Some developments on Dirichlet problems with discontinuous coefficients
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Let Ω be a bounded, open subset of $I\!R^N$, $N > 2$ and $M : \Omega \times I\!R \to I\!R^{N^2}$, be a bounded, elliptic and measurable matrix. If we assume that $E(x)$ is a vector field and $f(x)$ is a function such that

$$f \in L^m(\Omega), \;\; 1 \le m < \frac{N}{2}, \quad E \in (L^N(\Omega))^N,$$

and we consider the following Dirichlet problem [1]

$$(1) \qquad \begin{cases} -\operatorname{div}(M(x)\nabla u) = -\operatorname{div}(u\,E(x)) + f(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

existence and summability properties (depending on m) of weak or distributional solutions are proved in [2].

In [3], equations with coefficients E which do not belong to $(L^N(\Omega))^N$ are considered. The most important aim is the study of the case $E \in (L^2(\Omega))^N$, where the main point is the definition of solution, since the distributional definition of solution does not work.

References

[1] L. Boccardo: Some developments on Dirichlet problems with discontinuous coefficients; Boll. UMI, to appear.

[2] L. Boccardo, preprint.

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[1] related to the mathematical analysis of some models of flows in porous media (T. Gallouet)