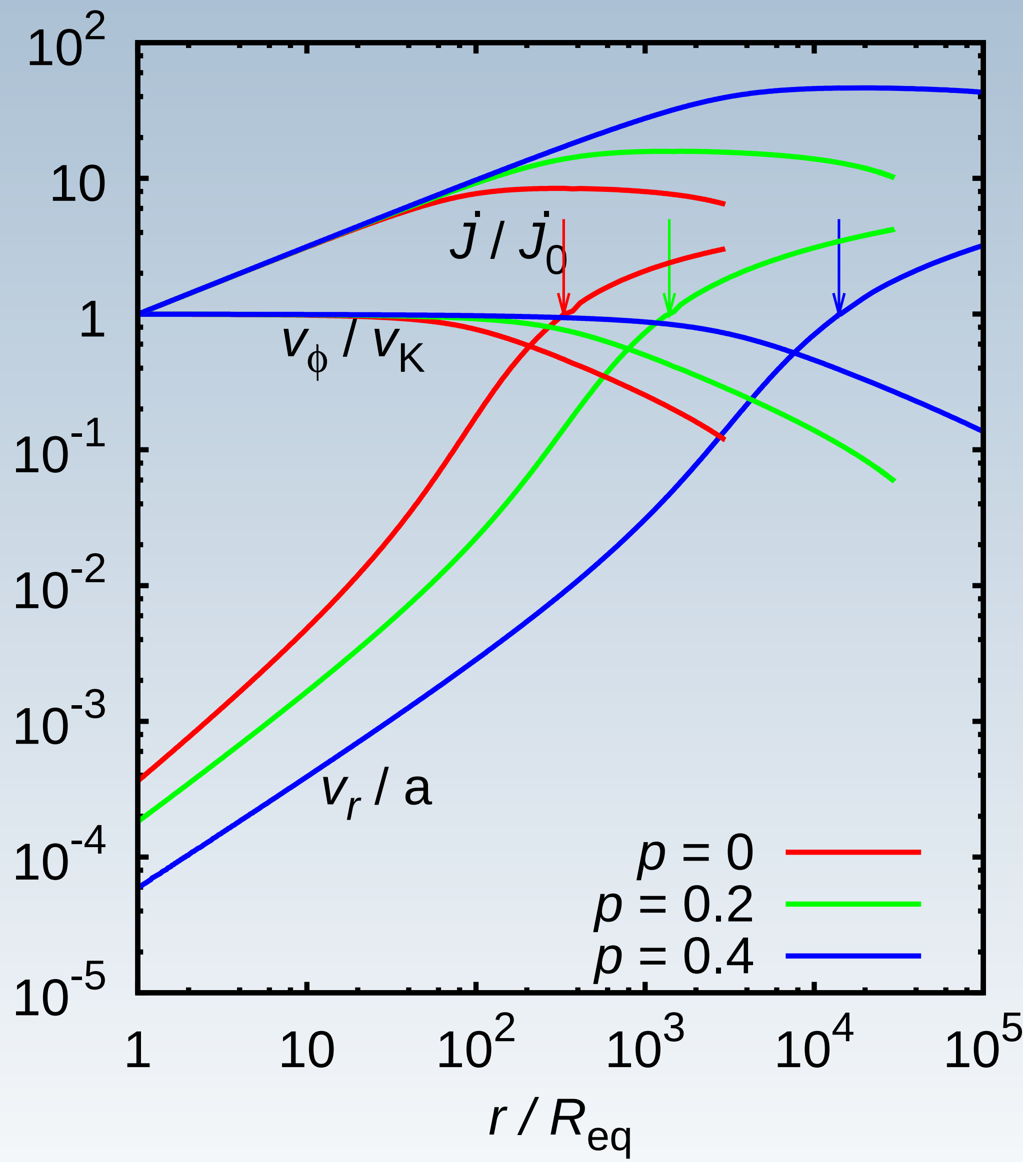
* in cooler disks the critical point at larger radii

Calculated disk models



* Keplerian disks nearly to the critical point

Calculated disk models



* $\dot{J}$ increases up to the critical point

Radiative ablation

∗ hot stars have radiatively-driven stellar winds

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∗ radiative force in the Sobolev approximation (Cranmer & Owocki 1995)

$$\mathbf{g}_{\mathrm{rad}} = \frac{c^{-2\alpha}}{1-\alpha}\left(\frac{\kappa_{\mathrm{e}}\bar{Q}}{c}\right)^{1-\alpha}\oint I(\mathbf{n})\left(\frac{\mathbf{n}\nabla\left(\mathbf{n}\mathbf{v}\right)}{\rho}\right)^{\alpha}\mathbf{n}\,\mathrm{d}\Omega$$

∗ where

- ⋆ κ_{e} is Thomson scattering cross-section
- ⋆ α, $\bar{Q}$ are force parameters (Gayley 1995)
- ⋆ $I(\mathbf{n})$ is emergent intensity

Radiative ablation

- ∗ hot stars have radiatively-driven stellar winds
- ⇒ radiative force may also ablate the disk
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- ∗ the emergent intensity $I(\mathbf{n})$ given by the stellar radiative flux reflected by the star

Disk wind mass-loss rate: an estimate

- classical CAK (Castor, Abbott & Klein 1975) wind mass-loss rate estimate

$$\dot{M}_{\mathrm{CAK}} = \frac{\alpha}{1-\alpha}\frac{L}{c^2}\left(\Gamma\bar{Q}\right)^{1/\alpha-1}$$

- $\bar{Q}$ and α are line force parameters

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$$\dot{M}_{\mathrm{CAK}} = \frac{\alpha}{1-\alpha}\frac{L}{c^2}\left(\Gamma\bar{Q}\right)^{1/\alpha-1}$$

- in the term of mass flux from a unit surface

$$\dot{m} = \frac{\alpha}{1-\alpha}\frac{\tilde{F}}{c^2}\left(\frac{\kappa_{\mathrm{e}}\tilde{F}\bar{Q}}{c\tilde{g}}\right)^{1/\alpha-1}$$

- $\tilde{F}$ is the driving flux and $\tilde{g}$ is local gravitational acceleration

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- assuming (F is flux from the star)

$$\tilde{F} = \frac{R}{r}F$$

Disk wind mass-loss rate: an estimate

* disk mass loss rate is given by

$$\dot{M}_{\rm dw}(R_{\rm out}) = 2 \times 2\pi \int_{R_{\rm eq}}^{R_{\rm out}} \dot{m} r \, \mathrm{d}r$$

Disk wind mass-loss rate: an estimate

* disk mass loss rate is given by

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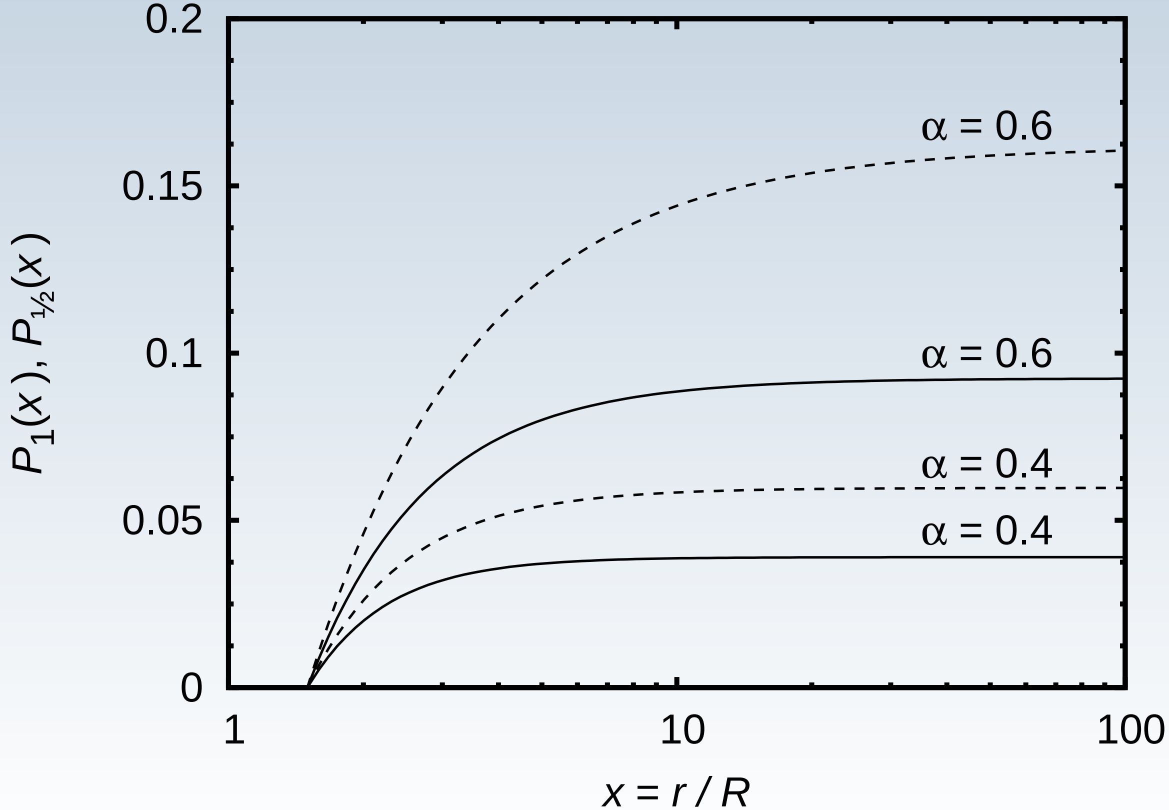
* after integration

$$\dot{M}_{\mathrm{dw}}(R_{\mathrm{out}}) = P_1 \left(\frac{R_{\mathrm{out}}}{R} \right) \dot{M}_{\mathrm{CAK}}$$

Disk wind mass-loss rate: better approximation

$$\dot{M}_{\rm dw}(R_{\rm out}) = P_1\left(\frac{R_{\rm out}}{R}\right)\,\dot{M}_{\rm CAK}$$

* $\dot{M}_{\rm CAK}$ is classical stellar wind mass-loss rate



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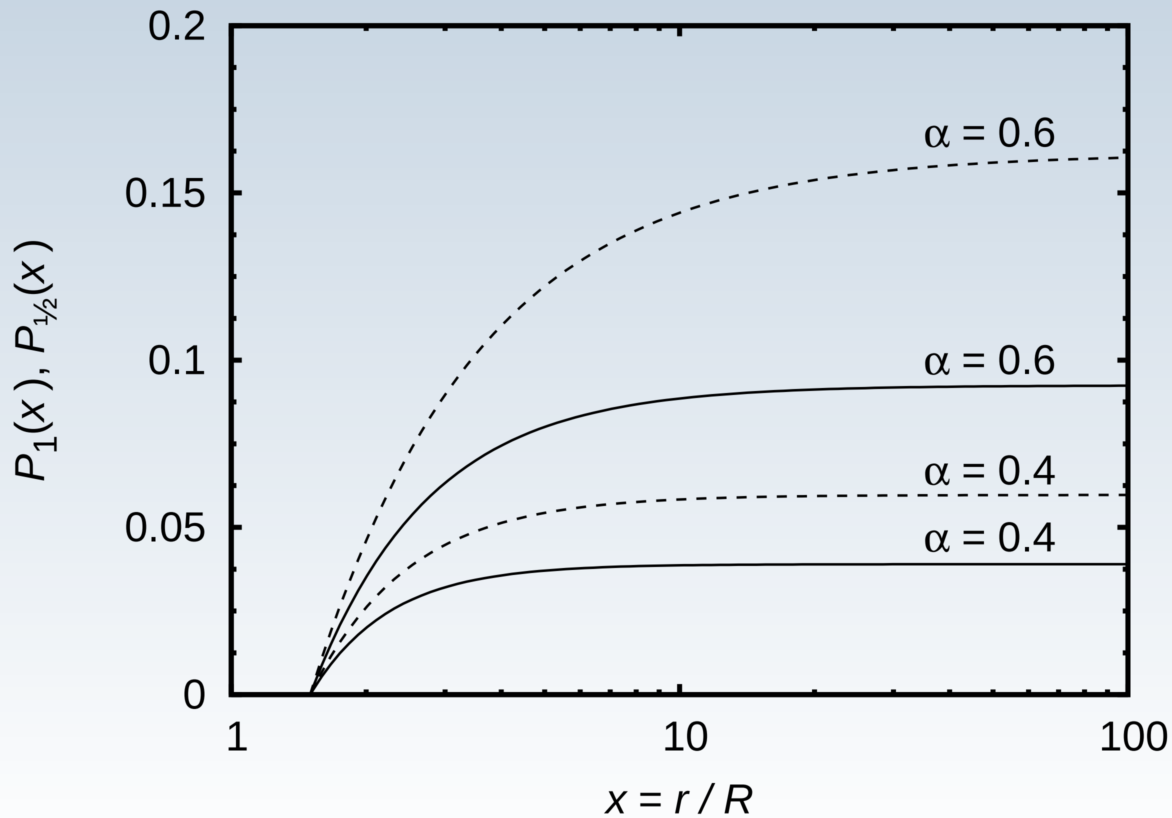
$*$ $\dot{M}_{\mathrm{CAK}}$ is classical stellar wind mass-loss rate

$\Rightarrow$ disk wind originates mainly from the regions close to the star

Disk wind angular momentum loss

$$\dot{J}_{\rm dw}(R_{\rm out}) = P_{\frac{1}{2}}\left(\frac{R_{\rm out}}{R}\right) R\, v_{\rm K}(R)\, \dot{M}_{\rm CAK}$$

* $R v_{\rm K}(R) \dot{M}_{\rm CAK}$ stellar wind loss



Open questions

* the source of the artificial viscosity
* precise calculation of disk ablation
* disk temperature distribution

Conclusions

- critically rotating stars may lose mass via decretion disk

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Conclusions

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- $\ast$ the mass-loss rate set by the angular momentum loss needed to keep the stellar rotation subcritical
- $\ast$ disk angular momentum loss rate depends on the outer disk radius $\dot{J} \sim R_{\mathrm{out}}^{1/2}$

$\Rightarrow$ mass-loss due to the disk $\dot{M} \sim R_{\mathrm{out}}^{-1/2}$

Conclusions

* critically rotating stars may lose mass via decretion disk
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* disk angular momentum loss rate depends on the outer disk radius $\dot{J} \sim R_{\rm out}^{1/2}$

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* disk wind mass-loss rate by order of magnitude lower than the stellar wind mass-loss rate