On Function $\mathfrak{F}$ and the Eigenvalue Problem for a Certain Class of Jacobi Matrices

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Abstract

We define a function $\mathfrak{F}$ with simple and nice algebraic properties on a subset of the space of complex sequences. Some special functions are expressible in terms of $\mathfrak{F}$, first of all the Bessel functions of the first kind. Function $\mathfrak{F}$ is also closely related to orthogonal polynomials, continued fractions, or it can be used as a tool for investigation of the spectrum of a certain class of tridiagonal operators. The latter is discussed in detail.

First, we show $\mathfrak{F}$ applied on a special type of sequence determines a complex function of one complex variable which zeros coincide with eigenvalues of a Jacobi matrix. Next, we construct a vector-valued function on the complex plain having the property that its values on spectral points of the Jacobi matrix are equal to corresponding eigenvectors. If time remains, we show how $\mathfrak{F}$ is related to the Weyl m-function or, more generally, to the Green function. At the end, we present several examples and applications of main results.