The abominable list of typos and omissions in Forcing Idealized

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After Definition 2.1.16, I am saying that nearly all definitions of σ-ideals considered in this book are ZFC-correct in this sense. The word "nearly" was introduced to the book in proof, after Pawlikowski noted that it is not clear how to argue that the Ramsey null ideal is ZFC-correct. Recently, Marcin Sabok showed that in fact this ideal is not $\mathbf{\Delta^1_2}$ on $\mathbf{\Sigma^1_1}$, and the ideal indeed is not ZFC-correct. This has some unpleasant repercussions if one wants to develop the theory of countable support iteration for Mathias forcing in ZFC. in particular, I do not know if the countable stages of that iteration are of the form Borel modulo a σ-ideal.

In Section 3.5, I did not state any ZFC results for $\mathbf{\Pi}^1_1$ on $\mathbf{\Sigma}^1_1$ σ-ideals, because I did not know that in such a case the Baire category game is determined. In fact, the game is determined in this case as proved below, and so all the theorems and corollaries in this section have a ZFC counterpart for such σ-ideals!

After Definition 3.7.1, I say that ergodicity does not imply c.c.c. or vice versa and I do not provide examples documenting this cryptic sentence. There is in fact a very instructive counterexample to the left-to-right implication. Consider the Turing equivalence E on 2^ω. It most certainly has countable classes. Every Borel E-invariant set is either disjoint from or contains a cone by a basic determinacy result. The collection I of all sets $B \subset 2^\omega$ disjoint from a cone forms a σ-ideal, and the ideal is ergodic with respect to E. At the same time, there is an uncountable antichain of pairwise disjoint sets in the quotient. Before I construct it, note that the quotient forcing adds a point $x \in 2^\omega$ such that all ground model reals are Turing reducible to it, and therefore the size of the continuum must be collapsed to $\aleph_0$. To construct the uncountable antichain, for every $n \in \omega$ and every $x \in 2^\omega$ let $f_n(x) = y$ if the n-th Turing machine outputs y on input x. The functions f_n are clearly Borel, and for every $y \in 2^\omega$ the set $\bigcup_n f_n^{-1}\{y\}$ contains a cone; therefore for some $n = n(y)$, the preimage $f_n^{-1}\{y\}$ must be I-positive. The sets $A_n = \{y : f_n^{-1}\{y\} \notin I\}$ together cover the whole space 2^ω, so one of them is uncountable, and the preimages associated to the points in it are pairwise disjoint.

The right-to-left implication fails as well, but the counterexample is not as instructive. Work in Section 3.7 shows that if I is ergodic, then there are countably many automorphisms $\{\pi_n : n \in \omega\}$ of P_I such that P_I forces the ground model version of P_I to be covered by $\bigcup_n \pi_n''\dot{G}$ where $\dot{G}$ is the generic filter; in other words, the ground model version of P_I becomes σ-centered. Now, [1, Section 3] provides an example of a c.c.c. Suslin forcing adding a single real such that it does not force the ground model version of itself to be σ-centered.

Regarding Proposition 3.7.10, it is instructive to compare the result with [1, Theorem 6.1]. That paper produces δ-proper forcing which is definable, not c.c.c. and has no perfect set of pairwise incompatible elements; this for every countable ordinal δ. The paper contains typos of its own, among others a statement that the forcings are $< \omega_1$-proper which they of course are not and cannot be.

Section 3.10.9, introducing the infinite game on a Boolean algebra which is related to preservation of Baire category, fails to mention an important ZFC determinacy result for the case that the ideal is $\mathbf{\Pi}^1_1$ on $\mathbf{\Sigma}^1_1$. The ZFC result is not mentioned because I only proved it recently.

Theorem 0.1. *Suppose that I is a $\mathbf{\Pi}^1_1$ on $\mathbf{\Sigma}^1_1$ σ-ideal on a Polish space X and the quotient forcing is proper. Then the Baire category game is determined.*

Proof. I need to unravel the game. Let $Z \subset \omega^\omega \times X$ be an analytic set whose vertical sections enumerate all analytic subsets of X, and let $W \subset (\omega^\omega)^\omega \times \omega^\omega$ be a closed set projecting into the analytic set $\{z \in (\omega^\omega)^\omega : \liminf Z_{z(n)} \notin I\}$. The latter set is analytic by the definability condition on the ideal I.

Consider the game G' in which Player II starts with a condition $B_{ini} \in P_I$ and then in each round n, first Player I plays $B_n \in P_I$, a subset of the set B_{ini}, then Player II plays $C_n \subset B_n$ in P_I, and after that Player I plays a point $z(n) \in \omega^\omega$ such that $Z_{z(n)} \subset C_n$ is a condition in P_I. Moreover, in the end of the round, Player I may or may not play a natural number $i(n)$ smaller than n. In the end, Player I obtains a sequence $z \in (\omega^\omega)^\omega$ consisting of the $z(n)$'s and a sequence w of natural numbers consisting of the $i(n)$'s. Player I wins if w is an infinite sequence and $(z, w) \in W$. This is a G_δ payoff condition for Player II, therefore the game is determined. The game G' is obviously harder for Player I than the Baire category game, so it will be enough to show that Player II has a winning strategy σ in G' implies that he has a winning strategy in the game G. The key point is that it is fairly easy to adjust the strategy σ so that it disregards the choice of the numbers $i(n)$. $\square$

In the proof of Theorem 3.10.24, introducing the infinite game on a Boolean algebra related to the preservation of category basis, I include a short paragraph containing the sentence: First I will argue that there must be a winning strategy for Player II which disregards the flag. After that, the (easy) argument fails to appear, probably due to editing error.

In the proof of Theorem 4.2.3 (3), in the definition of the rules of the integer game, at round n player II decides which of the first n sets of the set U belong to which sets $a_m : m \in n$. The last n of course cannot be ω as written in the book because then the game could not be coded as an integer game. The rest of the proof remains without change.

In Example 4.3.43, the definition of the pathological submeasure ϕ contains significant typos: it should read $\phi(b) = \min\{1, m(b)/n\}$. The last sentence of that example should assert that $\phi(C(x) \cap a) = 1/n$ is very small.

In the proof of Proposition 3.9.18, I am using "Steel's result" without saying which of the many Steel's results this may be. It is Fact 1.4.6 from the Prerequisites chapter, and it would be really impossible to prove the proposition without it.

In Section 5.4.2, regarding the ZFC treatment of the illfounded iterations, I restricted the treatment to σ-ideals generated by closed sets, because I did not know if in the general case of Baire category preserving forcings associated with $\mathbf{\Pi}^1_1$ on $\mathbf{\Sigma}^1_1$ ideals, the Baire category game is determined in ZFC. As zou can see above, the game really is determined, and therefore Theorem 5.4.11 can be improved to cover all $\mathbf{\Pi}^1_1$ on $\mathbf{\Sigma}^1_1$ ideals I such that their quotient P_I is provably proper and preserves Baire category.

References

[1] Haim Judah, Andrzej Roslanowski, and Saharon Shelah. Examples for souslin forcing. *Fundamenta Mathematicae*, 144:23–42, 1994. math.LO/9310224.