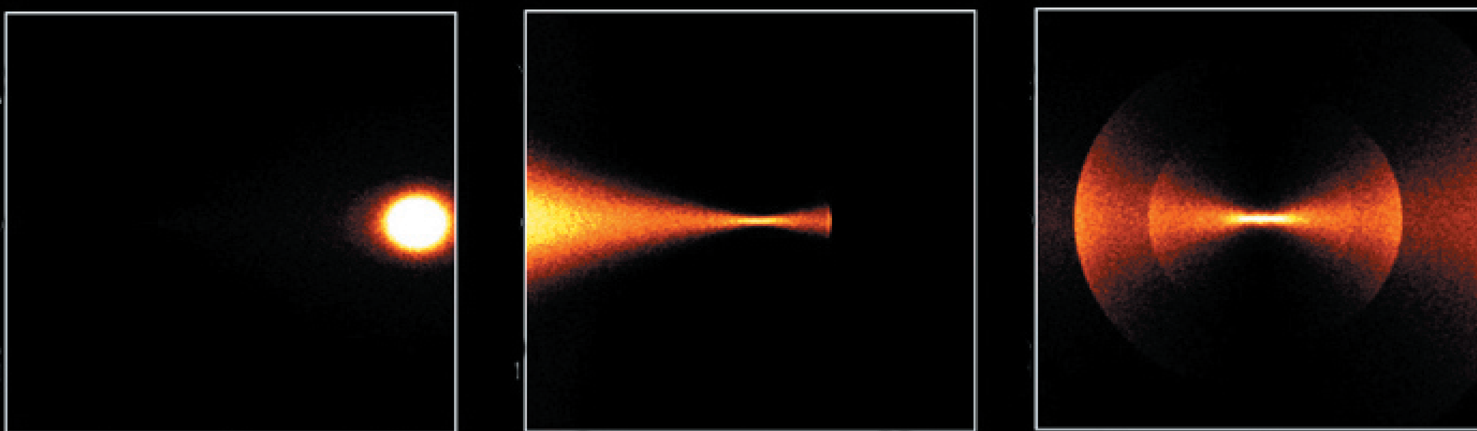
Simulations of Galactic Mergers Using Test Particles



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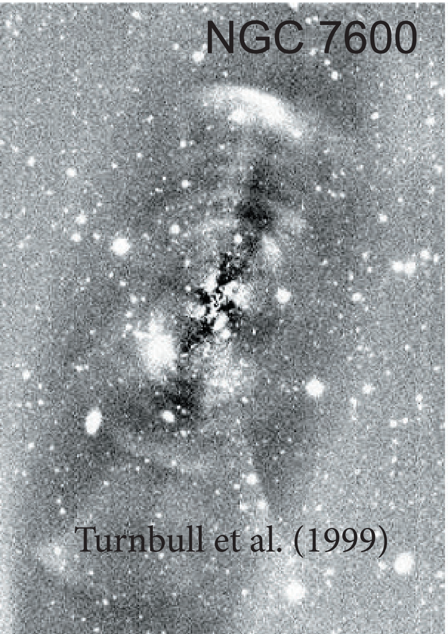
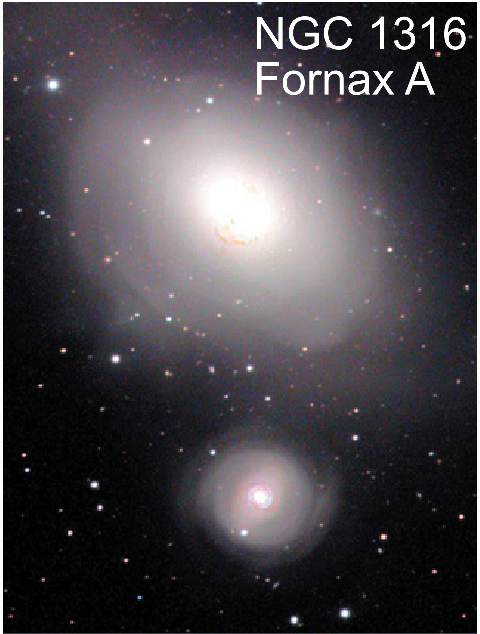
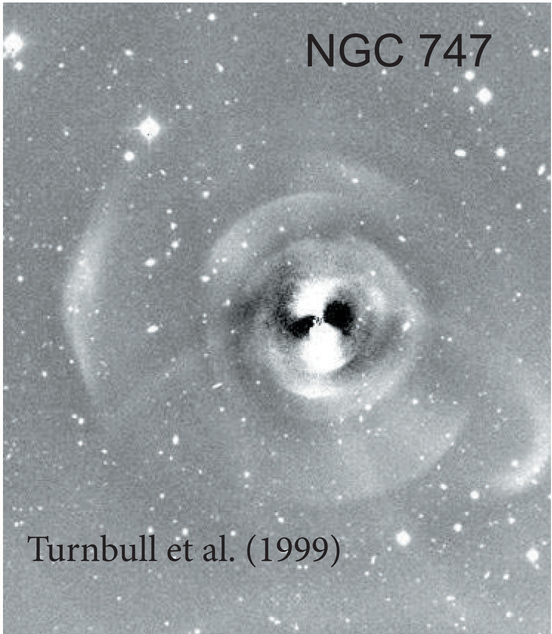
with kind leading of **Bruno Jungwiert** (Astronomical Institute of the Academy of Sciences of the Czech Republic)

Research of shell galaxies

Shell galaxies are mostly ellipticals which contain fine stellar structures that form open, (almost) concentric arcs which usually do not cross each other. Usually hard to detect

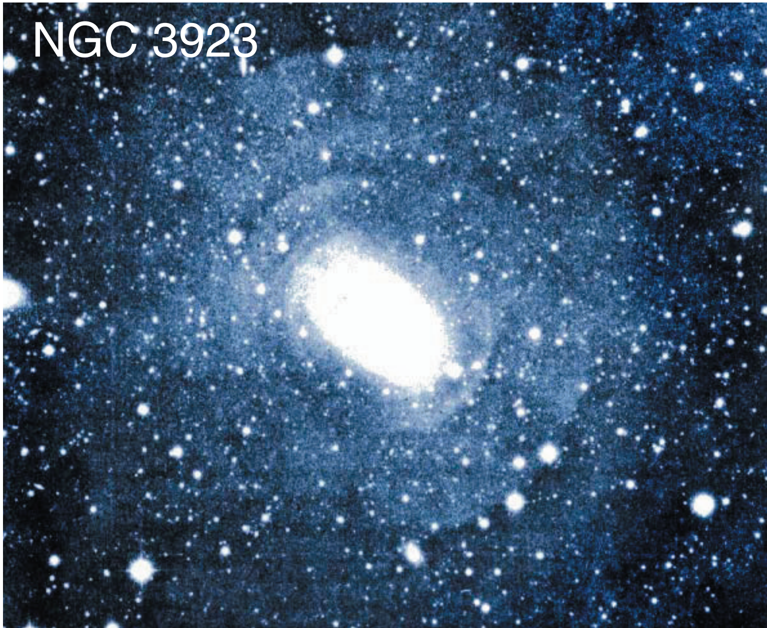






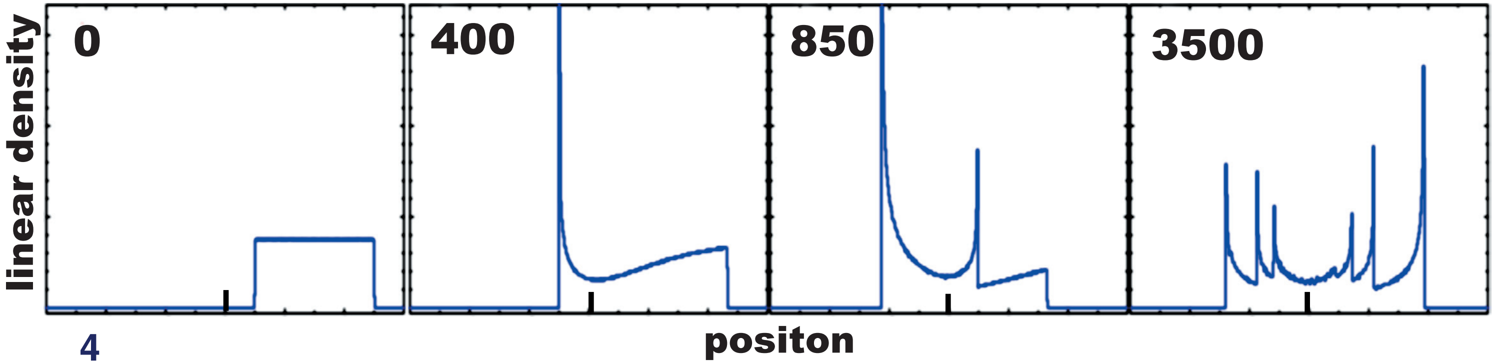
Shell Galaxies

- ~10% of E and S0 (up to 50% in regions of low galaxy density)
- from 1 to ~30 shells in a single galaxy (often less than 4)
- low contrast (0.1–0.2 mag)
- asocied with kinematically distnict cores and central dust features
- merger origin



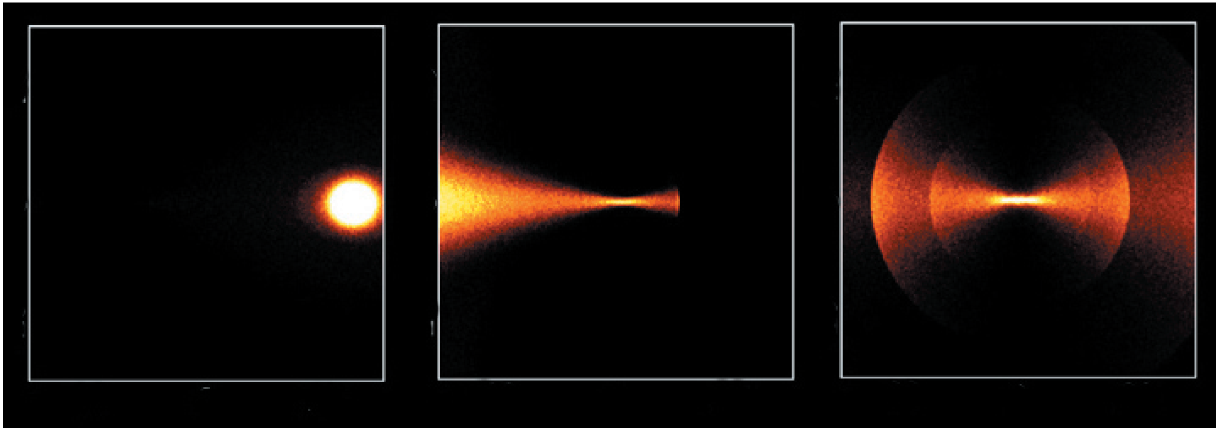
Scenario of origin

- (nearly) radial merger of a massive and significantly less massive galaxy developed by Quinn (1984)
- the secondary is disrupted and its stars oscillate back and forward in the potential of the primary
- the stars cumulate at their turning points and form shells
- the most tightly bound stars reach their turning points first and so on → density waves propagate outwards



Our setup

- two analytical spherical potentials of primary and secondary (plummer spheres) falling towards each other
- millions of test particles initially distributed to reproduce the potential of the secondary
- necessity to implement physical effects to test particles simulation. Somehow...



Implementation of friction

Dynamical friction

- a braking force of gravitational origin
- in a system of at least three gravitating bodies energy transfer is possible
- every such system tends to the thermodynamic equilibrium
- for instance a galaxy flying through a field of stars of a bigger galaxy is like a hot peace thrown into a colder sea (an important example for us) → slowdown of the secondary galaxy by the transfer of its energy to stars of the primary

Chandrasekhar formula

- an (analytical) formula for computing the dynamical friction, Chandrasekhar (1943)

$$\frac{d\mathbf{v}_M}{dt} = -16\pi^2 \ln\Lambda G^2 m(M+m) \frac{\int_0^{v_M} f(v_m) v_m^2 dv_m}{v_M^3} \mathbf{v}_M$$

M - mass of subject of the dynamical friction

m - mass of a single star

v_m, $\mathbf{v}_M$ - respective velocities

G - gravitational constant

$f(v_m)$ - distribution function of stars

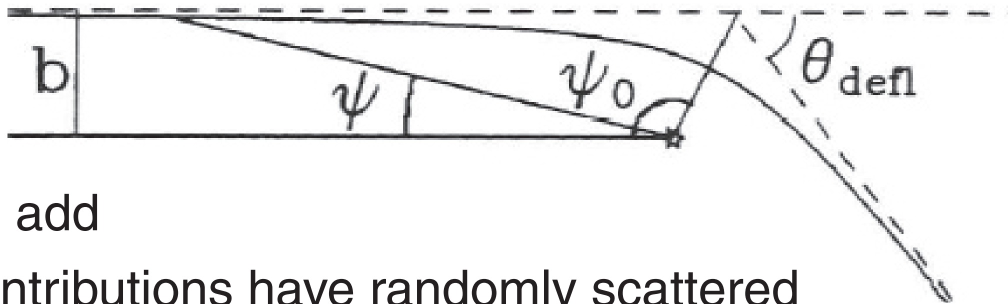
$$\Lambda \equiv \frac{b_{\max} V_0^2}{G(M+m)} , \quad V_0 = \mathbf{v}_m - \mathbf{v}_M$$

$b_{\max}$ - radius of field of stars

V_0 - relative velocity of frictioned subject

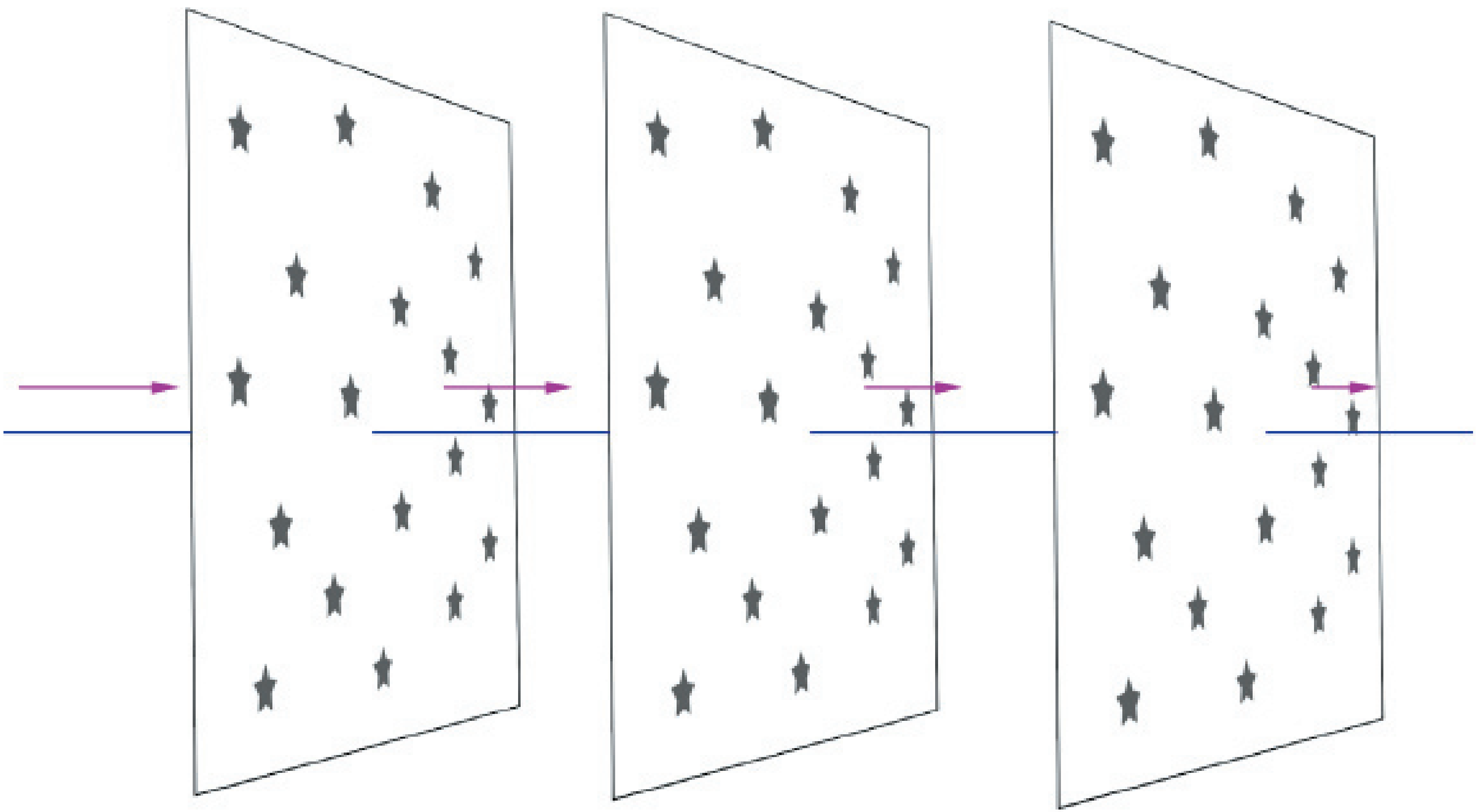
Depiction of derivation

- distant encounter with one star → deflected from the original direction → increase of the perpendicular component and loss of the velocity in the original direction
- many such encounters → the contributions add



- perpendicular contributions have randomly scattered azimuthal angles → add to zero
- contributions to the original direction of the velocity will always be opposite to it → the braking force

- in primary galaxy many stars acting on the massive body (secondary galaxy) in the same time (stars in the plane perpendicular to the direction of flight)



Towards the formula

- in the encounter of our object (scondary galaxy) with a single star the change of the component of velocity parallel to its original direction, between the times minus infinity and infinity would be:

$$\mid \triangle \mathbf{v}_{M\parallel} \mid = \frac{2\,m\,V_0}{M+m}\left[1+\frac{b^2 V_0^4}{G^2(M+m)^2}\right]^{-1}$$

- an object flying through a field with the phase-space number density of stars $f(\boldsymbol{v}_m, \boldsymbol{b})$
- the change in the parallel component of velocity in an infinitesimal time dt will be given by the integration of Eq. before multiplied by the density over the plane of $\boldsymbol{b}$ and the space $\boldsymbol{v}_m$
- $\boldsymbol{b}$ is measured from a given point in a plane, use the polar coordinates (b, φ)

$$\int\int\int f(\mathbf{v}_m, b, \varphi)\frac{2m\,V_0(\mathbf{v}_m)\,\mathbf{V}_0(\mathbf{v}_m)}{(M+m)\left[1+\frac{b^2V_0^4(\mathbf{v}_m)}{G^2(M+m)^2}\right]}\mathrm{d}^3\mathbf{v}_m\,b\mathrm{d}b\,\mathrm{d}\varphi$$

- no approximations so far

- assume the homogeneity of the field of stars, so as the distribution function of the stars does not depend on ***b***
- the remaining ***b***-dependent part can be easily integrated:

$$\int_0^{b_{max}} \frac{b\mathrm{d}b}{1+c^2b^2} = \left[\frac{\ln(1+c^2b^2)}{2\,c^2}\right]_{b=0}^{b=b_{max}}, \quad c = V_0^2/[G(M+m)]$$

- it is conventional to introduce the notation $\Lambda = \frac{b_{max}V_0^2}{G(M+m)}$
- Λ would typicaly be of the order of 10^3, thus

$\frac{1}{2}\ln(1+\Lambda^2) \cong \ln(\Lambda)$ so called Coulomb logarithm

- replacing V_0 (not the vector) in Λ by a typical speed, then the Coulomb logarithm does not depend on $\boldsymbol{v}_m$ and whole formula goes to:

$$4\pi \ln(\Lambda) G^2 m(M+m) \int f(\mathbf{v}_m) \frac{\mathbf{v}_m - \mathbf{v}_M}{|\mathbf{v}_m - \mathbf{v}_M|^3} \mathrm{d}^3\mathbf{v}_m$$

- the integral is of exactly the same form as in the Newton's law of gravity and if the stars move isotropically, the density distribution is spherical and by Newton's first theorem

$$-16\pi^2 \ln(\Lambda) G^2 m(M+m) \frac{\int_0^{v_M} f(v_m) v_m^2 \mathrm{d}v_m}{v_M^3} \mathbf{v}_M$$

- that is the formula from the begining and it is usually called the Chandrasekhar dynamical friction formula

- if $f(\boldsymbol{v}_m)$ is Maxwellian with dispersion σ

$$f = \frac{n_0}{(2\pi\sigma^2)^{3/2}} \exp(-\frac{1}{2} v^2/\sigma^2)$$

- the density of the stars is $\rho_0 = n_0 m$
- M much greater then m → $(M + m) \cong M$

Then...

$$\frac{\mathrm{d}\mathbf{v}_{M\|}}{\mathrm{d}t} = -\frac{4\pi \ln(\Lambda) G^2 \rho_0 M}{v_M^3} \left[\mathrm{erf}(X) - \frac{2X}{\sqrt{\pi}} \mathrm{e}^{-X^2}\right] \mathbf{v}_M$$

$$X \equiv v_M/(\sigma\sqrt{2})$$

$$\mathrm{erf}(X) \equiv \frac{2}{\sqrt{\pi}} \int_0^X \mathrm{e}^{-t^2} \mathrm{d}t$$

Principal inaccuracies of the Chandrasekhar formula

- the Chandrasekhar formula evaluates the change of the parallel component of the velocity after the flyby from infinity to infinity for every single star with the **same initial conditions** and then adds these changes and applies them to the secondary galaxy **in one moment**, the moment of the closest approach with these stars
- extenuating circumstances: During an encounter of two bodies, roughly one half of the velocity change takes place around the point of the closest approach on the scale of the impact parameter.

Inaccuracies of the Chandrasekhar formula coming from approximations

- the distribution function does not depend on position (is not true)
- the Coulomb logarithm is independent of velocity of the stars $\boldsymbol{v}_m$ (not true agin)
- go back to the equation before this approximations take e.g. the Maxwellian distribution plus plummer velocity dispersion, star density for the primary

Avoiding approximations

- **use only the radial symmetry** of our problem (transforming integral over velocity space to the spherical coordinates where the z-axis is parallel with the velocity of the secondary galaxy → no dependence on the azimuthal angle, integral over the other angular variable can be done analyticaly)

- Plummer potential →

$$\Phi(r) = -\frac{G\,M}{\sqrt{r^2+\varepsilon^2}}$$

M - mass of the galaxy

ε - Plummer radius

G - gravitational constant

and we a have beautiful...

...three-line integral over two parametres

$$\frac{\mathrm{d}\mathbf{v}_{M\|}}{\mathrm{d}t} \cdot \frac{\mathbf{v}_M}{v_M} = \frac{\sqrt{3\,M_p} G^{3/2} M_s \varepsilon_p^2}{2\sqrt{\pi} v_M^2} \int\limits_0^{\sqrt{R^2-d^2}} \int\limits_0^{\infty} \frac{b\mathrm{d}b\, v_m^2 \mathrm{d}v_m}{\left(\varepsilon_p^2 + d^2 + b^2\right)^{11/4} \left(G^2 M_s^2 + b^2 v_m^4\right)} \times$$

$$\times \left[e^{-3\,\frac{\sqrt{\varepsilon_p^2+d^2+b^2}\,(v_m - v_M)^2}{GM_p}} \left(GM_p - 6\,v_m v_M \sqrt{\varepsilon_p^2 + d^2 + b^2}\right) - \right.$$

$$\left. - e^{-3\,\frac{\sqrt{\varepsilon_p^2+d^2+b^2}\,(v_M + v_m)^2}{GM_p}} \left(GM_p + 6\,v_m v_M \sqrt{\varepsilon_p^2 + d^2 + b^2}\right) \right] ,$$

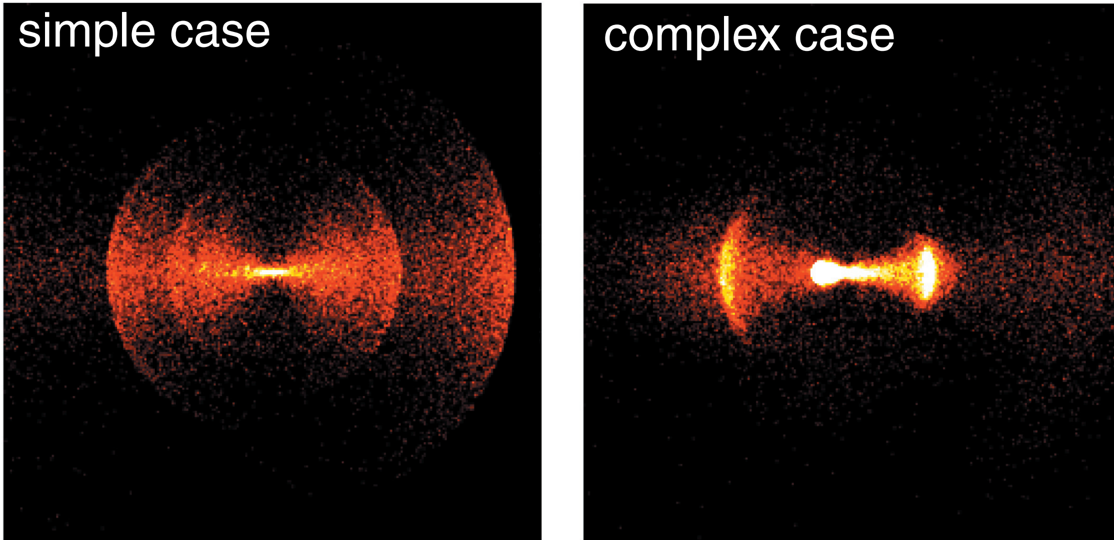
- can be numerically integrated using Maple
- we use it to modify values of Coulomb logarithm in one version of Chandrasekhar formula to have a good agreement with the numerical values of the integral

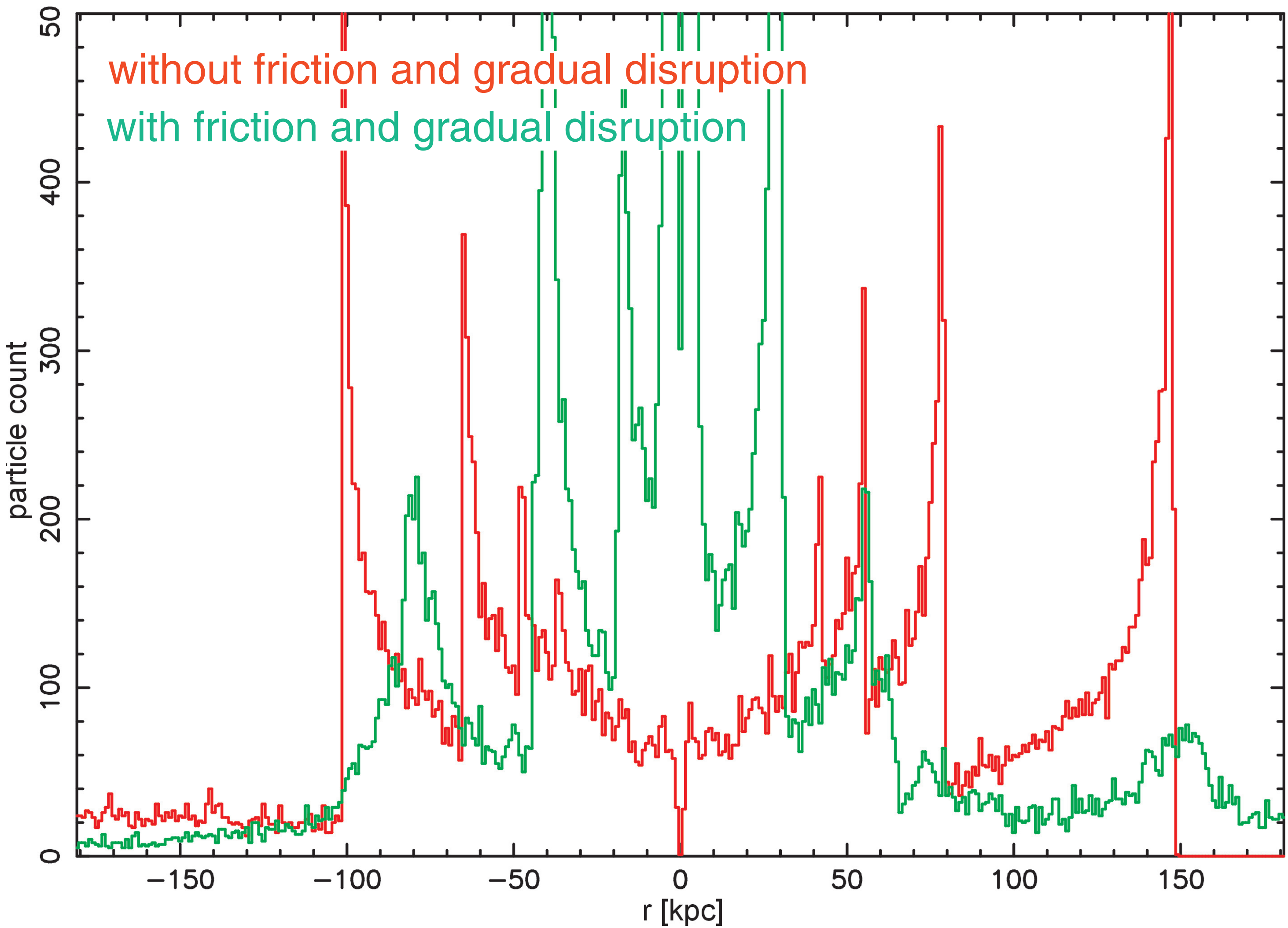
Gradual Decay

- in our case: progressive lowering of the overall mass of secondary galaxy
- computing tidal radius or escaping test particles and change the secondary mass accordingly
- at least the same magnitude of effect on a shell structure

Coupling of Both Effects

- including the dynamical friction and gradual disruption of satellite galaxy changes resulting structure
- coupling of these phenomena was never modeled in much detail before




without friction and gradual disruption
with friction and gradual disruption
particle count
r [kpc]
0
100
200
300
400
50
−150
−100
−50
0
50
100
150