~~Lecture 2~~ Lecture 3

24th Indian-Summer School
Understanding Neutrinos
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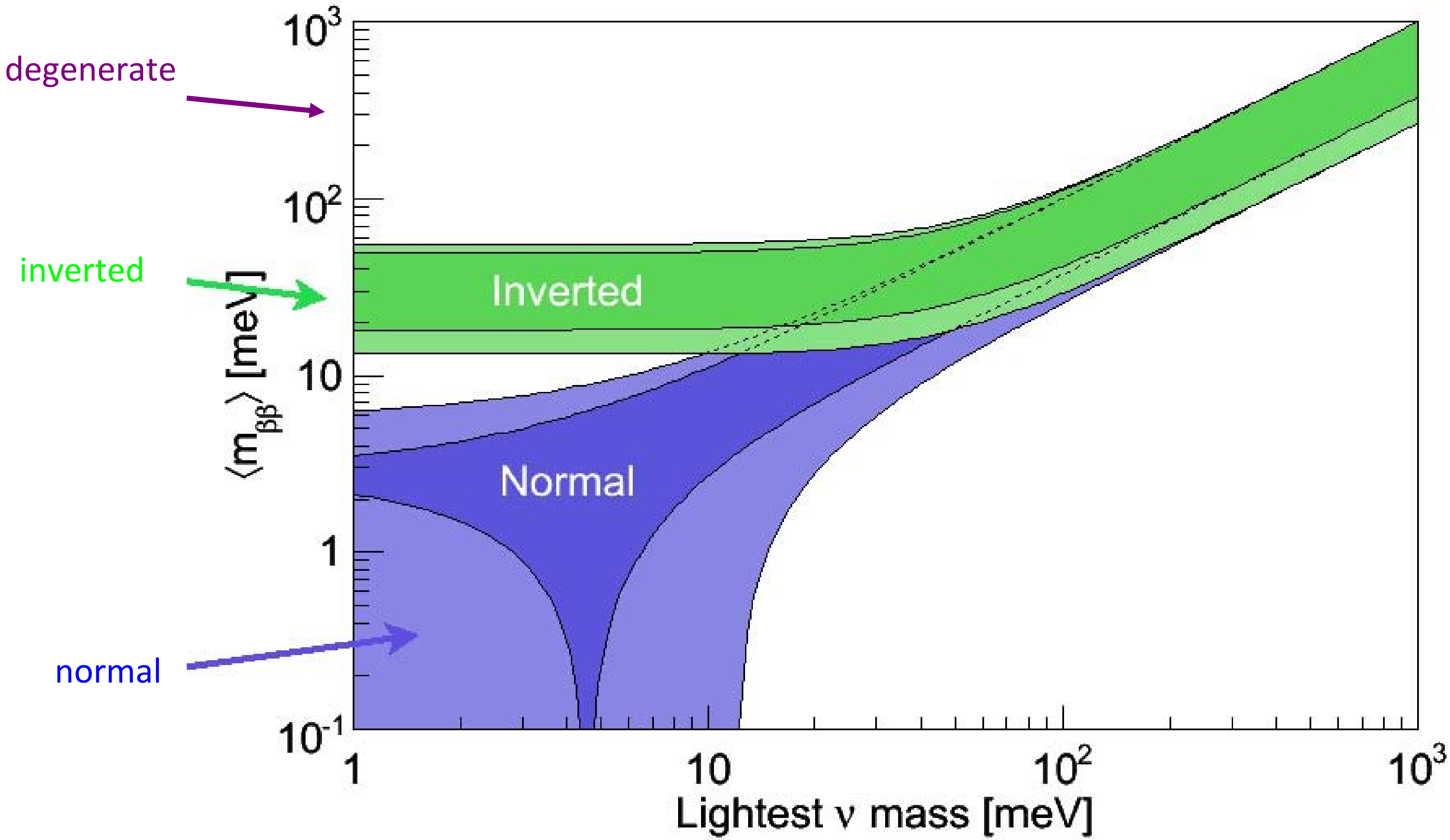
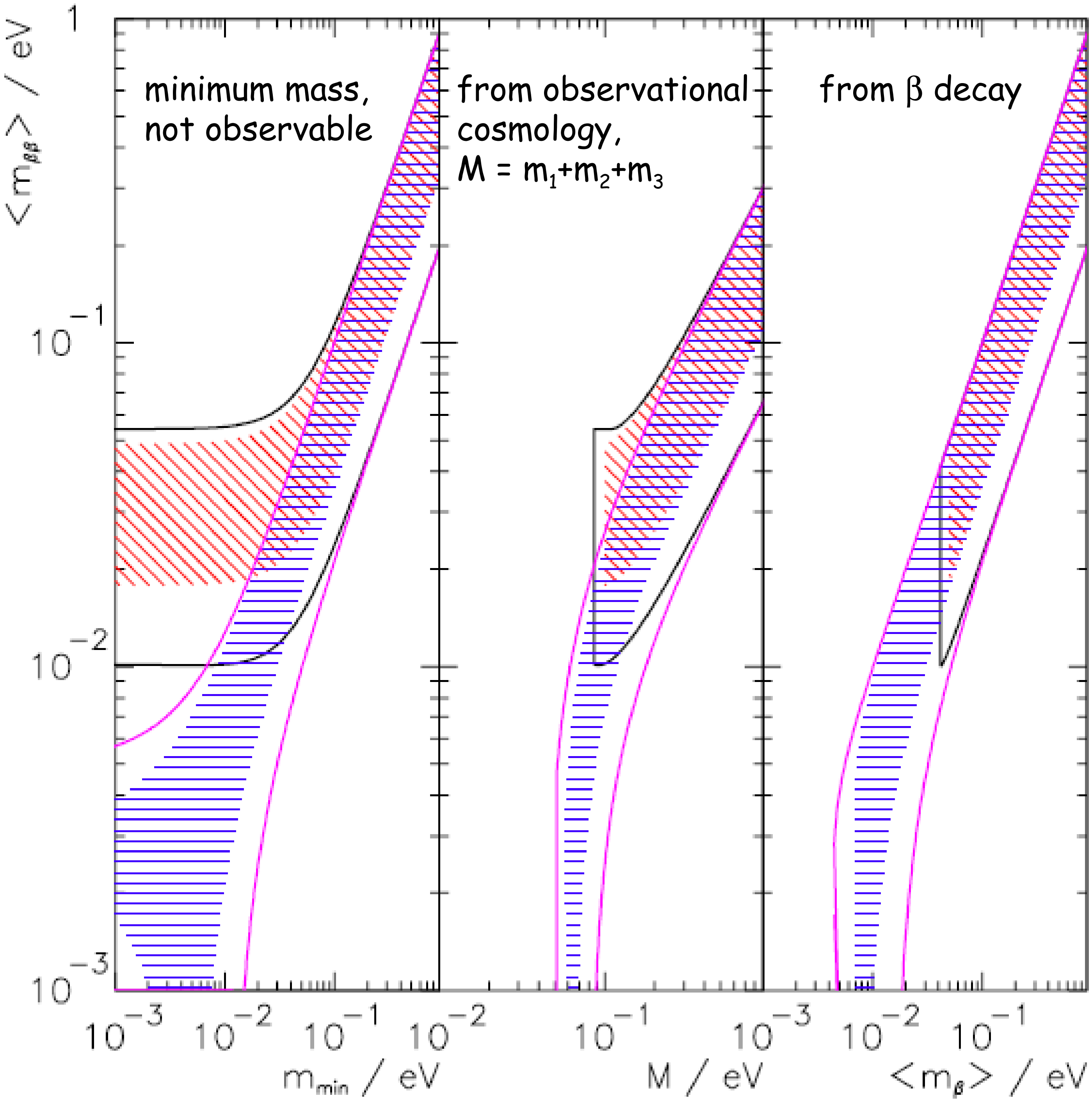
~~Thursday, Sep.6, 2012~~ Friday, Sep. 7, 2012

Double beta decay – continuation

Once again the usual representation of the relation between the $\langle m_{\beta\beta} \rangle$ and the actual neutrino mass . It shows that the $\langle m_{\beta\beta} \rangle$ axis can be divided into three distinct regions as indicated.
However, it creates the impression (**false**) that determining $\langle m_{\beta\beta} \rangle$ would decide between the two competing hierarchies.





$\langle m_{\beta\beta} \rangle$ vs. the absolute mass scales

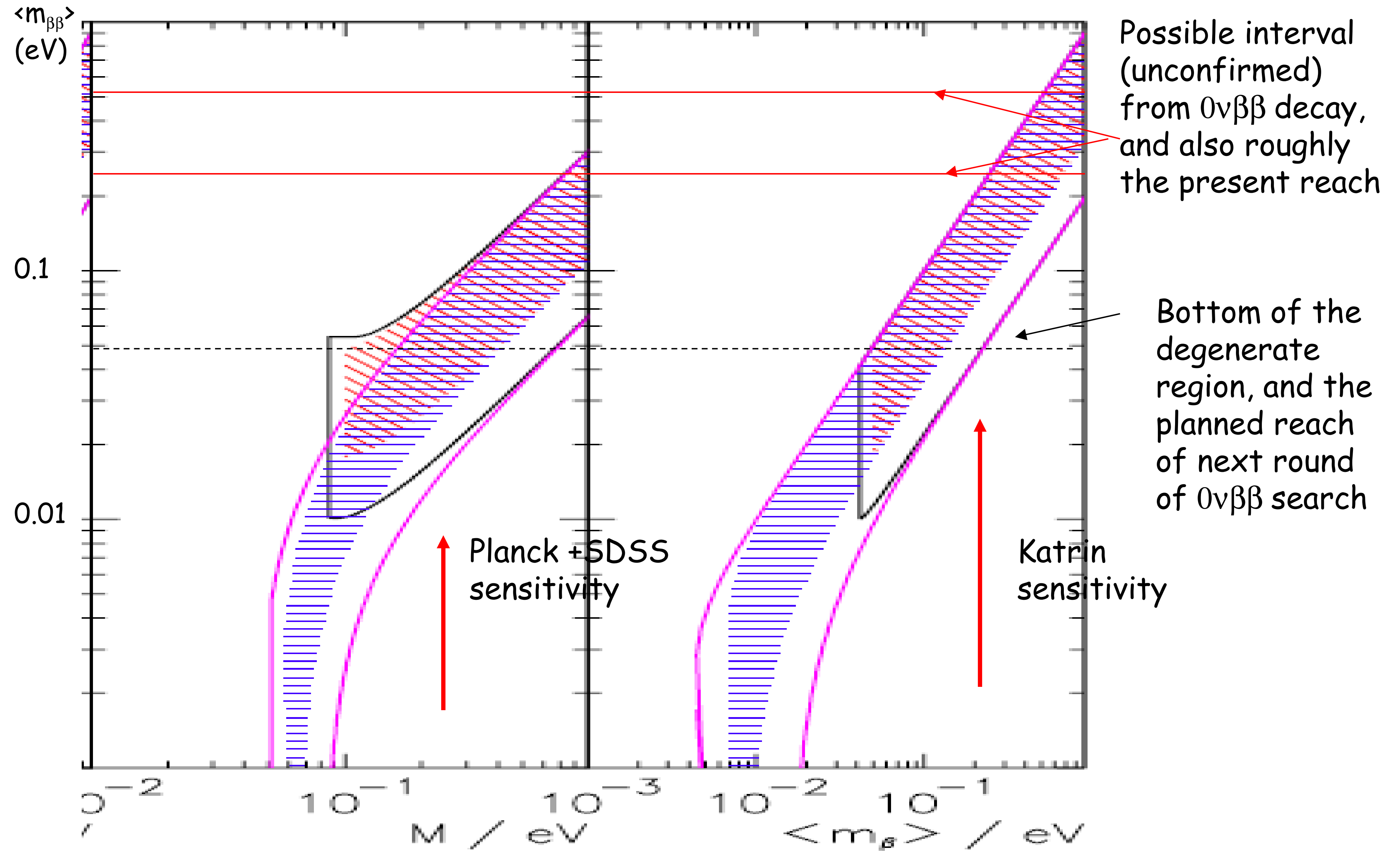
blue shading: normal hierarchy, $\Delta m^2_{31} > 0$.

red shading: inverted hierarchy $\Delta m^2_{31} < 0$

shading:best fit parameters, lines 95% CL errors.

Thanks to A. Piepke

The degenerate mass region will be explored by the next generation of $0\nu\beta\beta$ experiments and also probed by ways independent on Majorana nature of neutrinos.



ree regions of $\langle m_{\beta\beta} \rangle$ of interest:

egenerate mass region where all $m_i \gg |\Delta m_{31}^{2}|$. There $\langle m_{\beta\beta} \rangle > 0.05$ eV. $T_{1/2}$ for $0\nu\beta\beta$ decay < 10^{26-27} y in this region. This region will be xplored during the next 3-5 years with $0\nu\beta\beta$ decay experiments sing ~100 kg sources . Moreover, most if not all of that mass region ill be explored also by study of ordinary β decay and by the observational cosmology'. These latter techniques are independent of hether neutrinos are Majorana or Dirac particles.

Inverted hierarchy region where m_3 could be < Δm_{31}^2. However, quasidenegerate normal hierarchy is also possible for $\langle m_{\beta\beta} \rangle$ ~ 20-100 meV. $T_{1/2}$ for $0\nu\beta\beta$ decay is 10^{27-28} years here, and could be explored with ~ton size experiments. Proposals for such experiments, with timeline ~10 years, exist but are not funded as yet.

Normal mass hierarchy, $\langle m_{\beta\beta} \rangle$ < 20 meV. It would be necessary to use ~100 ton experiments. There are no realistic ideas how to do it.

Now lets add few general remarks regarding the neutrino mass determination.

The two-body decays, like $\pi^+ \to \mu^+ + \nu_\mu$ are very simple conceptually:
Consider pion decay in its rest frame, there

$$m_\nu^2 = m_\pi^2 + m_\mu^2 - 2m_\pi E_\mu \ ,$$

but the sensitivity is only to $m_\nu \sim 170$ keV with little hope of a substantial improvement.

Another conceptually simple methods of neutrino mass determination, like TOF, are not sensitive enough either

The time delay, with respect to massless particle, is

$\Delta t(E) = 0.514\ (m_\nu/E_\nu)^2 D$,

where m is in eV, E in MeV, D in 10 kpc, and Δt in sec. But there are no massless particles emitted by SN at the same time as neutrinos. Alternatively, we might look for a time delay between the charged current signal (i.e. ν_e) and the neutral current signal (dominated by ν_x). In addition , one might look for a broadening of the signal, and rearrangement according to the neutrino energy.

(see J. Beacom and P.V., Phys. Rev. D**58**, 05301 (1998)).

A necessary bit of nuclear structure theory

In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that again are bound in the ground state of the final nucleus.

The nuclear structure problem is therefore to evaluate, with a sufficient accuracy, **the ground state wave functions of both nuclei**, and evaluate the matrix element of the $0\nu\beta\beta$-decay operator connecting them.

This cannot be done exactly; some approximation and/or truncation is always necessary. Moreover, there is no other analogous observable that can be used to judge the quality of the result.

Can one use the 2νββ-decay matris elements for that?
What are the similarities and differences?

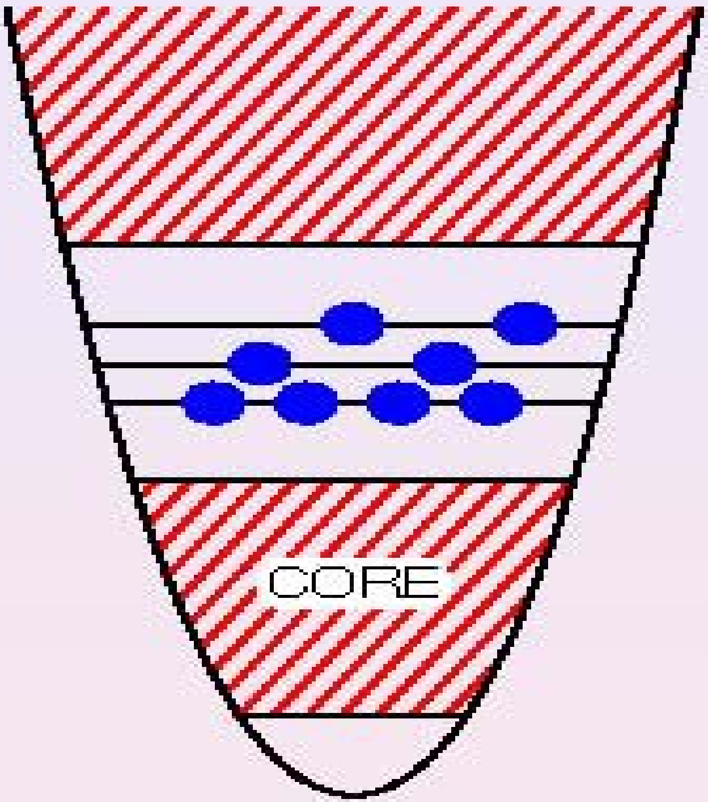
Both 2νββ and 0νββ operators connect the same states.
Both change two neutrons into two protons.

However, in 2νββ the momentum transfer q < few MeV
And thus $e^{iqr} \sim 1$, **long wavelength approximation** is valid, only the GT operator **στ** need to be considered.

In 0νββ q ~ 100-200 MeV, $e^{iqr} = 1 +$ **many terms**, there is no natural cutoff in that expansion.

Explaining 2νββ-decay rate is necessary but not sufficient

Basic procedures: Assume that the nucleus is made of interacting protons and neutrons bound in a confining potential; this is necessary.

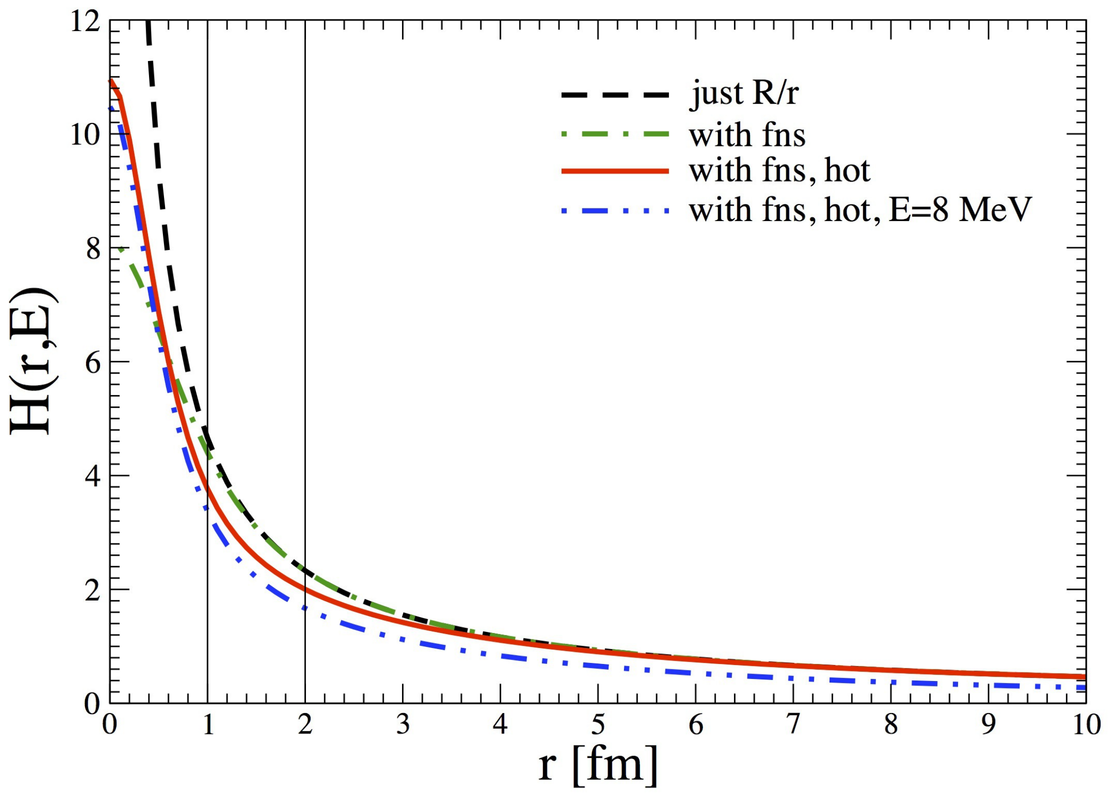


1) Define the valence space
2) Derive the effective hamiltonian H_{eff} using the nucleon-nucleon interaction plus some empirical nuclear data.
3) Solve the equations of motion to obtain the ground state wave functions

Note: Completely full or completely empty subshells in both the initial and final nuclei will not participate in the $\beta\beta$ decay.

Transition operator contains $\tau^+_1\tau^+_2$ that change neutrons into protons and in part $\sigma_1\sigma_2$ and the tensor operator S_{12}. Each of these parts in mutiplied by the `neutrino potential' (Fourier transform of the propagator) that introduces dependence on the radial distance between the nucleons.

$$H(r,E_m) = \frac{R}{2\pi^2}\int \frac{d\vec{q}}{\omega}\frac{1}{\omega + A_m} e^{i\vec{q}\cdot\vec{r}} = \frac{2R}{\pi r}\int_0^\infty dq \frac{q \sin(qr)}{\omega(\omega + A_m)} = \frac{2R}{\pi}\int_0^\infty dq \frac{j_0(qr)q}{q + A_m} .$$



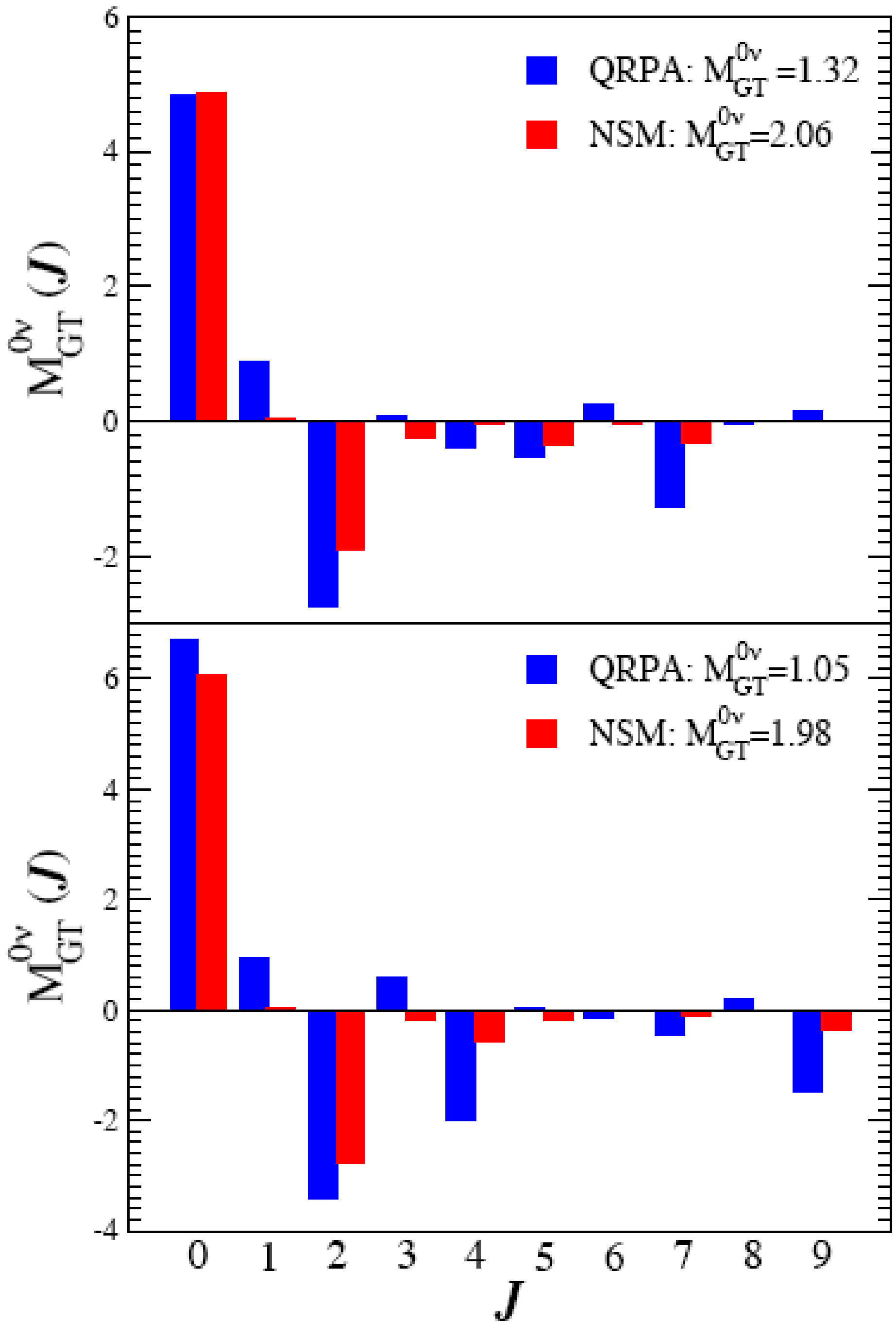
Various small additions to H(r,E) will be explained later.

Two complementary procedures are commonly used:

a) **Nuclear shell model (NSM)**
b) **Quasiparticle random phase approximation (QRPA)**

In NSM a l**imited** valence space is used but **all** configurations of valence nucleons are included. Describes well properties of low-lying nuclear states. Technically difficult, thus only few $0\nu\beta\beta$ calculations.

In QRPA a **large** valence space is used, but **only a class** of configurations is included. Describes collective states, but not details of dominantly few-particle states. Rather simple, thus many $0\nu\beta\beta$ calculations.



^{82}Se

^{130}Te

Why it is difficult to calculate the matrix elements accurately?

Contributions of different angular momenta J of the neutron pair that is transformed in the decay into the proton pair with the same J.

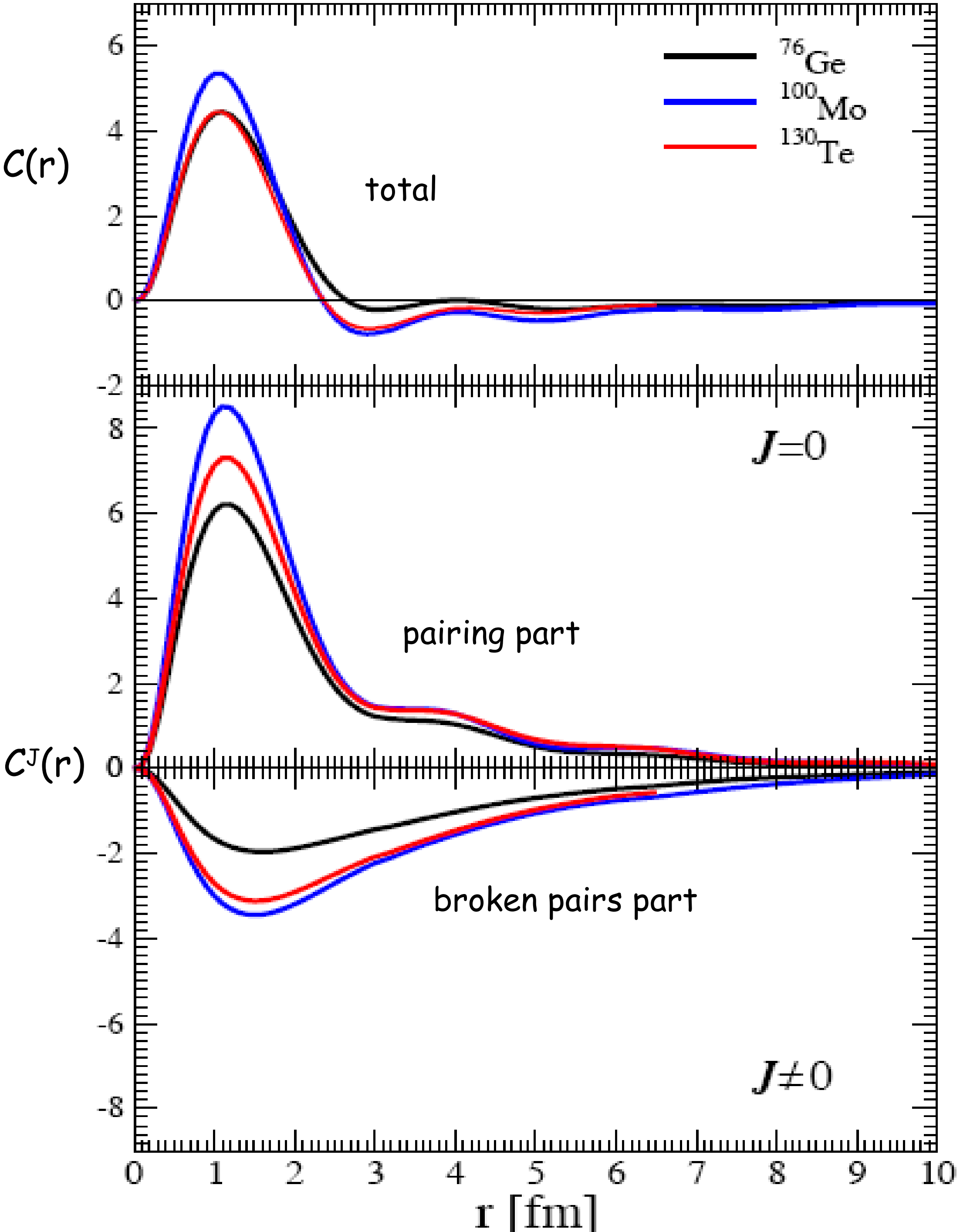
Note the opposite signs, and thus tendency to cancel, between the $J = 0$ (pairing) and the $J \neq 0$ (ground state correlations) parts.

The same restricted s.p. space is used for QRPA and NSM. There is a reasonable agreement between the two methods

Dependence of the $M^{0\nu}$ on the distance r between the two neutrons that are transformed into the two protons.

The "neutrino potential" is $H(r)= R/r\ \Phi(\omega r)$ where $\Phi(\omega r)$ is rather slowly varying function. This is a long range potential, more or less like a Coulomb potential. Thus, naively, one expect that the matrix element will get its main contribution from r ~ R, i.e. the mean distance between the nucleons in a nucleus.

This is not so. Due to the "pairing" and "broken pairs" competition, only distances r < 2-3 fm contribute, i.e., only nearest neighbors.



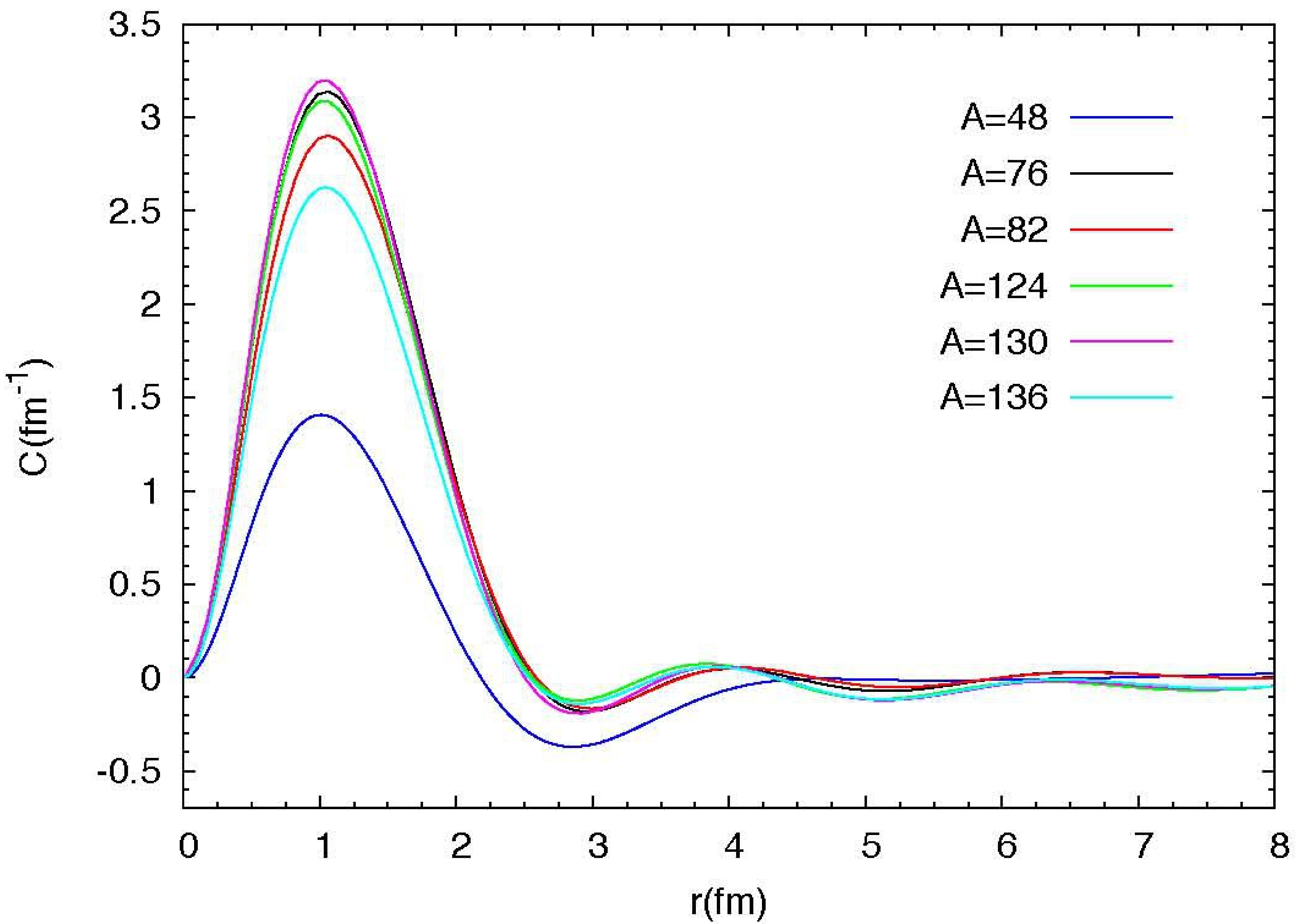
The radial dependence of $M^{0\nu}$ for the three indicated nuclei. The contributions summed over all components is shown in the upper panel. The `pairing' $J$ = 0 and `broken pairs' $J \neq 0$ parts are shown separately below.

Note that these two parts essentially cancel each other for r > 2-3 fm. This is a generic behavior. Hence the treatment of small values of *r* or large values of *q* are quite important.

$$M^{0\nu} = \int C(r)dr$$

The radial dependence of $M^{0\nu}$ for the indicated nuclei, evaluated in the **nuclear shell model**. (Menendes et al, arXiv:0801.3760).

Note the similarity to the QRPA evaluation of the same function.

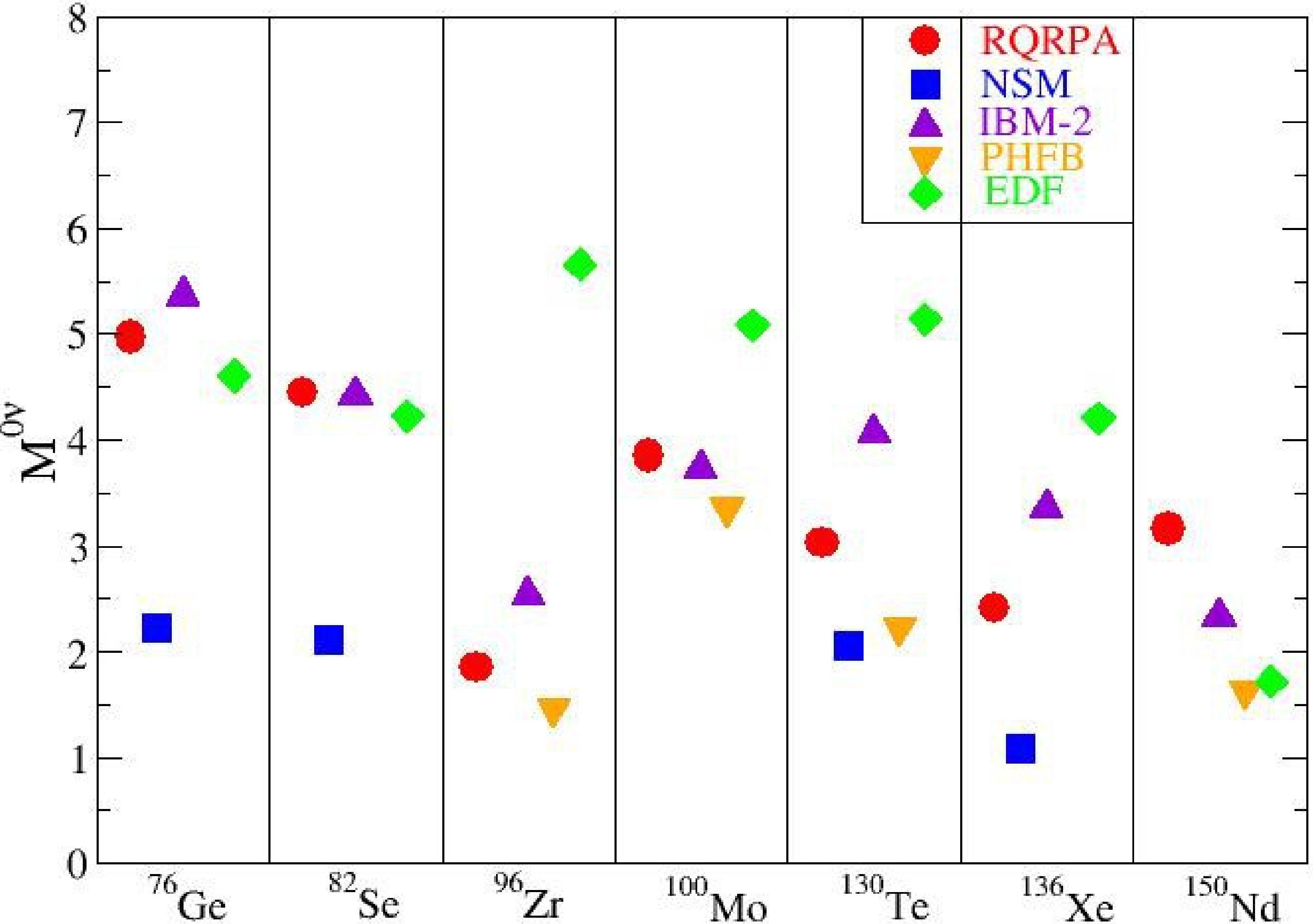


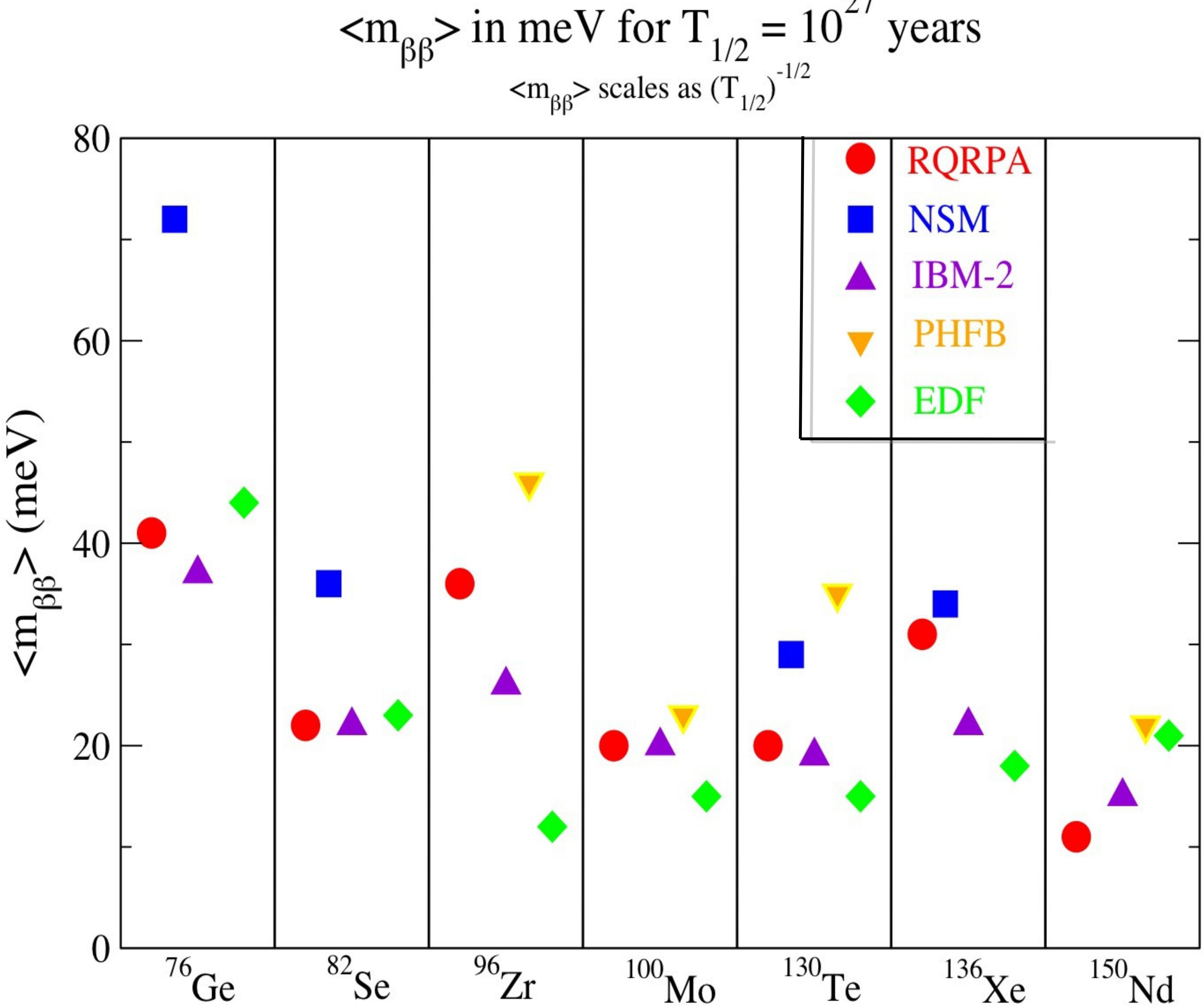
Conclusions so far:

- Various physics effects that influence the magnitude of the $0\nu\beta\beta$ nuclear matrix elements have been identified.
- The corresponding corrections, within QRPA, were estimated.
- In particular, the competition between the `pairing', ***J = 0***, and the `broken pairs', $J \neq 0$, contributions causes almost complete cancellation for the internucleon distance $r \geq$ 2-3 fm, hence making the short range behavior important.
- Thus the treatment of the nucleon finite size, induced weak currents and the short range nucleon-nucleon repulsion causes visible changes in the nuclear matrix elements.
- There is little independent information about such effects (for analogous charge-changing operators). Thus, the prudent approach is to include them in the corresponding systematic error.
- The total range, assuming the basic validity of QRPA, is reasonable, and the qualitative agreement with the ISM is encouraging.

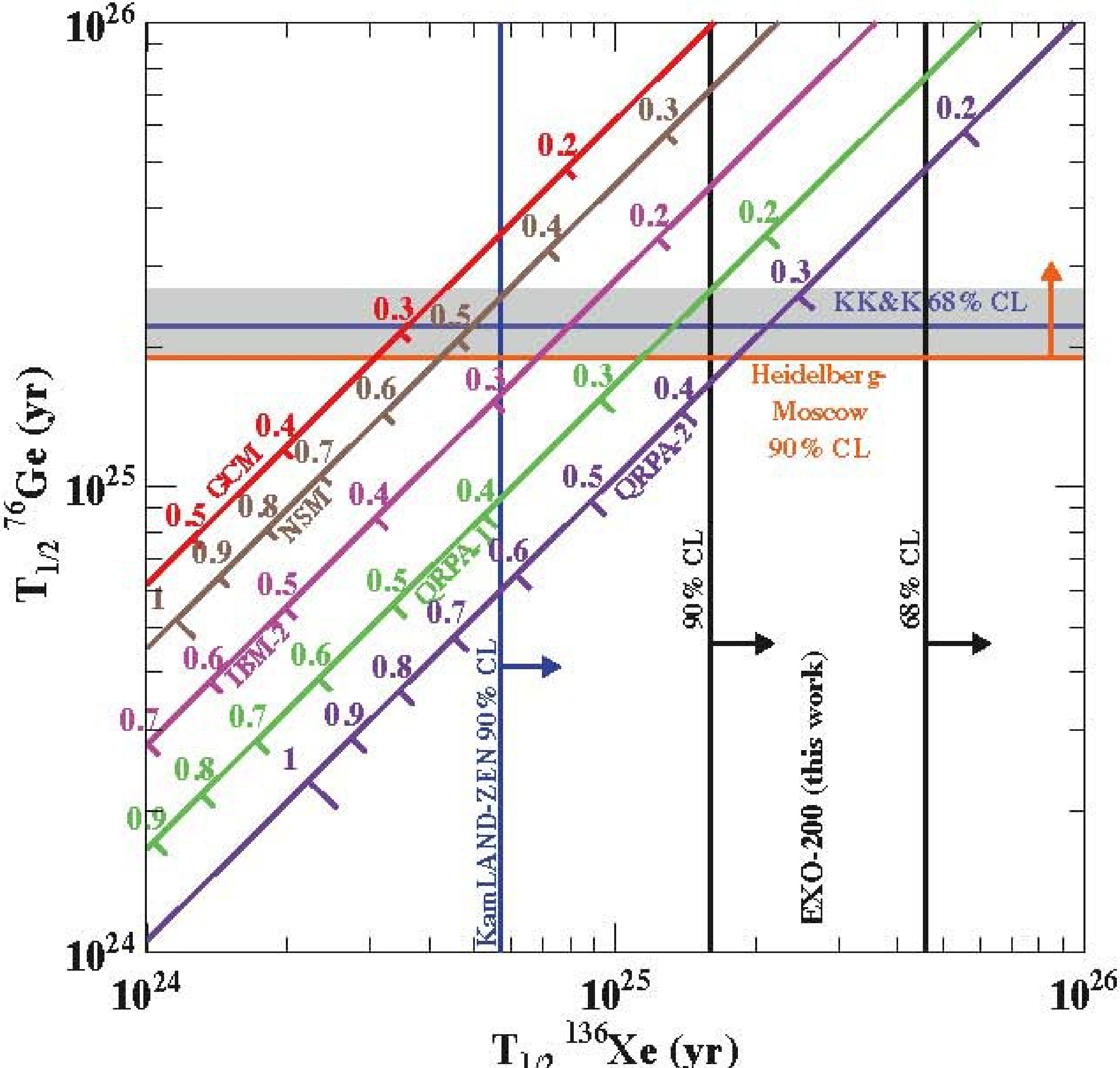
Nuclear matrix elements $M^{0\nu}$ for various methods: IBM-2 is Interactiong Boson Model -2, PHFB is Projected Hartree-Fock-Bogolyubov and EDF (or GCM) is Energy Density Functional or Generator Coordinate Method.

Note the relatively smooth dependence on A,Z in each method, but differences by the factor ~2 between the different methods. In particular, NSM is typically smaller and other methods agree with each other a bit better.





<m_ββ> in meV for T_1/2 = 10^27 years
<m_ββ> scales as (T_1/2)^-1/2
<m_ββ> (meV)
80
60
40
20
0
RQRPA
NSM
IBM-2
PHFB
EDF
76Ge
82Se
96Zr
100Mo
130Te
136Xe
150Nd



$T_{1/2}$ in ^{76}Ge versus $T_{1/2}$ in ^{136}Xe. The experimental limits are the horizontal and vertical lines. The theoretical results are represented by the diagonal lines. Their offset depends on matrix element ratio, and each point corresponds to a different $\langle m_{\beta\beta} \rangle$. The grey horizontal band represents the as yet unconfirmed claim of actual observation of the $0\nu\beta\beta$ decay in ^{76}Ge.

Auger *et al.* PRL **109**,032505(2012)