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Bounded Number of Parallel Productions in Scattered Context Grammars with Three Nonterminals

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Abstract. Scattered context grammars with three nonterminals are known to be computationally complete. So far, however, it was an open problem whether the number of parallel productions can be bounded along with three nonterminals. In this paper, we prove that every recursively enumerable language is generated by a scattered context grammar with three nonterminals and five parallel productions, each of which simultaneously rewrites no more than nine nonterminals.

Keywords: Scattered context grammars, parallel productions, descriptional complexity, generative power.

1. Introduction

Scattered context grammars (SCGs), introduced in [4] and also studied in e.g. [1, 3, 9, 12, 13, 16], are rewriting devices based on context-free productions which simultaneously rewrite a finite number of nonterminals in one derivation step. Although these grammars are originally introduced without erasing productions (so-called propagating or nonerasing, the latter of which is preferred in this paper) and shown to generate only context sensitive languages, it is known that allowing erasing productions makes SCGs computationally complete. In what follows, erasing productions are implicitly considered.

Concerning the descriptional complexity, Meduna [10] proved that SCGs with three nonterminals are computationally complete. In his construction, however, the number of parallel productions (those which simultaneously rewrite more than one nonterminal) and the number of nonterminals simultaneously rewritten in one derivation step depend on the structure of the generated language. Specifically, the number of parallel productions is not limited at all, while the constructed SCG simultaneously rewrites more than $2n + 4$ nonterminals in almost all derivation steps of any successful derivation, for some

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n strictly greater than the number of terminals of the generated language plus two. Later, Vaszil [15] presented a construction bounding the number of parallel productions to two and the number of simultaneously rewritten nonterminals to four. However, this construction requires five nonterminals. Although this construction has been improved since then (in the sense of the number of nonterminals, see [7]), three nonterminals along with the bounded number of parallel productions were not achieved. Recently, the number of nonterminals simultaneously rewritten by parallel productions has been bounded to nine along with three nonterminals (see [8]), while the number of parallel productions was still unbounded.

In this paper, we prove that every recursively enumerable language is generated by a SCG with three nonterminals and five parallel productions, each of which simultaneously rewrites no more than nine nonterminals. Note that these numbers are only upper bounds and the question of whether this result can be improved is open. On the other hand, the lower bound on the number of nonterminals and/or parallel productions required by SCGs to be computationally complete is not known. The only known result (see [11]) says that SCGs with one nonterminal are not able to generate the context sensitive language $\{a^{2^{2^n}} : n \geq 0\}$ (cf. Lemma 3.1).

2. Preliminaries and Definitions

In this paper, we assume that the reader is familiar with formal language theory (see [14]). For an alphabet (finite nonempty set) V, V^* represents the free monoid generated by V, where the unit is denoted by λ. Set $V^+ = V^* - \{\lambda\}$. For $w \in V^*$ and $a \in V$, let $|w|_a$ denote the number of occurrences of a in w and w^R denote the mirror image of w. Let **CF**, **CS**, and **RE** denote the families of context-free, context sensitive, and recursively enumerable languages, respectively.

A *scattered context grammar* (SCG) is a quadruple $G = (N, T, P, S)$, where N is the alphabet of nonterminals, T is the alphabet of terminals such that $N \cap T = \emptyset$, $S \in N$ is the start symbol, and P is a finite set of productions of the form $(A_1, A_2, \ldots, A_n) \to (x_1, x_2, \ldots, x_n)$, for some $n \geq 1$, where $A_i \in N$ and $x_i \in (N \cup T)^*$, for $i = 1, 2, \ldots, n$. If $n \geq 2$, the production is said to be *parallel*; otherwise, it is *context-free*. If for each $i = 1, 2, \ldots, n$, $x_i \neq \lambda$, the production is *nonerasing*; G is *nonerasing* if all its productions are nonerasing. For $u, v \in (N \cup T)^*$, $u \Rightarrow v$ provided that

1. $u = u_1 A_1 u_2 A_2 u_3 \ldots u_n A_n u_{n+1}$,
2. $v = u_1 x_1 u_2 x_2 u_3 \ldots u_n x_n u_{n+1}$, and
3. there is a production $(A_1, A_2, \ldots, A_n) \to (x_1, x_2, \ldots, x_n) \in P$,

where $u_i \in (N \cup T)^*$, for $i = 1, 2, \ldots, n+1$. The language of G is $L(G) = \{w \in T^* : S \Rightarrow^* w\}$, where $\Rightarrow^*$ is the reflexive and transitive closure of $\Rightarrow$. A (nonerasing) *scattered context language* is a language generated by a (nonerasing) SCG. By $\mathbf{SC}^\lambda(n, m, p)$ we denote the family of languages generated by SCGs with no more than n nonterminals and m parallel productions, each of which simultaneously rewrites no more than p nonterminals. Considering only nonerasing SCGs, λ is removed. If a bound is not considered or known, we write ∞ on the corresponding position.

3. Results

First, the following example presents a simple SCG generating a non-context-free language.

Example 3.1. Let $G = (\{S, A\}, \{a, b, c\}, \{(S) \to (AAA), (A, A, A) \to (aA, bA, cA), (A, A, A) \to (a, b, c)\}, S)$. As any successful derivation is of the form $S \Rightarrow AAA \Rightarrow^* a^{n-1}Ab^{n-1}Ac^{n-1}A \Rightarrow a^n b^n c^n$, we have $L(G) = \{a^n b^n c^n : n \geq 1\} \in \mathbf{SC}(2, 2, 3)$.

The following lemma shows a more complicated example of a nonerasing scattered context language. The proof can be found in [6] (a sketch of the proof can also be found in [8]).

Lemma 3.1. $\{a^{l^{k^n}} : n \geq 0\} \in \mathbf{CS} \cap \mathbf{SC}(12, 10, 4)$, for any $k, l \geq 2$.

Geffert [2] proved that for every $L \in \mathbf{RE}$, $L \subseteq T^*$, there are alphabets H and Γ with $T \subseteq \Gamma$, and homomorphisms $g, h : H^* \to \Gamma^*$ such that $L = \{w \in T^* : \text{there is } z \in H^+ \text{ such that } g(z) = wh(z)\}$. Latteux and Turakainen [5] proved that these homomorphisms can be nonerasing. By a technique analogous to the one used in the proof of Theorem 4 in [5], we code each $x \in \Gamma$ into $\varphi(x) = AB^iA$ such that $i > 0$ and $\varphi(x) \neq \varphi(y)$ whenever $x \neq y$; A and B are new symbols. Let $\overline{w}$ be a barred version of w, i.e., $\overline{w} = \bar{a}_1\bar{a}_2 \dots \bar{a}_n$ for $w = a_1a_2 \dots a_n$. Then, it is shown in [2, 5] that there is a grammar $G' = (\{S', A, B, \bar{A}, \bar{B}\}, T, P \cup \{A\bar{A} \to \lambda, B\bar{B} \to \lambda\}, S')$ with P containing the following types of context-free productions

$$S' \to aS'\overline{\varphi(a)}^R, \quad S' \to \varphi(g(y))S'\overline{\varphi(h(y))}^R, \quad S' \to \varphi(g(y))\overline{\varphi(h(y))}^R,$$

where $a \in T$ and $y \in H$, such that $L(G') = L$. In addition, every successful derivation of G' is of the form $S' \Rightarrow^* ww_1'S'w_2' \Rightarrow ww_1w_2$ generated by productions from P, where $w \in T^*$, $w_1 \in \{A, B\}^+$, $w_2 \in \{\bar{A}, \bar{B}\}^+$, and $ww_1w_2 \Rightarrow^* w$ by productions $A\bar{A} \to \lambda$ and $B\bar{B} \to \lambda$. Note that this form is similar to the one shown by Geffert [2] with the difference that in each production $S \to uSv$ or $S \to uv$, we have $u \neq \lambda \neq v$, which is important in what follows.

Theorem 3.1. $\mathbf{SC}^\lambda(3, 5, 9) = \mathbf{RE}$.

Proof:
Let $L \in \mathbf{RE}$, $L \subseteq T^*$, and $G' = (\{S', A, B, \bar{A}, \bar{B}\}, T, P' \cup \{A\bar{A} \to \lambda, B\bar{B} \to \lambda\}, S')$ be a grammar in the form mentioned above such that $L = L(G')$. Let $h : (\{A, B, \bar{A}, \bar{B}\} \cup T)^* \to (\{A, B\} \cup T)^*$ be a homomorphism defined as $h(A) = ABB$, $h(\bar{A}) = BBA$, $h(B) = h(\bar{B}) = BAB$, $h(a) = BaBA$, for $a \in T$. Define the SCG $G = (\{S, A, B\}, T, P, S)$ with P constructed as follows:

1. $(S) \to (SBBASABBSA)$
2. $(S) \to (h(a)Sh(u))$ if $S' \to aS'u \in P'$,
3. $(S) \to (h(v)Sh(u))$ if $S' \to vS'u \in P'$,
4. $(S, B, B, A, S, A, B, B, S) \to (\lambda, \lambda, \lambda, S, S, ABBS, \lambda, \lambda, \lambda)$,
5. $(S, B, B, A, S, A, B, B, S) \to (\lambda, \lambda, \lambda, S, S, S, \lambda, \lambda, \lambda)$,
6. $(S, A, B, B, S, B, B, A, S) \to (\lambda, \lambda, \lambda, S, S, S, \lambda, \lambda, \lambda)$,
7. $(S, B, A, B, S, B, A, B, S) \to (\lambda, \lambda, \lambda, S, S, S, \lambda, \lambda, \lambda)$,
8. $(S, S, S, A) \to (\lambda, \lambda, \lambda, \lambda)$.

To prove $L(G') \subseteq L(G)$, consider a successful derivation of G'. The beginning of such a derivation is of the form $S' \Rightarrow^* a_1a_2\dots a_nS'u \Rightarrow^* a_1a_2\dots a_n\bar{v}S'\bar{u}u \Rightarrow a_1a_2\dots a_nv'u'u$, for some $v' \in \{A,B\}^+$, $u'u \in \{\bar{A},\bar{B}\}^+$, and $a_i \in T$, for $i = 1,2,\dots,n$, $n \geq 0$. Let $v'u'u$ be of the form $Xv''YUu''V$, for some $v'' \in \{A,B\}^*$, $u'' \in \{\bar{A},\bar{B}\}^*$, $X,Y \in \{A,B\}$, and $U,V \in \{\bar{A},\bar{B}\}$. (Analogously for all strings of shorter length.) Then, the derivation proceeds by production $YU \to \lambda$, $YU \in \{A\bar{A}, B\bar{B}\}$, and $a_1a_2\dots a_nXv''YUu''V \Rightarrow a_1a_2\dots a_nXv''u''V$. Summarized, the derivation proceeds by a sequence $(YU \to \lambda)p_1p_2\dots p_r(XV \to \lambda)$ of productions $A\bar{A} \to \lambda$ and $B\bar{B} \to \lambda$, for some $r \geq 0$, i.e., $a_1a_2\dots a_nv'u'u = a_1a_2\dots a_nXv''YUu''V \Rightarrow a_1a_2\dots a_nXv''u''V \Rightarrow^* a_1a_2\dots a_nXV \Rightarrow a_1a_2\dots a_n$, which implies that $\overline{v'} = (u'u)^R$. Then, the derivation of the same string is simulated in G as follows. (Regular expressions appearing in the square brackets denote the productions applied in G.)

$$\begin{array}{rcl}
S & \Rightarrow & SBBASABBSA \quad [1] \\
 & \Rightarrow^* & SBBAh(a_1a_2\dots a_n)Sh(u)ABBSA \quad [2^*] \\
 & \Rightarrow^* & SBBAh(a_1a_2\dots a_n)h(v')Sh(u'u)ABBSA \quad [3^*] \\
 & \Rightarrow^* & a_1a_2\dots a_{n-1}SBa_nBAh(v')Sh(u'u)ABBSA \quad [4^*] \\
 & \Rightarrow & a_1a_2\dots a_nSh(v')Sh(u'u)SA \quad [5] \\
 & = & a_1a_2\dots a_nSh(X)h(v'')h(Y)Sh(U)h(u'')h(V)SA \\
 & \Rightarrow & a_1a_2\dots a_nSh(v'')h(Y)Sh(U)h(u'')SA \quad [6+7] \\
 & \Rightarrow^* & a_1a_2\dots a_nSh(Y)Sh(U)SA \quad [q_r\dots q_2q_1] \\
 & \Rightarrow & a_1a_2\dots a_nSSSA \quad [6+7] \\
 & \Rightarrow & a_1a_2\dots a_n \quad [8]
\end{array}$$

where for each $i = 1,2,\dots,r$,

$$q_i = \begin{cases} (S,A,B,B,S,B,B,A,S) \to (\lambda,\lambda,\lambda,S,S,S,\lambda,\lambda,\lambda) & \text{if } p_i = A\bar{A} \to \lambda, \\ (S,B,A,B,S,B,A,B,S) \to (\lambda,\lambda,\lambda,S,S,S,\lambda,\lambda,\lambda) & \text{otherwise.} \end{cases}$$

On the other hand, to prove $L(G) \subseteq L(G')$, let $S \Rightarrow^* x$ be a derivation of $x \in (\{S,A,B\} \cup T)^*$, and let i and j be numbers of applications of production 1 and 8, respectively, in that derivation. With respect to h and the form of productions, it can be seen that there exists $k \geq 0$ such that

$$|x|_B = 2k, \quad |x|_A = k + i - j, \quad |x|_S = 1 + 2i - 3j\,.$$

Assume that $x \in T^*$, then $|x|_A = |x|_B = |x|_S = 0$, which implies $2k = 0$ and $i = j = 1$, i.e., each of productions 1 and 8 is applied exactly once in each successful derivation. Clearly, production 8 is applied as the last production.

To prove that production 1 is applied as the first production, assume that a production constructed in 2 or 3 of the form $(S) \to (h(v)Sh(u))$, where $v \in \{A,B\}^+ \cup T$ and $u \in \{\bar{A},\bar{B}\}^+$, is applied first. As production 1 has to be applied to introduce two other Ss, consider the derivation from the beginning to the first application of production 1, i.e., $S \Rightarrow h(v)Sh(u) \Rightarrow^* h(v)h(v')SBBASABBSAh(u')h(u)$. As $h(v) \in \{ABB, BAB, BaBA : a \in T\}^+$ and $h(u) \in \{BBA, BAB\}^+$ are nonempty, and there is no production removing symbols occurring before the first or after the last S, neither $h(v)$ nor $h(u)$ can be eliminated—a contradiction; the derivation is not successful. Thus, any successful derivation of G is of the form

$$S \Rightarrow SBBASABBSA \Rightarrow^* w_1Sw_2Sw_3Sw_4A \Rightarrow w_1w_2w_3w_4\,,$$

for some terminal strings $w_1, w_2, w_3, w_4 \in T^*$.

Consider the beginning of a successful derivation of G of the form $S \Rightarrow SBBASABBSA \Rightarrow^* u_1Su_2Su_3Su_4A$. We need to prove that $u_1 \in T^*$, $u_2 \in (BBA+\lambda)\{BaBA : a \in T\}^*\{ABB, BAB\}^*$, $u_3 \in \{BAB, BBA\}^*(ABB + \lambda)$, and $u_4 = \lambda$. To do this, examine all the possible derivation steps from a sentential form

$$w_1Sw_2Sw_3SA\,, \tag{1}$$

for $w_1 \in T^*$, $w_2 \in (BBA+\lambda)\{ABB, BAB, BaBA : a \in T\}^*$, and $w_3 \in \{BAB, BBA\}^*(ABB+\lambda)$.

If a production constructed in 2 or 3 of the form $(S) \to (h(v)Sh(u))$ is applied, then there are the following possibilities: (i) $w_1Sw_2Sw_3SA \Rightarrow w_1h(v)Sh(u)w_2Sw_3SA$, and the derivation is not successful because $h(v) \in \{ABB, BAB, BaBA : a \in T\}^+$ cannot be eliminated; (ii) $w_1Sw_2Sw_3SA \Rightarrow w_1Sw_2h(v)Sh(u)w_3SA$, and the proof proceeds by induction because the sentential form is of the form (1); (iii) $w_1Sw_2Sw_3SA \Rightarrow w_1Sw_2Sw_3h(v)Sh(u)A$, and the derivation is not successful because $h(u) \in \{BAB, BBA\}^+$ cannot be eliminated. Thus, productions 2 and 3 can only be applied to the middle S.

Productions 4 to 7 are applied: Production 4 implies that $w_2 = BaBAw_2'$, $w_3 = w_3'ABB$, and $w_1Sw_2Sw_3SA \Rightarrow w_1aSw_2'Sw_3SA$; production 5 implies that $w_2 = BaBAw_2'$, $w_3 = w_3'ABB$, and $w_1Sw_2Sw_3SA \Rightarrow w_1aSw_2'Sw_3'SA$; production 6 implies that $w_2 = ABBw_2'$, $w_3 = w_3'BBA$, and $w_1Sw_2Sw_3SA \Rightarrow w_1Sw_2'Sw_3'SA$; and production 7 implies that $w_2 = BABw_2'$, $w_3 = w_3'BAB$, and $w_1Sw_2Sw_3SA \Rightarrow w_1Sw_2'Sw_3'SA$; for some $w_2' \in \{ABB, BAB, BbBA : b \in T\}^*$, $a \in T \cup \{\lambda\}$, and $w_3' \in \{BAB, BBA\}^*$; otherwise, A or B is moved before the first or after the last S, and the derivation is not successful. In all cases, the proof proceeds by induction.

Thus, we have shown that if $S \Rightarrow SBBASABBSA \Rightarrow^* u_1Su_2Su_3Su_4A$, then $u_1 \in T^*$, $u_2 \in (BBA + \lambda)\{ABB, BAB, BaBA : a \in T\}^*$ and $u_3 \in \{BAB, BBA\}^*(ABB + \lambda)$ because if BBA (ABB) is once removed from the prefix $SBBA$ (suffix $ABBSu_4A$), then it can never be generated again between the first (second) and the second (third) S in any successful derivation, and $u_4 = \lambda$.

To complete the proof, it remains to show that $u_2 \in (BBA+\lambda)\{BaBA : a \in T\}^*\{ABB, BAB\}^*$. Consider the longest part of the successful derivation generated by production 1 followed by a sequence of productions 2 and 3, which is, according to the previous results, of the form

$$S \Rightarrow SBBASABBSA \Rightarrow^* wSBBAvSuABBSA\,,$$

where $w \in T^*$ (clearly, $w = \lambda$ here), $v \in \{ABB, BAB, BaBA : a \in T\}^*$, and $u \in \{BAB, BBA\}^*$. For the same reason as above, only productions 4 and 5 are applicable:

$$wSBBAvSuABBSA \;\Rightarrow\; wSvSuABBSA \quad [4] \tag{2}$$

$$wSBBAvSuABBSA \;\Rightarrow\; wSvSuSA \quad [5] \tag{3}$$

As productions 2 and 3 can be applied now, let

$$wSvSuABBSA \;\Rightarrow^*\; wSvv'Su'uABBSA \quad [(2+3)^*] \tag{4}$$

$$\text{resp.} \quad wSvSuSA \;\Rightarrow^*\; wSvv'Su'uSA \quad [(2+3)^*] \tag{5}$$

be the longest parts of the successful derivation generated by productions 2 and 3, i.e., the application of one of productions 4 to 8 follows. In addition, u, v, w are as above, and from the form of productions 2 and 3 we have that $v' \in \{ABB, BAB, BaBA : a \in T\}^*$ and $u' \in \{BAB, BBA\}^*$.

I. In derivation (4), each of productions 6, 7, and 8 leads to an incorrect sentential form because after the application of any of these productions, BA or B is the suffix of the sentential form. Thus, either production 4 or 5 has to be applied. It implies that vv' is of the form $BaBAv''$, for some $a \in T$ and $v'' \in \{ABB, BAB, BbBA : b \in T\}^*$, i.e.,

$$wSBaBAv''Su'uABBSA \ \Rightarrow\ waSv''Su'uABBSA \quad [4] \tag{6}$$

and the derivation proceeds as in (4), or

$$wSBaBAv''Su'uABBSA \ \Rightarrow\ waSv''Su'uSA \quad [5] \tag{7}$$

and the derivation proceeds as in (5). By induction,

$$wSBBAvSuABBSA \ \Rightarrow^*\ ww'Sv'''Su'''SA \quad [(2+3+4)^*5]\,, \tag{8}$$

for $ww' \in T^*$, $v \in \{BaBA : a \in T\}^*\{v'''\}$, $v''' \in \{ABB, BAB, BaBA : a \in T\}^*$, and $u''' \in \{BAB, BBA\}^*$.

II. In derivation (5), each of productions 4 and 5 leads to an incorrect sentential form, and production 8 finishes the derivation, i.e., $vv' = u'u = \lambda$. Assume that either production 6 or 7 is applied. Then, either $vv' = ABBv''$ and $u'u = u''BBA$, or $vv' = BABv''$ and $u'u = u''BAB$, i.e.,

$$wSABBv''Su''BBASA \ \Rightarrow\ wSv''Su''SA \quad [6] \tag{9}$$

$$\text{or}\qquad wSBABv''Su''BABSA \ \Rightarrow\ wSv''Su''SA \quad [7] \tag{10}$$

and the derivation proceeds as in (5).

Note that the application of production 2 would lead, in its consequence, to an incorrect sentential form because the derivation would reach one of the sentential forms $wSBaBAxSyBABSA$ or $wSBaBAxSyBBASA$, and productions 6 and 7 would move B to the left of the first S.

By induction, the successful derivation proceeds as

$$wSvSuSA \ \Rightarrow^*\ wSSSA \Rightarrow w \quad [(3+6+7)^*8]\,. \tag{11}$$

Thus, we have shown that the sequence $(6+7)(2+3)^*(4+5)$ of productions cannot be applied in any successful derivation of G. Therefore, all applications of productions 4 and 5 precede any application of productions 6 and 7, which means that $u_2 \in (BBA + \lambda)\{BaBA : a \in T\}^*\{ABB, BAB\}^*$.

Finally, skipping all productions 6 and 7 in the considered successful derivation $S \Rightarrow^* w$, we have

$$\begin{aligned} S &\Rightarrow SBBASABBSA \quad [1] \\ &\Rightarrow^* wSh(v)Sh(u)SA \quad [(2+3+4)^*53^*] \\ &\Rightarrow wh(vu) \quad [8]\,, \end{aligned}$$

where $w \in T^*$, $h(v) \in \{ABB, BAB\}^+$, $h(u) \in \{BAB, BBA\}^+$, and $h(v) = h(u)^R$ (see **II** above). Then, by applications of the corresponding productions constructed in 2 and 3, ignoring productions 4 and 5, and applying $S' \to xy$ in the last application of production 3 of the form $(S) \to (h(x)Sh(y))$, we have that $S' \Rightarrow^* wvu$ in G'. As $h(v) = h(u)^R$, we have (by the definition of h) that $\overline{v} = u^R$, and therefore $wvu \Rightarrow^* w$ by productions $A\bar{A} \to \lambda$ and $B\bar{B} \to \lambda$, which completes the proof. □

4. Conclusion and Discussion

We have improved the descriptional complexity of scattered context grammars with three nonterminals by showing that $\mathbf{SC}^{\lambda}(3,5,9) = \mathbf{RE}$. However, we have not proved the optimality, so it is open whether this result can be improved. In what follows, we give a brief overview of the latest results and open problems concerning the descriptional complexity of SCGs.

1. It is shown in [11] that $\{a^{2^{2^n}} : n \geq 0\} \notin \mathbf{SC}^{\lambda}(1,\infty,\infty)$. On the other hand, it is open (due to erasing productions) whether $\mathbf{SC}^{\lambda}(1,\infty,\infty) - \mathbf{CS} = \emptyset$.
2. So far, we only know $\mathbf{CF} \subset \mathbf{SC}^{\lambda}(2,\infty,\infty) \subseteq \mathbf{RE}$. The proper inclusion is shown in Example 3.1.
3. It is shown in [7] that $\mathbf{SC}^{\lambda}(4,3,6) = \mathbf{RE}$.
4. It is shown in [15] that $\mathbf{SC}^{\lambda}(5,2,4) = \mathbf{RE}$.
5. What is the generative power of SCGs with only one parallel production?
6. We know $\mathbf{SC}(\infty,\infty,\infty) \subseteq \mathbf{CS}$. However, is this inclusion proper? Are there n, m, p such that $\mathbf{SC}(\infty,\infty,\infty) \subseteq \mathbf{SC}(n,m,p)$?

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