On the Descriptional Complexity of Scattered Context Grammars

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Abstract

This paper proves that every recursively enumerable language is generated by a scattered context grammar with no more than four nonterminals and three non-context-free productions. In its conclusion, it gives an overview of the results and open problems concerning scattered context grammars and languages.

Key words: scattered context grammar; descriptional complexity.

1 Introduction

The family of propagating scattered context languages, defined by Greibach and Hopcroft in [3], is a subset of the family of context-sensitive languages. However, the equality of these two language families is an open problem. Allowing erasing productions, the family of scattered context languages equals to the family of recursively enumerable languages (see [5]).

Besides the theoretical aspects, the motivation to study the descriptional complexity of scattered context grammars with respect to numbers of nonterminals and non-context-free productions is the recently started work on parsers and compilers based on these grammars, and the problems concerning them (for more details see Rychnovský [8]).

Over its history, some interesting results have been achieved in the descriptional complexity of scattered context grammars, however, some questions remain open. Specifically, Meduna [7] proved that scattered context grammars with only one

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nonterminal are not able to generate the exponential language $\{a^{2^{2^n}} : n \geq 0\}$. However, this language is a propagating scattered context language (see [4]). In addition, Meduna [6] proved that scattered context grammars with no more than three nonterminals characterize the family of recursively enumerable languages. In this case, the number of non-context-free productions—productions with more than one nonterminal on the left-hand side—is not limited for the whole family of languages (and thus it depends on the generated language). Later, Vaszil [10] limited the number of non-context-free productions by showing that the family of recursively enumerable languages is characterized by scattered context grammars with no more than five nonterminals and two non-context-free productions. Finally, the previous result has been improved with respect to the number of nonterminals; see [4] for a proof that the family of recursively enumerable languages is characterized by scattered context grammars with no more than four nonterminals and four non-context-free productions.

This paper proves that every recursively enumerable language is generated by a scattered context grammar with no more than four nonterminals and three non-context-free productions. Furthermore, this paper summarizes the results and open problems concerning scattered context grammars and languages.

2 Preliminaries and Definitions

We assume that the reader is familiar with formal language theory (see [1,9]). For an alphabet (finite nonempty set) V, V^* represents the free monoid generated by V. The unit of V^* is denoted by ε. Set $V^+ = V^* - \{\varepsilon\}$. For $w \in V^*$, w^R denotes the mirror image of w. Denote the families of recursively enumerable languages and context-sensitive languages by $\mathscr{L}_{RE}$ and $\mathscr{L}_{CS}$, respectively.

A *scattered context grammar* is a quadruple $G = (N, T, P, S)$, where N is a nonterminal alphabet, T is a terminal alphabet such that $N \cap T = \emptyset$, $S \in N$ is the start symbol, and P is a finite set of productions of the form $(A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n)$, for some $n \geq 1$, where $A_i \in N$ and $x_i \in (N \cup T)^*$, for all $i = 1, 2, \ldots, n$. If $n \geq 2$, the production is said to be *non-context-free*. If for each $i = 1, \ldots, n$, we have $x_i \neq \varepsilon$, the production is said to be *propagating*. G is *propagating* if all its productions are propagating.

For $u, v \in (N \cup T)^*$, $u \Rightarrow v$ in G provided that

(1) $u = u_1 A_1 u_2 \ldots u_n A_n u_{n+1}$,
(2) $v = u_1 x_1 u_2 \ldots u_n x_n u_{n+1}$, and
(3) $(A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n) \in P$,

where $u_i \in (N \cup T)^*$, for all $i = 1, \ldots, n+1$.

The language generated by G is defined as $L(G) = \{w \in T^* : S \Rightarrow^* w\}$, where $\Rightarrow^*$ denotes the reflexive and transitive closure of $\Rightarrow$. A language L is a (propagating) scattered context language if there is a (propagating) scattered context grammar, G, such that $L = L(G)$.

Let $m,n \in \{1,2,3,\ldots\} \cup \{\infty\}$. Define the family of languages $\mathscr{L}_{SC}(m,n)$ so that $L \in \mathscr{L}_{SC}(m,n)$ if and only if there is a scattered context grammar $G = (N,T,P,S)$ with no more than m nonterminals and n non-context-free productions such that $L(G) = L$.

For example, it is shown in [4] that for any integers k, $l \geq 2$, there is a propagating scattered context grammar G such that $L(G) = \{a^{l^{k^n}} : n \geq 0\} \in \mathscr{L}_{SC}(12,10)$.

3 Main Results

The main result of this section proves that every recursively enumerable language is generated by a scattered context grammar with no more than four nonterminals and three non-context-free productions none of which has more than six nonterminals on its left-hand side.

Recall that Geffert [2] proved that every recursively enumerable language is generated by a grammar $G_1 = (\{S,A,B,C,D\},T,P \cup \{AB \rightarrow \varepsilon, CD \rightarrow \varepsilon\},S)$, where P contains only context-free productions of the forms

(1) $S \rightarrow uSa$,
(2) $S \rightarrow uSv$,
(3) $S \rightarrow \varepsilon$,

for $u \in \{A,C\}^*$, $v \in \{B,D\}^*$, and $a \in T$. In addition, any terminal derivation of G_1 is of the form $S \Rightarrow^* w_1w_2w$ by productions from P, where $w_1 \in \{A,C\}^*$, $w_2 \in \{B,D\}^*$, $w \in T^*$, and $w_1w_2w \Rightarrow^* w$ by $AB \rightarrow \varepsilon$ and $CD \rightarrow \varepsilon$.

Lemma 1 *Let $G_1 = (\{S,A,B,C,D\},T,P \cup \{AB \rightarrow \varepsilon, CD \rightarrow \varepsilon\},S)$ be a grammar in Geffert normal form. Then, there is a grammar*

$$G' = (\{S',A,B,\$\},T,P' \cup \{AB\$BA \rightarrow \$, A\$A \rightarrow \$, \$ \rightarrow \varepsilon\},S')$$

such that $L(G') = L(G_1)$ and P' contains only context-free productions.

PROOF. Let $G_1 = (\{S,A,B,C,D\},T,P \cup \{AB \rightarrow \varepsilon, CD \rightarrow \varepsilon\},S)$ be a grammar in Geffert normal form. Define the homomorphism $h : \{A,B,C,D\}^* \rightarrow \{A,B\}^*$ so that $h(A) = AB$, $h(B) = BA$, $h(C) = A$, and $h(D) = A$. Construct the grammar $G' = (\{S', A, B, \$\}, T, P' \cup \{AB\$BA \to \$, A\$A \to \$, \$ \to \varepsilon\}, S')$ with

$$
\begin{aligned}
P' = &\{S' \to h(u)S'a : S \to uSa \in P\} \\
&\cup \{S' \to h(u)S'h(v) : S \to uSv \in P\} \\
&\cup \{S' \to \$\}\,.
\end{aligned}
$$

Then, any terminal derivation of G' is of the form $S' \Rightarrow^* w_1\$w_2w$ by productions from P', where $w_1 \in \{AB, A\}^*$, $w_2 \in \{BA, A\}^*$, $w \in T^*$, and $w_1\$w_2w \Rightarrow^* \$w \Rightarrow w$ by $AB\$BA \to \$$ (simulating $AB \to \varepsilon$ in G_1), $A\$A \to \$$ (simulating $CD \to \varepsilon$), and finished by $\$ \to \varepsilon$. $\square$

The main result follows.

Theorem 2 $\mathscr{L}_{RE} = \mathscr{L}_{SC}(4,3)$.

PROOF. Let L be a recursively enumerable language. Then, there is a grammar G_1 in Geffert normal form such that $L(G_1) = L$. Let $G' = (\{S', A, B, \$\}, T, P' \cup \{AB\$BA \to \$, A\$A \to \$, \$ \to \varepsilon\}, S')$ be a grammar constructed from G_1 by the construction given in Lemma 1.

Define $G = (\{S, A, B, \$\}, T, P, S)$ with P constructed as follows:

(1) $(S) \to (BaBSu)$ if $S' \to uS'a \in P'$,
(2) $(S) \to (vSu)$ if $S' \to uS'v \in P'$,
(3) $(S) \to (BB\$\$BB)$ if $S' \to \$ \in P'$,
(4) $(\$) \to (\varepsilon)$,
(5) $(B, B, \$, \$, B, B) \to (\$, \varepsilon, \varepsilon, \varepsilon, \varepsilon, \$BB)$,
(6) $(B, \$, \$, B) \to (\$, \varepsilon, \varepsilon, \$)$,
(7) $(A, \$, \$, A) \to (\$, \varepsilon, \varepsilon, \$)$.

To prove that $L(G') \subseteq L(G)$, consider a terminal derivation of G'. Such a derivation is of the form

$$
\begin{aligned}
S' &\Rightarrow u_1S'v_1 \\
&\Rightarrow u_1u_2S'v_2v_1 \\
&\Rightarrow^* u_1u_2\ldots u_nS'v_n\ldots v_2v_1 \\
&\Rightarrow u_1u_2\ldots u_n\$v_n\ldots v_2v_1\,,
\end{aligned}
$$

by a sequence of productions $p'_1p'_2\ldots p'_np'_{n+1}$, for some $n \geq 1$, where $p'_i \in P'$, $u_i \in \{AB, A\}^*$, $v_i \in (\{BA, A\} \cup T)^*$, for all $i = 1, \ldots, n$, and $p'_{n+1} \in \{S' \to \$\}$.

In G, by the sequence of productions $p_n \dots p_2 p_1 p_{n+1}$, where for all $i = 1, \dots, n+1$, p_i is constructed from p'_i as shown in (1) through (3) of the construction, we have

$$\begin{aligned} S &\Rightarrow h(v_n)Su_n \\ &\Rightarrow^* h(v_n \dots v_2)Su_2 \dots u_n \\ &\Rightarrow h(v_n \dots v_2 v_1)Su_1 u_2 \dots u_n \\ &\Rightarrow h(v_n \dots v_2 v_1)BB\$\$BBu_1 u_2 \dots u_n\,, \end{aligned}$$

where $h : (\{A,B\} \cup T)^* \to (\{A,B\} \cup T)^*$ is a homomorphism defined as $h(A) = A$, $h(B) = B$, and for all $a \in T$, $h(a) = BaB$.

Let $v_n \dots v_2 v_1$ be of the form $va_1 \dots a_k$, for some $k \geq 0$, where $v \in \{BA, A\}^*$ and $a_i \in T$, for all $i = 1, \dots, k$ ($k = 0$ implies that there is no terminal symbol). Then, $h(v_n \dots v_2 v_1) = vBa_1 B \dots Ba_k B$. Let $u = u_1 \dots u_n$. As the derivation continues in G' by $AB\$BA \to \$$ and $A\$A \to \$$, finished by $\$ \to \varepsilon$, i.e., $u\$va_1 \dots a_k \Rightarrow^* a_1 \dots a_k$, we have $u = v^R$.

In G, however, the simulation continues as follows. By a sequence of production 5, finished by two applications of production 6,

$$\begin{aligned} vBa_1B \dots Ba_{k-1}BBa_kBBB\$\$BBu &\Rightarrow vBa_1B \dots Ba_{k-1}BBa_kB\$\$BBu \\ &\Rightarrow vBa_1B \dots Ba_{k-1}B\$a_k\$BBu \\ &\;\;\vdots \\ &\Rightarrow vBa_1B\$ \dots a_{k-1}a_k\$BBu \\ &\Rightarrow vBa_1\$ \dots a_{k-1}a_k\$Bu \\ &\Rightarrow v\$a_1 \dots a_{k-1}a_k\$u\,. \end{aligned}$$

Then, as $u = v^R$, by a sequence of productions 6 and 7, finished by two applications of production 4,

$$\begin{aligned} v\$a_1 \dots a_{k-1}a_k\$u &\Rightarrow^* \$a_1 \dots a_{k-1}a_k\$ \\ &\Rightarrow^2 a_1 \dots a_{k-1}a_k\,. \end{aligned}$$

Thus, it proves that if there is a terminal derivation $S' \Rightarrow w$ of G', $w \in T^*$, then there is a derivation $S \Rightarrow^* w$ of G.

To prove the other inclusion, $L(G) \subseteq L(G')$, consider a terminal derivation of G. Such a derivation is of the form

$$\begin{aligned} S &\Rightarrow^* v_n \dots v_2 v_1 BB\$\$BBu_1 u_2 \dots u_n \text{ (by productions 1–3)} \\ &\Rightarrow^* a_1 \dots a_k \text{ (by productions 4–7)}\,, \end{aligned}$$

where, for some $n \geq 1$, $k \geq 0$ ($k = 0$ implies $a_1 \dots a_k = \varepsilon$), $v_i \in \{BA,A\}^* \cup \{B\}T\{B\}$, $u_i \in \{AB,A\}^*$ and $a_i \in T$, for all $i = 1,\dots,n$.

By a sequence of productions corresponding to productions applied in the derivation of G but in the inverted order, we have

$$S' \Rightarrow^* u_1 u_2 \dots u_n \$ h^{-1}(v_n \dots v_2 v_1)$$

in G'. To prove that $u_1 u_2 \dots u_n \$ h^{-1}(v_n \dots v_2 v_1) \Rightarrow^* a_1 a_2 \dots a_k$ and $h^{-1}(v_n \dots v_1) \in \{AB,A\}^* T^*$, examine the form of $v_n \dots v_1$.

Notice first that if a nonterminal occurs between two \$s, then it can never be removed. In addition, from now on, we do not consider production 4 because after this production, none or only production 4 is applicable. Thus, we say that an applicable production is *feasible* if it is not production 4 and it does not introduce any nonterminal between two \$s.

(A) If $v_n \dots v_1 = \varepsilon$, the sentential form $v_n \dots v_2 v_1 BB\$\$BB u_1 u_2 \dots u_n$ is of the form $BB\$\$BB u_1 u_2 \dots u_n$, and only productions 5 and 6 are feasible. By production 5 followed by the only applicable production 4, however, the derivation is blocked; $BB\$\$BB u_1 u_2 \dots u_n \Rightarrow \$\$BB u_1 u_2 \dots u_n \Rightarrow^2 BB u_1 u_2 \dots u_n$. Thus, only production 6 is feasible in the derivation, followed by the only applicable production 4, i.e.,

$$\begin{aligned} BB\$\$BB u_1 u_2 \dots u_n &\Rightarrow B\$\$B u_1 u_2 \dots u_n \\ &\Rightarrow \$\$ u_1 u_2 \dots u_n \\ &\Rightarrow^2 u_1 u_2 \dots u_n\,, \end{aligned}$$

which means that $u_1 \dots u_n = \varepsilon$ because $u_1 \dots u_n \in \{AB,A\}^*$. Thus, if $BB\$\$BB \Rightarrow^* \varepsilon$ in G, then $\$ \Rightarrow \varepsilon$ in G'. Clearly, $h^{-1}(v_n \dots v_1) \in \{AB,A\}^* T^*$.

(B) If $v_n \dots v_1 = vBaB$, for some $a \in T$ and $v \in (\{BA,A\} \cup \{B\}T\{B\})^*$, the sentential form is $vBaBBB\$\$BB u_1 u_2 \dots u_n$, where $u_1 \dots u_n \in \{AB,A\}^*$. The only feasible productions are 5 and 6. However, production 6 blocks the derivation; clearly, $vBaBBB\$\$BB u_1 u_2 \dots u_n \Rightarrow vBaBB\$\$B u_1 u_2 \dots u_n$ and only productions 6 is feasible because $u_1 \dots u_n \in \{AB,A\}^*$, i.e., $vBaBB\$\$B u_1 u_2 \dots u_n \Rightarrow vBaB\$\$ u_1 u_2 \dots u_n$. On the other hand, by production 5,

$$vBaBBB\$\$BB u_1 u_2 \dots u_n \Rightarrow vBaB\$\$BB u_1 u_2 \dots u_n$$

and only productions 5 and 6 are feasible.

Consider a more general sentential form $vBaB\$w\$BB u_1 u_2 \dots u_n$, $v \in (\{BA,A\} \cup \{B\}T\{B\})^*$, $a \in T$, $u_1 \dots u_n \in \{AB,A\}^*$, and $w \in T^*$. Then, only productions 5 and 6 are feasible.

(B1) Assume that $v \in \{BA,A\}^*$, then production 5 blocks the derivation because $vBaB\$w\$BBu_1u_2 \dots u_n \Rightarrow v\$aw\$BBu_1u_2 \dots u_n$ and any of productions 5, 6, 7 adds a nonterminal between \$s. Thus, production 6 has to be applied twice, and we have

$$vBaB\$w\$BBu_1u_2 \dots u_n \Rightarrow^2 v\$aw\$u_1u_2 \dots u_n\,.$$

(B2) If v contains a substrings BcB, for some $c \in T$, i.e., $v = v_1BcBv_2$, for some $v_1 \in (\{BA,A\} \cup \{B\}T\{B\})^*$, $v_2 \in \{BA,A\}^*$, and the sentential form is

$$v_1BcBv_2BaB\$w\$BBu_1u_2 \dots u_n\,,$$

then we prove that $v_2 = \varepsilon$. Clearly, by production 5,

$$v_1BcBv_2BaB\$w\$BBu_1u_2 \dots u_n \Rightarrow v_1BcBv_2\$aw\$BBu_1u_2 \dots u_n$$

and if $v_2 \neq \varepsilon$, the derivation is blocked; we can either remove \$s or get a nonterminal between \$s. By production 6,

$$v_1BcBv_2BaB\$w\$BBu_1u_2 \dots u_n \Rightarrow v_1BcBv_2Ba\$w\$Bu_1u_2 \dots u_n$$

and only production 6 is feasible because $u_1 \dots u_n \in \{AB,A\}^*$, i.e.,

$$v_1BcBv_2Ba\$w\$Bu_1u_2 \dots u_n \Rightarrow v_1BcBv_2\$aw\$u_1u_2 \dots u_n\,.$$

Consider a sentential form $v_1\gamma v_2\$w\$u$, where $v_1 \in (\{BA,A\} \cup \{B\}T\{B\})^*$, $\gamma \in \{B\}T\{B\} \cup \{\varepsilon\}$ and $\gamma = \varepsilon$ if and only if there is no terminal symbol in v_1, $v_2 \in \{BA,A\}^*$, $w \in T^*$, and $u \in \{AB,A\}^*$. Examine the form of v_2.

(i) If $v_2 = v_3BA$, $v_3 \in \{BA,A\}^*$, then only production 7 is feasible. Thus, $u \in \{Au', ABu' : u' \in \{AB,A\}^*\}$. Assume that $u'' \in \{u', Bu'\}$, then $v_1\gamma v_3BA\$w\$Au'' \Rightarrow v_1\gamma v_3B\$w\$u''$ and $u'' = Bu'$. By the only feasible production 6, $v_1\gamma v_3B\$w\$Bu' \Rightarrow v_1\gamma v_3\$w\$u'$. Thus, it proves that if $h^{-1}(v_n \dots v_1)$ is of the form vBA, for some $v \in (\{BA,A\} \cup T)^*$, then u is of the form ABu', for some $u' \in \{AB,A\}^*$.

(ii) If $v_2 = v_3XA$, for some $v_3 \in \{BA,A\}^*$, $X \in \{A, \varepsilon\}$ and $X = \varepsilon$ if and only if $v_2 = A$, then only production 7 is feasible, i.e., $u \in \{Au', ABu' : u' \in \{AB,A\}^*\}$. Let $u'' \in \{u', Bu'\}$, then $v_1\gamma v_3XA\$w\$Au'' \Rightarrow v_1\gamma v_3X\$w\$u''$. Assume that $u'' = Bu'$, then the sentential form is either $(\{BA,A\} \cup \{B\}T\{B\})^*A\$w\$B\{AB,A\}^*$, or $(\{BA,A\} \cup \{B\}T\{B\})^*BaB\$w\$B\{AB,A\}^*$. In both cases, however, we get a nonterminal between \$s, i.e., $u'' = u'$. Thus, it proves that if $h^{-1}(v_n \dots v_1)$ is of the form vA, for some $v \in (\{BA,A\} \cup T)^*$, then u is of the form Au', for some $u' \in \{AB,A\}^*$.

By induction, the nonterminal string v_3 (v_3X) can be eliminated, i.e.,

$$v_1\gamma v_2\$w\$u \Rightarrow^* v_1\gamma\$w\$u',$$

where $u' \in \{AB,A\}^*$, which proves that if $h^{-1}(v_n\dots v_1)$ is of the form vw, for some $v \in \{BA,A\}^*$ and $w \in T^*$, then $u_1u_2\dots u_n = v^R$.

By the above, the derivation eliminates v_2, i.e.,

$$v_1BcBv_2\$aw\$u_1u_2\dots u_n \Rightarrow^* v_1BcB\$aw\$u,$$

for some $u \in \{AB,A\}^*$. Then, the derivation is blocked because BB is not a substring of u. Therefore, $v_2 = \varepsilon$.

Note that the case $v_n\dots v_1 \in \{vBA, vXA : v \in \{BA,A\}^*, X \in \{A,\varepsilon\}$ and $X = \varepsilon$ if and only if $v = \varepsilon\}$ has been examined above.

Thus, we have proved that $h^{-1}(v_n\dots v_1) \in \{AB,A\}^*T^*$ and that if there is a terminal derivation of G,

$$\begin{aligned} S &\Rightarrow^* v_n\dots v_2v_1BB\$\$BBu_1u_2\dots u_n \text{ (by productions 1–3)} \\ &\Rightarrow^* a_1\dots a_k \text{ (by productions 4–7)}, \end{aligned}$$

for some $n \geq 1$, $k \geq 0$ ($k = 0$ implies $a_1\dots a_k = \varepsilon$), where $u_i \in \{AB,A\}^*$, $a_i \in T$, for all $i = 1,\dots,n$, and $v_n\dots v_1 \in \{BA,A\}^*(\{B\}T\{B\})^*$, then

$$\begin{aligned} S' &\Rightarrow^* u_1u_2\dots u_n\$h^{-1}(v_n\dots v_2v_1) \\ &\Rightarrow^* a_1a_2\dots a_k \end{aligned}$$

is a terminal derivation of G'. □

4 Summary

The following results are proved in [7], [6], Theorem 2, and [10], respectively.

Theorem 3

(1) $\mathscr{L}_{CS} \not\subseteq \mathscr{L}_{SC}(1,\infty) \subset \mathscr{L}_{RE}$.
(2) $\mathscr{L}_{RE} = \mathscr{L}_{SC}(3,\infty)$.
(3) $\mathscr{L}_{RE} = \mathscr{L}_{SC}(4,3)$.
(4) $\mathscr{L}_{RE} = \mathscr{L}_{SC}(5,2)$.

Open Problems

(1) $\mathscr{L}_{SC}(1,\infty) \subset \mathscr{L}_{CS}$?
(2) $\mathscr{L}_{SC}(2,\infty) = \mathscr{L}_{RE}$?
(3) $\mathscr{L}_{SC}(\infty,1) = \mathscr{L}_{RE}$?
(4) Is there $m \geq 0$ such that $\mathscr{L}_{RE} = \mathscr{L}_{SC}(3,k)$, for some $k \leq m$?
(5) Can some analogous results be proved for propagating scattered context grammars?
(6) Is the generative power of propagating scattered context grammars equal to the power of context-sensitive grammars?
(7) Are propagating scattered context grammars closed under complement?

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