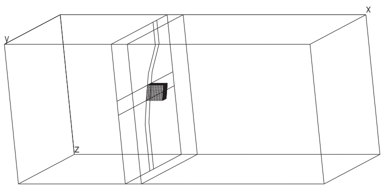
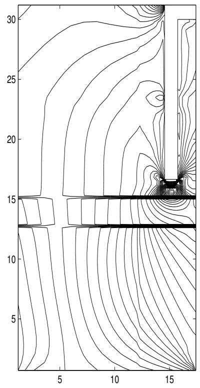
Project COPERNICUS KIT 977006

High Performance Computing in Geosciences II

HIPERGEOS II

Final report





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1 Introduction

The INCO KIT 977006 project *High Performance Computing in Geosciences II* (HIPERGEOS II) is a continuation, *Keep-in-touch (KIT)* of COPERNICUS 940820 project. The previous (COPERNICUS) project falls in the framework of the theme *High Performance Computing and Networking (HPCN)* which aims at exploitation of the opportunities of HPCN and expansion of the HPCN potential towards industry.

The KIT project is oriented towards research and development of numerical solution methods suitable for applications in geosciences, namely, research in the finite element technology and solutions of large scale linear and nonlinear systems arising from the finite element analysis. The methods under investigation are implemented in computer programs and tested on various large-scale real-life problems of geosciences. The testing makes use of benchmark problems reflecting the practical requirements of the industrial partners, who were directly involved in the COPERNICUS 940820 project.

The KIT project aims at continuing the research in efficient numerical methods as well as at keeping the scientific contacts among the participating academic groups, which in our case include University of Nijmegen (head O. Axelsson), Free University of Brussels (head R. Beauwens), Central laboratory for Parallel Computing of the Bulgarian Academy of Sciences Sofia (head S. Margenov) and two institutes of the Academy of Sciences of the Czech Republic: Institute of Computer Sciences, Prague (head J. Nedoma) and Institute of Geonics, Ostrava (head R. Blaheta).

The collaboration of the participating groups was supported by mutual visits, participation at conferences as well as by organizing special meetings and workshops, listed below:

- HIPERGEOS Workshop at Sozopol, Bulgaria: June 5, 1999
- HIPERGEOS Workshop at Pilsen/Nečtiny, CR: July 7, 1999
- HIPERGEOS Workshop at Lausanne, Switzerland: August 22, 2000
- HIPERGEOS Workshop at Sozopol, Bulgaria: June 6-10, 2001
- HIPERGEOS Workshop at Pilsen, Czech Republic: June 25-29, 2001

1.1 Project objectives

Many human activities in the Earth, such as underground constructions, mining of raw materials, underground storage of oil, gas or waste, require deep understanding of various phenomena, which are caused by these activities or exist in the Earth. Therefore, there is a strong interest in simulation or modelling these phenomena, in order to better understand and predict deformations and stresses in geological media, flow of underground water and propagation of contaminants or heat transfer.

Modelling is also required for various natural processes in the Earth, such as movements of lithospheric plates, which result in tectonic stresses and determine seismicity. The understanding of these tectonic and geodynamic processes is very important for the design of large underground structures like mines, radioactive and toxic industrial waste repositories, bridges. The possibility of their destruction by earthquake or tectonic stresses, which are not properly treated, endangers to a great extent large areas of the Earth.

Large-scale simulations are also required in foundation engineering, where the understanding of the phenomena in soils is important for a safe and economical design of foundations of bridges and large constructions.

Reliable numerical simulations of the described phenomena require to (re)formulate and improve the presently used mathematical models and to develop efficient numerical methods for their solution. In order to get a reasonable turnaround time of large-scale and/or very complex models, the application of very powerful solution methods with a nearly optimal computational complexity (i.e. proportional to the degrees of freedom) and their efficient implementation on high performance computers is crucial.

This project is a platform for interaction between experts in numerical mathematics, computer science and mathematical modelling in geosciences on both theoretical and applied level to achieve a progress in large-scale simulations and taking advantage of the synergetic effect. The major objective of the HIPERGEOS II project is to *develop high performance and robust numerical solution methods* applicable to the solution of problems, which arise from mathematical modelling in geosciences. The development of these numerical methods is accompanied by their implementation on the advanced high performance computer systems including the parallel ones.

1.2 Structure of this report

This report is organized as follows. The next section includes a description of the research work that has been accomplished during the project. Subsections 2.1 - 2.5 follow the structure of the suggested final deliverables from the project proposal. Each subsection contains its own references, which list the main publications prepared in the course of the solution of the project.

As appendices, we include three technical reports, which describe the application of the selected methods to the solution of the defined benchmark problems. These appendices are directly related to Deliverables 2.1, 2.2, 2.4 and 2.5. We include also a report entitled "Applications of large-scale models of processes in the lithosphere: Engineering analysis" by R. Beauwens and J. Nedoma, which is tied to Deliverable 2.3 which is not directly included in the final report due to its large size (65 pages).

The conclusions and final remarks are given in Section 3.

2 Final deliverables and summary of project results

In this section, we summarize the project results. More details can be found in the references and in Appendices 1 - 3.

2.1 Performance analysis of solvers for nonlinear problems

In many applications of mathematical modelling in geosciences, it is necessary to consider nonlinear effects and solve large nonlinear systems arising from discretization of the corresponding boundary value problems. Such a situation is quite common in geomechanics, where it may be necessary to consider nonlinear behaviour of soils and rocks. For the solution of the nonlinear systems, we need suitable solvers, which are efficient, robust and parameter free or only weekly depending on a few control parameters.

In this project we consider mostly the solution of the nonlinear problems by numerical methods based on linearization. Namely, we consider damped inexact Newton methods with a suitable damping strategy or continuation methods with an adaptive procedure when defining the continuation steps. Such methods are studied in [1], which presents a unified background for Newton type methods and, in particular, gives a survey of approximate (inexact) Newton (continuation) methods. Methods of this type are required when solving ill-conditioned problems where it is difficult (or even impossible) to find an initial approximation, which satisfies the Kantorovich convergence conditions.

A generalization of a damped inexact Newton method has been considered which has been implemented via a two-level Newton method and applied for problems arising in multi-layer soil mechanics ([2]). The two-level method utilizes first a much coarser initial mesh where the nonlinear equation can be solved more accurately with relatively little costs. In order to improve the discretization error, the solution of the problem is next interpolated to the fine mesh to serve as a good initial approximation and then solved using Newton linearization. It can be proven that within this framework it suffices with only one or two Newton steps on the fine mesh to recover the discretization error on this fine mesh. Numerical tests suggest that when we are interested in the minimization of the least-squares (L_2-norm) error, rather than in minimization of the residual norm, this method is much more efficient than if we solve the corresponding nonlinear system directly (using general Newton linearization), without utilizing a coarse mesh. Moreover, the two-level method framework is especially efficient for quadratic (or higher order) finite element methods, the use of which leads to a further reduction of the degrees of freedom. See also the report [3] prepared within the COPERNICUS 940820 project.

The solvers based on the damped inexact Newton method and continuation method with an adaptive procedure for defining the size of the continuation steps were applied to the solution of benchmark problems arising from the bridge foundations. The tested solvers also utilized a new condition of sufficient accuracy of the linearization. The testing, which is described in [4] (see also Appendix 1), shows efficiency and robustness of this type of solvers.

Another types of nonlinearities can be found in the modelling of movements of lithospheric plates and related problems. In this field, one must deal with coercive and semicoercive contact problems with friction in linear elasticity as well as contact problems in non-linear elastic rheology. The analysis of the mathematical formulations (existence and properties of the weak formulation and its FE approximation can be found in [7], [14] and [15]. For the solution of the unilateral contact problems with small contact zone in 2D elasticity, a domain decomposition method suitable for parallel implementation on a cluster computer system or other parallel computers was developed, see [9] and [13]. This method was found also applicable to bridge engineering problems, see [12].

Besides contact problems, an intensive work was carried out at the ICS AS CR, on the numerical solution of the Stefan problems and coupled Stefan-like problem in Bingham rheology and linear thermo-elasticity. The described work at the ICS AS CR results in development of 2D and 3D finite element packages for contact problems and for the coupled contact-Stefan-like problems. All software modules are written in Fortran and/or Pascal languages and can be run on various PC computers.

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2.2 Performance analysis of the developed parallel algorithms and codes

The power of the contemporary computers is a combination of both the raw power of the individual processing units and the possibility to perform parallel computing on those, connected in a multi-processor system. The parallel approach, which is widely used in the area of HPCN, requires specific numerical methods and techniques. Within this project, we investigate several classes of iterative methods suitable for parallel computations. Besides suitability for parallelization, another important question concerning iterative methods is their robustness, i.e. high efficiency for a sufficiently large class of target problems. We touch on this question later.

The first class of iterative methods, which was investigated and tested, are iterative methods based on space decomposition - subspace correction strategy. The space decomposition is performed via displacement decomposition, domain decomposition or decomposition of composite mesh FE spaces. The subproblems on the subspaces are allowed to be solved by inner iterations, which is suitable in many respects including balancing of the ratio between computations and communications during the parallel processing. The space decomposition preconditioners were implemented within the framework of the standard or generalized preconditioned conjugate gradient (CG) methods. The application of these methods to solve the benchmark problems from the large-scale modelling of the effects caused by mining is described in Appendix 2, see also Section 2.4 and the references therein.

Another parallel iterative method is described and tested in [12]. The method is applied to elasticity problems and uses a block diagonal preconditioner constructed by the displacement decomposition and circulant block factorization. The method was implemented and tested on the solution of 3D pile foundation problem with up to 786 thousands of DOF. The tests run on several parallel computers.

For the efficient solution of equations on parallel computers it is also crucial to make a *good* (well-balanced) partitioning of the mesh in substructures of about equal size enabling in this way a good load balance for the processors working in parallel. This task is far from trivial when one deals with adaptively constructed and highly nonuniform grids. An attempt to solve this can also be found in [8]. There various partitioning strategies were tested on the grids obtained from some of the real-life problems studied in [3, 4].

Much research effort during this project has been devoted to the construction of robust preconditioning methods, that is, methods which reduce the condition number of the problem (resp. the system matrix) significantly and for which bounds hold, such that they neither depend on size and shape of elements used, nor on problem parameters, for instance jump in material coefficients, anisotropy, near incompressibility, etc.

One such class of robust preconditioning methods is based on subspace projection method, whereby a set of vectors (forming a rectangular matrix) is constructed and used in an approximate projection method. The latter is defined in such a way that the subspace spanned by those vectors (approximately) includes the subspace spanned by the eigenvectors of the given matrix (or of an a priori preconditioned matrix) corresponding to the smallest eigenvectors. This enables us to "move" the eigenvalues to a cluster of bigger eigenvalues, in this way significantly reducing the condition number.

This method has been analysed and tested on a set of large scale real-life benchmark problems, see [2] and [3, 4].

In a separate publication, the method has been analysed theoretically and shown to belong to a class of methods which minimize the K-condition number (i.e., the nth power of the quotient of the arithmetic and geometric averages of the eigenvalues), see [6].

Further, a two-level preconditioning method has been analysed for three-dimensional elasticity problems, see [2]. A robust version of such a method has been constructed and analysed in [7].

A continuation and further development of the research on constructing robust preconditioning techniques for the iterative solution of systems of equations with matrices obtained by finite element method discretization of linear elasticity problems has been accomplished. The obtained results were reported at 3rd International Conference on "Large-Scale Scientific Computations", Sozopol, Bulgaria, June 6-10, 2001 and later published in [1]. The approach there fits in the general *coarse-fine mesh* framework, it is generally applicable and suitable for implementation on high performance parallel computer platforms. The improved preconditioning techniques were tested and compared with the already reported results in already reported in the Second KUN Progress report from September 2000 and in [3, 4] on the same collection of benchmark problems and showed a considerable improvements in computer performance.

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2.3 Applications of large-scale models of processes in the lithosphere

The main objective of this part of the research is to develop complete geodynamical models, which incorporate coupled geodynamical, geomechanical and geothermal effects. Geothermal effects are found to be important for lithospheric formations because the geothermal diffusion produces stress alterations in upper and deeper parts of the lithosphere, which occur relatively quickly and meaningfully contribute to the stress field. These effects are observed namely in anomalous geothermal zones characterized e.g. by the great volcanism. On the contrary, the deformations of rocks and the friction acting on the contact boundaries among geological bodies represent an important heat source, which can also remarkably influence the quality of rocks, which can recrystallize, for instance. Consequently, we have to formulate and solve fully coupled thermo-mechanical problems with contact conditions among separated geological bodies. Moreover, the phase transitions due to produced heat need to formulate Stefan-like problems.

One type of the considered applications is modelling of the global tectonic processes. In the last three years, we investigated the interaction between the South American and Nazca plates, which form one of the most typical Wadati-Benioff zone existing on the Earth. A detailed examination of this zone shows some peculiarities that cause a controversy in the interpretation of its own structure, both in the depth and lateral directions, and in a satisfactory explanation of some of the related phenomena, such as the origin and the mechanism responsible for the deep earthquakes in South America. There are also important controversies about the seismotectonic characteristics in the Andean region, in relation to the morphology of the Wadati-Benioff zone, all along the interaction regions between the plates. Knowledge of movements of both global lithospheric plates as well as of their sub-blocks are very important information for mathematical modelling and mathematical simulations of plate tectonic processes in the investigated region. The numerical analyses brings recognition and new information about the tectonic taking place in this active subduction zone. Namely the mathematical modelling can reveal interplate stresses and aseismic zones observed in the Wadati-Benioff zone as well as the origin of the volcanism and the mechanism of the earthquakes, see Berrocal, Nedoma, Kestřánek (2000), (2001).

The model problem, which was formulated for the investigation of the origin of volcanism in the region of Central Aleutian, is based on the contact-Stefan-like problem. The model problem is essentially simplified. We assumed that there are heat sources in the lithosphere and no heat flux among the neighbouring outside areas and the investigated region. The deformed lithosphere evoked a dissipative heat and a temperature increase, which result in the interplate stresses of the extension type in the lithospheric rocks. First results obtained through this model were published on the conference GEONICS'2001, see Nedoma (2001).

Applications in Civil Engineering. The modelling of the global tectonic and geomechanical processes in the upper parts of the Earth can be exploited for the assessment of behaviour and safety of large structures, as bridges, dams, tunnels or e.g. underground nuclear waste repositories, because all these structures are significantly influenced by the global tectonic and geomechanical processes in the upper parts of the Earth. For the construction of mathematical models of such structures, the influence of critical conditions, global tectonics and tectonic stresses must be taken into the account. The assessment of the influences is again based on the computation of the stress-strain fields in the studied regions, which demand the solution of large 2D and 3D contact problems in elasticity and thermo-elasticity with necessity of local refinement, see Nedoma (2001). It is also important to evaluate the uncertainties in input data. One method for such an evaluation is investigated in Hlaváček and Nedoma (2001), (2002).

The numerical methods for the solution of contact problems can be also exploited for the direct investigation of various civil engineering problems. As an example, we can mention the investigation of the tunnel for water conveying from the water dam to the hydroelectric power stations, which was performed in the Institute of Computer Sciences AS CR in cooperation with CMRI-CBRI Roorkee, India. The considered tunnel is crossed by active deep faults and this situation can be modelled as a contact problem in elasticity. Another analyses clear up the properties and behaviours of the loaded bridge. This modelling leads to the solution of large scale 2D contact problems in elasticity. For such problems the domain decomposition solver was developed, see Hlaváček (1999), Daněk (2001a,b).

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2.4 Application of a large-scale 3D code for the design of new extraction technologies in mining

Mathematical modelling of geomechanical effects induced by mining is mostly based on the finite element solution of boundary value problems. These FE computations are frequently characterized by considerable demands on computing resources, mainly due to the following reasons:

- the need to consider large 3D domains for the solution of far field problems as well as for proper specification of boundary conditions induced by the virgin stress in rocks or soils,
- the need to solve of a number of variants corresponding to the various construction stages, different input data (mostly given with a considerable uncertainty) as well as to optimization of the design.

For the testing of the developed numerical methods, we formulated three types of benchmarks.

a) FOOT - a scalable benchmark problem with an adjustable number of DOF representing footing problems. It has a simple geometry, nonuniform rectangular grid and homogeneous isotropic elastic material, see [5], [9].

b) BM1 - representing a large scale linear system (about 4 million of DOF), which arise from the modelling of stress changes due to mining at the deposit Rožná, see [1], [2] and the Appendix 2.

c) XDR - a refinement of the BM1 problem resulting in about 8 million DOF.

The testing was performed on the following computer platforms:

1. parallel multicomputer : IBM SP at TU Ostrava, see [1], [2]
2. symmetric multiprocessors : SUN HPC 6500 with 18 processors at EPCC Edinburgh, see [9], IBM RS/6000 with 4 processors at TU Ostrava,
3. clusters of PC : TU Ostrava, EPCC Edinburgh, SARA Amsterdam, IGAS Ostrava.

The recent tests performed on a Beowulf cluster newly assembled at the Institute of Geonics AS CR are described in Appendix 2. These results confirm the great computing power and the usefulness of this new computer platform based on commodity (PC) components, which is additionally inexpensive and widely available, thus ensure very good price-performance ratio.

The linear systems were solved by the displacement decomposition and domain decomposition methods with inexact solution of the arising subproblems. The solution of subproblems was carried out through their replacement by incomplete factorization or through their solution by inner PCG iterations. In the latter case, we used the generalized PCG in the outer iterations. More details can be found in [6], [8], [12] and [10], see also Appendix 2.

Besides the problems arising from a FE discretization with a global FE mesh, we considered also the composite mesh technique, which turns out to be very appropriate for the solution of multi-scale geomechanical problems. The iterative composite mesh methods are investigated in [4], [7]. The theory of two-level composite mesh methods exploits estimates of the C.B.S. constant, which were studied in [11]. A benchmark concerning the use of the composite meshes, made of a global problem with about 4 million DOF and a local problem with about 1 million DOF, is described in [10] see also Appendix 2.

An additional effort was also devoted to 3D finite element mesh generation and aposteriori evaluation of the discretization error, see [3].

The tested methods were used not only for solving the benchmark problems but also for complete modelling of the stress changes induced by mining at the deposit Rožná. This modelling was ordered by the mining company GEAM and the results of this modelling were found to be useful and novel from the point of view of practical geomechanical engineering. It is confirmed by a continuing interest of the GEAM company in the modelling, which were performed during the COPERNICUS 940820 project, secondly in the year 1999 and which will continue during this year.

A further development will be more focused on methods, which are efficient for computing on Beowulf clusters. The existing experience will be also utilized and extended for developing efficient methods for the solution of thermo-mechanical problems within a newly started project of the Grant Agency of the CR "Modelling of thermo-mechanical processes connected to the underground deposition of the spent nuclear fuel".

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2.5 Application of large-scale modelling in bridge engineering

The three dimensional computer simulation of the stressed-strained field around pile foundation systems in weak multi-layer soil media is often a bottleneck phase in the design of foundation systems for large-scale engineering constructions. The impact of the large-scale computer simulation on the improvement of the reliability and cost-efficiency level of the design process is expressed in the terms of: a) better understanding of the spatial behaviour of the zones of active interactions between the piles and the surrounding soil layers; b) abilities for multiple solutions of different variants applying also adaptive refinement strategies in subdomains where the stresses are near to their critical levels.

The computer simulation of foundation systems was considered during the period of this project by nearly all of the involved research groups. Our test problems are taken from the bridge engineering practice. Some of the obtained results were published in a number of papers and reports, see e.g. [1, 2, 3, 4, 5, 6, 7, 8]. Here we describe the results presented in the last deliverable report [2] which is directly related to the topic of this section.

Nonconforming finite elements based on *rotated* multilinear shape functions were first introduced by Rannacher and Turek as a class of robust elements for the Stokes problem. More generally, the recent activities in the development of efficient solution methods for nonconforming finite element systems are inspired by their attractive properties as a stable discretization tools for ill-conditioned problems. The application of nonconforming elements for numerical simulation of foundation systems is motivated by the purpose better to approximate the solution of the Lame system of partial differential equations with storing coefficient jumps. Note that a jump of the elasticity modulus of order $10^3 - 10^4$ is the typical case when pile systems in weak soil layers are considered. Two variants of rotated trilinear elements have been implemented, where the nodal basis functions correspond respectively to midpoint and integral mid-value interpolation operators. Some accuracy advantages of the second approach has been observed.

The so-called Reduced and Selective Integration (RSI) FEM discretization of the problem is implemented in the developed algorithm. The key point here is the introduced L_2-orthogonal projection onto Q^H, the space of piecewise constant functions on a certain coarser decomposition Ω^H of Ω. It is important to note that the straightforward low order non-conforming FEM discretization of the elasticity problem is ill posed since the obtained discrete problem does not satisfy the Korn's second inequality. This disadvantage is overcome by the $I_L^{Q^H}$ projection which is equivalent to a mixed formulation with respect to the space pair $(V_E^h \times Q^H)$.

A new code in Fortran was created. It was evaluated to be a a better variant than the already available one for three-dimensional problems of linear and nonlinear elasticity using trilinear conforming brick elements. The computational efficiency of the code is due to a very careful handling of the sparsity structure of the corresponding stiffness matrix. It is important to note that, as a general property, the number of the nonzero entries of the global stiffness matrix is very large when RSI FEM is implemented. The variant we have proposed avoids this disadvantage which is confirmed by the derived computational complexity estimates. The performance the code is clearly illustrated by the real large-scale problems solved where the degrees of freedom are above $18 * 10^6$. This is approximately $2 * 100^2$ times more than the size of the largest benchmark related to the pile simulation, which we have considered at the end of the first HIPERGEOS Project. This progress was achieved mainly through a series of improvements of the algorithm.

The following benchmarks are included in the report [2]:

Benchmark 1. Computer simulation of the stressed-strained state of a soil body under a *square footing* is considered. The vertical external load is uniformly distributed across a square seal of the soil body surface. The mechanical characteristics $E_s = 10MPa$ and $\nu_s = 0.35$ correspond to a *softly plastic clay* layer. The behaviour of the solver is examined varying the size of the discrete problem and the ratio of mesh anisotropy.

Benchmark 2. The interaction between a soil media and a foundation element with varying elasticity modulus is considered. The model characteristics of the foundation are taken as $E_p \in \{10MPa, 100MPa, 1000MPa, 10\,000MPa\}$ and $\nu_p = 0.2$ while these of the soil are as in Benchmark 1. A vertical external load is applied uniformly distributed across the top side of the foundation. In the finest grid case , and when $E_p = 10\,000 MPa$, this benchmark could be associated with a computer simulation of a single pile in a weak soil layer. The main goal of this benchmark is to verify the robustness of the code with respect to both the coefficient jumps and the increasing size of the subdomain where we have this jumps. It is important to note that the cross section of the foundation element in this benchmark is the minimal one as as determined by size of the mesh parameter.

Benchmark 3. Computer simulation of a single pile in a weak soil layer. The domain $\Omega = 34.8m \times 34.8m \times 31m$ and the size of the pile is $1.2m \times 1.2m \times 15m$. The related coefficients of the soil and of the pile are respectively: $E_s = 19.88MPa, \nu_s = 0.42$; and $E_p = 31.5GPa, \nu_p = 0.2$. This is one of the approved benchmarks within the frame of the first HIPERGEOS Project. The new point of the results presented in [2] is to show the efficiency of the locally refined mesh around the pile. To make such a conclusion we have compared the numerical results obtained on three consecutive meshes where the discrete problem on the finest one has more than $18 * 10^6$ unknowns.

Benchmark 4. Computer simulation of a pile foundation system in a multi-layer soil media. A system of four piles connected on the top by a linking plate is considered. This is a fully real-life engineering problem well illustrating the increased potential of the developed code. The symmetry of the construction is taken into account allowing for reduced computations of a pair of piles equipped with certain boundary conditions. The nonhomogeneous Neumann boundary conditions on the top-sides of the piles represent the loads of V_1=2000 kN, V_2=4000 kN and $H_1 = H_2$=150 kN, coming from the upper bridge construction. The soil media consists of four layers determined by the following mechanical characteristics: $E_{s1} = 20MPa$, $\nu_{s1} = 0.4$; $E_{s1} = 11MPa$, $\nu_{s1} = 0.35$; $E_{s1} = 7MPa$, $\nu_{s1} = 0.25$; $E_{s1} = 4.6MPa$, $\nu_{s1} = 0.2$. The mesh is properly refined around the piles. The presented selected set of plots well represent the complex spatial behaviour of the computed stressed-strained state of the interaction between the pile system and the surrounding multi-layer soil media.

Concluding Remarks: The analysis of the numerical tests confirms the robustness of the developed new algorithms and codes. The research team considers the obtained results as very prospective. One of the important steps in this field is to implement the accumulated *know how* to the numerical solution of coupled thermo-hydro-mechanical problems. Such a proposition is strongly motivated by the stable approximation properties of the nonconforming FEM with respect to all of the included phenomena. Such a complex treatment could be of a serious importance for various large scale engineering problems including such related to the foundation systems under complicated geological conditions. Another practical problem of a great importance, described by the same mathematical model, is related to the long time study of the behaviour of nuclear waste repositories.

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3 Conclusions and final remarks

As planned, the project contributed to the development, implementation and testing of numerical methods, which enable to perform large-scale simulations in geosciences and geoengineering. The list of our recent contributions includes the efficient implementation of high-performance algorithms for: a) new classes of finite element discretization methods; b) improved iterative solution methods for coupled linear and nonlinear problems on locally refined and composed grids. Throughout the project, also special purpose 3D codes were created, which are based on improved mathematical methods for the analysis of linear and nonlinear elasticity and plasticity problems.

Being one of the main aims of the project, the parallel implementation follows the inherently parallelizable steps of the algorithms based on the hierarchical structure of the applied sparse matrix computations. A special attention has been devoted to the development of efficient portable codes for computer systems with cluster architecture.

The presented results of benchmarking demonstrate well the potential of the developed codes. The considered tests are taken from the recent engineering practice and demonstrate our abilities to solve large-scale real-life problems.

The results presented in this report clearly illustrate the achievements of the research team obtained after the end of the first Copernicus HIPERGEOS I Project. The project brings together groups with outstanding results in numerical analysis and people with a strong interest in applications in geosciences. Such tight collaboration gives new results, which open new possibilities for the mathematical modelling of the Earth and environmental problems as well as modelling of industrial problems arising from mining or civil engineering.

The accumulated *know how* opens possibilities for further collaboration and applications of the presently developed mathematical models, numerical methods and portable program implementations. Especially, the following two areas of applications seems to be important:

- Computer simulation of multi-scale engineering and environmental problems. The efficient treatment of such problems will be based on our experience with the analysis of coupled problems on composite grids by using hierarchical structured solution methods and algorithms.

- Numerical solution of coupled thermo-hydro-mechanical problems, which seems to be of a primal importance for various large scale engineering applications including geodynamical problems, constructions of foundation systems under complicated geological conditions and long time computer simulations of performance of nuclear waste repositories.

  Some experience in the solution of the coupled problems was already gained. Moreover, the properties of the nonconforming FEM seems to be convenient with respect to all the included physical phenomena. Finally, the considered iterative methods can be extended to very efficient solvers for the coupled problems but also some difficulties with the robustness can be met and should be overcome here.

The above problems are typically interdisciplinary and large scale. Their formulation represents the new level of requirements corresponding to the most recent engineering and technological needs. The solution of such problems could be realized only with the use of the new instruments for funding research, such as Integrated Projects and Networks of Excellence. Our expectations are that a progress in the solution of such complex problems will be possible within the new Sixth Framework Programme (FP6).

The results obtained during the solution of the HIPERGEOS project were presented in many papers and conference contributions, see the list in Section 2. Some information about the project COPERNICUS 94008 and KIT 977006 projects can be found on the web pages http://www-math.sci.kun.nl/math/Copernicus/KIT/index.html.