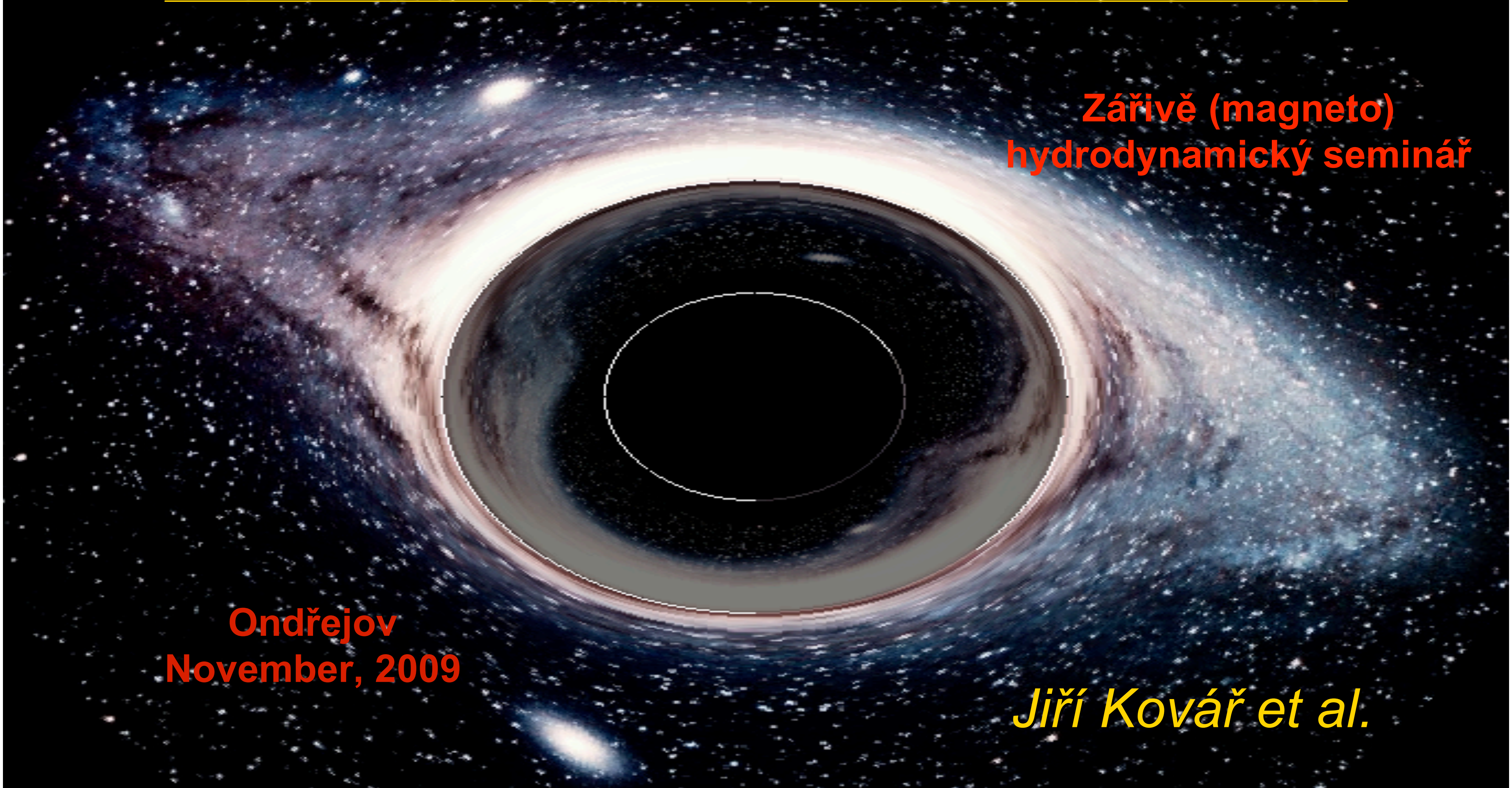
ON SOME GENERAL RELATIVISTIC PROBLEMS IN THE VICINITY OF COMPACT OBJECTS



Zářivě (magneto) hydrodynamický seminář

Ondřejov
November, 2009

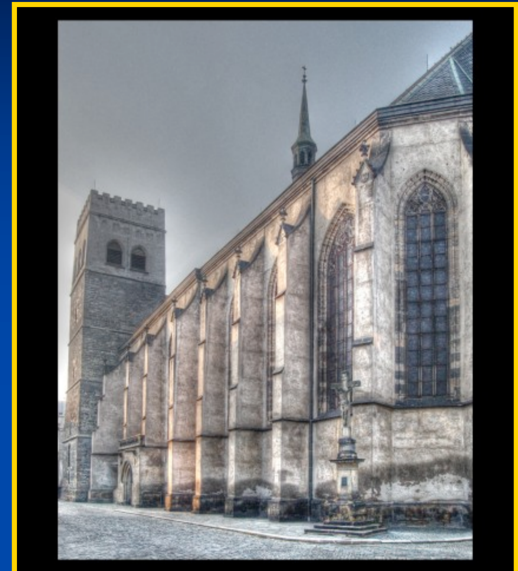
Jiří Kovář et al.

INFLUENCE OF COSMOLOGICAL CONSTANT ON BLACK-HOLE PHYSICS

OFF-EQUATORIAL MOTION OF CHARGED PARTICLES NEAR COMPACT OBJECTS

Olomouc – Czech Republic

- founded in 10^{th} century
- 110 000 citizens
- Palacký University (1573)



Institute of Physics – Silesian University in Opava

UNIVERSITAS SILESIANA OPAVIENSIS MCMXCI

SCIENTIAE SCEPTRA PERMANENT

http://www.physics.cz

- founded in 1991
- main field of interest

  Relativistic and particle physics
  and its applications in astrophysics and cosmology

- areas of interest

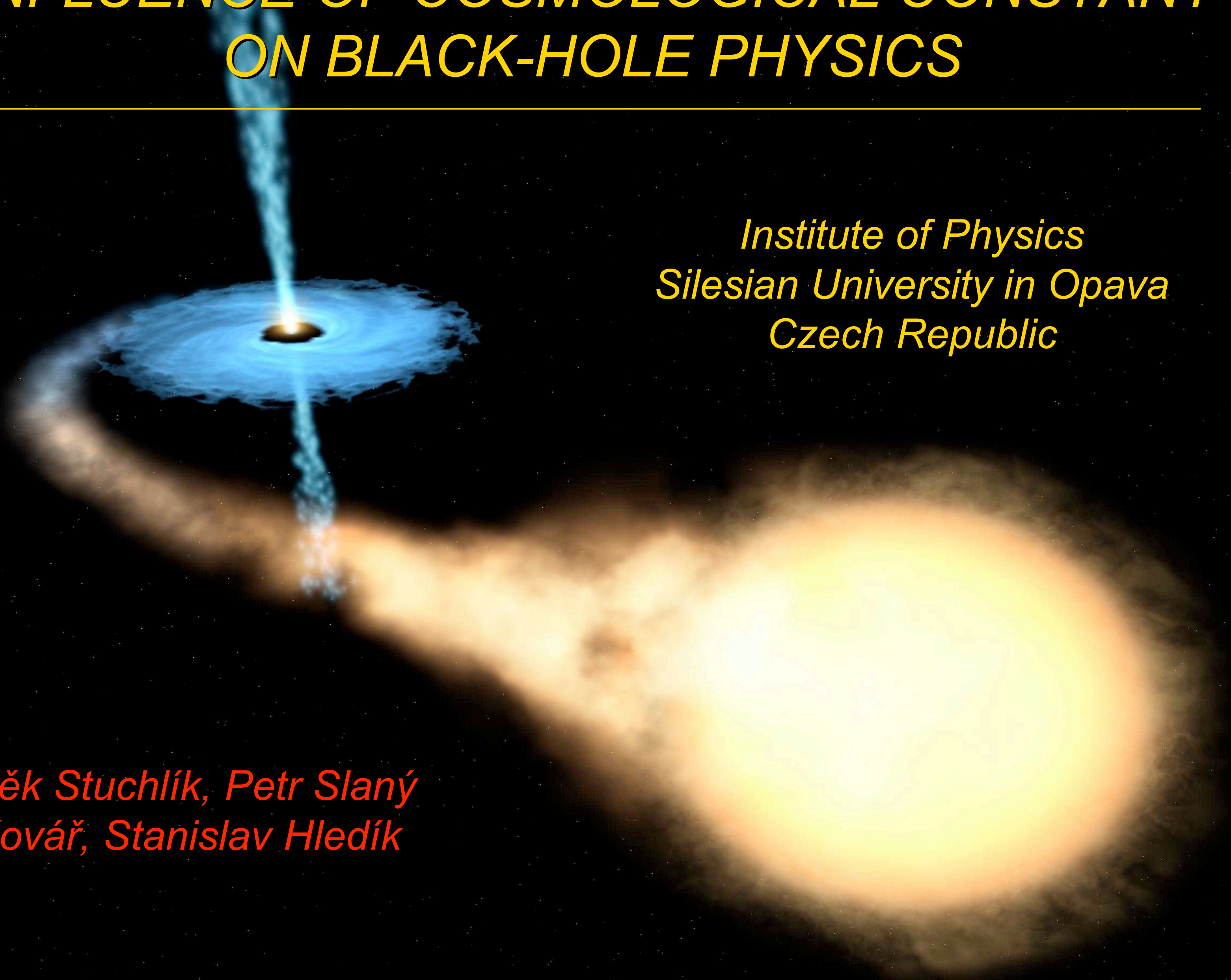
  1) quasi periodic oscilations
  2) relevance of Λ in astrophysics and cosmology
  3) oscillation models of accretion discs
  4) properties of neutron and quark stars
  5) advanced scientific computing and visualization

Cooperation and RAGTime workshops

1) Astronomical Institute, Czech Academy of Sciences, Prague
2) Nicolaus Copernicus Astronomical Center, Warsaw, Poland
3) Centre for the Study of Radiation in Space, Toulouse, France
4) University of Gothenburg, Sweden
5) Massachusetts Institute of Technology, MA, USA
6) University of California, Santa Barbara, CA, USA
7) University of Oxford, UK
8) International School for Advanced Studies, Trieste, Italy

RAGTime workshops: http://www.ragtime11.cz

INFLUENCE OF COSMOLOGICAL CONSTANT ON BLACK-HOLE PHYSICS



Institute of Physics
Silesian University in Opava
Czech Republic

Zdeněk Stuchlík, Petr Slaný
Jiří Kovář, Stanislav Hledík

Introduction

- recent cosmological observations indicate an accelerating universe (generated by some appropriate form of the so-called dark energy)

- large variety of possible candidates for the dark energy

  - standard possibility represented by non-zero vacuum energy, which can be involved into the GR by the cosmological term in Einstein's equations, allowing to get accelerated expansion of universe solution

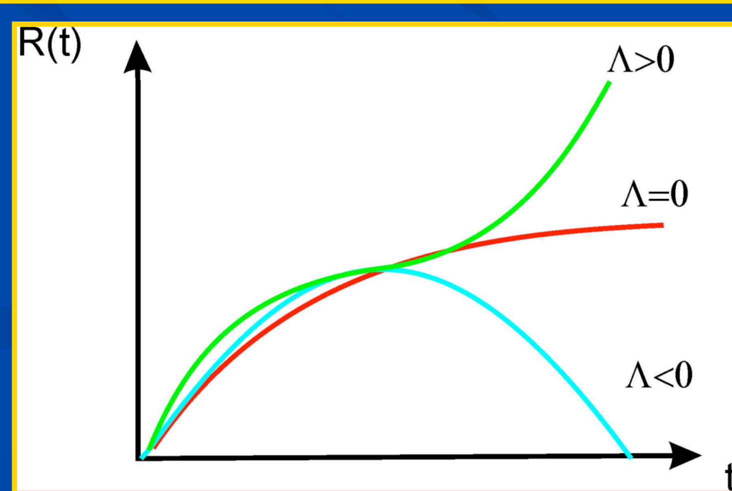
Einstein's equations

$$R_{\alpha\beta} - \frac{1}{2} R\, g_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi\, T_{\alpha\beta}$$

Friedmann–Robertson–Walker solution

$g_{\alpha\beta}$

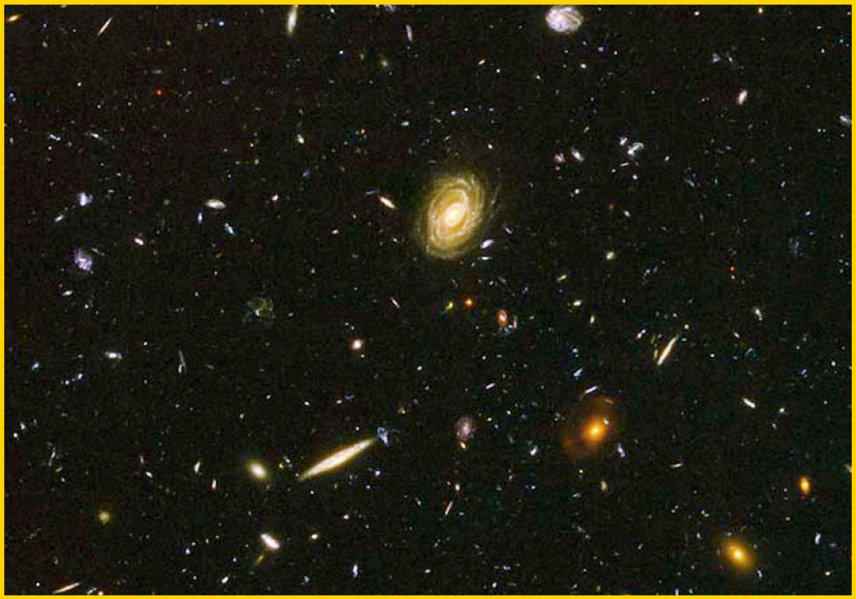
Friedman's models of universe



Introduction

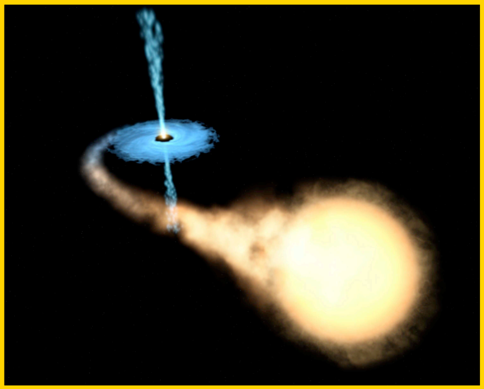
Repulsive cosmological constant influences

universe dynamics



Friedmann–Robertson–Walker solution

astrophysics ?



Kerr-de Sitter solution

1) test particle motion
2) spinning test particle motion
3) perfect fluid equilibrium configuration

A) standard GR approach
B) inertial forces GR approach
C) pseudo-Newtonian approach

Kerr-de Sitter geometry

- line element

$$ds^2 = g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{\phi\phi}d\phi^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2$$

- metric coefficients

$$\begin{aligned} g_{tt} &= \frac{a^2\Delta_\theta \sin\theta^2 - \Delta_r}{I^2\rho^2} \\ g_{t\phi} &= \frac{a[\Delta_r - (a^2+r^2)\Delta_\theta]\sin\theta^2}{I^2\rho^2} \end{aligned}$$

$$\begin{aligned} g_{\phi\phi} &= \frac{[(a^2+r^2)^2\Delta_\theta - a^2\Delta_r\sin\theta^2]\sin\theta^2}{I^2\rho^2} \\ g_{rr} &= \frac{\rho^2}{\Delta_r}, \quad g_{\theta\theta} = \frac{\rho^2}{\Delta_\theta} \end{aligned}$$

$$\begin{aligned} I &= 1 + \frac{1}{3}\Lambda a^2 \\ \rho^2 &= r^2 + a^2\cos^2\theta \end{aligned}$$

$$\begin{aligned} \Delta_r &= r^2 - 2Mr + a^2 - \frac{1}{3}\Lambda r^2(r^2+a^2) \\ \Delta_\theta &= 1 + \frac{1}{3}\Lambda a^2\cos^2\theta \end{aligned}$$

- dimensionless cosmological parameter $y = \frac{1}{3}\Lambda M^2$

Kerr-de Sitter geometry

Limit cases

a, y .... Kerr-de Sitter geometry

rotating black holes in the universe with $\Lambda>0$

---

a, y=0 .... Kerr geometry

rotating black holes in the universe with $\Lambda=0$

---

a=0, y .... Schwarzschild-de Sitter geometry

static black holes in the universe with $\Lambda>0$

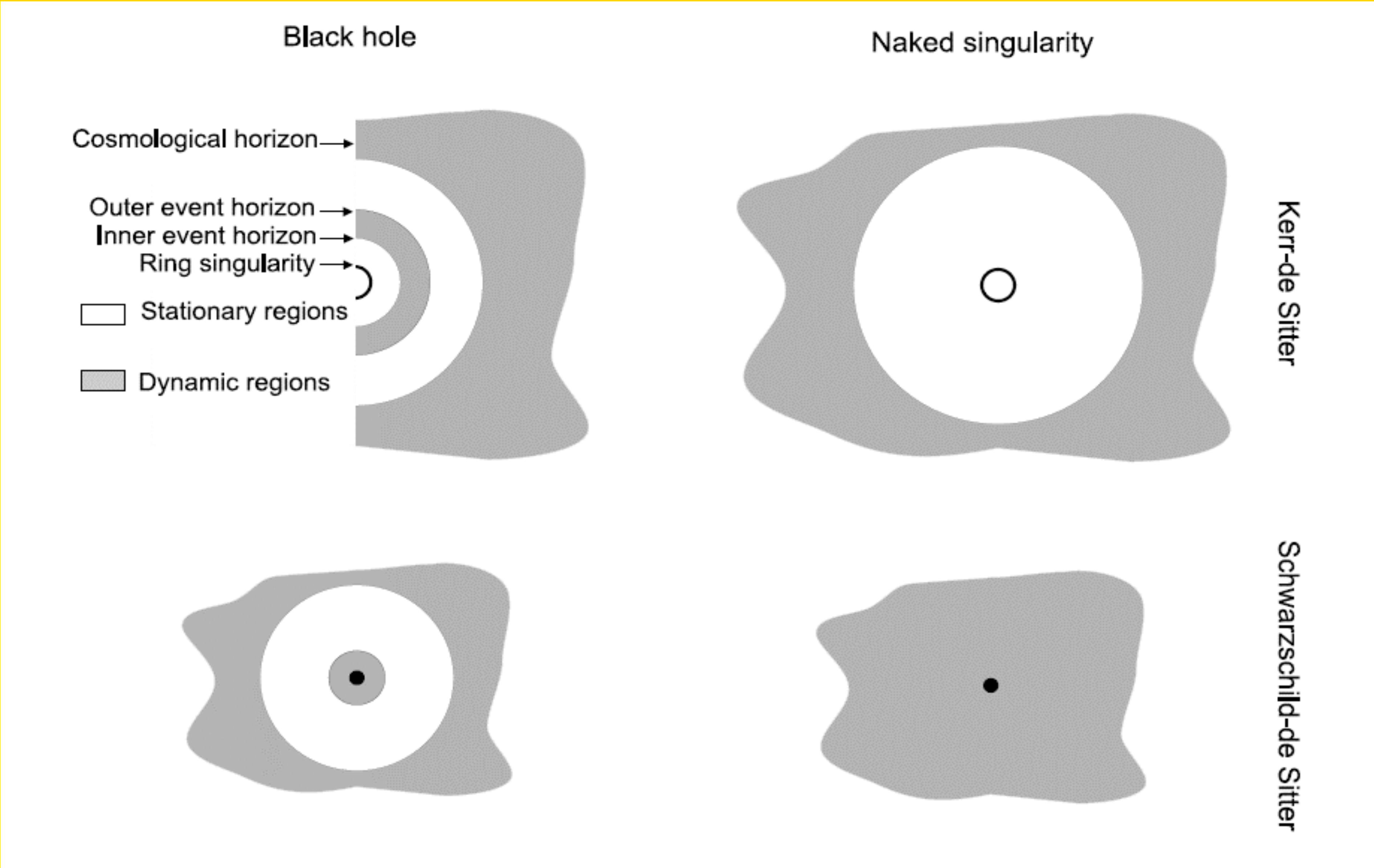
---

a=0, y=0 .... Schwarzschild geometry

static black holes in the universe with $\Lambda=0$

Kerr-de Sitter geometry

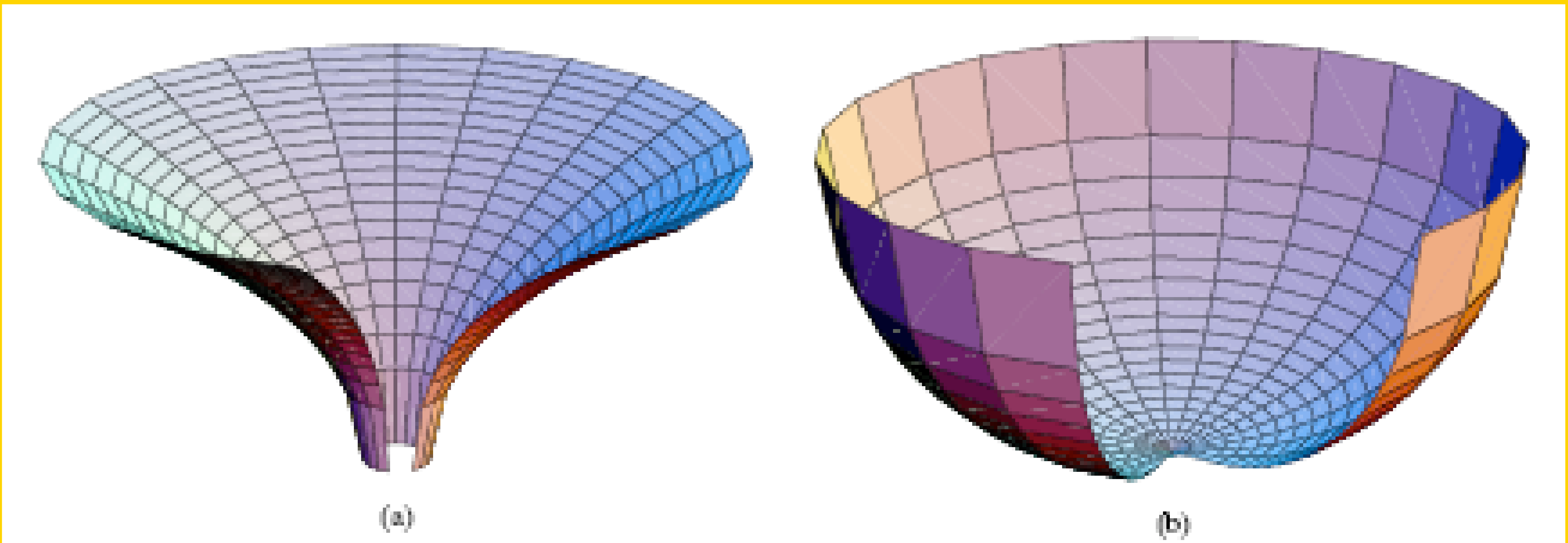
Equatorial plane



Embedding diagrams

$\Lambda=0$

$\Lambda>0$



Schwarzschild geometry

Schwarzschild-de Sitter geometry

Test particle motion

Standard GR approach

[Stuchlik and Hledik, Physical Review D (60), 1999]
[Stuchlik and Slany, Physical Review D (69), 2004]

- Carter's equations (geodetical motion)

$$r^2\frac{\mathrm{d}r}{\mathrm{d}\lambda} = \pm R^{1/2}(r),$$
$$r^2\frac{\mathrm{d}\varphi}{\mathrm{d}\lambda} = -IP_\theta + \frac{aIP_r}{\Delta_r},$$
$$r^2\frac{\mathrm{d}t}{\mathrm{d}\lambda} = -aIP_\theta + \frac{(r^2+a^2)IP_r}{\Delta_r},$$

$$R(r) = P_r^2 - \Delta_r\left(m^2r^2+K\right),$$
$$P_r = I\mathcal{E}\left(r^2+a^2\right) - Ia\Phi,$$
$$P_\theta = I(a\mathcal{E}-\Phi),$$
$$K = I^2(a\mathcal{E}-\Phi)^2.$$

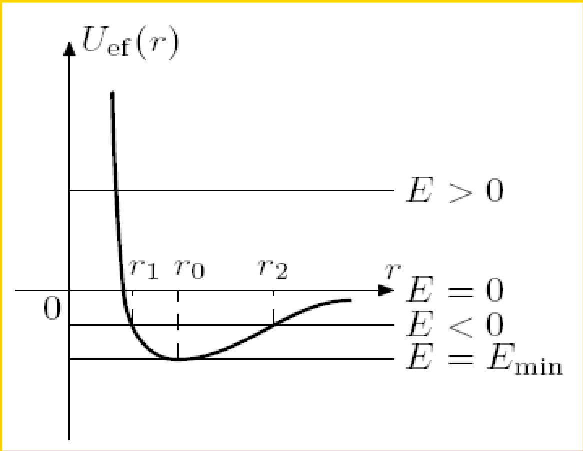
- effective potential

$$V_{\text{eff}}(r;L,y) \equiv \left(1-\frac{2}{r}-yr^2\right)\left(1+\frac{L^2}{r^2}\right)$$

$$V_{\text{eff}}(r;L,a,y) \equiv \frac{a\left[yr\left(r^2+a^2\right)+2\right]L \pm \Delta_r^{1/2}\left\{r^2L^2+r\left[\left(1+ya^2\right)r\left(r^2+a^2\right)+2a^2\right]\right\}^{1/2}}{\left[\left(1+ya^2\right)r\left(r^2+a^2\right)+2a^2\right]}.$$

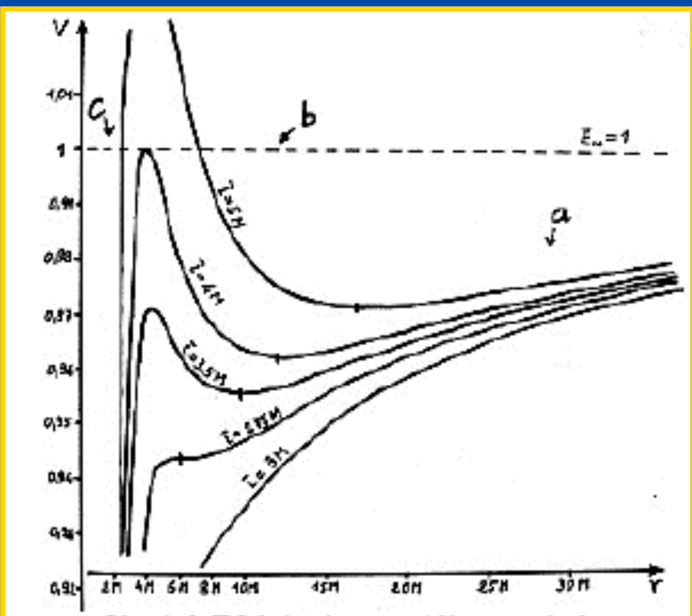
Test particle motion

Effective potential

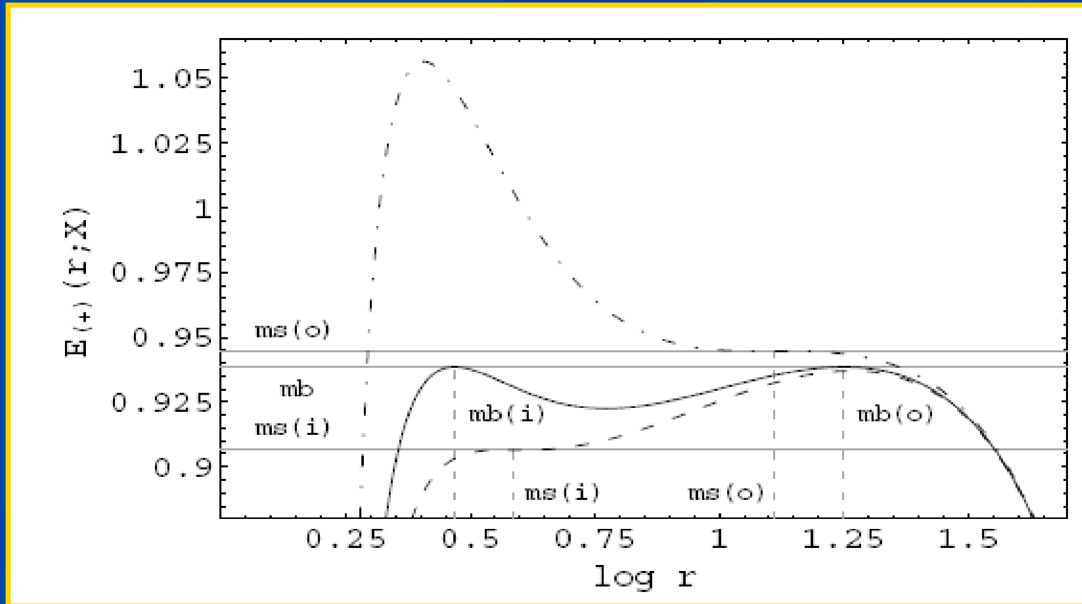


Newtonian physics

Schwarzschild

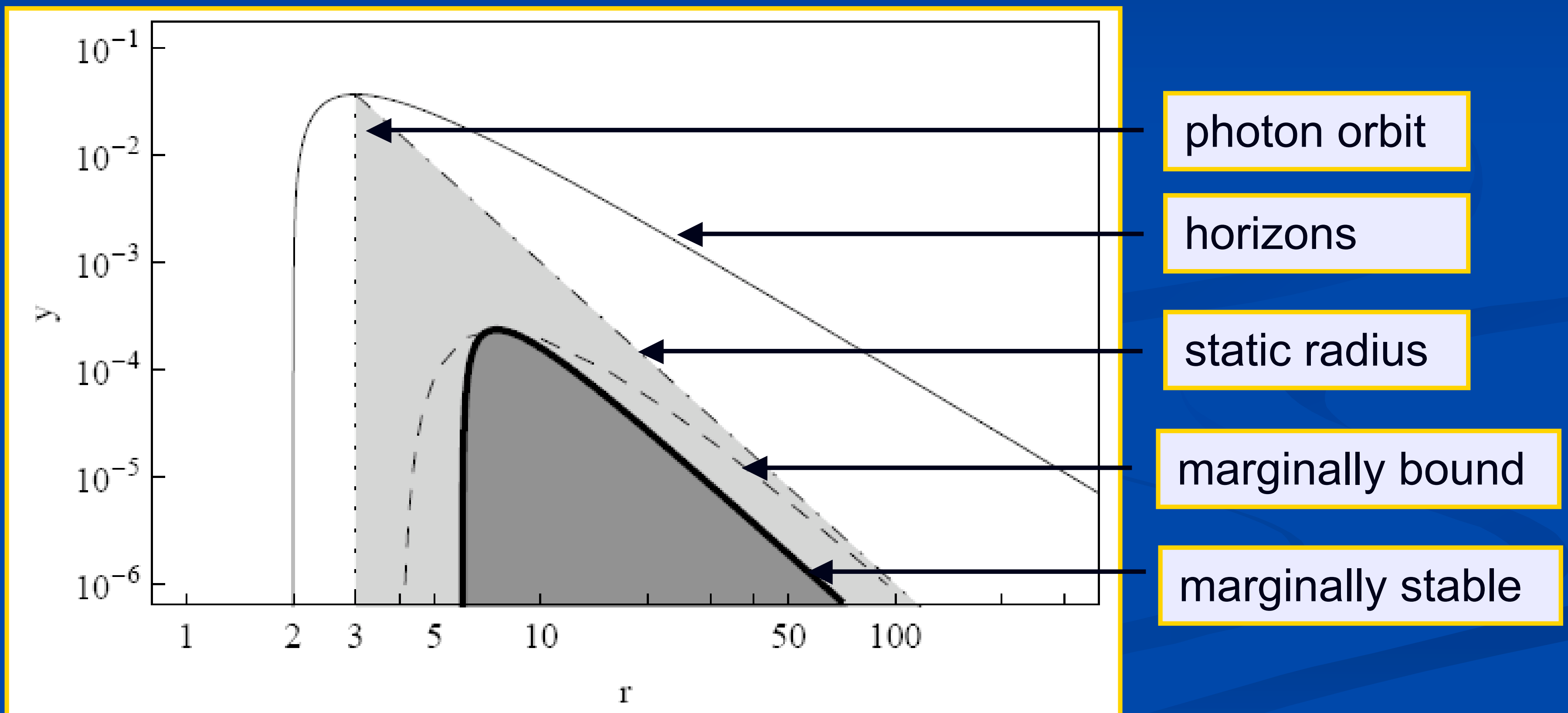


Kerr (Schwarzschild)-de Sitter



Test particle motion

Schwarzschild-de Sitter



Test particle motion

Inertial forces formalism

[Kovar and Stuchlik, Int. Journal of Modern Phys. A (21), 2006]
[Kovar and Stuchlik, Class. Quantum Grav. (24), 2007]

- special observers

$$n^i = \mathrm{e}^{-\Phi}(\eta^i + \Omega_{LNRF}\xi^i),$$
$$\Phi = \tfrac{1}{2}\ln[-(\eta^i + \Omega_{LNRF}\xi^i)(\eta_i + \Omega_{LNRF}\xi_i)]$$

- geodetical motion

$$G_k + Z_k + C_k + E_k = 0$$

- inertial forces

$$\begin{aligned} G_k^{\perp} &= -m\nabla_k\Phi \\ Z_k^{\perp} &= -m(\gamma v)^2\tilde{\tau}^i\tilde{\nabla}_i\tilde{\tau}_k \\ C_k^{\perp} &= -m\gamma^2 v X_k \\ E_k^{\perp} &= -m\dot{V}\tilde{\tau}_k \end{aligned}$$

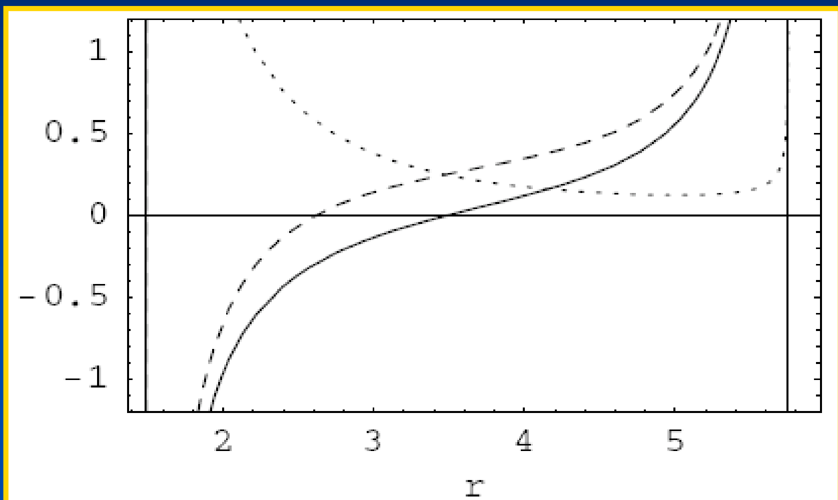
$$G_k = -m\nabla_k\Phi,$$
$$Z_k = m\tilde{v}^2\tilde{R}^{-1}\nabla_k\tilde{R},$$
$$C_k = -m(1-\tilde{v}^2)^{1/2}\tilde{v}\tilde{R}\nabla_k\Omega_{LNRF},$$
$$E_k = m(1-\tilde{v}^2)^{3/2}\mathrm{e}^{\Phi}\tilde{R}u^i\nabla_i(\Omega)\tilde{\tau}_k.$$

$$\tilde{R} = (\xi^i\xi_i)^{1/2}\mathrm{e}^{-\Phi}, \quad \tilde{\Omega} \equiv \Omega - \Omega_{LNRF}, \quad \tilde{v} = \gamma v,$$

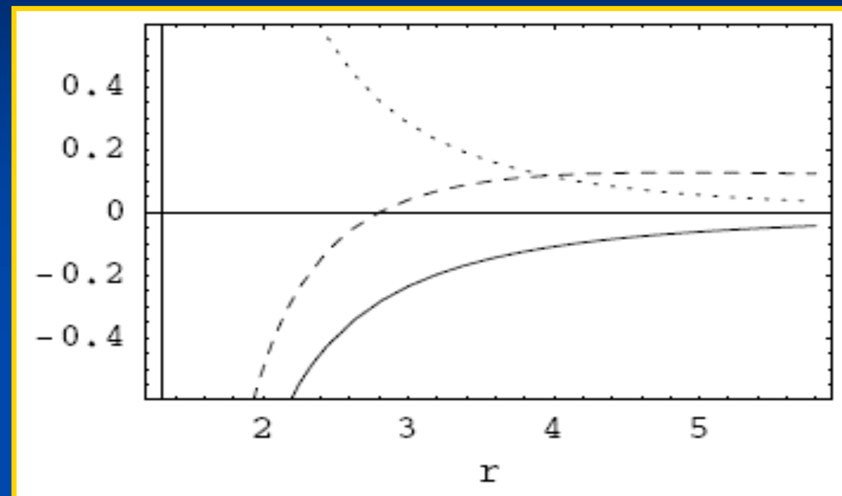
Test particle motion

Inertial forces formalism

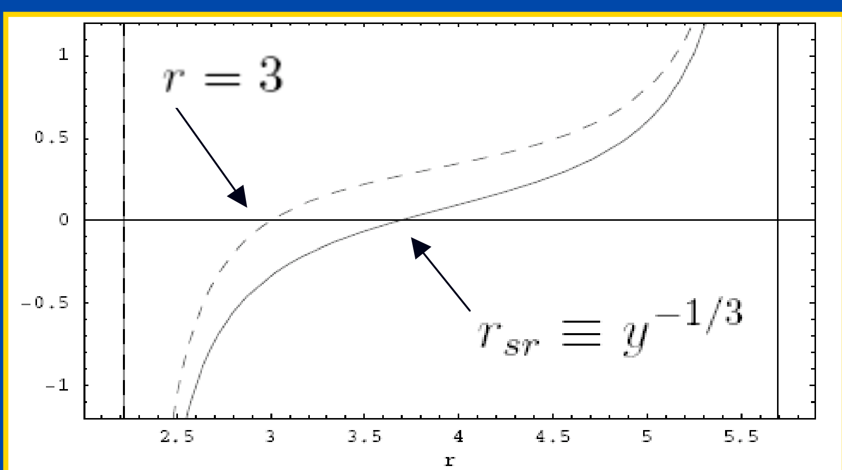
Kerr-de Sitter



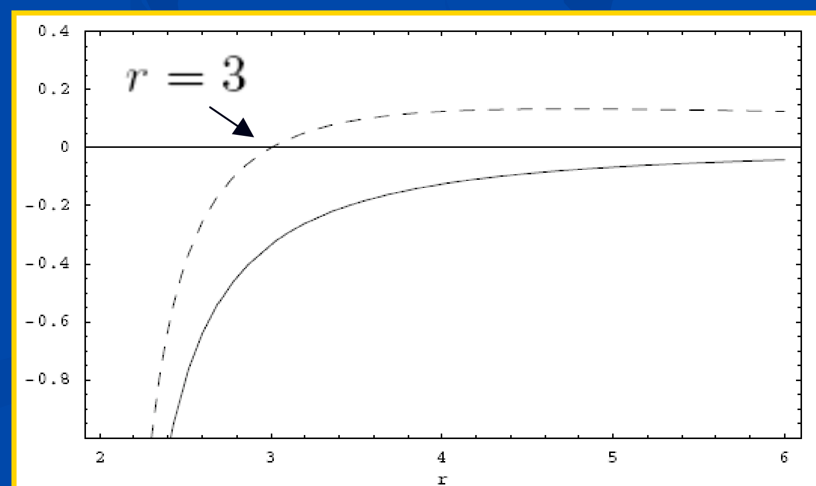
Kerr



Schwarzschild-de Sitter



Schwarzschild



Test particle motion

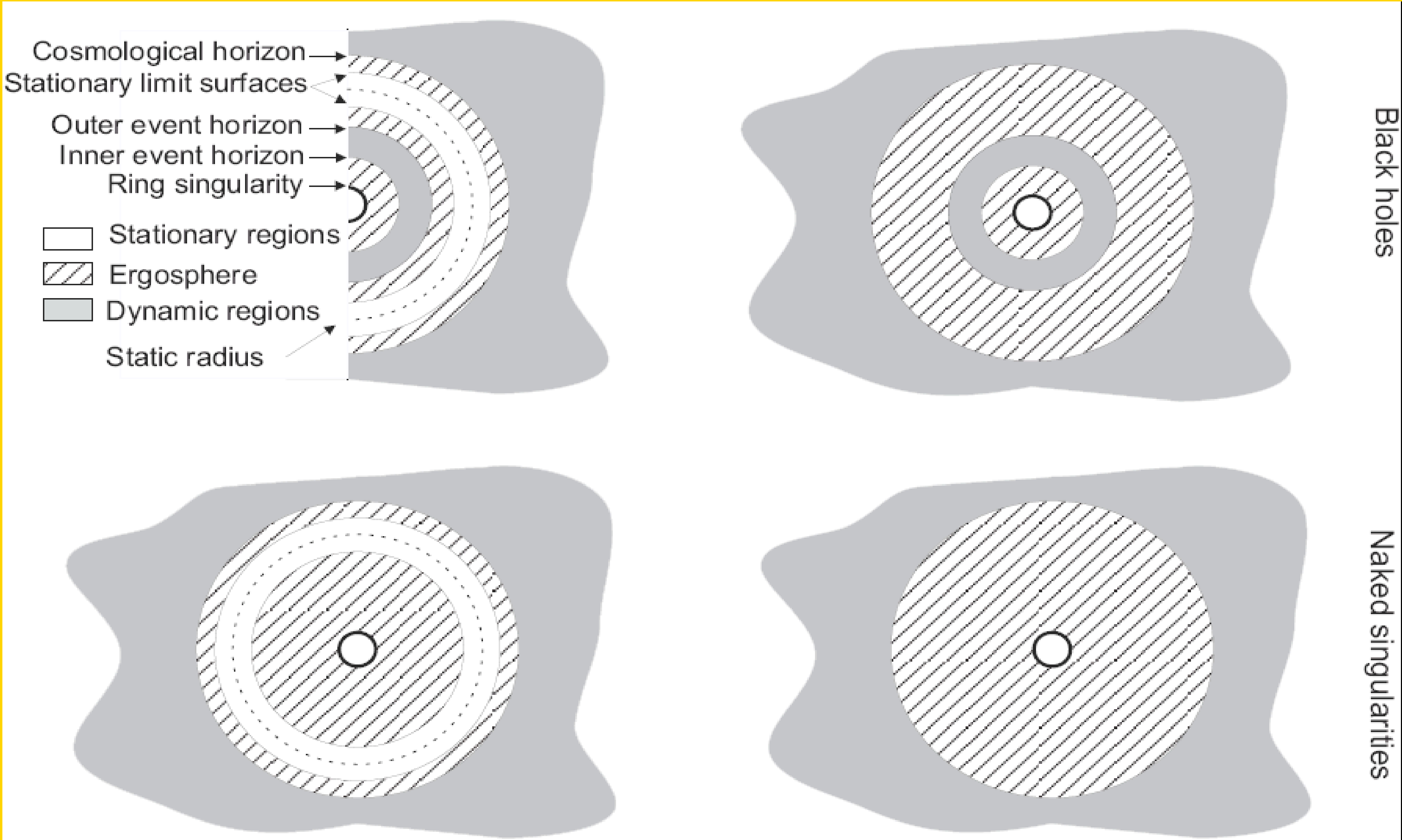
- ergosphere $$g_{tt} \leq 0$$

- static radius $$u^i \nabla_i u^j = 0$$

$$u^i = \frac{1}{\sqrt{-g_{tt}}} \delta^i_t$$

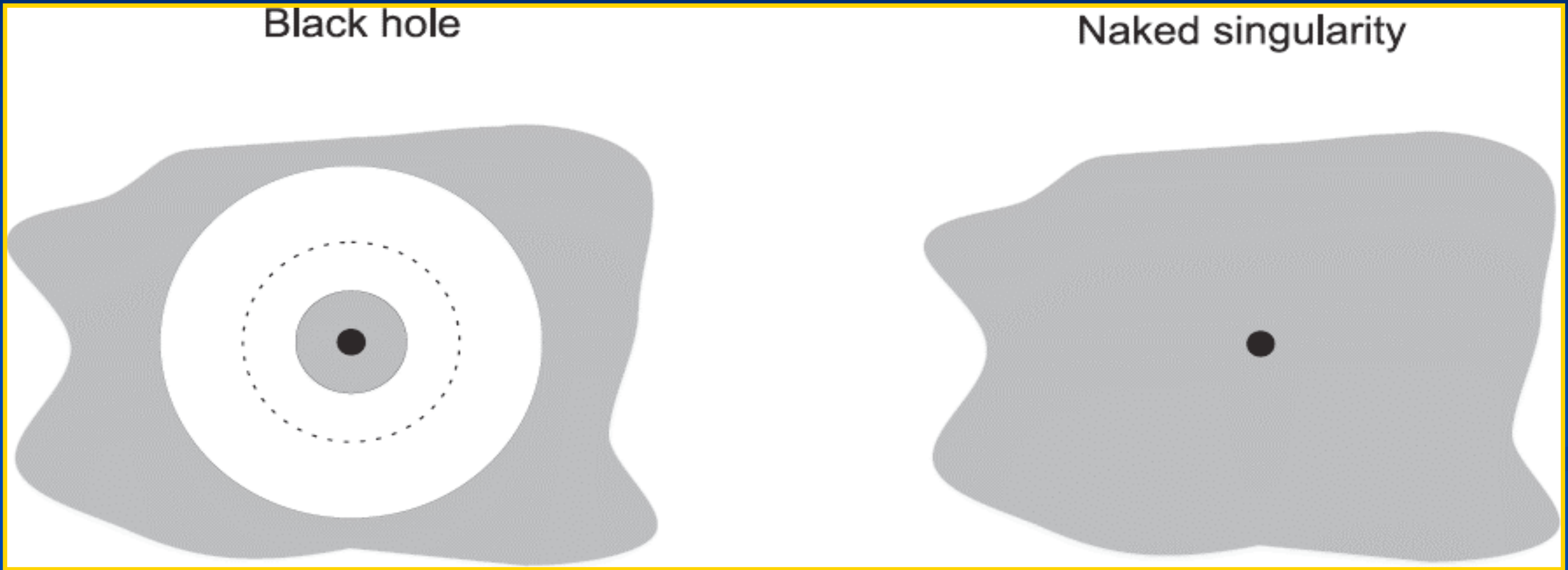
Test particle motion

Kerr-de Sitter



Test particle motion

Schwarzschild-de Sitter



Spinning particle motion

[Stuchlik, Acta Phys. Slovaca (49),1999]
[Stuchlik and Kovar, Class. Quantum Grav. (23), 2006]

- motion of spinning particles

$$m\frac{Du^\alpha}{d\tau} = -\epsilon^{\alpha\mu\nu\beta}\frac{D^2u_\beta}{d\tau^2}S_\mu u_\nu + \frac{1}{2}\epsilon^{\lambda\mu\rho\sigma}R^\alpha_{\nu\lambda\mu}u^\nu u_\sigma S_\rho$$

- spin vector dynamics given by Fermi-Walker transport equation

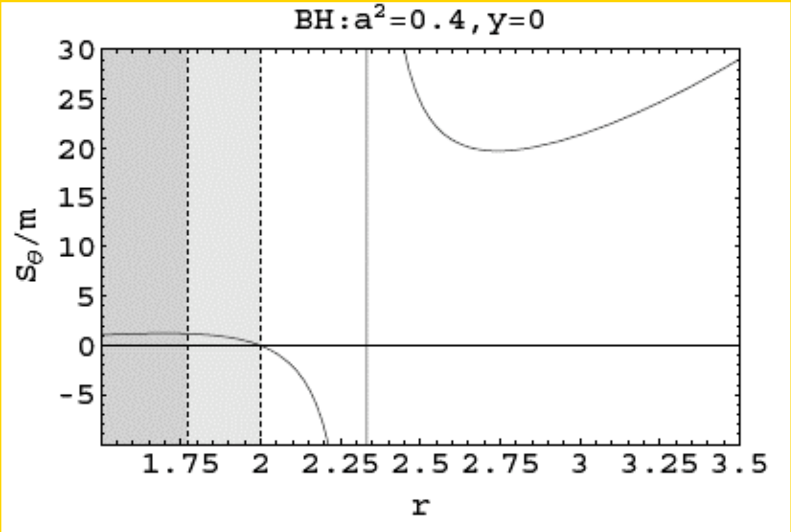
$$\frac{DS_\alpha}{d\tau} = u_\alpha\frac{Du^\beta}{d\tau}S_\beta$$

- 4-velocity of particle at rest

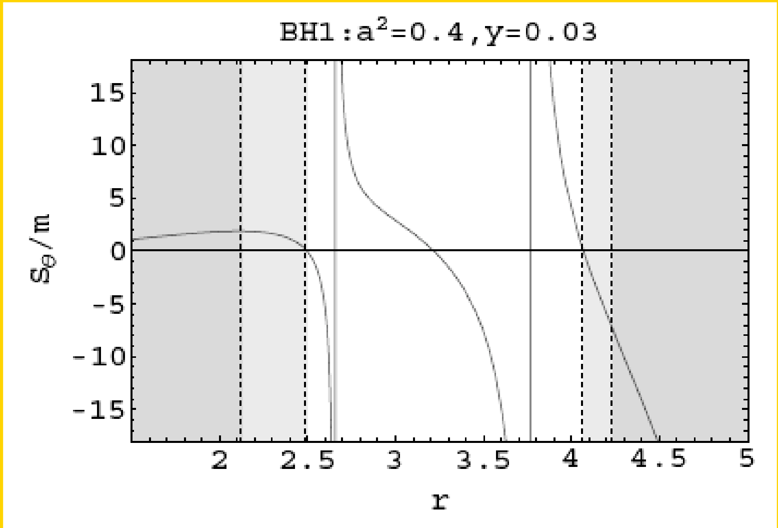
$$u^\alpha = \frac{1}{\sqrt{-g_{tt}}}\delta^t_\alpha, \quad \frac{du^\alpha}{d\tau} = u^\beta\partial_\beta u^\alpha = 0$$

Spinning particle motion — *Equilibrium conditions in equatorial plane*

S_r	S_φ	S_θ	S	SdS	Kerr	KdS
$=0$	$=0$	$=0$	—	$r_{sr}=y^{-1/3}$	—	$r_{sr}=y^{-1/3}$
		$\neq 0$	—	$r_{sr}=y^{-1/3}$	$r=r(S_\theta,a)(\infty)$	$r=r(S_\theta,a,y)(\infty)$
	$\neq 0$	$=0$	—	$r_{sr}=y^{-1/3}$	—	$r_{sr}=y^{-1/3}$
		$\neq 0$	—	$r_{sr}=y^{-1/3}$	—	—
$\neq 0$	arbitr.	arbitr.	—	$r_{sr}=y^{-1/3}$	—	—



Kerr



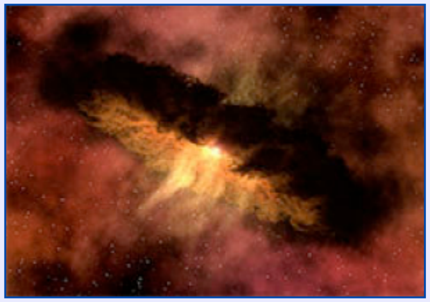
Kerr-de Sitter

Perfect fluid tori

Basic equations

[Stuchlik, Slany and Hledik, Astron. and Astroph. (363), 2000]
[Slany and Stuchlik, Class. Quantum Grav. (22), 2005]
[Stuchlik, Slany and Kovar, Class. Quantum Grav., 2009 (in print)]

- rotating perfect fluid



$$T_{ik} = (p + \epsilon) U_i U_k + p g_{ik}$$

- solution of relativistic Euler equation

$$\int_0^p \frac{dp}{p + \epsilon} = W_{in} - W,$$

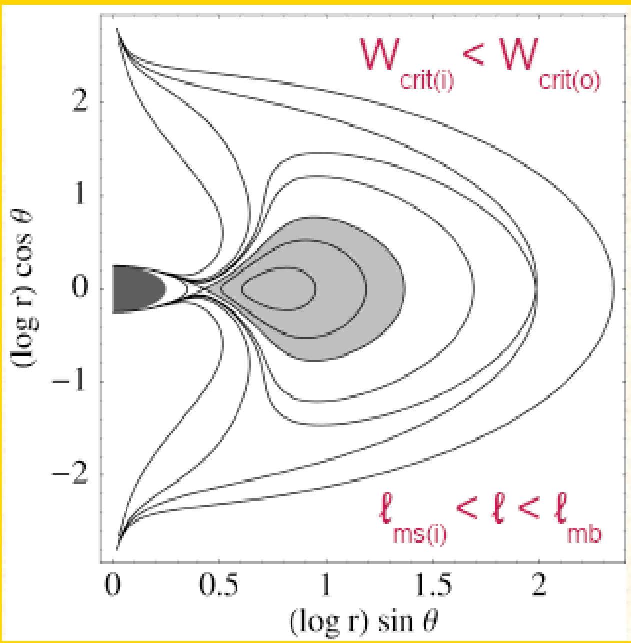
$$W_{in} - W = \ln (U_t)_{in} - \ln (U_t) + \int_{l_{in}}^{l} \frac{\Omega dl}{1 - \Omega l}$$

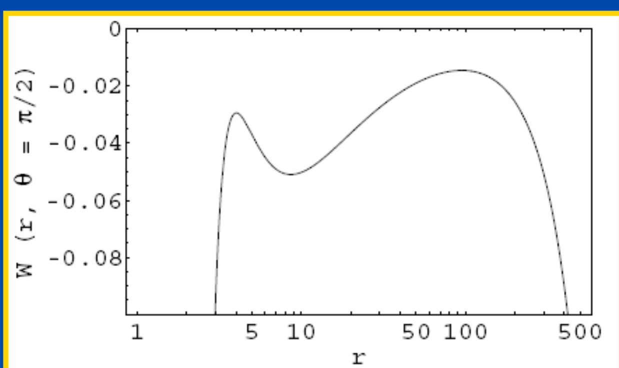
- rotating fluid of $l = const$

Perfect fluid tori

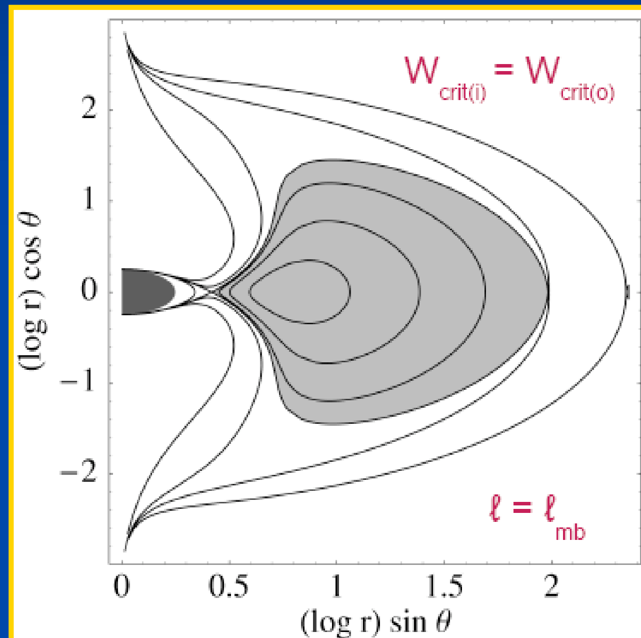
Configurations

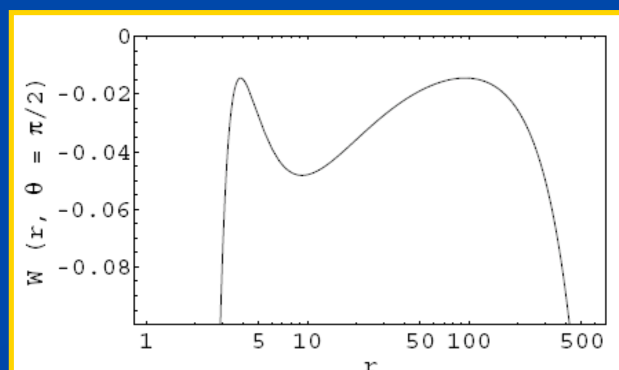
Accretion



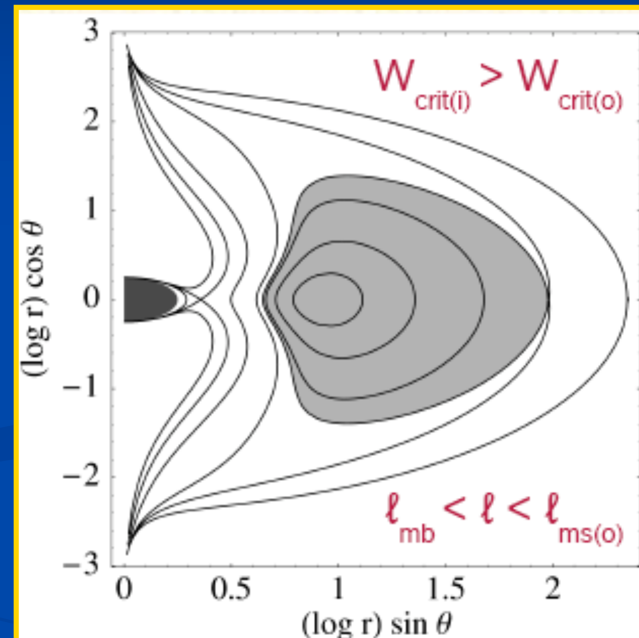


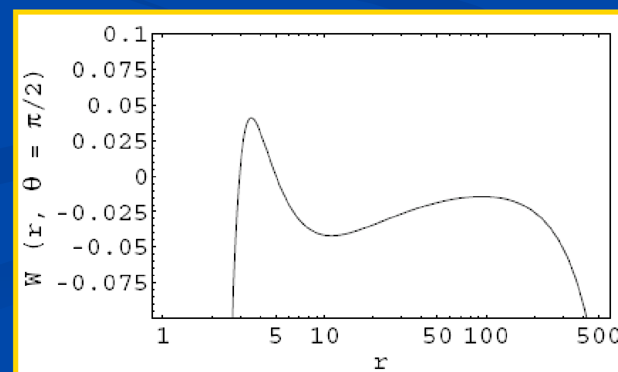
Marginal bound





Excretion

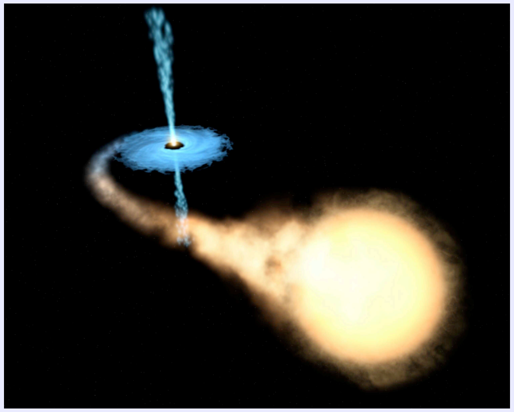


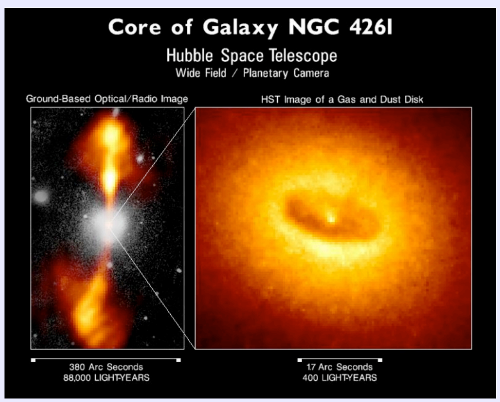


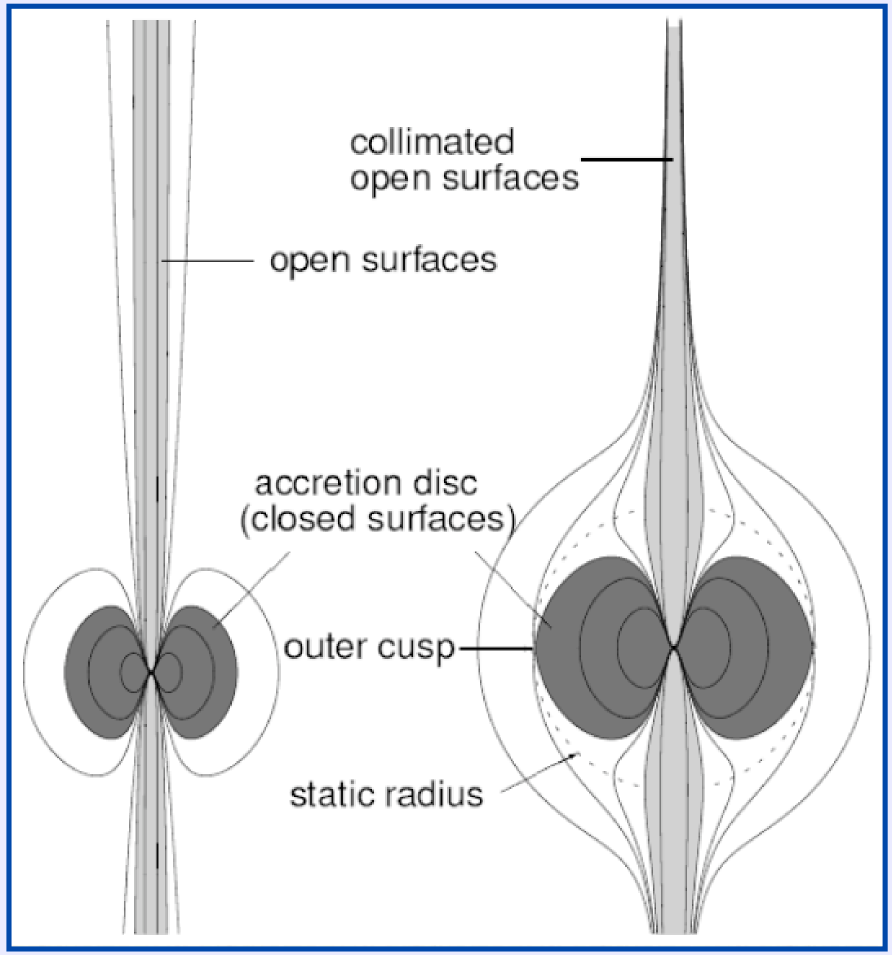
Perfect fluid tori

Other calculations

Collimation of surfaces







Mass density and temperature profiles

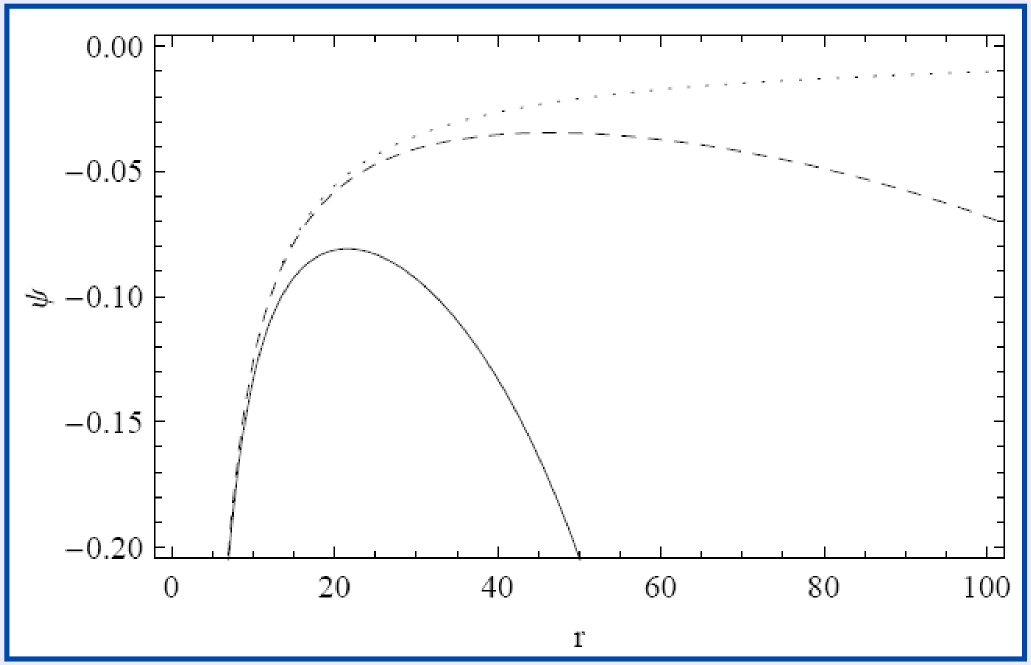
Pseudo-Newtonian approach

Gravitational potential

[Stuchlik and Kovar, Int. Journal of Modern Physics D (17), 2008]
[Stuchlik, Slany and Kovar, Class. Quantum Grav. (26), 2009]

- pseudo-Newtonian gravitational potential

$$\psi = -\frac{1 + yr^3/2}{r - 2 - yr^3}$$



- approximative approach
- Newtonian routines + relativistic effects (cosmological repulsion)

Pseudo-Newtonian approach *Summary*

- exact determination of
  - horizons
  - static radius
  - marginally stable circular orbits
  - marginally bound circular orbits
  - cusps of tori
  - critical equipressure surfaces

- small differences when determining
  - effective potential (energy) barriers
  - density and temperature profiles

Conclusions

Summary

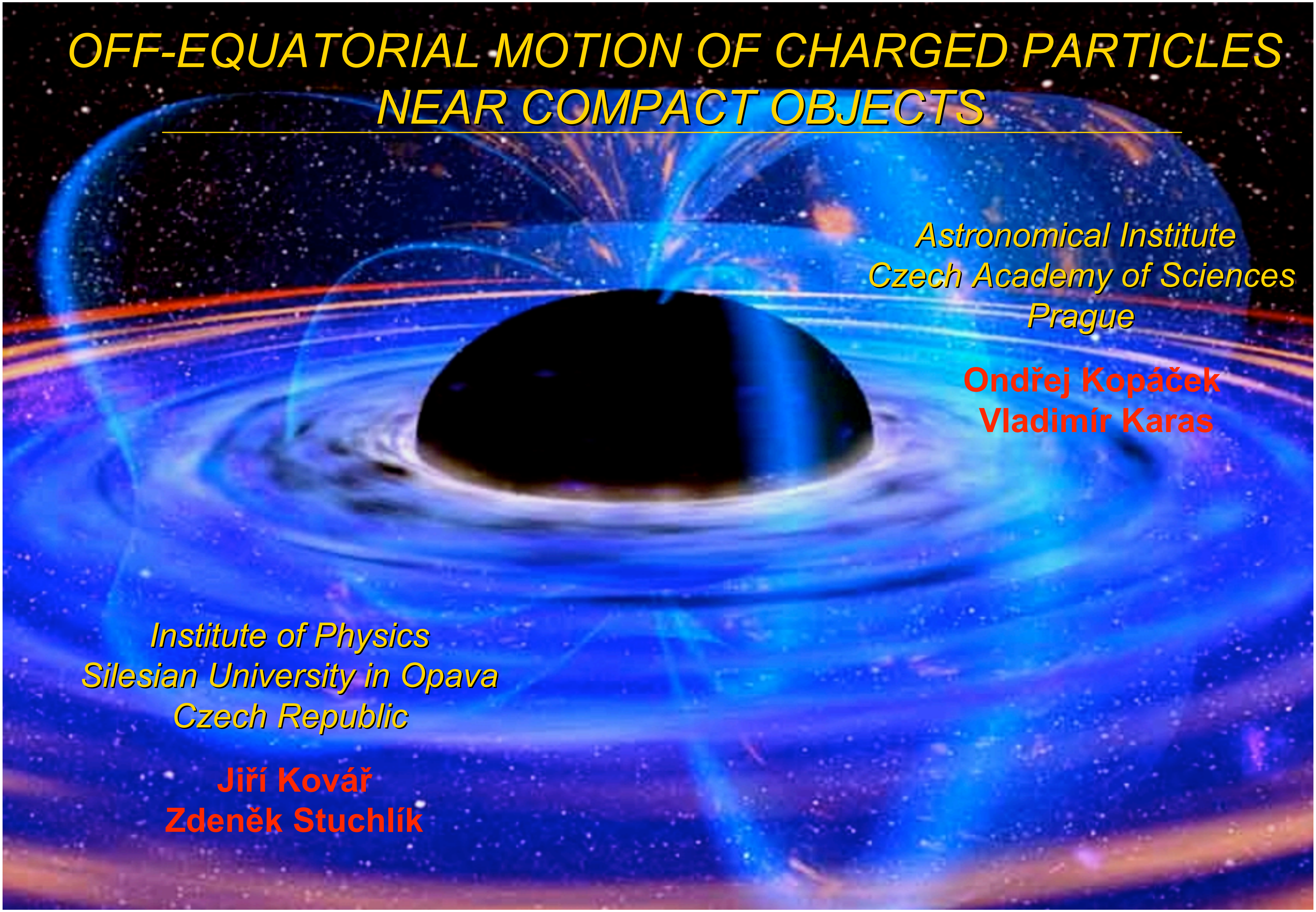
$\Lambda_0 \sim 10^{-56}\ cm^{-2}$

y	M $[M_\odot]$	r_s [kpc]	$r_{ms(o)+}$ [kpc]	$r_{mb(o)+}$ [kpc]
10^{-46}	1	0.1	0.07	0.1
10^{-44}	10	0.2	0.15	0.2
10^{-42}	100	0.5	0.3	0.5
10^{-34}	10^6	10	7	10
10^{-30}	10^8	50	30	50
10^{-28}	10^9	110	70	110
10^{-26}	10^{10}	250	150	250
10^{-22}	10^{12}	1100	700	1100

binary systems: $\sim 10^{-3}$ pc

large galaxies: Sa–Sc $\sim$ 100 kpc, E $\sim$ 200 kpc, cD $\sim$ 1000 kpc

OFF-EQUATORIAL MOTION OF CHARGED PARTICLES NEAR COMPACT OBJECTS



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Silesian University in Opava
Czech Republic

Jiří Kovář
Zdeněk Stuchlík

Introduction

- 'Halo orbits' – off-equatorial circular orbits of constant r and θ
  (stable orbits)

- problems to deal with

  - halo orbits existence and basic features

  [Dullin, Horányi and Howard, 1999, 2002] (weak GF)
  [Kovář, Stuchlík and Karas, 08, Class. Quantum Gravity] (strong GF)
  [Calvani, de Felice, Fabbri, Turolla, 82, Nuevo Cimento] (strong GF)
  [Kovář, Kopáček, Karas and Stuchlík, 09, in preparation] (strong GF)

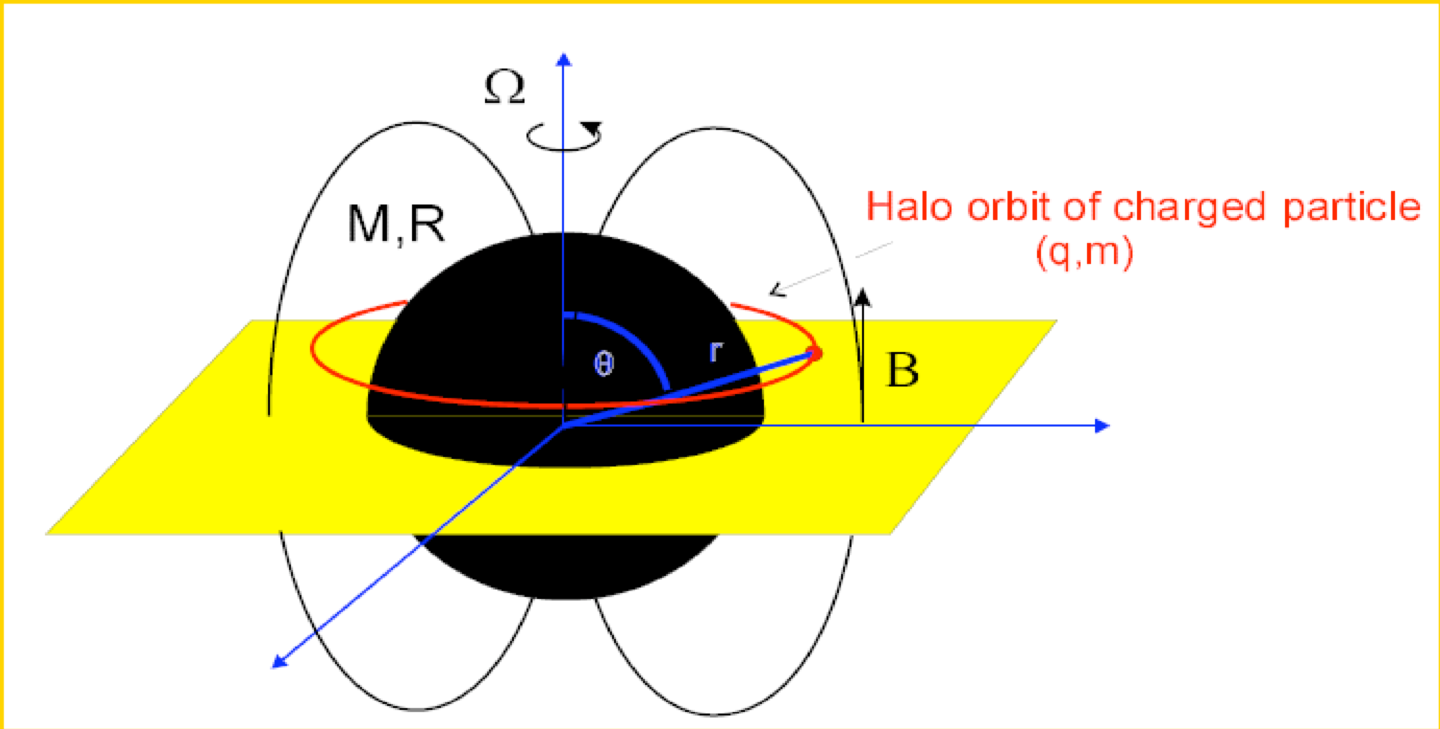
  - related off-equatorial motion

  [Karas, Vokrouhlický,92, General Relativity and Gravitation]
  [Kopáček, Kovář, Karas and Stuchlík, 09, in preparation]

  - astrophysical consequences

  [?]

Existence of halo orbits



Schwarzschild geometry + rotating dipole MF	slowly rotating neutron star
Kerr geometry + dipole MF	Kerr BH with plasma ring
Kerr geometry + uniform magnetic field	Kerr BH in galactic MF
Kerr-Newman geometry	Kerr-Newman BH and NS

Existence of halo orbits

Basic equations

- Einstein-Maxwell's equations

$$R_{\alpha\beta} - \frac{1}{2} R\, g_{\alpha\beta} + \Lambda\, g_{\alpha\beta} = 8\pi\, T_{\alpha\beta}$$

$$T_{\alpha\beta} = T^{\rm MAT}_{\alpha\beta} + T^{\rm EM}_{\alpha\beta}$$

$$F^{\alpha\beta}{}_{;\beta} = 4\pi J^{\alpha}$$

$$F_{\alpha\beta;\gamma} + F_{\beta\gamma;\alpha} + F_{\gamma\alpha;\beta} = 0$$

$$T^{\rm EM}_{\alpha\beta} = \frac{1}{4\pi}\left(F_{\alpha\mu} F^{\mu}_{\beta} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} g_{\alpha\beta}\right)$$

$$g_{\alpha\beta}$$

$$F^{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu}$$

Schwarzschild geometry + rotating dipole MF	slowly rotating neutron star
Kerr geometry + dipole MF	Kerr BH with plasma ring
Kerr geometry + uniform magnetic field	Kerr BH in galactic MF
Kerr-Newman geometry	Kerr-Newman BH and NS

Existence of halo orbits

Backgrounds

- Geometry

$$ds^2 = -\frac{\Delta}{\rho^2}[dt - a\sin\theta d\phi]^2 + \frac{\sin^2\theta}{\rho^2}[(r^2+a^2)d\phi - a dt]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

Schwarzschild	M	$\Delta = r^2 - 2Mr + a^2 + Q^2$
Kerr	M, a	$\rho^2 = r^2 + a^2\sin^2\theta$
Kerr-Newmann	M, a, Q	

- Electromagnetic field

$$A_t = A_t(r,\theta; p_1, p_2)$$
$$A_\phi = A_\phi(r,\theta; p_1, p_2)$$

test rotating dipole in Schwarzschild	$p_1 \equiv \Omega,\ p_2 \equiv \mu(B,R)$
test static dipole in Kerr	$p_1 \equiv a,\ p_2 \equiv \mu(I,R)$
test uniform in Kerr	$p_1 \equiv a,\ p_2 \equiv B$
Kerr-Newmann	$p_1 \equiv a,\ p_2 \equiv Q$

Existence of halo orbits

Effective potential

- Hamiltonian

$$\mathcal{H} = \frac{1}{2} g^{\mu\nu}(\pi_\mu - qA_\mu)(\pi_\nu - qA_\nu)$$

- Hamilton's equations

$$dx^\mu/d\lambda = \partial\mathcal{H}/\partial\pi_\mu, \; d\pi_\mu/d\lambda = -\partial\mathcal{H}/\partial x^\mu$$

$$dx^\mu/d\lambda = \pi^\mu - qA^\mu \equiv p^\mu$$

Axially symmetric and stationary

- constants of motion

$$\pi_\phi = p_\phi + qA_\phi \equiv L$$

$$\pi_t = p_t + qA_t \equiv -E$$

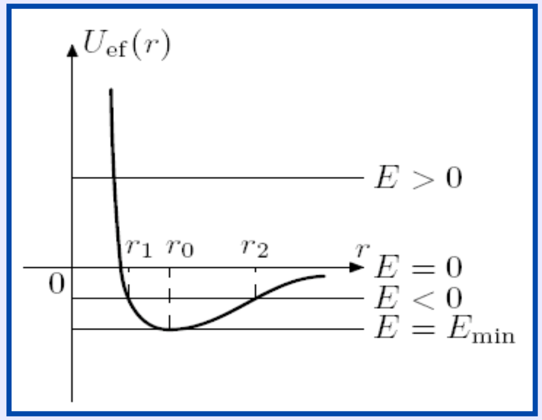
$$g^{\mu\nu} p_\mu p_\nu = -m^2$$

K ?

- effective potential

$$W_{eff} = \frac{\beta + \sqrt{\beta^2 - \alpha\gamma}}{\alpha}$$

$$\alpha = -g^{tt},$$
$$\beta = 2[g^{t\phi}(\tilde{L} - \tilde{q}A_\phi) - g^{tt}\tilde{q}A_t],$$
$$\gamma = -g^{\phi\phi}(\tilde{L} - \tilde{q}A_\phi)^2 - g^{tt}\tilde{q}^2 A_t^2 + 2g^{t\phi}\tilde{q}A^t(\tilde{L} - \tilde{q}A_\phi) - 1$$

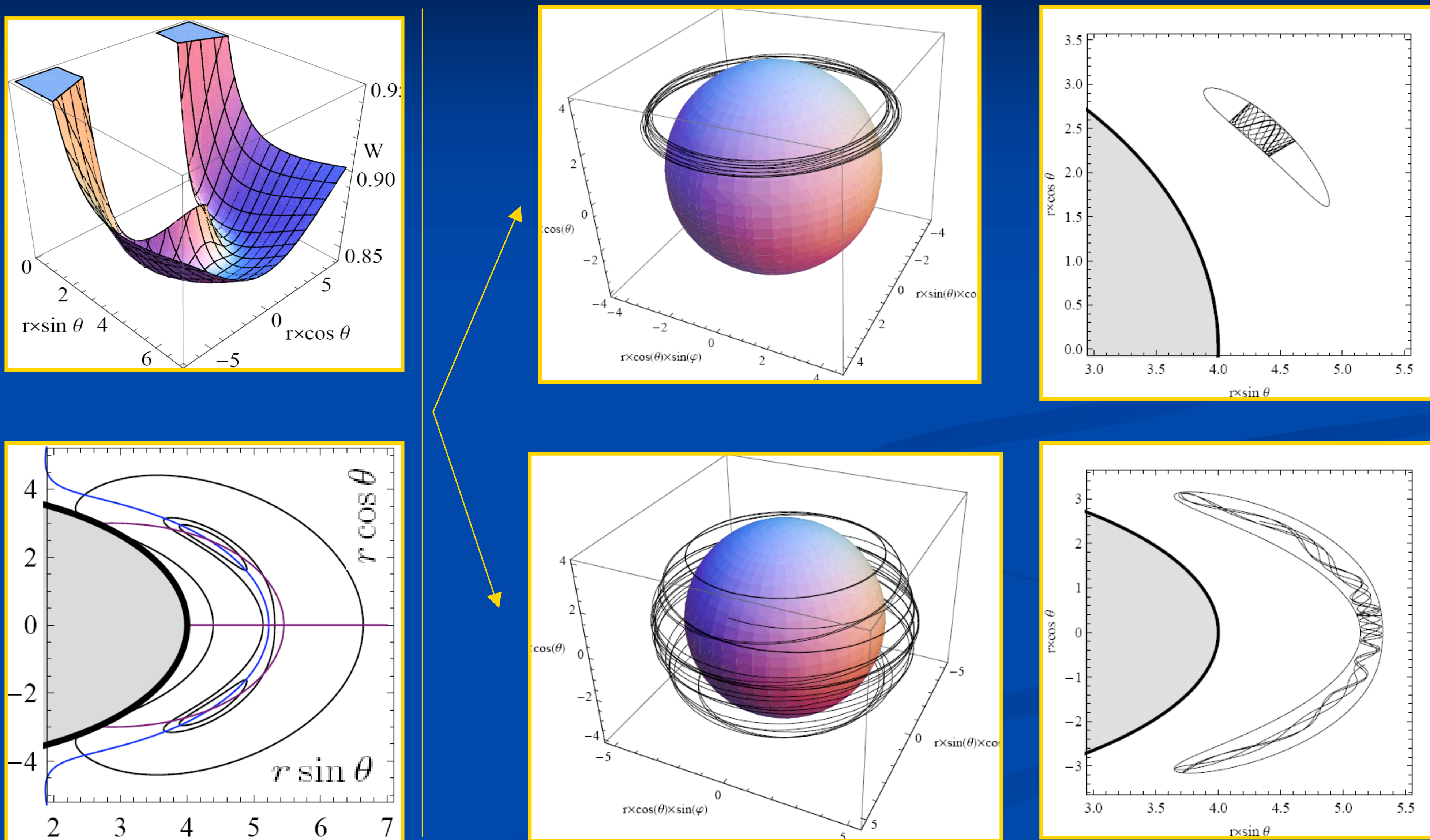


$$\tilde{E} = E/m, \; \tilde{L} = L/m, \; \tilde{q} = q/m$$

$$r/M \rightarrow r, \; t/M \rightarrow t$$

Existence of halo orbits

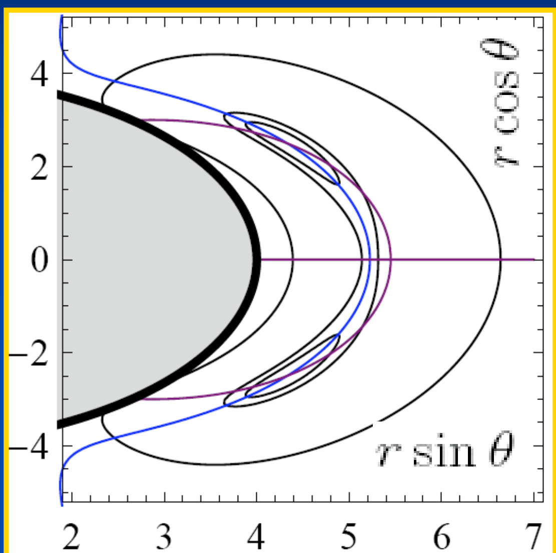
Neutron stars

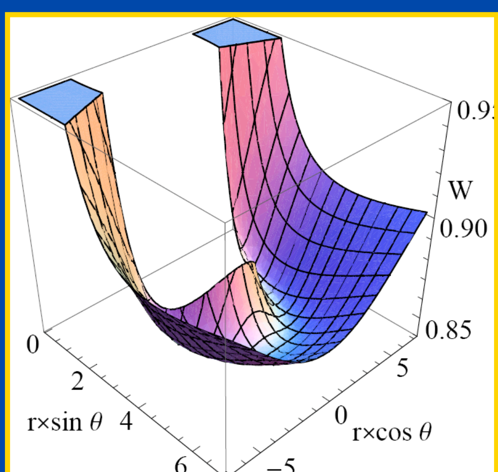


Existence of halo orbits

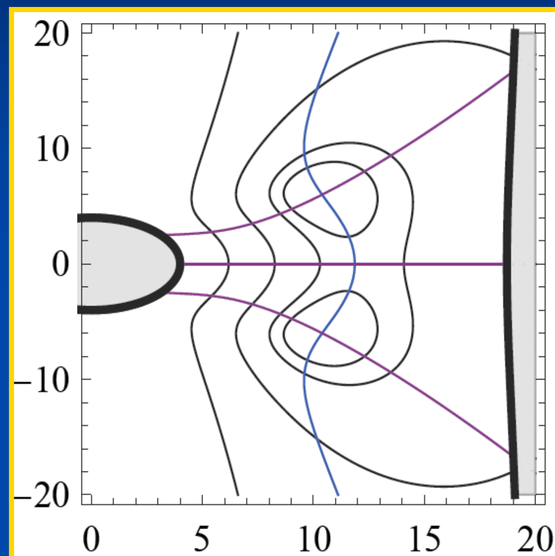
Neutron stars

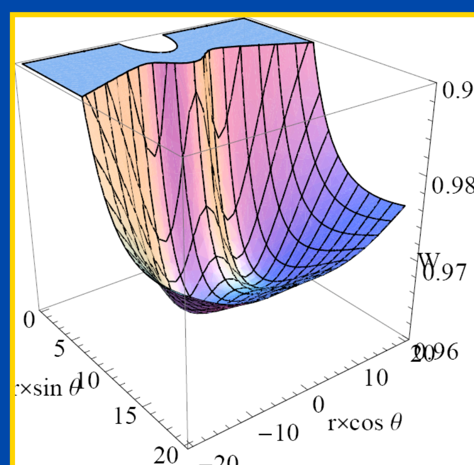
Co-rotating negative



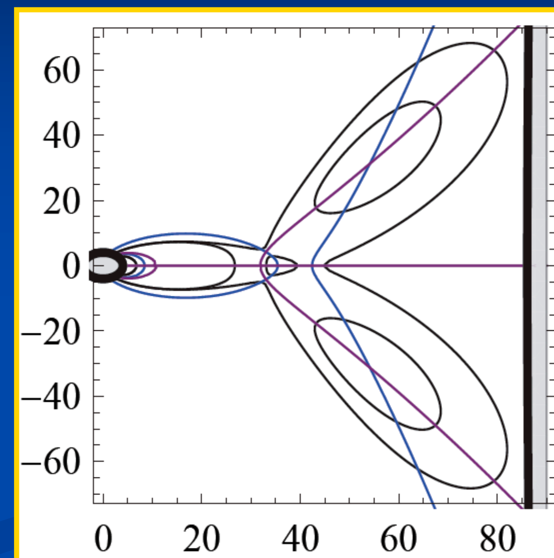


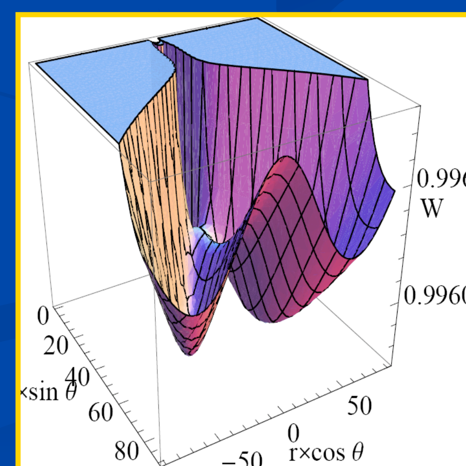
Counter-rotating positive





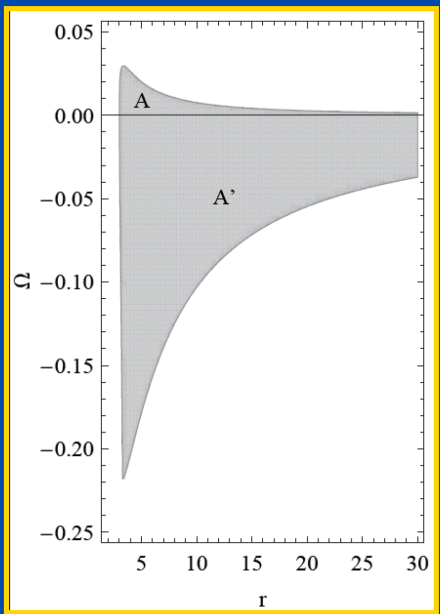
Co-rotating positive

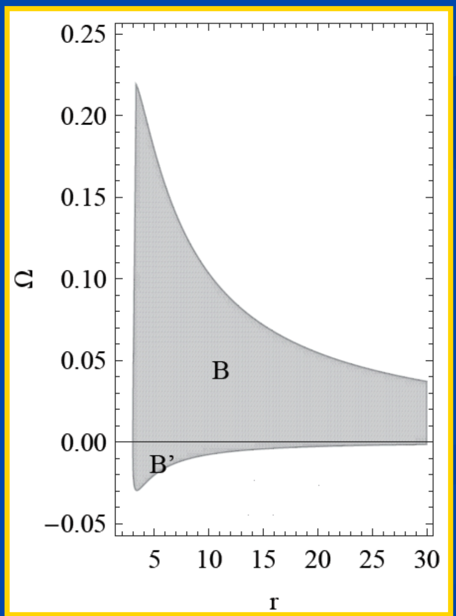


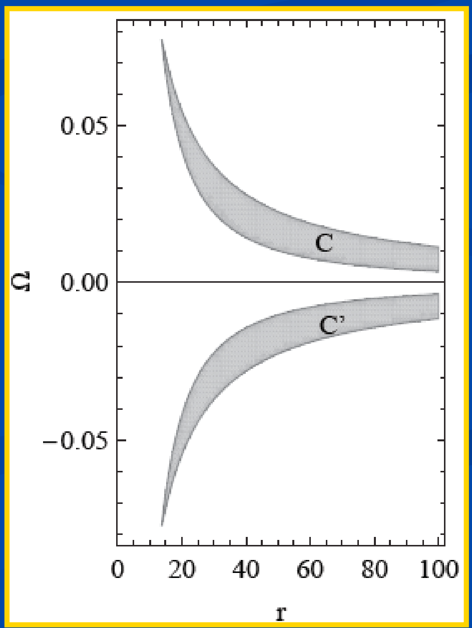


Existence of halo orbits

MF	Charge	Halo orbits
static	+ −	counter-rotating co-rotating
rotating	+ + −	counter-rotating co-rotating co-rotating

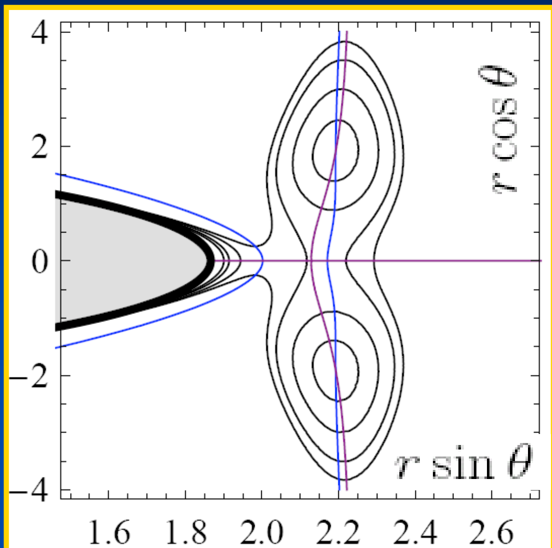


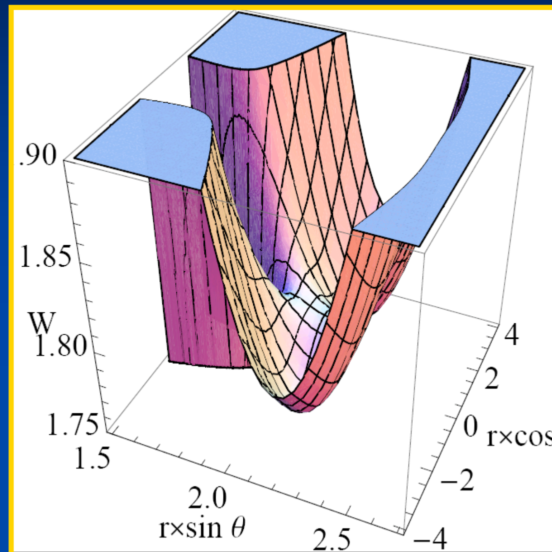




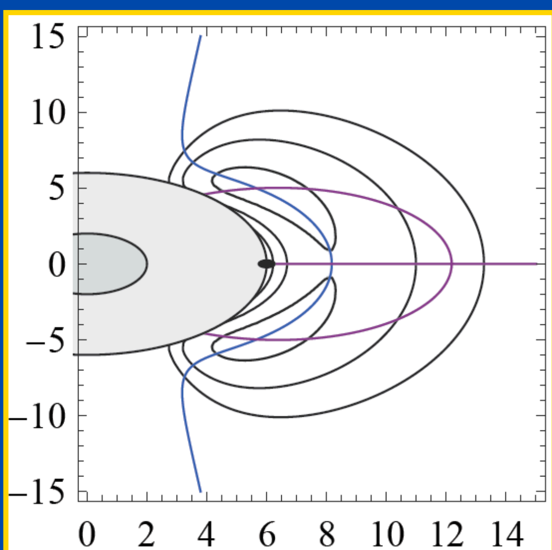
Existence of halo orbits

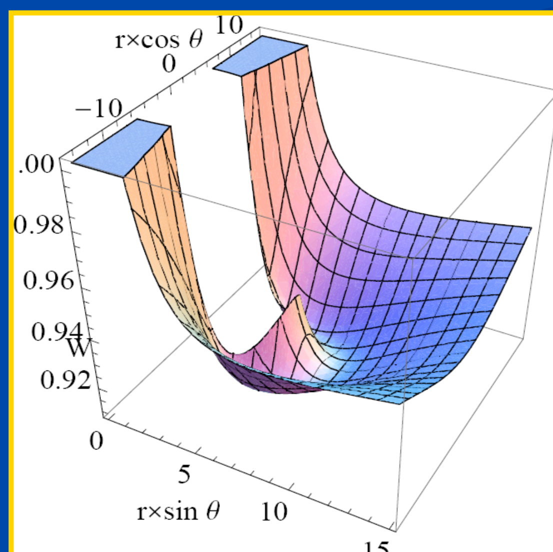
Kerr BH with magnetic fields





galactic uniform MF



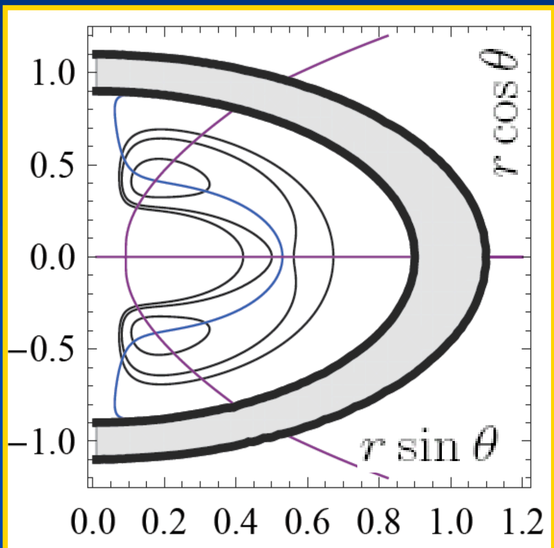


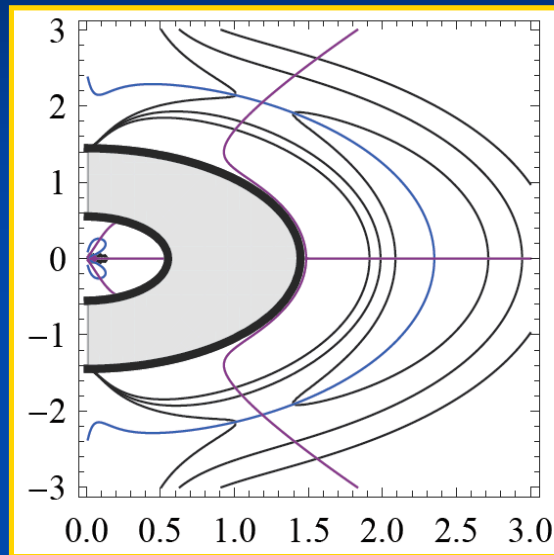
current ring

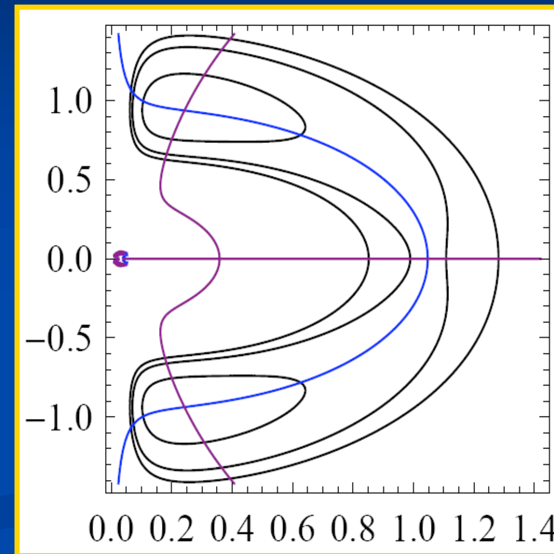
Existence of halo orbits

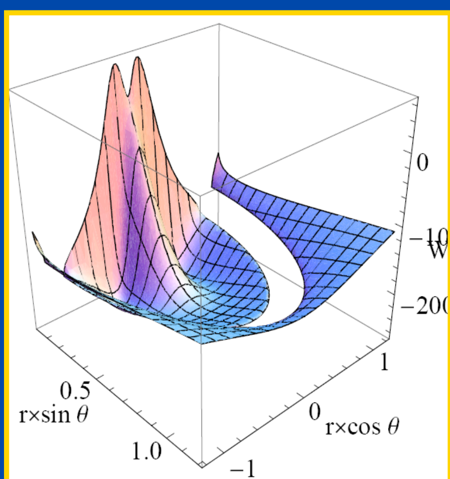
Kerr –Newmann BH and NS

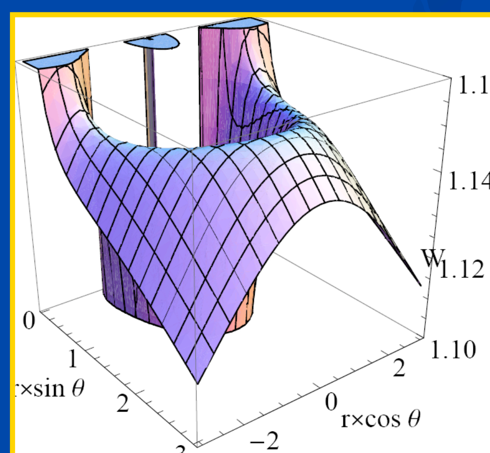
Black hole - inner	Black hole – outer	Naked singularity

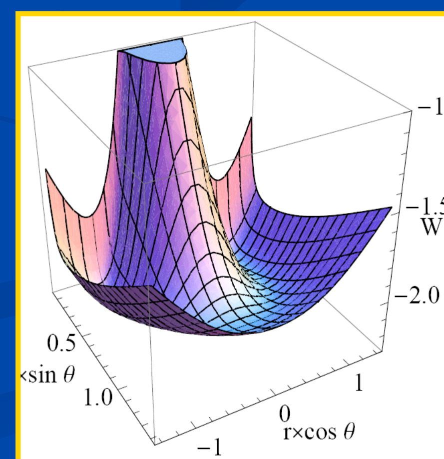












Related trajectories

- Equation of motion

$$\frac{d^2x^\alpha}{d\lambda^2} + \Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\lambda}\frac{dx^\gamma}{d\lambda} = qF^\alpha_\beta\frac{dx^\beta}{d\lambda}$$

$$\lambda = \tau/m$$

- Separability of equation and searching for constants of motion

$$m \qquad p_t + qA_t \equiv -E \qquad p_\phi + qA_\phi \equiv L \qquad K$$

- numerical integration

$$t = t(\tau), \quad \phi = \phi(\tau), \quad r = r(\tau), \quad \theta = \theta(\tau)$$

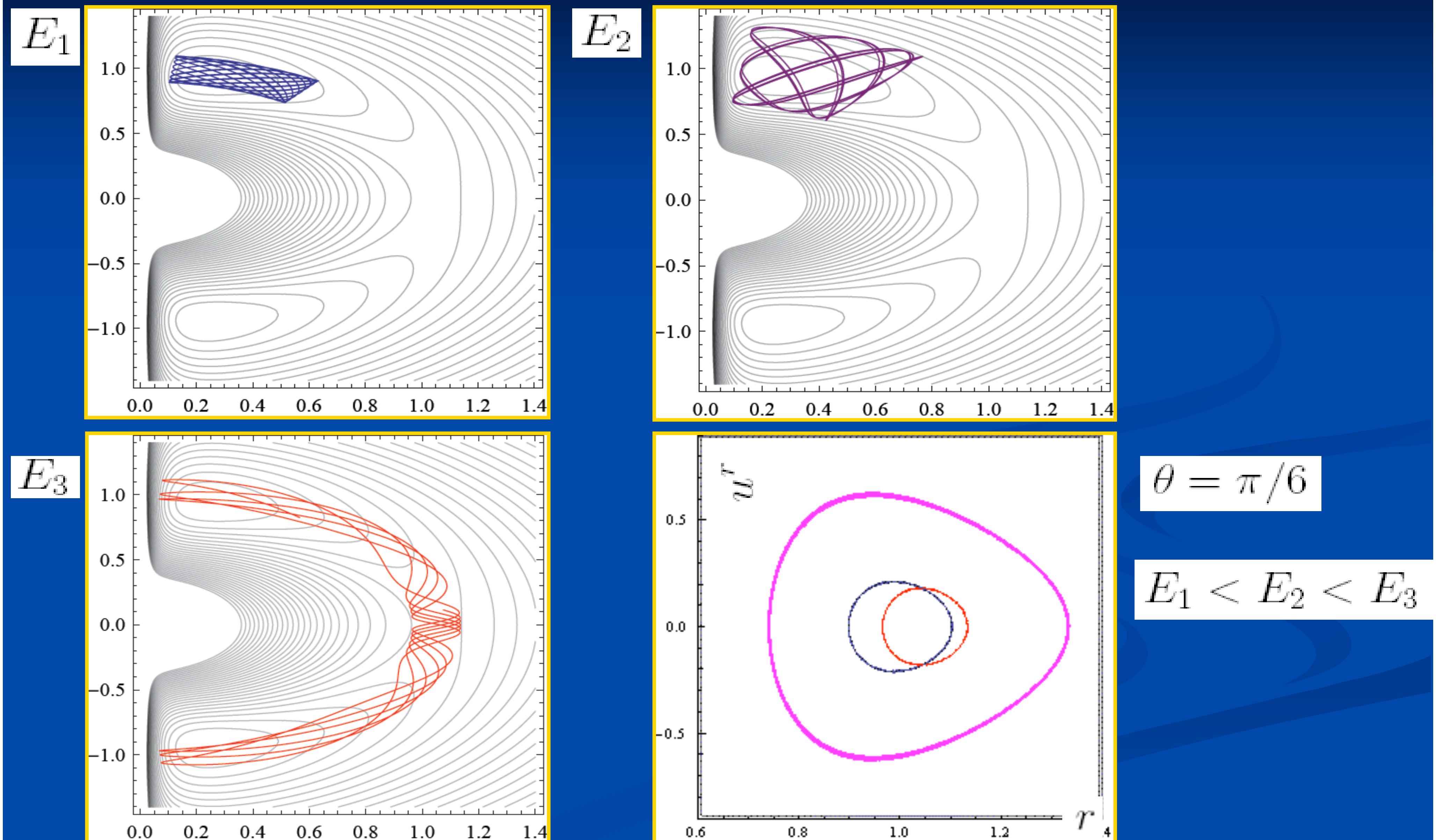
- Poincaré surfaces of section
  - surface of phase-space $\theta = \mathrm{const}$
  - cross sections of the trajectory with another two coordinates

$$r = r(\tau) \qquad u^r(\tau) = \frac{\mathrm{d}r(\tau)}{\mathrm{d}\tau}$$

- fuzzy structure $\Rightarrow$ chaotic motion $\Rightarrow$ no additional constant
- curve $\Rightarrow$ regular motion $\Rightarrow$ additional constant

Related trajectories

Kerr-Newmann BH and NS

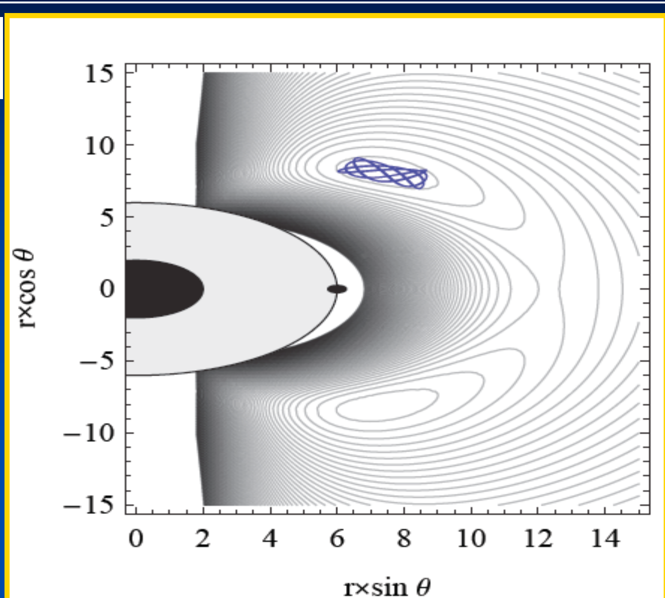


Related trajectories

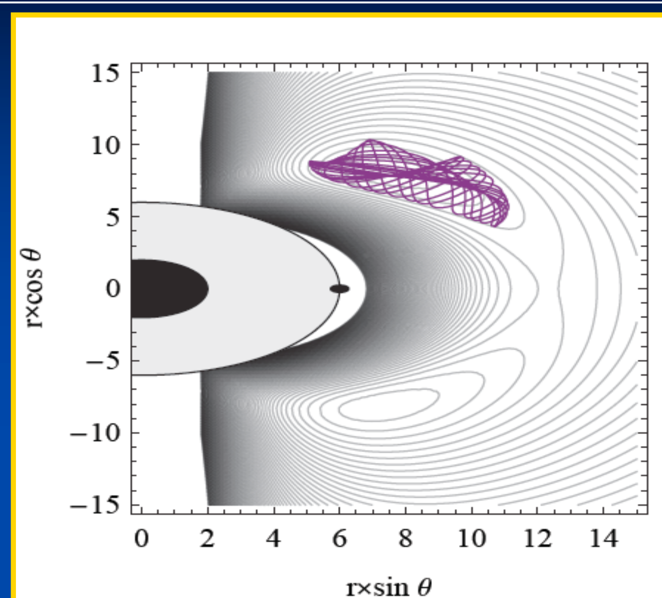
Kerr BH + dipole MF

Takahashi, Koyama ApJ,2009

E_1



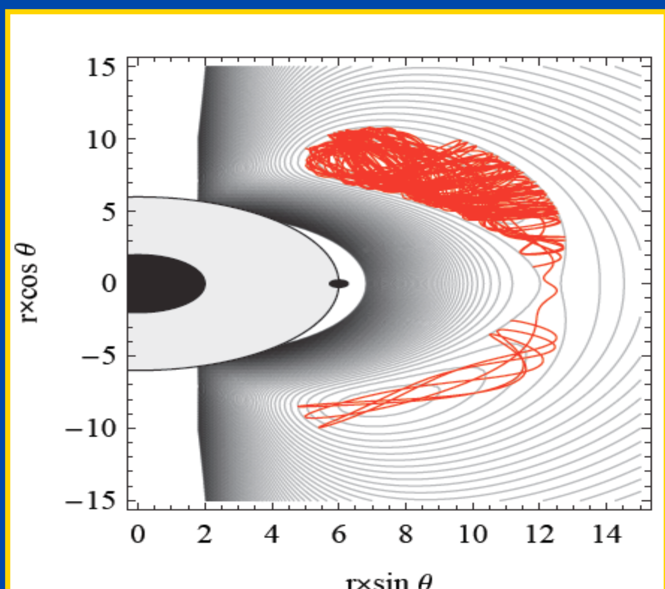
E_2

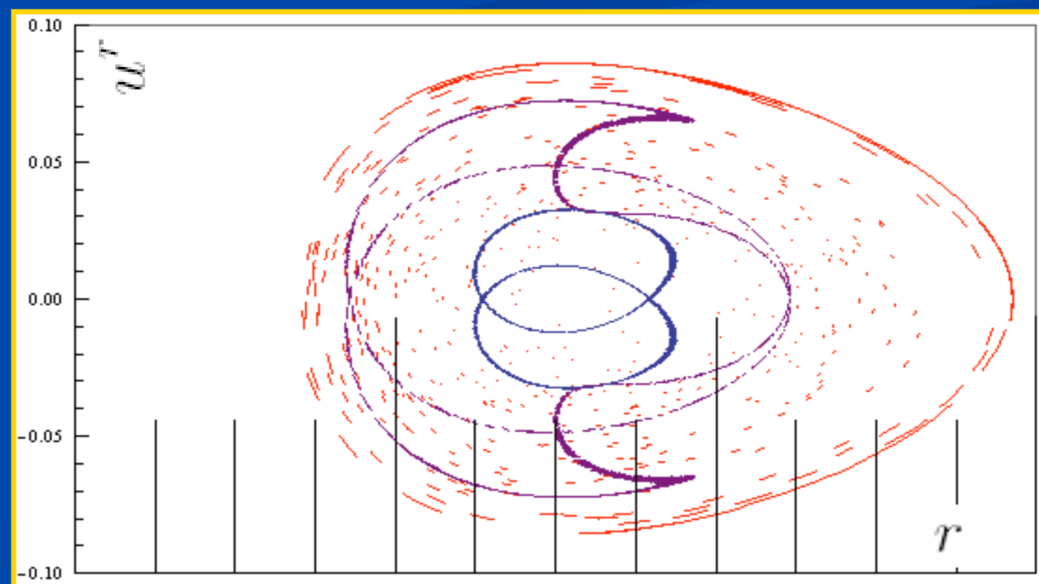


$E_1 < E_2 < E_3$

$\theta = \pi/6$

E_3





Related trajectories

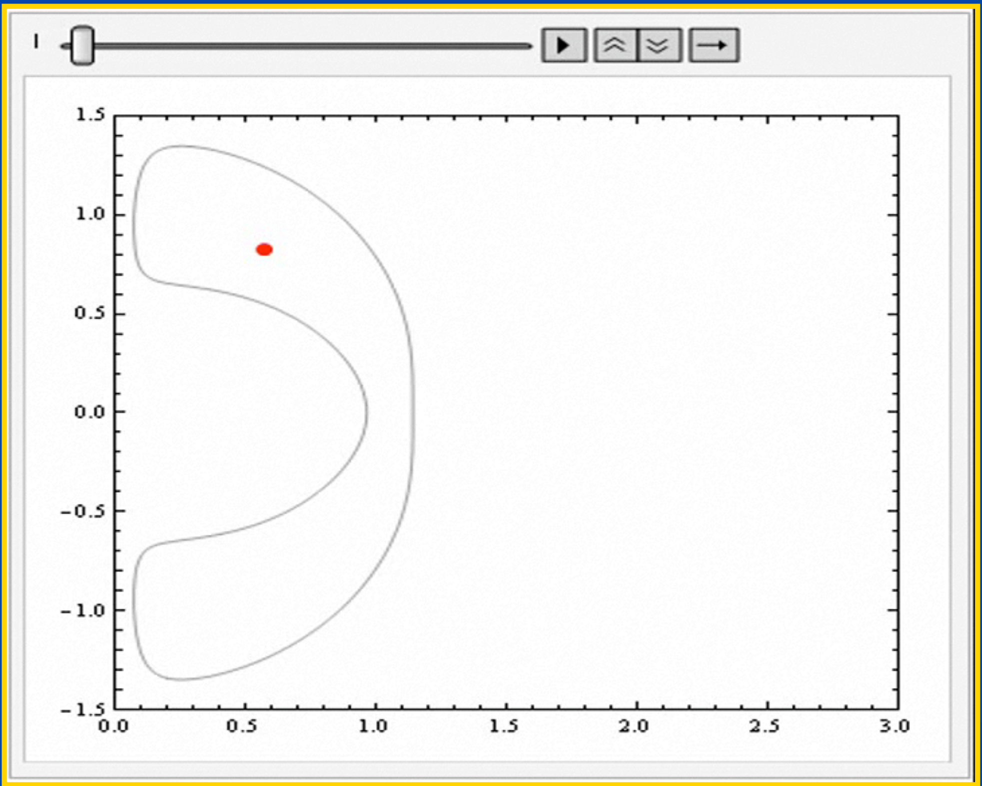
Charged particles motion

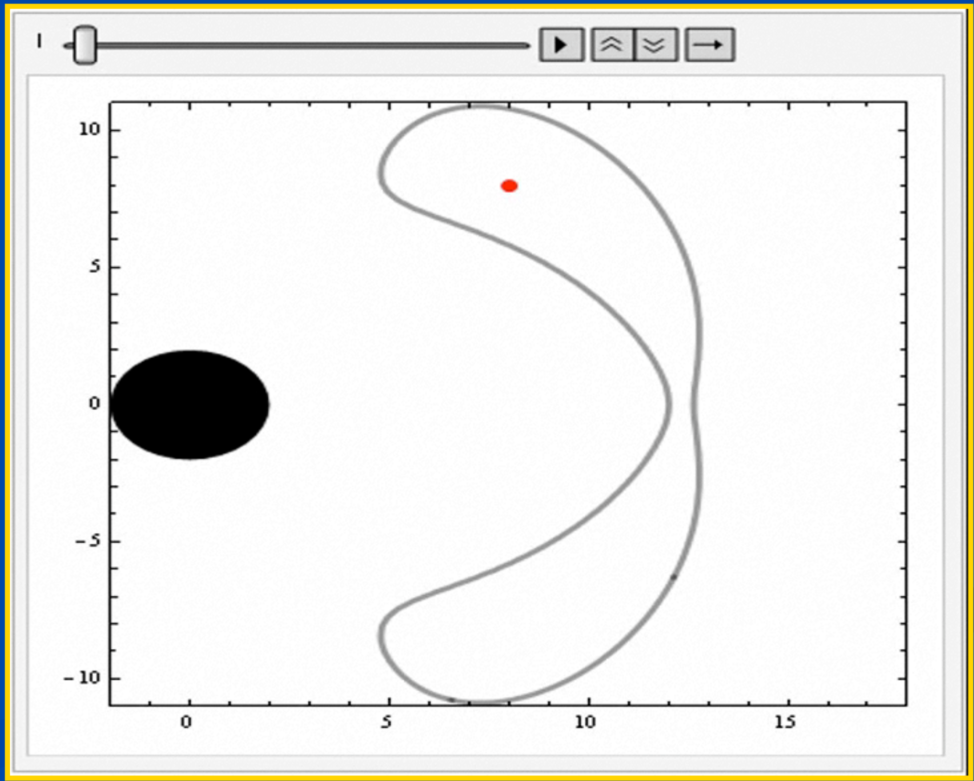
Regular motion

Kerr-Newmann NS

Chaotic motion

Kerr BH and dipole MF





Summary

1. We have proved the existence of stable halo (off-equatorial) orbits of charged particles near all of the investigated models of the compact objects.

2. Except the unique Kerr-Newmann case, the motion of particles along the halo orbits in the studied cases is chaotic, with the degree of chaoticness growing with the growing energy of particles, especially when both the halo lobes are joined in the equatorial plane or allow the inflow of particles into the BH

3. We expect halo clouds of charged particles to exist near compact objects, regardless the "rough" used models (the single test particle approximation and approximative background description).

Acknowledgement

Díky Vám za pozvání a pozornost