Neutrino Phenomenology

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Part 1

What Are Neutrinos Good For?

Energy generation in the sun starts with the reaction —

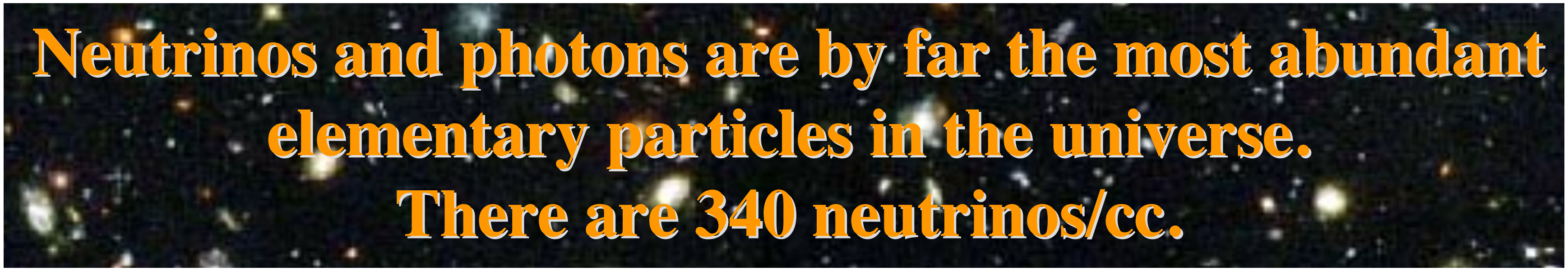
$$p + p \rightarrow d + e^{+} + \nu$$

Spin: $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$

Without the neutrino, angular momentum would not be conserved.

Uh, oh ……

The Neutrinos



The neutrinos are spin – 1/2, electrically neutral, leptons.

The only known forces they experience are the weak force and gravity.

This means that their interactions with other matter have very low strength.
Thus, neutrinos are difficult to detect and study.

Their weak interactions are successfully described by the Standard Model.

The Neutrino Revolution
(1998 – …)

Neutrinos have nonzero masses!

Leptons mix!

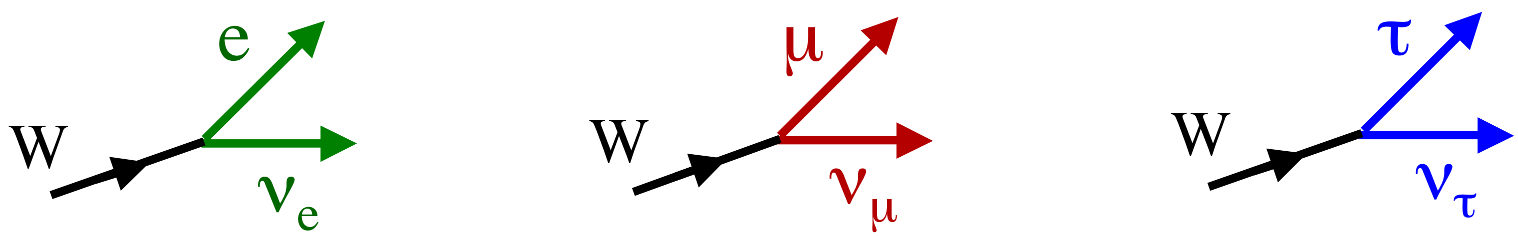
These discoveries come from the observation of

neutrino flavor change (neutrino oscillation).

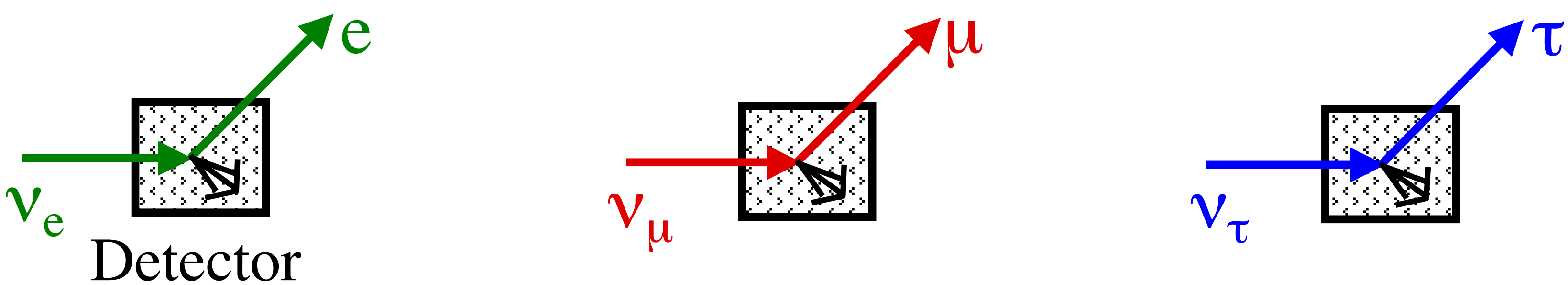
The Physics of Neutrino Oscillation — Preliminaries

The Neutrino Flavors

We ***define*** the three known flavors of neutrinos, ν_e, ν_μ, ν_τ, by W boson decays:



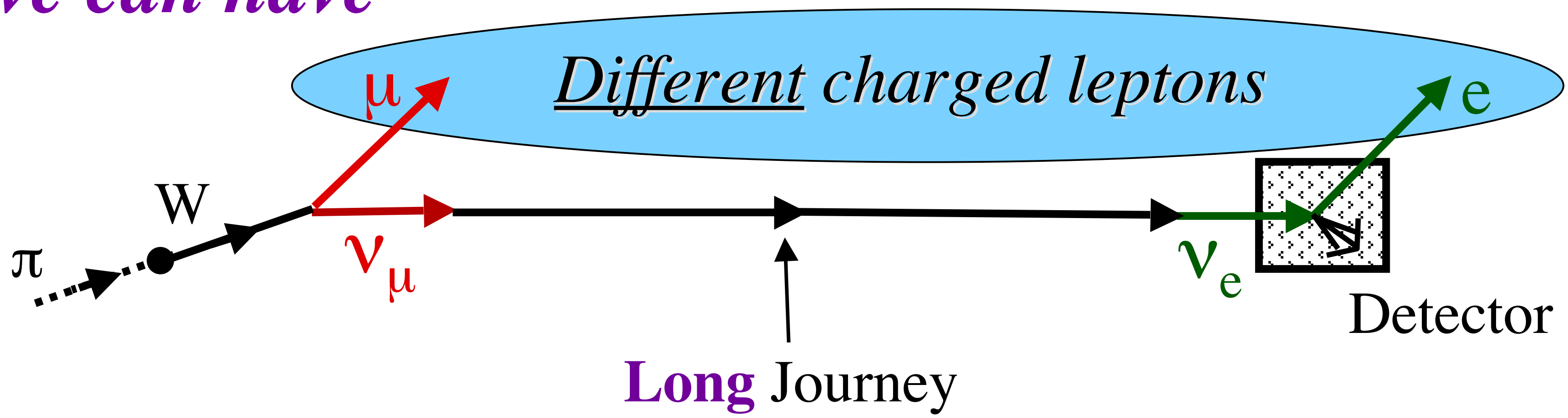
As far as we know, when interacting, a neutrino of given flavor creates only the charged lepton of the same flavor:



With $\alpha = e, \mu, \tau$, ν_α makes only ℓ_α ($\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$).

Neutrino Flavor Change

If neutrinos have masses, and leptons mix, we can have —



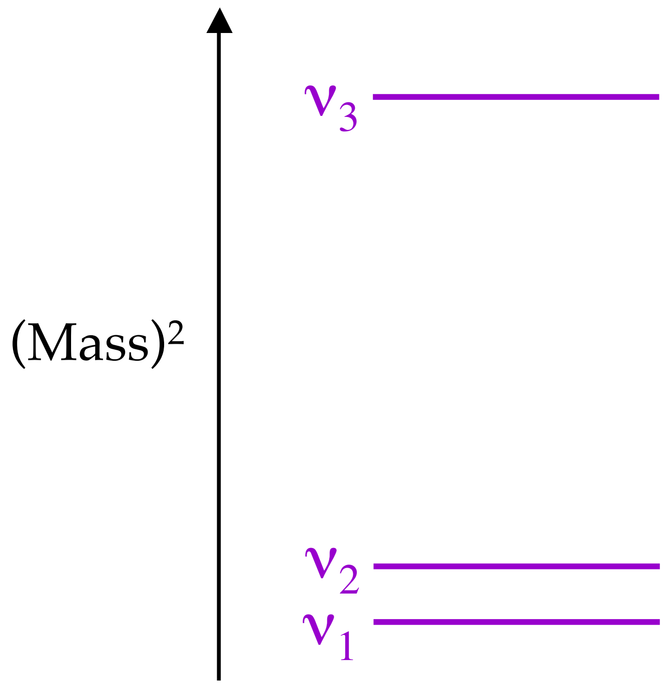
Given time (travel distance), a ν can change its flavor.

$$\nu_\mu \longrightarrow \nu_e$$

The last 15 years have brought us compelling evidence that such flavor changes actually occur.

Flavor Change Requires ***Neutrino Masses***

There must be some spectrum of neutrino mass eigenstates ν_i:



Mass (ν_i) $\equiv$ m_i

Flavor Change Requires ***Leptonic Mixing***

The neutrinos $\nu_{e,\mu,\tau}$ of definite flavor

$(W \rightarrow e\nu_e$ or $\mu\nu_\mu$ or $\tau\nu_\tau)$

must be superpositions of the mass eigenstates:

$$|\nu_\alpha> = \sum_i U^*_{\alpha i} \, |\nu_i> \quad .$$

Neutrino of flavor
$\alpha = e, \mu,$ or τ

PMNS Leptonic Mixing Matrix

Neutrino of definite mass m_i

There must be ***at least 3*** mass eigenstates ν_i,
because there are 3 orthogonal neutrinos
of definite flavor ν_α.

This *mixing* is easily incorporated into the Standard Model (SM) description of the $\ell\nu W$ interaction.

For this interaction, we then have —

Semi-weak coupling

Left-handed

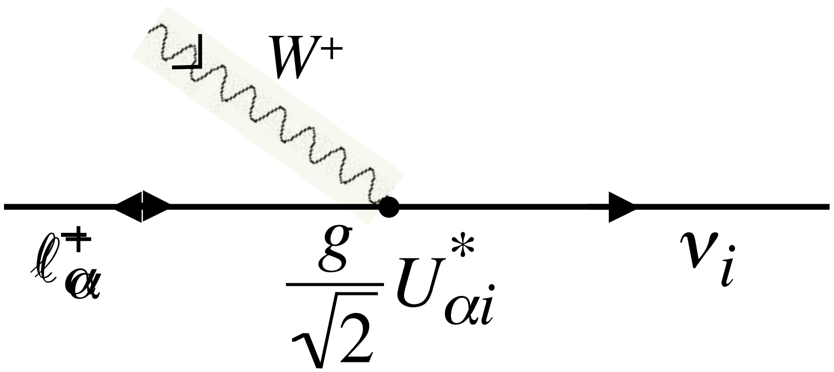
$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} \nu_{L\alpha} W_{\lambda}^{-} + \bar{\nu}_{L\alpha} \gamma^{\lambda} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

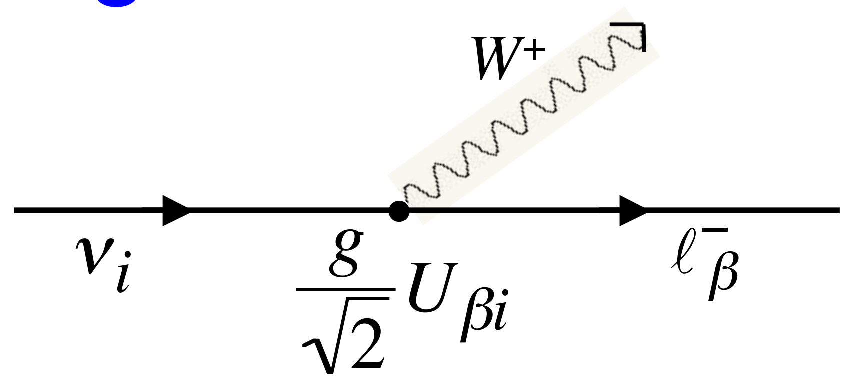
$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \bar{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

Taking mixing into account

We can use this form of the SM $\ell\nu W$ interaction to derive the probability for neutrino oscillation.

The Meaning of *U*





$$U = \begin{array}{c c} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \end{array}$$

The e row of U: The linear combination of neutrino mass eigenstates that couples to e.

The ν_1 column of U: The linear combination of charged-lepton mass eigenstates that couples to ν_1 .

In some models, such as the See-Saw model, if there are no new leptons such as sterile neutrinos, then *U* is *unitary*, or at least nearly so.

Then —

$$\mathcal{A}mp\left(W \to \ell_\alpha^+ + \nu \,;\; \nu \to \ell_\beta^- + W\right) \propto \sum_{i=1}^{3} U_{\alpha i}^* U_{\beta i} = \delta_{\beta\alpha}\text{, as observed.}$$

However, note that there may be additional leptons, and the 3 x 3 mixing matrix may ***not*** be unitary.

We will assume that ***U*** is unitary unless otherwise stated.

Slides on

The Physics of Neutrino Oscillation

go here.

Evidence For Flavor Change

Neutrinos	Evidence of Flavor Change
Solar	Compelling
Reactor (Long-Baseline)	Compelling
Atmospheric	Compelling
Accelerator (Long-Baseline)	Compelling
Accelerator & Reactor (Short-Baseline)	“Interesting”

KamLAND Evidence for Oscillatory Behavior

The KamLAND detector studies $\overline{\nu}_e$ produced by Japanese nuclear power reactors ~ 180 km away.

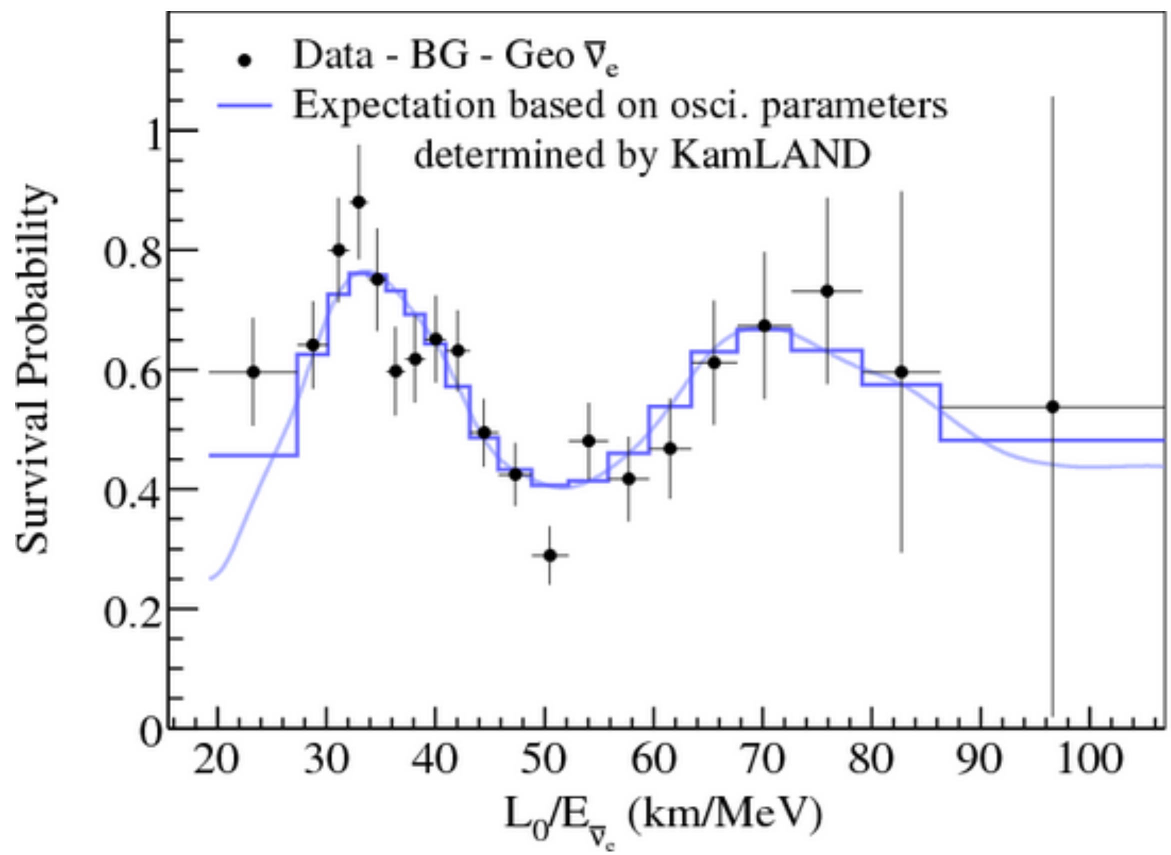
For KamLAND, $x_{\mathrm{Matter}} < 10^{-2}$. Matter effects are negligible.

The $\bar{\nu}_e$ survival probability, $P(\bar{\nu}_e \to \bar{\nu}_e)$, should oscillate as a function of L/E following the vacuum oscillation formula.

In the two-neutrino approximation, we expect —

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta \sin^2\left[1.27\Delta m^2\left(eV^2\right)\frac{L(km)}{E(GeV)}\right].$$

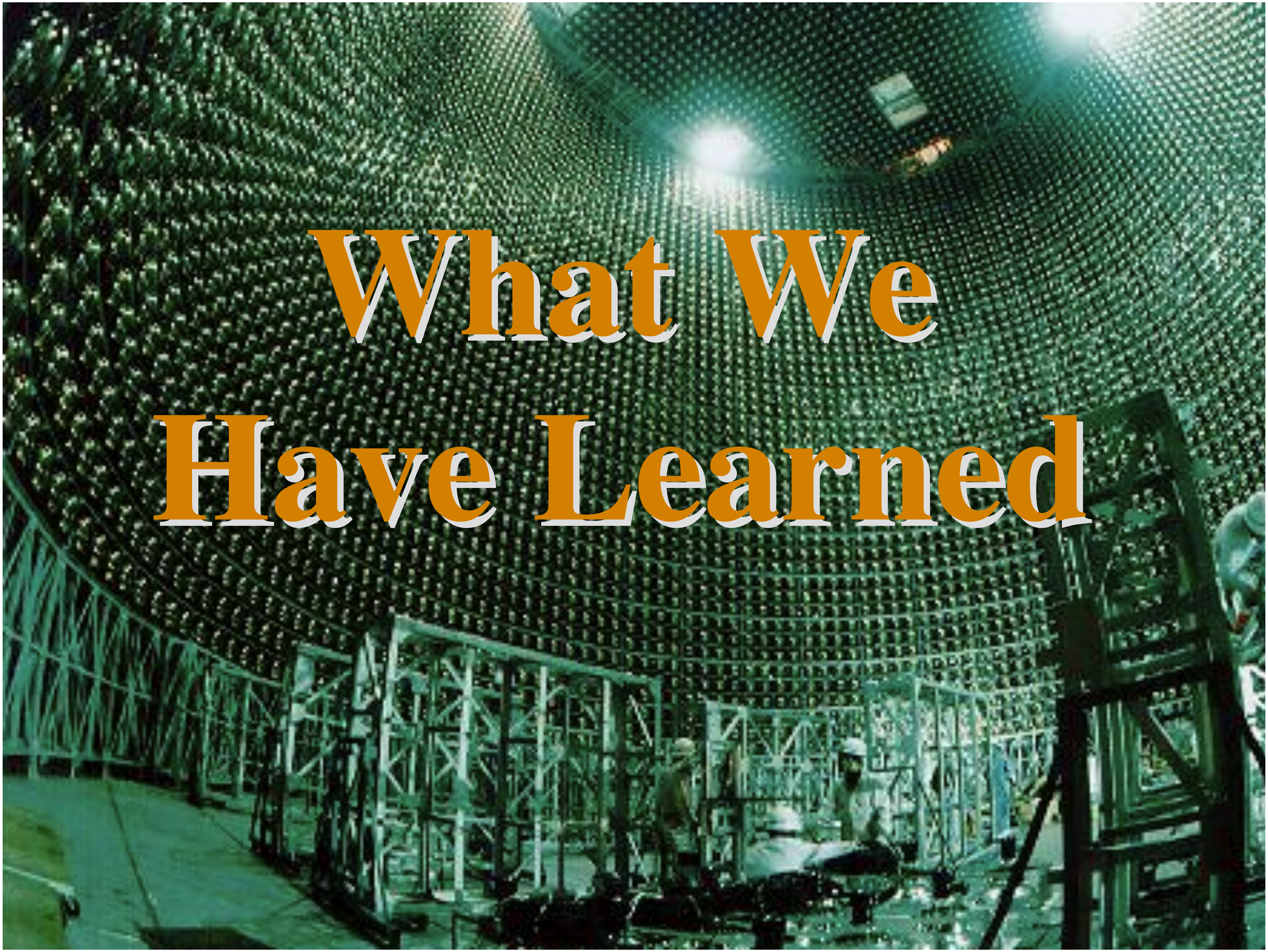
Survival probability $P(\overline{\nu}_e \rightarrow \overline{\nu}_e)$ of reactor $\overline{\nu}_e$



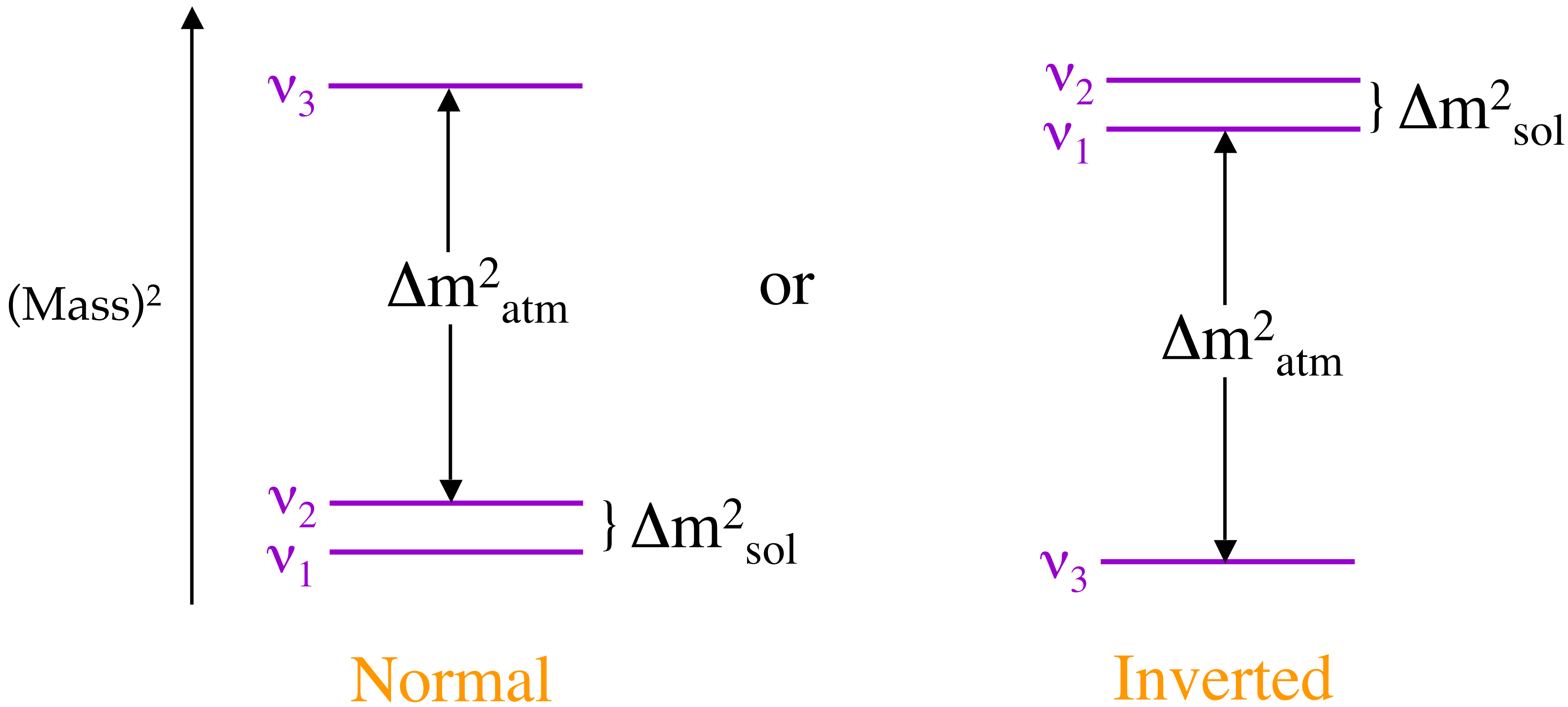
L_0 = 180 km is a flux-weighted average travel distance.

$P(\overline{\nu}_e \rightarrow \overline{\nu}_e)$ actually oscillates!

What We Have Learned

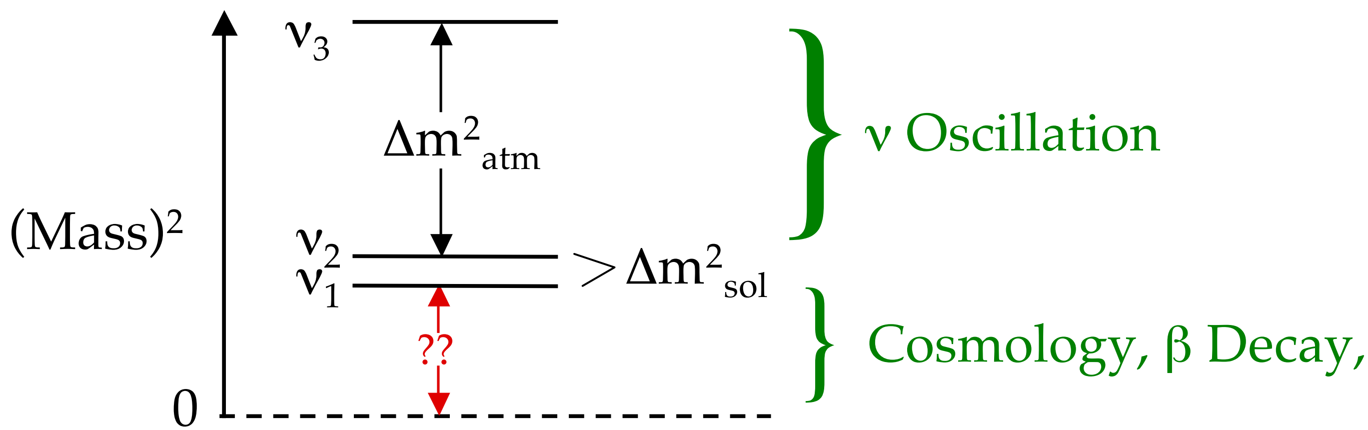


The $(\text{Mass})^2$ Spectrum



$\Delta m^2_{sol} \cong 7.5 \times 10^{-5}\ \mathrm{eV}^2$, $\quad \Delta m^2_{atm} \cong 2.4 \times 10^{-3}\ \mathrm{eV}^2$

The Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{atm}} <$ Mass[Heaviest ν_i]

Mass[Heaviest ν_i] $\gtrsim 0.04$ eV

The Upper Bound From Cosmology

Neutrino mass affects large scale structure and the CMB.

Cosmological Data + Cosmological Assumptions $\Rightarrow$

$$\underbrace{\Sigma\, m_i}_{\text{Mass}(\nu_i)} < (0.17 - 1.0)\ \text{eV}\ .$$

(Seljak, Slosar, McDonald
Hannestad; Pastor)

If there are only 3 neutrinos,

$$\underbrace{0.04\ \text{eV}}_{\sqrt{\Delta m^2_{atm}}} \lesssim \text{Mass[Heaviest } \nu_i] < \underbrace{(0.07 - 0.4)\ \text{eV}}_{\text{Cosmology}}$$

The Upper Bound From Tritium

(Christian Weinheimer)

Cosmology is wonderful, but there are known loopholes in its argument concerning neutrino mass.

The absolute neutrino mass can in principle also be measured by the kinematics of β decay.

Tritium decay: $\quad {}^3H \to {}^3He + e^- + \overline{\nu_i}\ ;\ i = 1, 2, \text{or } 3$

$$BR\left({}^3H \to {}^3He + e^- + \overline{\nu_i}\right) \propto \left|U_{ei}\right|^2$$

In ${}^3H \to {}^3He + e^- + \overline{\nu_i}$, the bigger m_i is, the smaller the maximum electron energy is.

There are 3 separate thresholds in the β energy spectrum.

The β energy spectrum is modified according to —

$$(E_0 - E)^2 \Theta[E_0 - E] \Rightarrow \sum_i |U_{ei}|^2 (E_0 - E)\sqrt{(E_0 - E)^2 - m_i^2}\, \Theta[(E_0 - m_i) - E]$$

Maximum β energy when there is no neutrino mass

β energy

Present experimental energy resolution is insufficient to separate the thresholds.

Measurements of the spectrum bound the average neutrino mass —

$$\langle m_\beta \rangle = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

$\boxed{\text{Presently: } \langle m_\beta \rangle < 2 \text{ eV}}$

Mainz & Troitzk

Leptonic Mixing

$$|\nu_\alpha\rangle = \sum_{i=1,2,3} U^*_{\alpha i}\, |\nu_i\rangle$$

Neutrino of flavor
α = e, μ, or τ

Neutrino of definite mass m_i

Unitary Leptonic Mixing Matrix

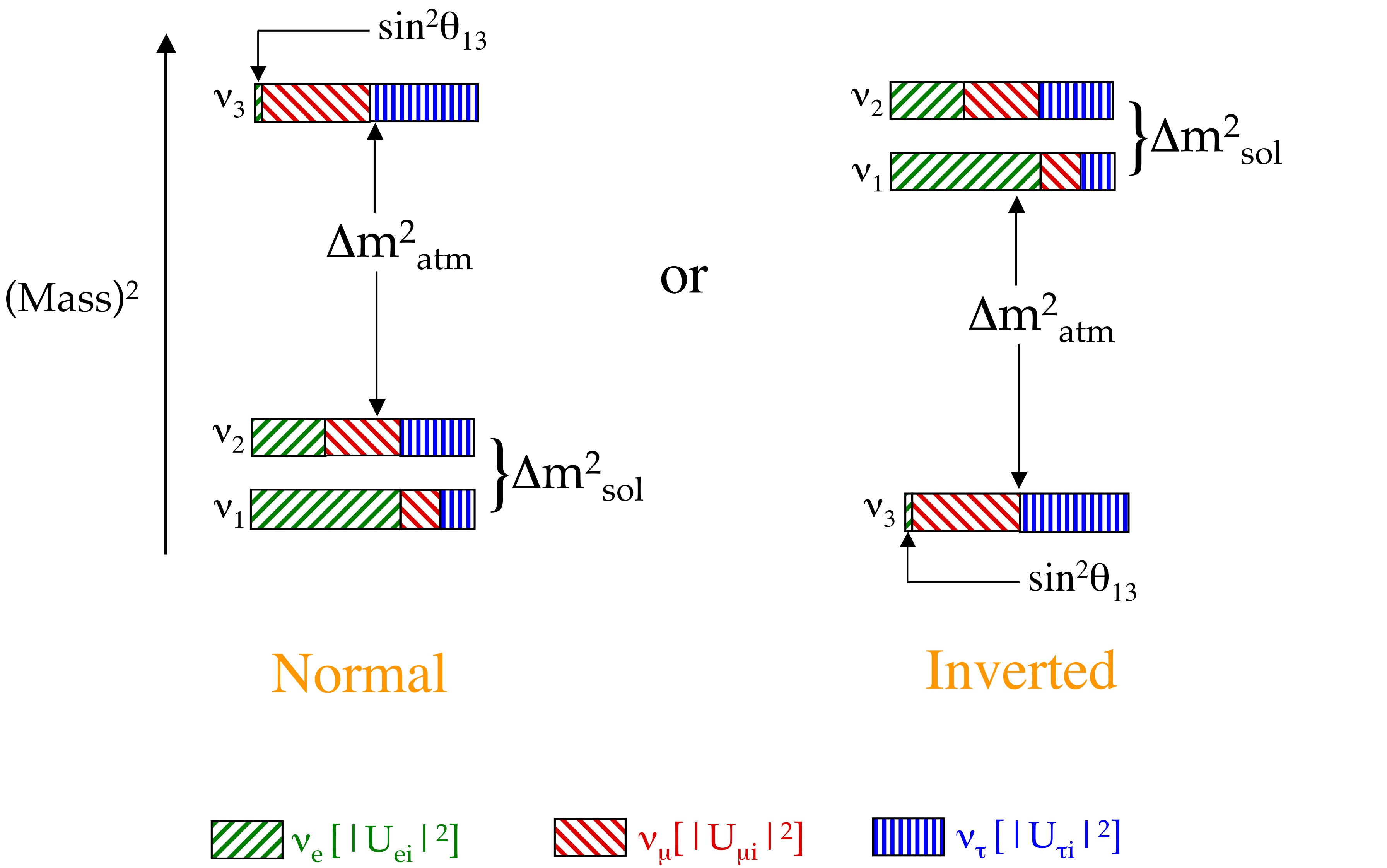
has the inverse:

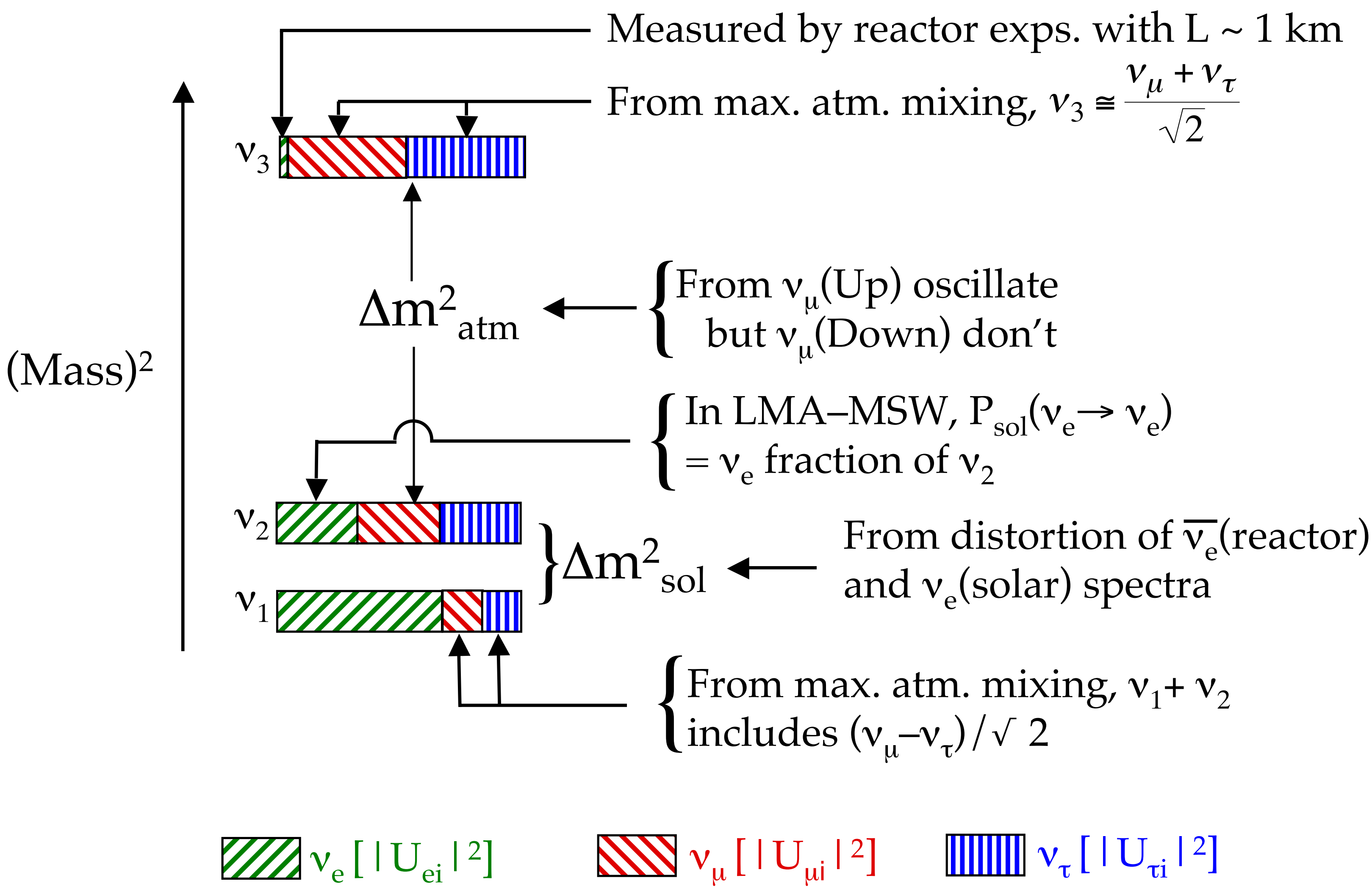
$$|\nu_i\rangle = \sum_{\beta = e, \mu, \tau} U_{\beta i}\, |\nu_\beta\rangle$$

Flavor-β fraction of $\nu_i = |U_{\beta i}|^2$.

When a ν_i interacts and produces a charged lepton, the probability that this charged lepton will be of flavor β is $|U_{\beta i}|^2$.

The spectrum, showing its approximate flavor content, is




Measured by reactor exps. with L ~ 1 km
From max. atm. mixing, $\nu_3 \cong \frac{\nu_\mu + \nu_\tau}{\sqrt{2}}$
ν_3
Δm^2_{atm}
From ν_μ(Up) oscillate
but ν_μ(Down) don't
(Mass)^2
In LMA–MSW, $P_{sol}(\nu_e \to \nu_e)$
= ν_e fraction of ν_2
ν_2
Δm^2_{sol}
From distortion of $\overline{\nu_e}$(reactor)
and ν_e(solar) spectra
ν_1
From max. atm. mixing, $\nu_1 + \nu_2$
includes $(\nu_\mu - \nu_\tau)/\sqrt{2}$
ν_e [$|U_{ei}|^2$]
ν_μ [$|U_{\mu i}|^2$]
ν_τ [$|U_{\tau i}|^2$]

The 3 X 3 Unitary Mixing Matrix

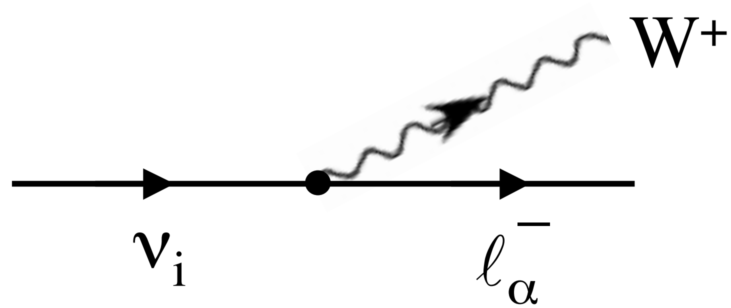
Caution: We are *assuming* the mixing matrix U to be 3 x 3 and unitary.

$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}{}^{-} + \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}{}^{*} \ell_{L\alpha} W_{\lambda}{}^{+} \right)$$

$$(CP)\left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}{}^{-} \right)(CP)^{-1} = \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i} \ell_{L\alpha} W_{\lambda}{}^{+}$$

Phases in U will lead to CP violation, unless they are removable by redefining the leptons.

$U_{\alpha i}$ describes —



$$U_{\alpha i} \sim \langle \ell_\alpha^- W^+ | H | \nu_i \rangle$$

When $|\nu_i\rangle \rightarrow |e^{i\varphi}\nu_i\rangle$, $U_{\alpha i} \rightarrow e^{i\varphi}\, U_{\alpha i}$, all α

When $|\ell_\alpha^-\rangle \rightarrow |e^{i\varphi}\,\ell_\alpha^-\rangle$, $U_{\alpha i} \rightarrow e^{-i\varphi}\, U_{\alpha i}$, all i

Thus, one may multiply any column, or any row, of U by a complex phase factor without changing the physics.

Some phases may be removed from U in this way.

Exception: If the neutrino mass eigenstates are their own antiparticles, then —

Charge conjugate

$$\nu_i = \nu_i^{\,c} = C\overline{\nu_i}^{\,T}$$

One is no longer free to phase-redefine ν_i without consequences.

U can contain additional CP-violating phases.

How Many Mixing Angles and $\not{CP}$ Phases Does U Contain?

Real parameters before constraints:	18
Unitarity constraints — $\sum_i U_{\alpha i}{}^* U_{\beta i} = \delta_{\alpha\beta}$	
Each row is a vector of length unity:	– 3
Each two rows are orthogonal vectors:	– 6
Rephase the three ℓ_α :	– 3
Rephase two ν_i , if $\overline{\nu}_i \neq \nu_i$:	– 2
Total physically-significant parameters:	4
Additional (Majorana) $\not{CP}$ phases if $\overline{\nu}_i = \nu_i$:	2

How Many Of The Parameters Are Mixing Angles?

The *mixing angles* are the parameters in U when it is *real*.

U is then a three-dimensional rotation matrix.

Everyone knows such a matrix is described in terms of 3 angles.

Thus, U contains 3 mixing angles.

Summary

Mixing angles	$\not{CP}$ phases if $\bar{\nu}_i \neq \nu_i$	$\not{CP}$ phases if $\bar{\nu}_i = \nu_i$
3	1	3

The Mixing Matrix U

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\times \underbrace{\begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Majorana phases}}$$

$c_{ij} \equiv \cos\theta_{ij}$
$s_{ij} \equiv \sin\theta_{ij}$

$\theta_{12} \approx 34°$, $\theta_{23} \approx 39\text{-}51°$, $\theta_{13} \approx 8\text{-}10°$ ***No more worry!***

δ would lead to $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$. *CP violation*

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

The Majorana $\cancel{CP}$ Phases

The phase α_i is associated with neutrino mass eigenstate ν_i:

$U_{\alpha i} = U^0{}_{\alpha i} \exp(i\alpha_i/2)$ for all flavors α.

$$\mathrm{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}{}^* \exp(-\, i m_i{}^2 L/2E)\, U_{\beta i}$$

is insensitive to the Majorana phases α_i.

Only the phase δ can cause CP violation in neutrino oscillation.

There Is Nothing Special About θ_{13}

All mixing angles must be nonzero for $\cancel{CP}$ in oscillation.

For example —

$$P\left(\bar{\nu}_\mu \to \bar{\nu}_e\right) - P\left(\nu_\mu \to \nu_e\right) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta$$
$$\times \sin\left(\Delta m^2{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{21}\frac{L}{4E}\right)$$

In the factored form of U, one can put δ next to θ_{12} instead of θ_{13}.