On Edge-Elements Reproducing the Discrete Kernel of the Curl Operator

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Time-harmonic Maxwell's equations

$$\mathbf{curl}\left(\mu_r^{-1}\,\mathrm{curl}\,\mathbf{E}\right) - \kappa^2 \epsilon_r \mathbf{E} = \mathbf{F} \quad \text{in } \Omega$$
$$\mathbf{E}\cdot\tau = 0 \quad \text{on } \partial\Omega$$

where

- $\Omega \subset \mathbb{R}^2$
- $\mathbf{curl} = (\partial/\partial x_2, -\partial/\partial x_1)^\top$
- $\mathrm{curl}\,\mathbf{E} = \partial E_2/\partial x_1 - \partial E_1/\partial x_2$
- $\tau = (-\nu_2, \nu_1)^\top$ positively oriented unit tangent vector
- $\mu_r = \mu_r(x) \in \mathbb{R}$ relative permeability
- $\epsilon_r = \epsilon_r(x) \in \mathbb{C}^{2\times 2}$ relative permittivity
- $\mathbf{E} = \mathbf{E}(x) \in \mathbb{C}^2$ phaser of the electric field intensity
- $\mathbf{F} = \mathbf{F}(x) \in \mathbb{C}^2$
- $\kappa \in \mathbb{R}$ the wave number

Weak formulation

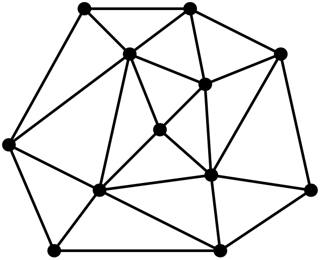
$$\mathbf{E} \in W : \quad a(\mathbf{E}, \mathbf{\Phi}) = \mathcal{F}(\mathbf{\Phi}) \quad \forall \mathbf{\Phi} \in W$$

- $W = \{\mathbf{E} \in \mathbf{H}(\mathrm{curl}, \Omega) : \mathbf{E} \cdot \tau = 0 \text{ on } \partial\Omega\}$
- $a(\mathbf{E}, \mathbf{\Phi}) = (\mu_{\mathrm{r}}^{-1} \,\mathrm{curl}\, \mathbf{E}, \mathrm{curl}\, \mathbf{\Phi}) - \kappa^2(\epsilon_{\mathrm{r}} \mathbf{E}, \mathbf{\Phi})$
- $\mathcal{F}(\mathbf{\Phi}) = (\mathbf{F}, \mathbf{\Phi}) = \int_\Omega \mathbf{F} \cdot \overline{\mathbf{\Phi}} \,\mathrm{d}x$

Lowest-order edge elements

$$\mathbf{E}_h \in W_h : \quad a(\mathbf{E}_h, \mathbf{\Phi}_h) = \mathcal{F}(\mathbf{\Phi}_h) \quad \forall \mathbf{\Phi}_h \in W_h$$

- $W_h = \{\mathbf{E}_h \in W : \mathbf{E}_h|_K \in [P^1(K)]^2 \ \forall K \in \mathcal{T}_h$
  and $\mathbf{E}_h|_K \cdot \tau_e = \mathbf{E}_h|_{K^*} \cdot \tau_e = \text{const.} \ \forall e \in \mathcal{E}, \ e = K \cap K^*\}$
- $\mathcal{T}_h$ ... triangulation
- τ_e ... fixed tangent to $e \in \mathcal{E}$
- $\mathcal{E}$ ... edges in $\mathcal{T}_h$
- $\mathcal{E} = \mathcal{E}_\mathrm{b} \cup \mathcal{E}_\mathrm{i}$
- $\mathcal{V}$ ... vertices in $\mathcal{T}_h$
- $\mathcal{V} = \mathcal{V}_\mathrm{b} \cup \mathcal{V}_\mathrm{i}$
- $N_v = N_{bv} + N_{iv}$
- $N_e = N_{be} + N_{ie}$



- Whitney functions: $\psi_e \in W_h$ for all $e \in \mathcal{E}_\mathrm{i}$
  $\psi_e \cdot \tau_e = 1/|e|$ on $e \in \mathcal{E}_\mathrm{i}$
  $\psi_e \cdot \tau_{e^*} = 0$ on $e^* \neq e, \ e^* \in \mathcal{E}$

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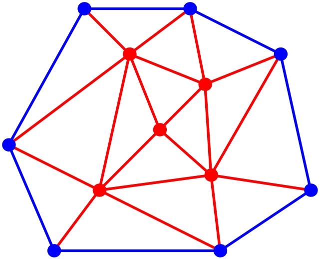

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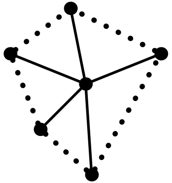
Courant FE



- $V_h = \{\varphi_h \in H_0^1(\Omega) : \varphi_h|_K \in P^1(K)\ \forall K \in \mathcal{T}_h\}$
- Courant functions: $\varphi_i \in V_h$ for all $B_i \in \mathcal{V}_\mathrm{i}$
  $\varphi_i(x) = 1$ at $x = B_i$, $B_i \in \mathcal{V}_\mathrm{i}$
  $\varphi_i(x) = 0$ at $x = B_j$, $B_j \neq B_i$, $B_j \in \mathcal{V}$
- $\nabla\varphi_i \in W_h$
- $\nabla\varphi_i \cdot \tau_e = \dfrac{\sigma_{ie}}{|e|}$, where $\sigma_{ie} = \begin{cases} +1 & \tau_e \text{ aims towards } B_i \\ -1 & \text{otherwise} \end{cases}$
- $\nabla\varphi_i = \displaystyle\sum_{e\in\omega(B_i)} \sigma_{ie}\psi_e$ … unique way
  - $e^* \in \omega(B_i)$:
    $$\nabla\varphi_i \cdot \tau_{e^*} = \frac{\sigma_{ie^*}}{|e^*|} = \sigma_{ie^*}\psi_{e^*} \cdot \tau_{e^*} = \sum_{e\in\omega(B_i)} \sigma_{ie}\psi_e \cdot \tau_{e^*} \quad \text{on } e^*$$
  - $e^* \notin \omega(B_i)$: $\displaystyle \nabla\varphi_i \cdot \tau_{e^*} = 0 = \sum_{e\in\omega(B_i)} \sigma_{ie}\psi_e \cdot \tau_{e^*} \quad \text{on } e^*$

De Rham diagram

$$
\begin{array}{ccccccccccc}
\mathbb{R} & \xrightarrow{\text{id}} & H^1 & \xrightarrow{\nabla} & \mathbf{H}(\text{curl}) & \xrightarrow{\text{curl}} & \mathbf{H}(\text{div}) & \xrightarrow{\text{div}} & L^2 & \xrightarrow{0} & 0 \\
 & & \downarrow & & \downarrow & & & & & & \\
 & & V_h & \xrightarrow{\nabla} & W_h & \xrightarrow{\text{curl}} & & & & &
\end{array}
$$

- $\mathbb{R} = \ker(\nabla)$
- $\mathcal{R}(\nabla) = \ker(\text{curl})$
- $\mathcal{R}(\text{curl}) = \ker(\text{div})$
- $\mathcal{R}(\text{div}) = L^2$

$$\text{curl}\,\boldsymbol{\psi}_e \neq 0 \quad \forall e \in \mathcal{E}_{\mathrm{i}}$$

Discrete kernel of curl

Theorem

$$S_\varphi = \operatorname{span}\{\nabla\varphi_i,\ B_i \in \mathcal{V}_{\mathrm{i}}\} = \{\boldsymbol{\psi} \in W_h : \operatorname{curl}\boldsymbol{\psi} = 0\} = \mathcal{K}_h$$

Proof.

(a) $S_\varphi \subset \mathcal{K}_h \quad \Leftarrow \quad \operatorname{curl}\nabla\varphi = 0$

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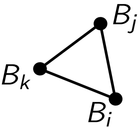

Proof.

(b) $S_\varphi \supset \mathcal{K}_h$

- $\boldsymbol{\psi} \in \mathcal{K}_h$, $\mathrm{curl}\,\boldsymbol{\psi} = 0$
- $e \in \mathcal{E}$, $e = \overline{B_iB_j}$, $\alpha_j = \alpha(B_j)$ given by $\alpha_j - \alpha_i = \int_e \boldsymbol{\psi} \cdot \tau_{ij}$
- $\overline{B_iB_jB_k} = K \in \mathcal{T}_h \quad \Rightarrow \quad \widetilde{\alpha}_i - \alpha_i = \widetilde{\alpha}_i \mp \alpha_k \mp \alpha_j - \alpha_i$
$$= \int_{\partial K} \boldsymbol{\psi} \cdot \tau_{\partial K} = \int_K \mathrm{curl}\,\boldsymbol{\psi} = 0$$
- Define $\Phi \in V_h : \ \Phi(B_i) = \alpha_i \ \forall B_i \in \mathcal{V} \quad \Rightarrow \quad \nabla\Phi \in W_h$
- $\nabla\Phi|_e \cdot \tau_e \,|e| = \int_e \nabla\Phi \cdot \tau_e = \int_{B_i}^{B_j} \Phi' = \Phi(B_j) - \Phi(B_i) =$
$$= \int_e \boldsymbol{\psi} \cdot \tau_e = \boldsymbol{\psi}|_e \cdot \tau_e \,|e| \quad \Rightarrow \quad \boldsymbol{\psi} = \nabla\Phi$$

□

Discrete kernel of curl

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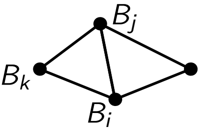

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□

Basis reproducing the discrete kernel of curl

- $\mathcal{B}_W = \{\psi_e : e \in \mathcal{E}_{\rm i}\}$ ... basis in W_h, $\dim W_h = \#\mathcal{E}_{\rm i} = N_{ie}$
- $\mathcal{B}_\mathcal{K} = \{\nabla\varphi_i : B_i \in \mathcal{V}_{\rm i}\}$ ... basis in $\mathcal{K}_h$, $\dim \mathcal{K}_h = \#\mathcal{V}_{\rm i} = N_{iv}$
- $N_{iv} \leq N_{ie}$

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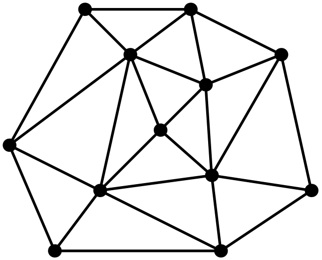
- $N_{iv} \le N_{ie}$
  - Euler's formula: $N_t - N_e + N_v = 1$
  - $3N_t = 2N_{ie} + N_{be}$
  - $N_{be} = N_{bv}$
  - $N_{be} \ge 3$
  - $\Rightarrow \quad 3N_{iv} = 3 + N_{ie} - N_{be} \le N_{ie} \le 3N_{ie}$



$N_t = 15$
$N_e = 26$
$N_v = 12$

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- $N_{iv} \leq N_{ie}$
- $\mathcal{E}_\mathrm{i} = \mathcal{E}_\mathrm{rem} \cup \mathcal{E}_\mathrm{keep} \quad \#\mathcal{E}_\mathrm{rem} = N_{iv} \quad \#\mathcal{E}_\mathrm{keep} = N_{ie} - N_{iv}$
- Is $\mathcal{B} = \mathcal{B}_\mathcal{K} \cup \{\psi_e : e \in \mathcal{E}_\mathrm{keep}\}$ a basis of W_h ?

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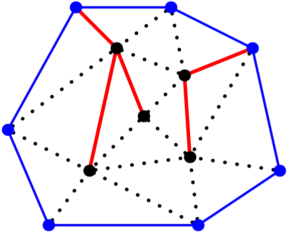
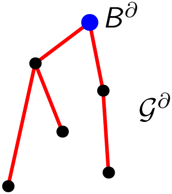
Theorem

$\mathcal{B}$ is a basis of W_h $\Leftrightarrow$ *the only cycle in $(\mathcal{V}, \mathcal{E}_\mathrm{b} \cup \mathcal{E}_\mathrm{rem})$ is $(\mathcal{V}_\mathrm{b}, \mathcal{E}_\mathrm{b})$*

$\Leftrightarrow$ *$\mathcal{G}^\partial = (\mathcal{V}_\mathrm{i} \cup \{B^\partial\}, \mathcal{E}^\partial_\mathrm{rem})$ is a spannig tree*





Basis reproducing the discrete kernel of curl

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- $N_{iv} \leq N_{ie}$
- $\mathcal{E}_\mathrm{i} = \mathcal{E}_\mathrm{rem} \cup \mathcal{E}_\mathrm{keep}$ $\quad \#\mathcal{E}_\mathrm{rem} = N_{iv}$ $\quad \#\mathcal{E}_\mathrm{keep} = N_{ie} - N_{iv}$
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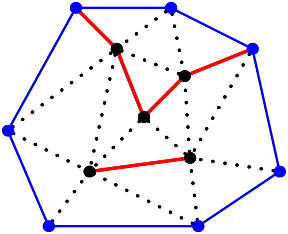
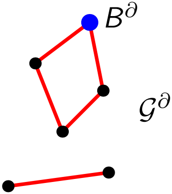




Proof

"$\Rightarrow$" : by contradiction:
$\mathcal{B}$ a basis in W_h and $\mathcal{G}^\partial$ not a spanning tree

- $\mathcal{G}^\partial$ has an isolated component $(\mathcal{V}_{\rm isol}, \mathcal{E}_{\rm isol})$:
$$\mathcal{V}_{\rm isol} \subset \mathcal{V}_{\rm i},\ \mathcal{E}_{\rm isol} \subset \mathcal{E}_{\rm rem}$$
- $\varphi = \sum_{B_i \in \mathcal{V}_{\rm isol}} \varphi_i$
- $\nabla\varphi = \sum_{B_i \in \mathcal{V}_{\rm isol}} \nabla\varphi_i = \sum_{B_i \in \mathcal{V}_{\rm isol}} \sum_{e \in \omega(B_i)} \sigma_{ie}\psi_e = \sum_{e \in \omega^0(\mathcal{V}_{\rm isol})} \sigma_{ie}\psi_e$
  - $\omega^0(\mathcal{V}_{\rm isol}) = \left\{ e = \overline{B_i B_j} : B_i \in \mathcal{V}_{\rm isol},\ \ B_j \notin \mathcal{V}_{\rm isol} \right\}$
  - $e = \overline{B_i B_j} \in \mathcal{E}_{\rm isol}, \qquad B_i \in \mathcal{V}_{\rm isol},\ \ B_j \in \mathcal{V}_{\rm isol}$
  - $\nabla\varphi = \cdots + \underbrace{\sigma_{ie}\psi_e + \sigma_{je}\psi_e}_{=0} + \cdots$
- $(\mathcal{V}_{\rm isol}, \mathcal{E}_{\rm isol})$ isolated
$e \in \omega^0(\mathcal{V}_{\rm isol}) \quad \Rightarrow \quad e \notin \mathcal{E}_{\rm isol} \quad \Rightarrow \quad e \notin \mathcal{E}_{\rm rem} \quad \Rightarrow \quad \psi_e \in \mathcal{B}$

Proof

"$\Leftarrow$" : by contradiction:

$\mathcal{G}^\partial$ spanning tree and $\mathcal{B}$ not a basis in W_h

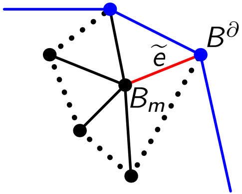
- $\nabla\varphi = \sum_{B_i \in \mathcal{V}_\mathrm{i}} c_i \nabla\varphi_i = \sum_{e \in \mathcal{E}_\mathrm{keep}} d_e \boldsymbol{\psi}_e = \boldsymbol{\psi}$ $\quad \exists B_k \in \mathcal{V}_\mathrm{i} : c_k \neq 0$, $\quad \exists e_k \in \mathcal{E}_\mathrm{keep} : d_{e_k} \neq 0$
- $e^* \in \mathcal{E}_\mathrm{rem}, \quad e^* = \overline{B_i B_j}, \quad B_i, B_j \in \mathcal{V}_\mathrm{i}$

  $\nabla\varphi \cdot \tau_{e^*}\big|_{e^*} = \sigma_{ie^*} \dfrac{c_i - c_j}{|e^*|} = \sigma_{je^*} \dfrac{c_j - c_i}{|e^*|} = 0$
  - $\boldsymbol{\psi}_e \cdot \tau_{e^*} = 0 \quad \forall e \in \mathcal{E}_\mathrm{keep}$
- $c_i = c_j = c^* \quad \forall B_i, B_j \in \mathcal{V}_\mathrm{i}$
- $\mathcal{G}^\partial$ is a spanning tree

  $\exists\, \widetilde{e} \in \mathcal{E}_\mathrm{rem},\ \widetilde{e} = \overline{B_m B^\partial}$: $B_m \in \mathcal{V}_\mathrm{i},\ B^\partial \in \mathcal{V}_\mathrm{b}$

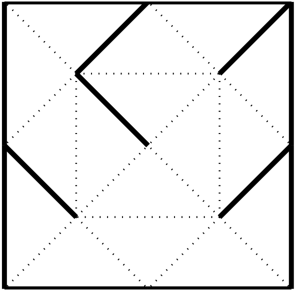
  $0 = \boldsymbol{\psi} \cdot \tau_{\widetilde{e}}\big|_{\widetilde{e}} = \nabla\varphi \cdot \tau_{\widetilde{e}}\big|_{\widetilde{e}} = \sigma_{m\widetilde{e}} \dfrac{c_m}{|\widetilde{e}|}$

  $\Rightarrow \quad c_m = 0$
- $c_i = c^* = 0 \quad \forall B_i \in \mathcal{V}_\mathrm{i}$

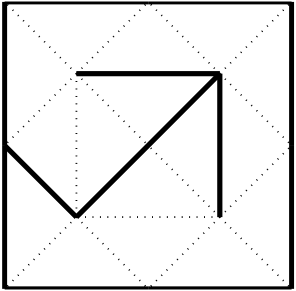


What choice of $\mathcal{E}_{\rm rem}$ is the best?

- $\mu_{\rm r} = \epsilon_{\rm r} = I, \quad \kappa = 1$
- $a(\mathbf{E}, \mathbf{\Phi}) = (\operatorname{curl} \mathbf{E}, \operatorname{curl} \mathbf{\Phi}) - (\mathbf{E}, \mathbf{\Phi})$
- $S_{ij} = a(\mathbf{\Phi}_i, \mathbf{\Phi}_j) \quad \mathbf{\Phi}_i, \mathbf{\Phi}_j \in \mathcal{B} \qquad \mathbf{\Phi}_i = \begin{cases} \nabla \varphi_k & B_k \in \mathcal{V}_{\rm i} \\ \boldsymbol{\psi}_e & e \in \mathcal{E}_{\rm keep} \end{cases}$
- $\operatorname{cond}(S)$



min cond(S)



max cond(S)

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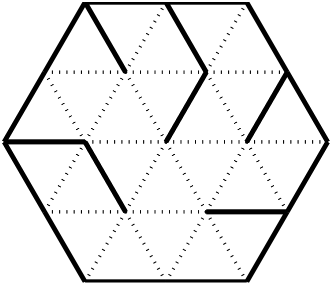
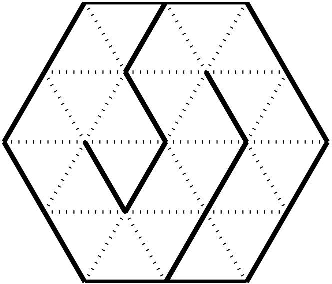


min cond(S)



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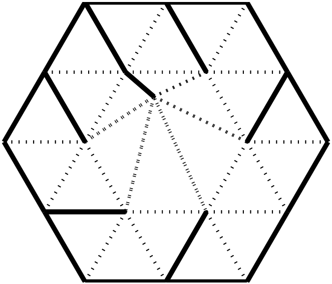
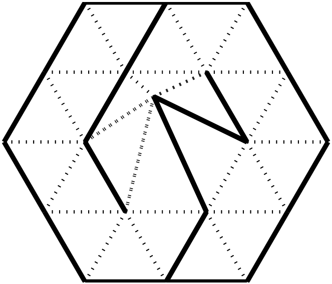


min cond(S)



max cond(S)

Linear algebraic solver

$$S = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & 0 \end{pmatrix} - \kappa^2 \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

Thank you for your attention

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