Subspace Approximated Matrices in Numerical Linear Algebra

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For $n \times n$ matrices A and A_0 and a sequence of subspaces $\{0\} = \mathcal{V}_0 \subset \cdots \mathcal{V}_n = \mathbb{R}^n$ with $\dim(\mathcal{V}_k) = k$, the k-th subspace approximated matrix A_k is defined as

$$A_k = A + \Pi_k(A_0 - A)\Pi_k,$$

where Π_k is the orthogonal projection on $\mathcal{V}_k^\perp$. As a consequence, both $A_k v = Av$ and $v^* A_k = v^* A$ for all $v \in \mathcal{V}_k$, and thus A_k gradually changes from A_0 into A. Moreover, in practice, $\mathcal{V}_{k+1}$ may depend on A_k, in order to enforce A_{k+1} to be closer to A in some sense. By choosing A_0 as a simple approximation of A, this turns the subspace approximated matrices into interesting preconditioners for linear algebra problems involving A.

In this presentation we will discuss the use of subspace approximated matrices in eigen value computations, and in solving linear systems of equations.