25th Indian-Summer School of Physics
Understanding Hot & Dense QCD Matter
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Jets

Lecture 2

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LPTHE Paris

Outline

- Lecture 1: jet clustering
  - *Basic requirements and properties*
  - *Jet clustering algorithms*
- Lecture 2: jet properties
  - *Jet areas*
  - *Corrections to a jet transverse momentum*
    - *non-perturbative effects, UE, pileup,...*
    - *background estimation and subtraction*
  - *Generalised pileup subtraction*
- Lecture 3: jet substructure
  - *Jet grooming*
  - *Boosted objects: taggers*
  - *Analytical understanding*

These lectures include many ideas and contributions by Gavin Salam and Gregory Soyez

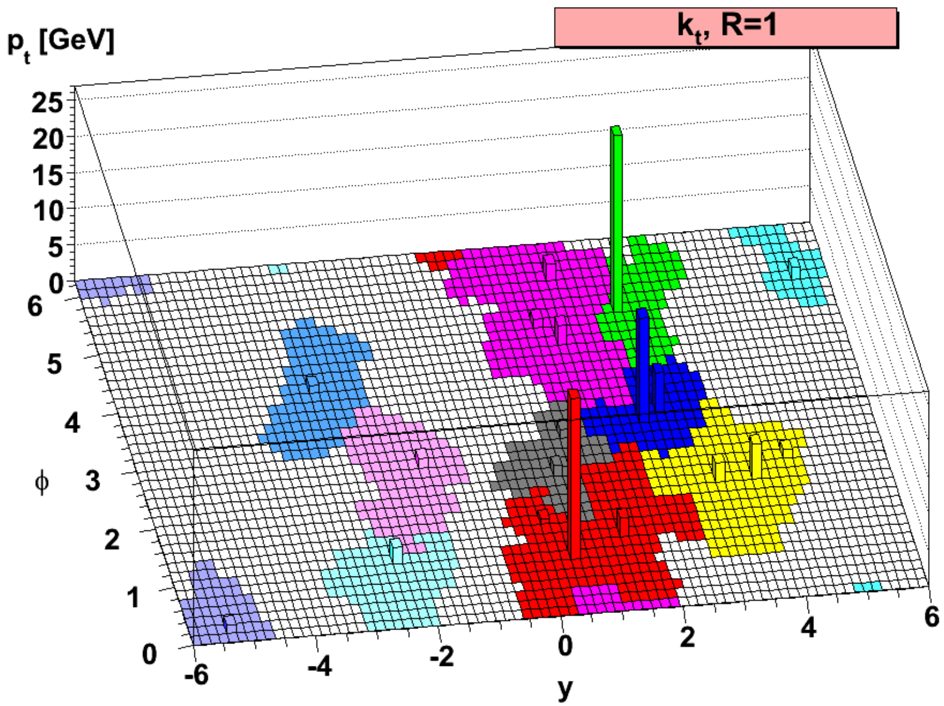
Jets 'reach'

Algorithmically, a jet is simply a collection of particles

For a number of reasons, it is however useful to consider its **spatial extent**, i.e. given the position of its axis, up to where does it collect particles? What is its shape?

These details are important for a number of corrections of various origin: perturbative, non-perturbative (hadronisation), pileup, detector related, etc

Note that the intuitive picture of a jet being a cone (of radius R) is sometimes **wrong.**
This is what k_t jets can look like:
(more later about what this plot really means)



From jet 'reach' to jet areas

MC, Salam, Soyez, 0802.1188

Not one, but three **definitions** of a jet's size:

- **Passive** area

  Place a single very soft particle (a '**ghost**') in the event, measure the extent of the region where it gets clustered within a given jet

  Reach of jet for **pointlike** radiation

- **Active** area

  Fill the events with many very soft particles ('**ghosts**'), cluster them together with the hard ones, see how many get clustered within a given jet

  Reach of jet for **diffuse** radiation

- **Voronoi** area

  Sum of areas of intersections of Voronoi cells of jet constituents with circle of radius R centred on each constituent

  Coincides with passive area for k_t algorithm

(In the large number of particles limit all areas converge to the same value)

Active Area

Add **many** ghost particles in random configurations to the event.
Cluster many times. *Allow ghosts to cluster among themselves too.*
Count how many ghosts on average get clustered into a given jet J.

$$A(J\,|\,\{g_i\}) = \frac{N_g(J)}{\nu_g} \equiv A_g N_g(J)$$

- $N_g(J)$: Number of ghosts in jet J
- $A(J\,|\,\{g_i\})$: Active area of jet J for a single ghosts configuration
- ν_g: Ghost density
- A_g: Area of a single ghost

$$A(J) = \lim_{\nu_g \to \infty} \langle A(J\,|\,\{g_i\}) \rangle_g$$

- $A(J)$: Active area of jet J

Jet active area

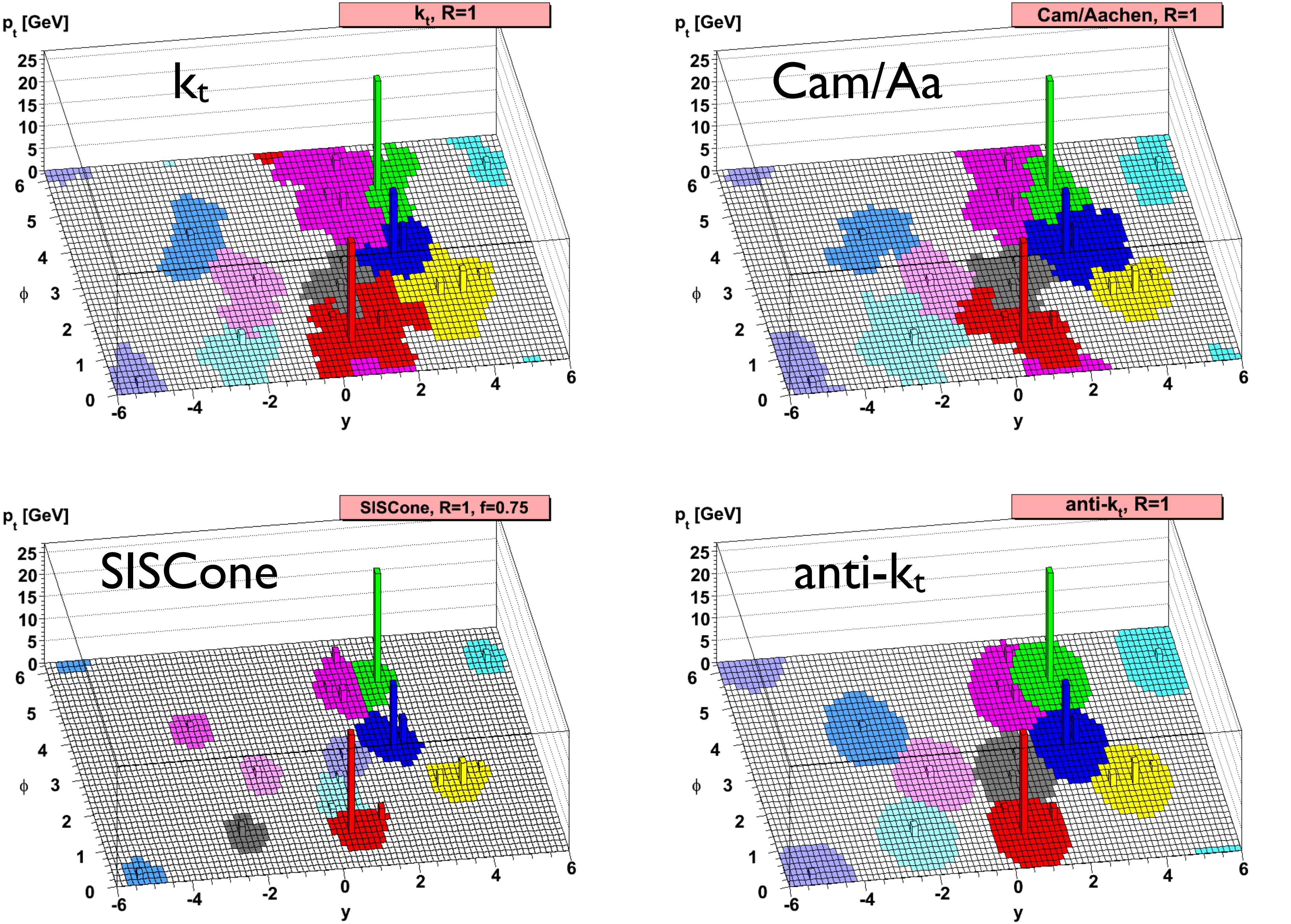
The definition of **active area** mimics the behaviour of the jet-clustering algorithms in the presence of a **large number of randomly distributed soft particles,** like those due to **pileup or underlying event**

Tools needed to implement it

1. An **infrared safe jet algorithm** (the ghosts should not change the jets)
2. A reasonably **fast implementation** (we are adding thousands of ghosts)

Both are available

As a bonus, active areas also allow for a **visualisation** of a jet's reach



kt, R=1
kt
Cam/Aachen, R=1
Cam/Aa
SISCone, R=1, f=0.75
SISCone
anti-kt, R=1
anti-kt
pt [GeV]
φ
y

Jet areas: the single hard particle case

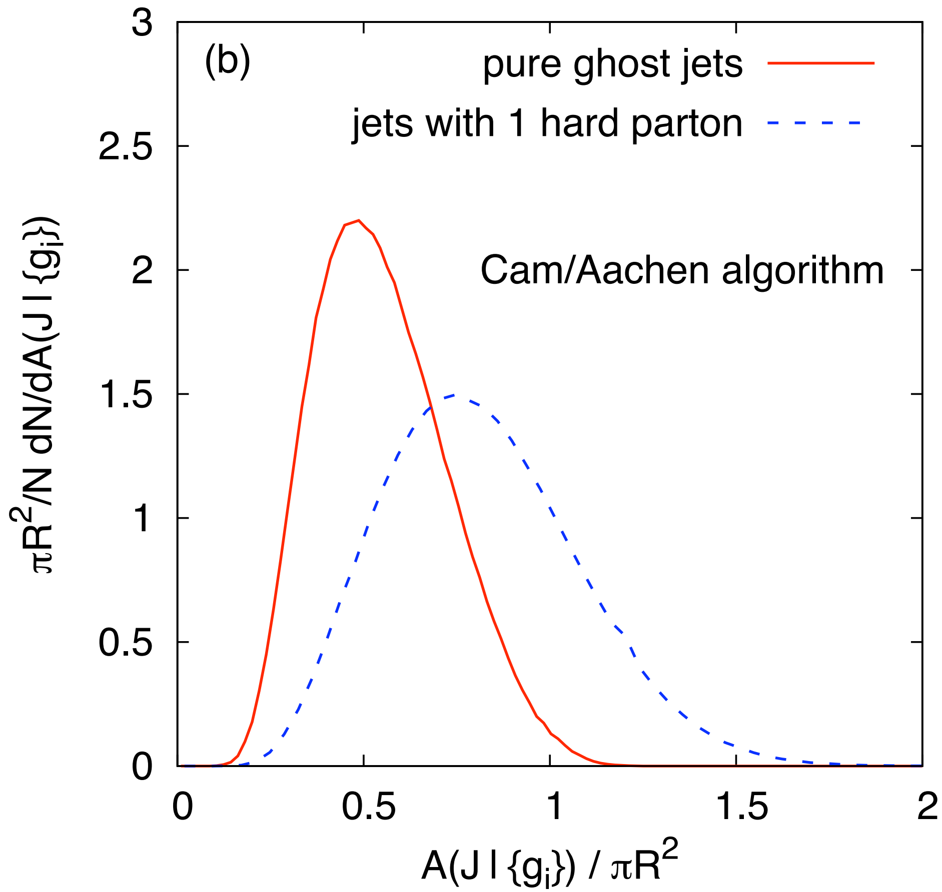
It is worth noting that, for a jet made of a **single hard particle**, while **passive** areas are indeed **πR^2, active** areas are **not**

Active areas	k_t	Cam/Aa	SISCone	anti-k_t
$\langle A \rangle / \pi R^2$	0.81	0.81	1/4	1

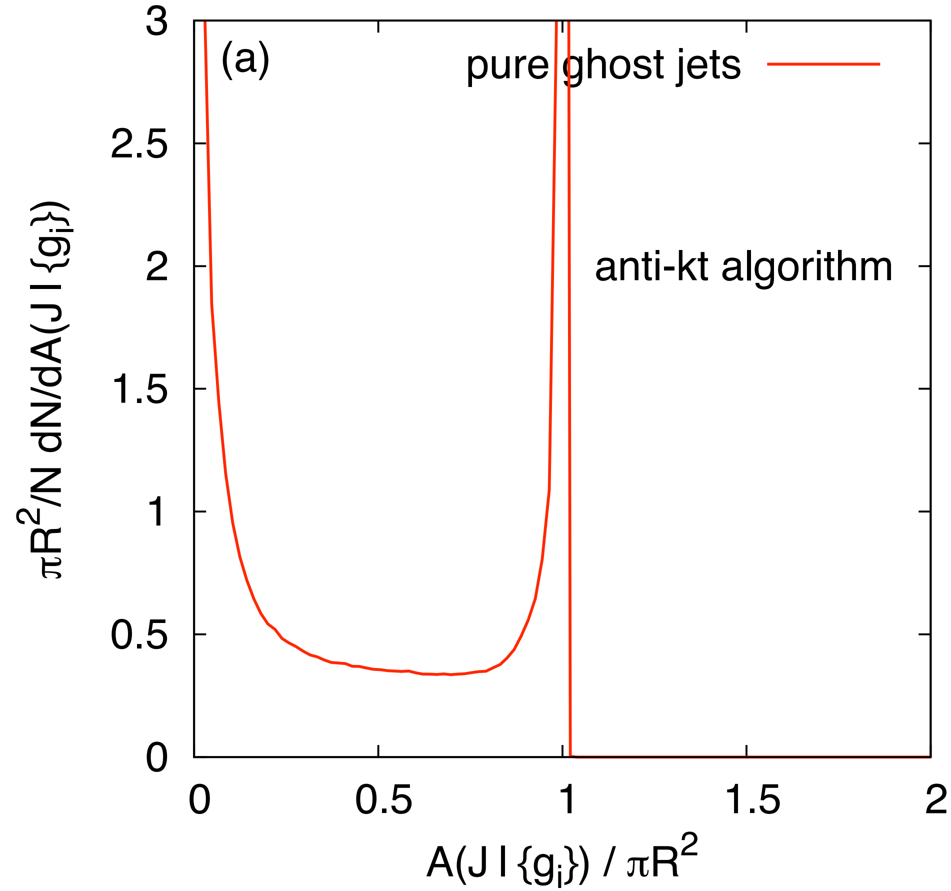
Only anti-k_t has the behaviour one would naively expect, i.e. area = πR^2

Active area distributions

k_t and Cam/AA



anti-k_t



For a roughly uniformly soft background, anti-k_t gives many small jets and many large ones

(you can't fill a plane with circles!)

Areas as a dynamical jet property

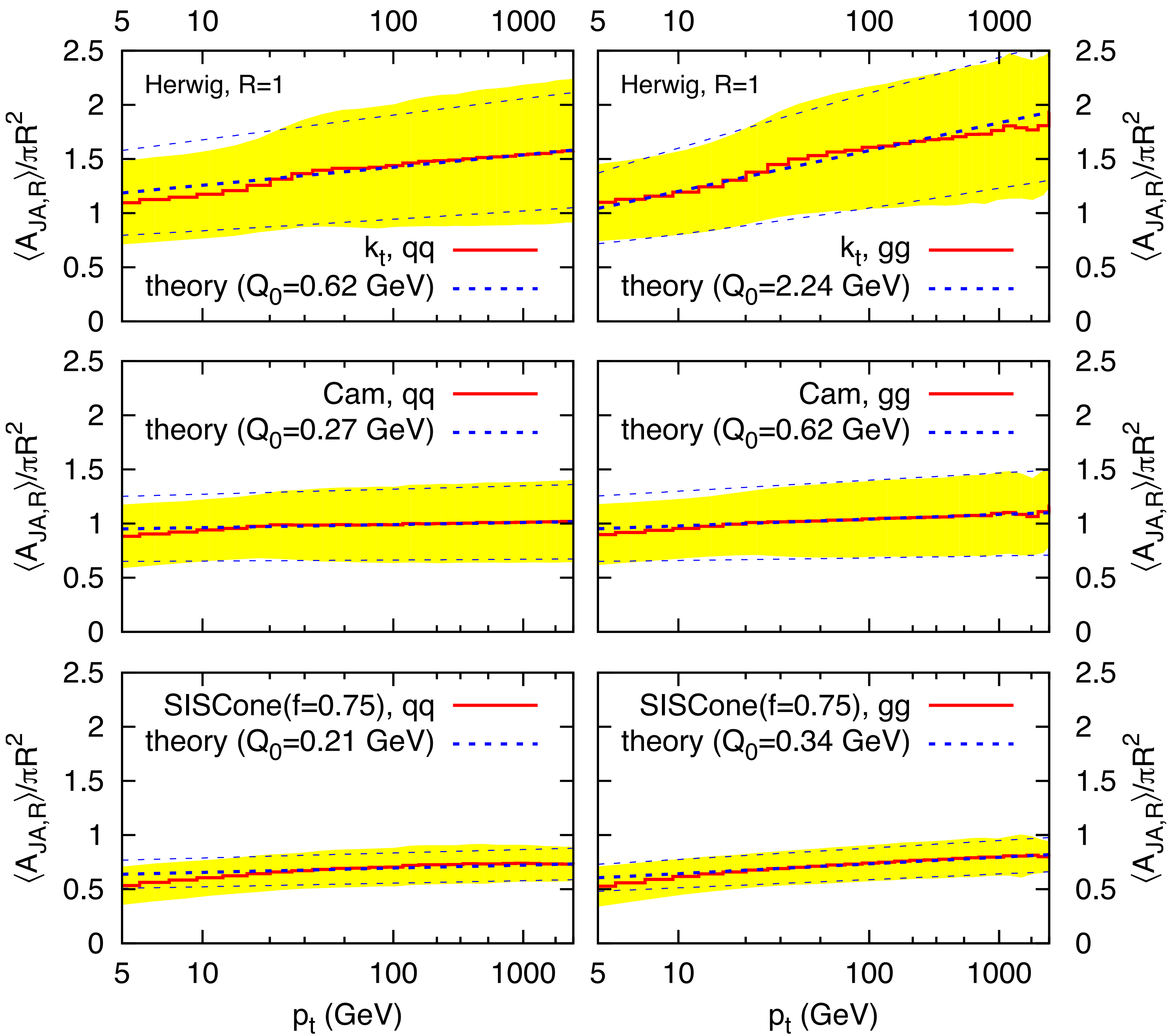
The average area of a jet can change with its p_t:

$$\langle \Delta A \rangle = D \, \frac{C_1}{\pi b_0} \ln \frac{\alpha_s(Q_0)}{\alpha_s(R p_{t1})}$$

	k_t	Cam/Aa	SISCone	anti-k_t
D	0.52	0.08	0.12	0

Again, only anti-k_t has a typical area that does **not** increase with p_t

Jet areas scaling violations



Averages and dispersions evolution from Monte Carlo simulations (dijet events at LHC) in good agreement with simple LL calculations

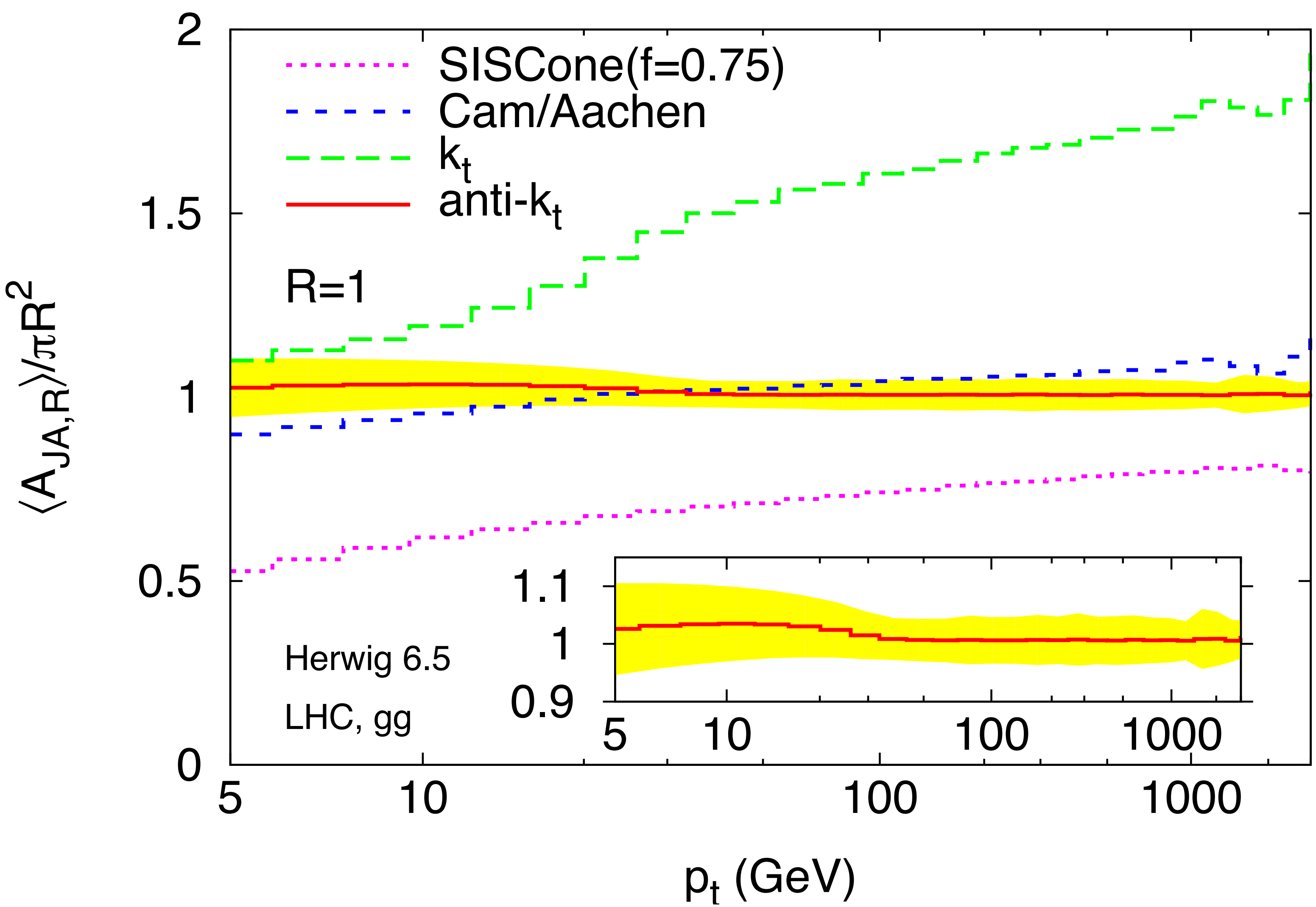
Area scaling violations are a legitimate observable.

(Though they might not be the best place where to measure α_s ....)

Jet areas scaling violations

MC, Salam, Soyez, arXiv:0802.1189

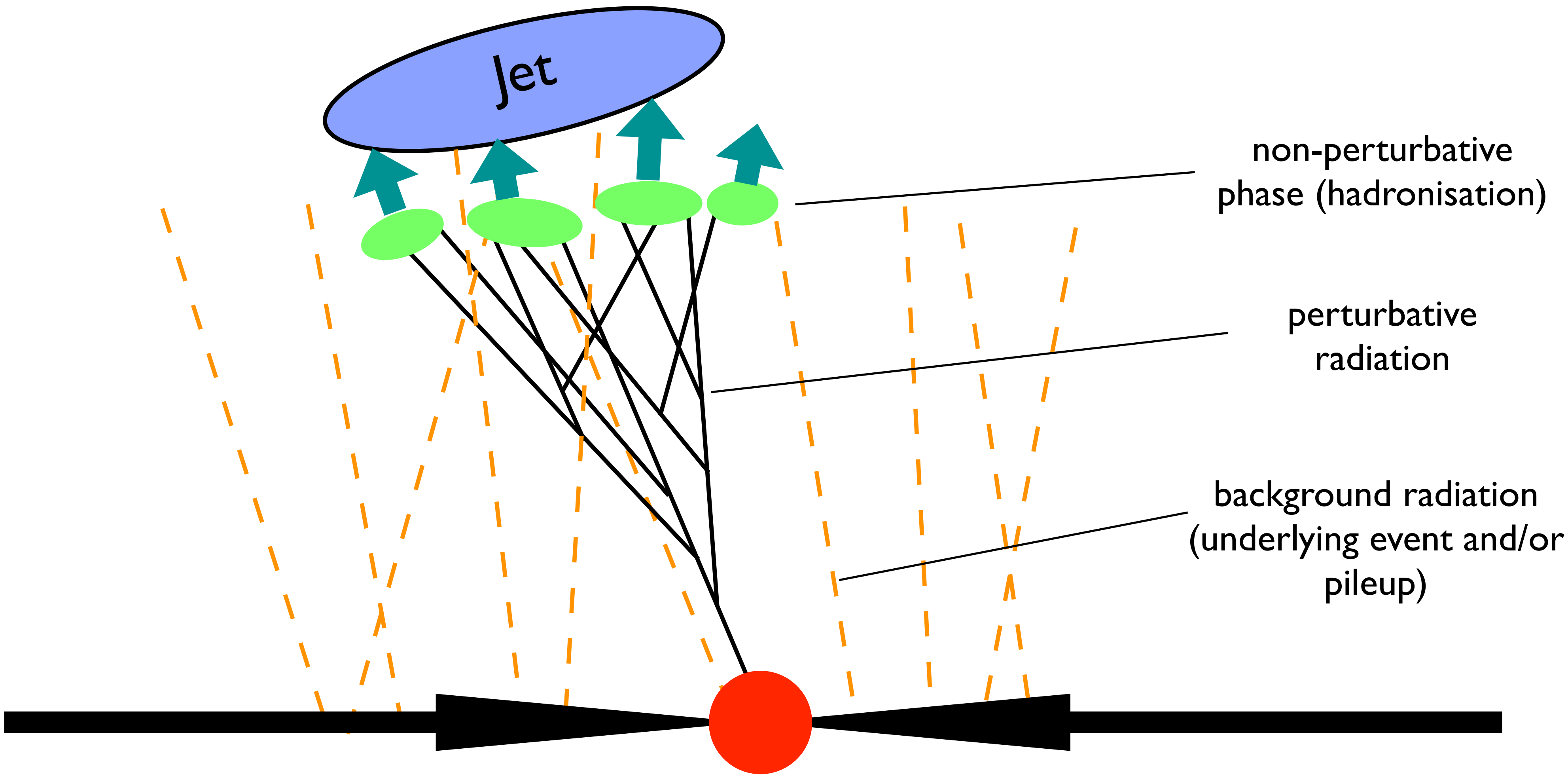
Check anti-k_t behaviour: scaling violations indeed absent, as predicted



Jet area: summary

- Jets CAN have an area, but one must define it
- Different jet algorithms can have very different area properties:
  - Jet areas in many algorithms can fluctuate significantly from a jet to another. *Isolated hard jets in anti-k_t are an exception*
  - Jet areas can depend on a jet's p_t, driven by a (calculable) anomalous dimension that is specific to each jet algorithm. *Anti-k_t jets are again an exception: the anomalous dimension is zero*

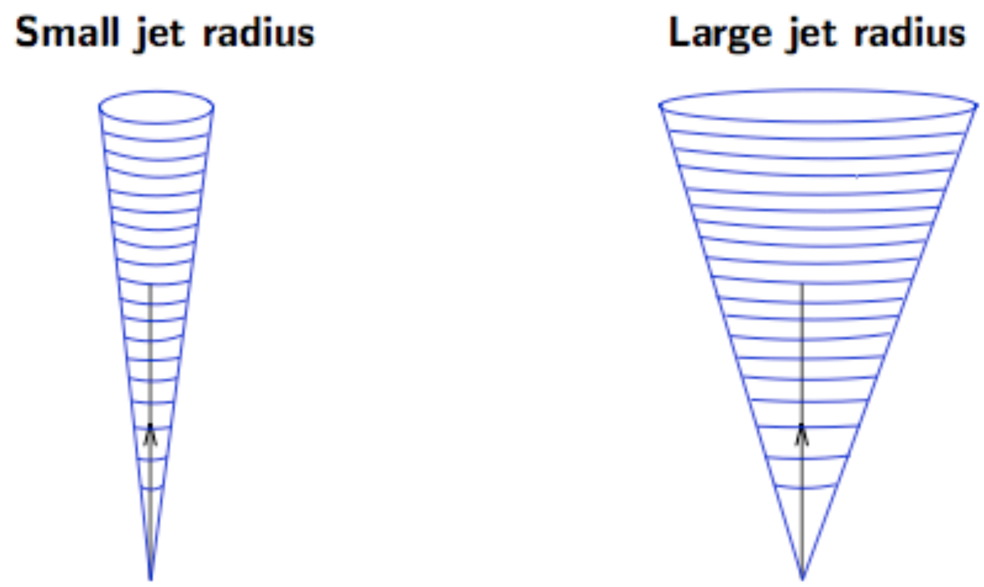
What contributes to a jet's transverse momentum?



Contributions to a jet p_t

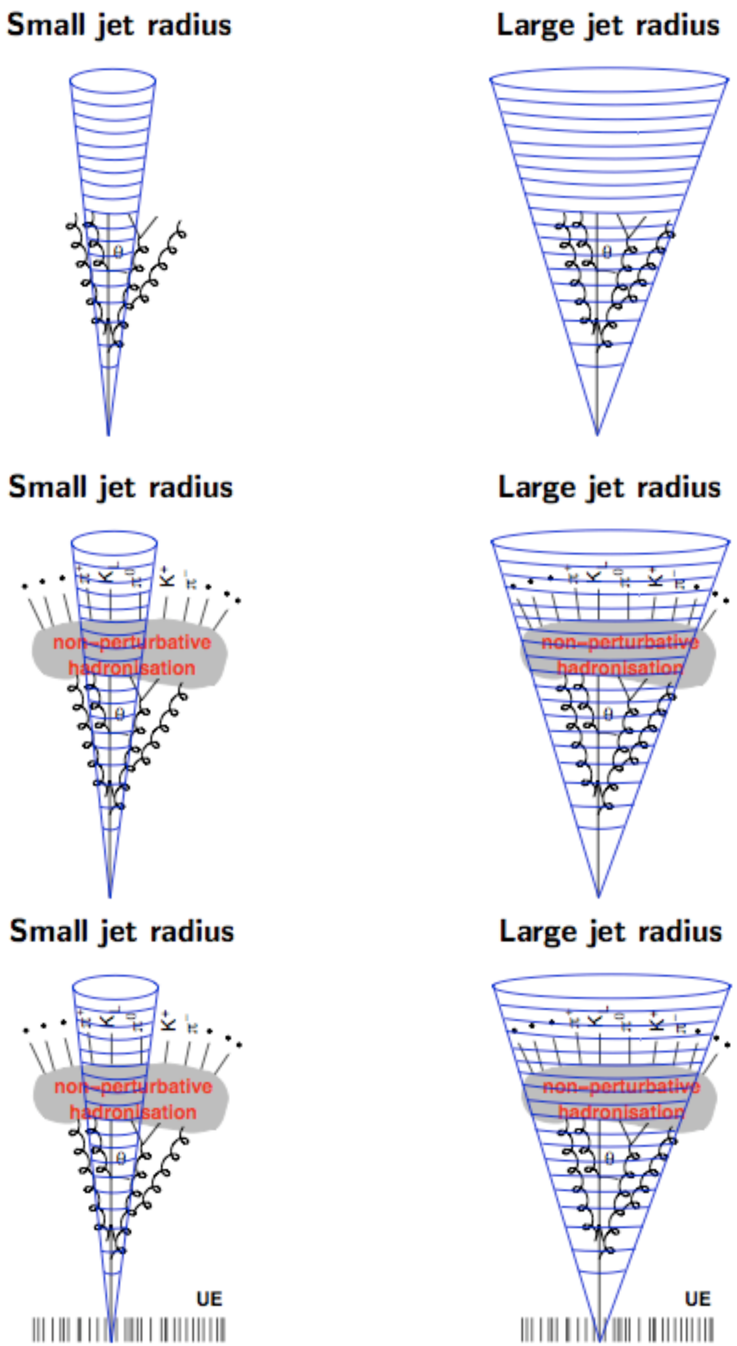
- Ideally, we'd like a jet to contain all radiation emitted by a parton, i.e. reproduce exactly the parton. This is impossible for a number of reasons:
  - First and foremost, *a parton is not a physical observable, while a jet is*. There cannot be exact equivalence between them
  - A *jet has finite extent*, and part of the radiation (perturbative or non-perturbative) emitted by the parton may end up beyond the jet's boundaries, leading to a decrease of the jet momentum (with respect to the parton's)
  - A jet does not fragment in a vacuum. *Background radiation* (underlying event and/or pileup) can affect its momentum in at least two ways:
    - Some background radiation will be clustered with the jet, increasing its momentum
    - Some hard particles (from the parton fragmentation process) may not cluster with the rest of the jet because of the disturbing presence of other nearby particles (they will form other jets with them). Alternatively, hard particles that in the vacuum would not have clustered with the jet will

Effects of jet ‘radius’



Irrelevant for a single-particle jet

Effects of jet 'radius'



perturbative radiation:
large radius better (lose less)

non-perturbative hadronisation:
large radius better (lose less)

underlying event:
large radius worse (capture more)

R-dependent effects

Perturbative radiation: $$\Delta p_t \simeq \frac{\alpha_s(C_F, C_A)}{\pi} p_t \ln R$$

Hadronisation: $$\Delta p_t \simeq -\frac{(C_F, C_A)}{R} \times 0.4 \text{ GeV}$$

Underlying Event: $$\Delta p_t \simeq \frac{R^2}{2} \times (2.5 \text{ — } 15 \text{ GeV})$$

Tevatron LHC

(small-R limit results)

Analytical estimates: Dasgupta, Magnea, Salam, arXiv:0712.3014

Which R to choose?

The value of R matters because it affects,
in opposite ways, a number of things:

Small R:

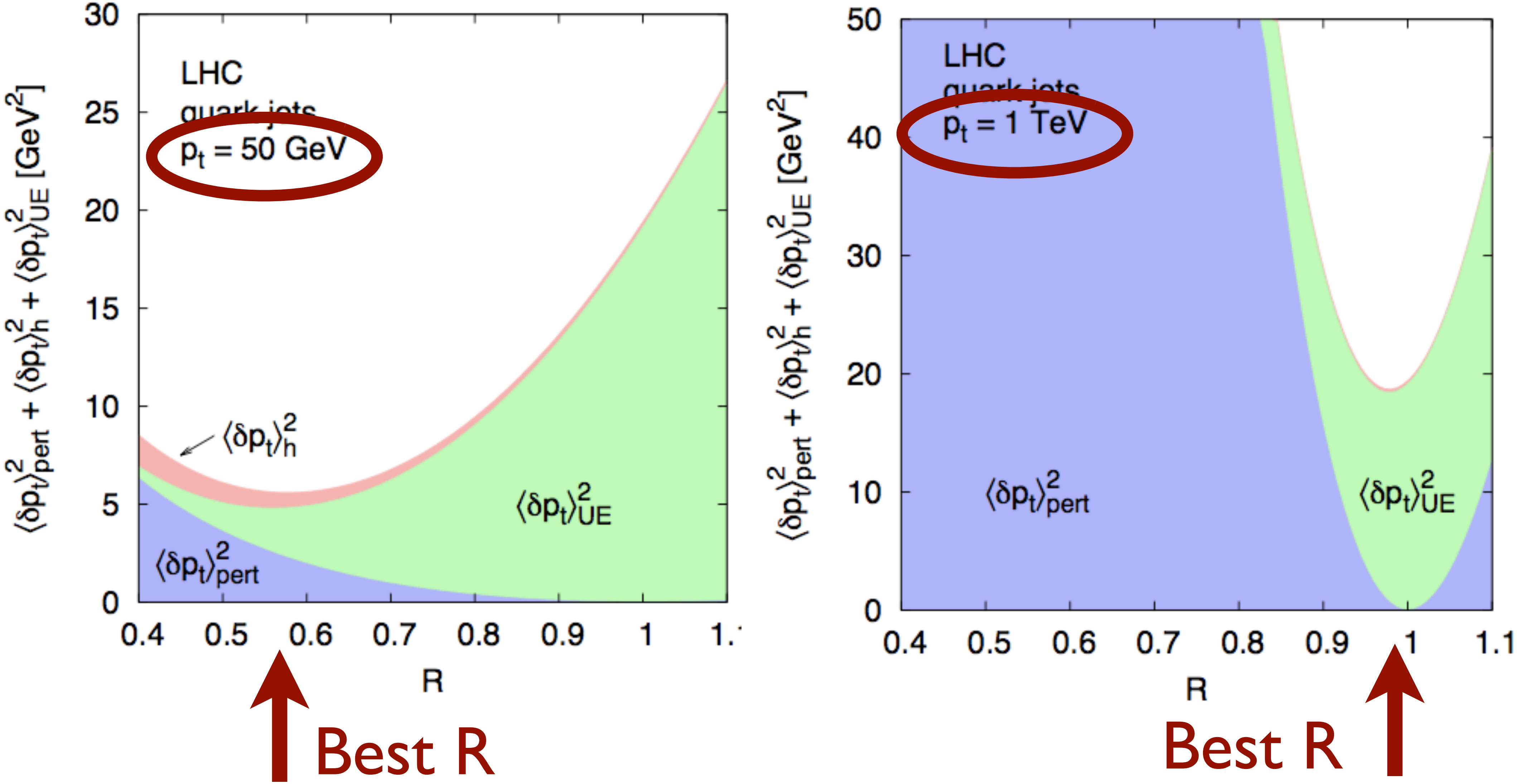
- ✔ Limit underlying event and pileup contamination
- ✔ Better resolve many-jets events
- ✖ Perturbative radiation loss, larger hadronisation effects

Large R:

- ✔ Limit non-perturbative hadronisation effects
- ✔ Limit perturbative radiation loss
- ✖ More sensitive to EU and pileup

The best compromise will in general depend on the specific observable

Minimize $\Sigma(\Delta p_t)^2$



Dasgupta, Magnea, Salam, arXiv:0712.3014

Effect of background

How are the hard jets modified by the background?

(Can be underlying event and/or pileup)

Susceptibility (how much bkgd gets picked up)	**Jet areas**
Resiliency (how much the original jet changes)	**Backreaction**

Jet areas physical meaning

A jet's active area expresses the susceptibility of that jet's transverse momentum to contamination from a uniform background

Consider a jet of transverse momentum p_t, made up of N_g ghosts, each with transverse momentum $p_{t,g}$.

It holds
$$\frac{\partial p_t}{\partial p_{t,g}} = \frac{\partial (N_g p_{t,g})}{\partial p_{t,g}} = N_g$$

Recalling the definition of active jet area, $A_J = A_g N_g$, we can then rewrite

$$A_J = A_g \frac{\partial p_t}{\partial p_{t,g}} = \frac{\partial p_t}{\partial \rho}$$

Transverse momentum density, $\rho = p_{t,g}/A_g$

The jet area is therefore the susceptibility of a jet's p_t to contamination, because for a generic background transverse momentum density ρ it will hold

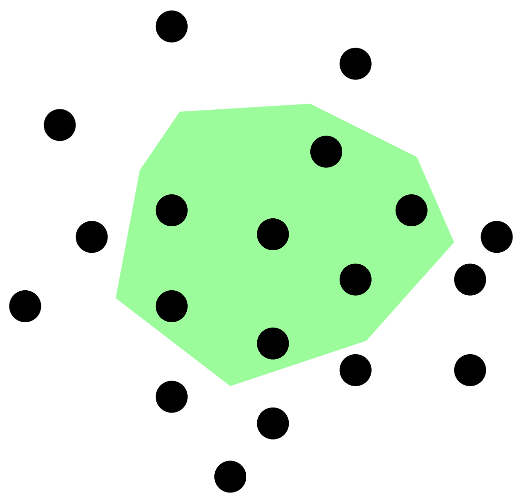
$$\Delta p_t = \rho \frac{\partial p_t}{\partial \rho} = \rho A_J$$

Susceptibility

Resiliency: backreaction

"How (much) a jet changes when immersed in a background"

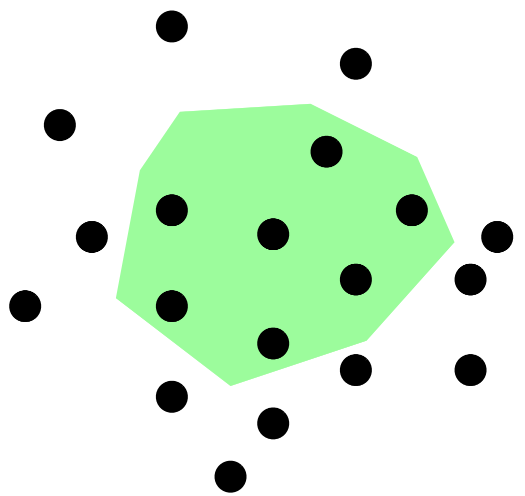
Without background

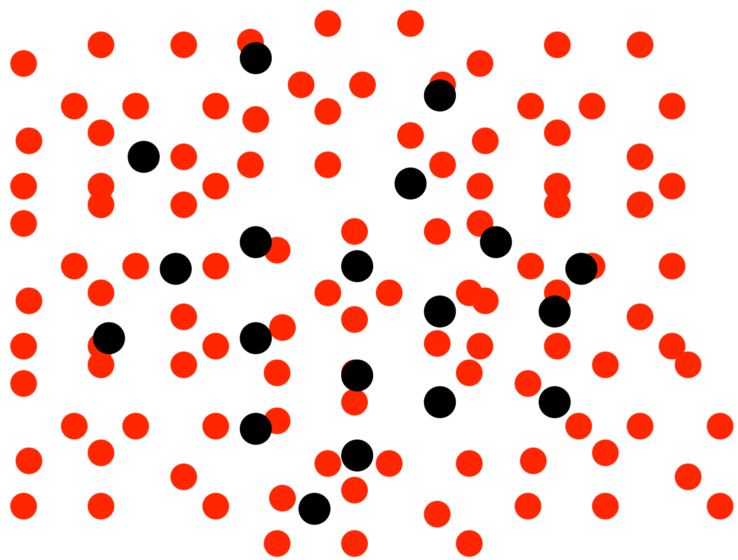


Resiliency: backreaction

"How (much) a jet changes when immersed in a background"

Without background





Resiliency: backreaction

"How (much) a jet changes when immersed in a background"

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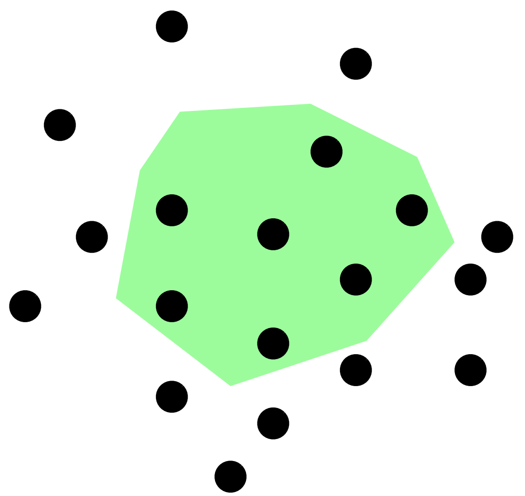
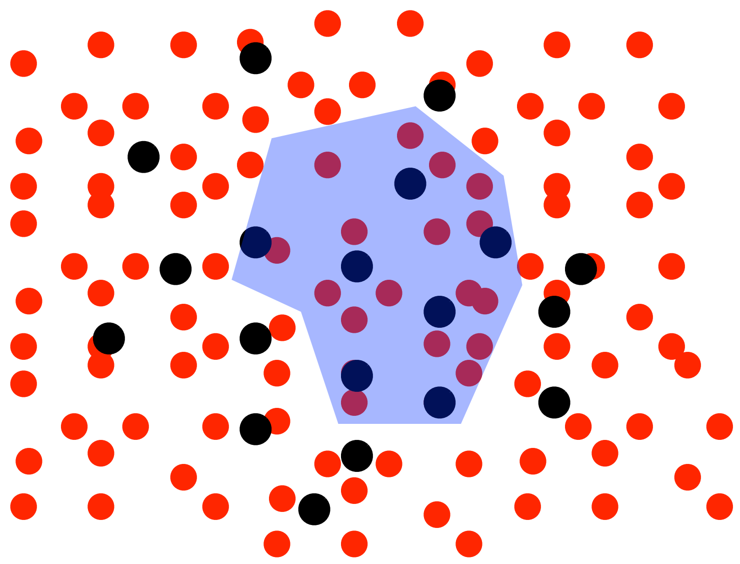
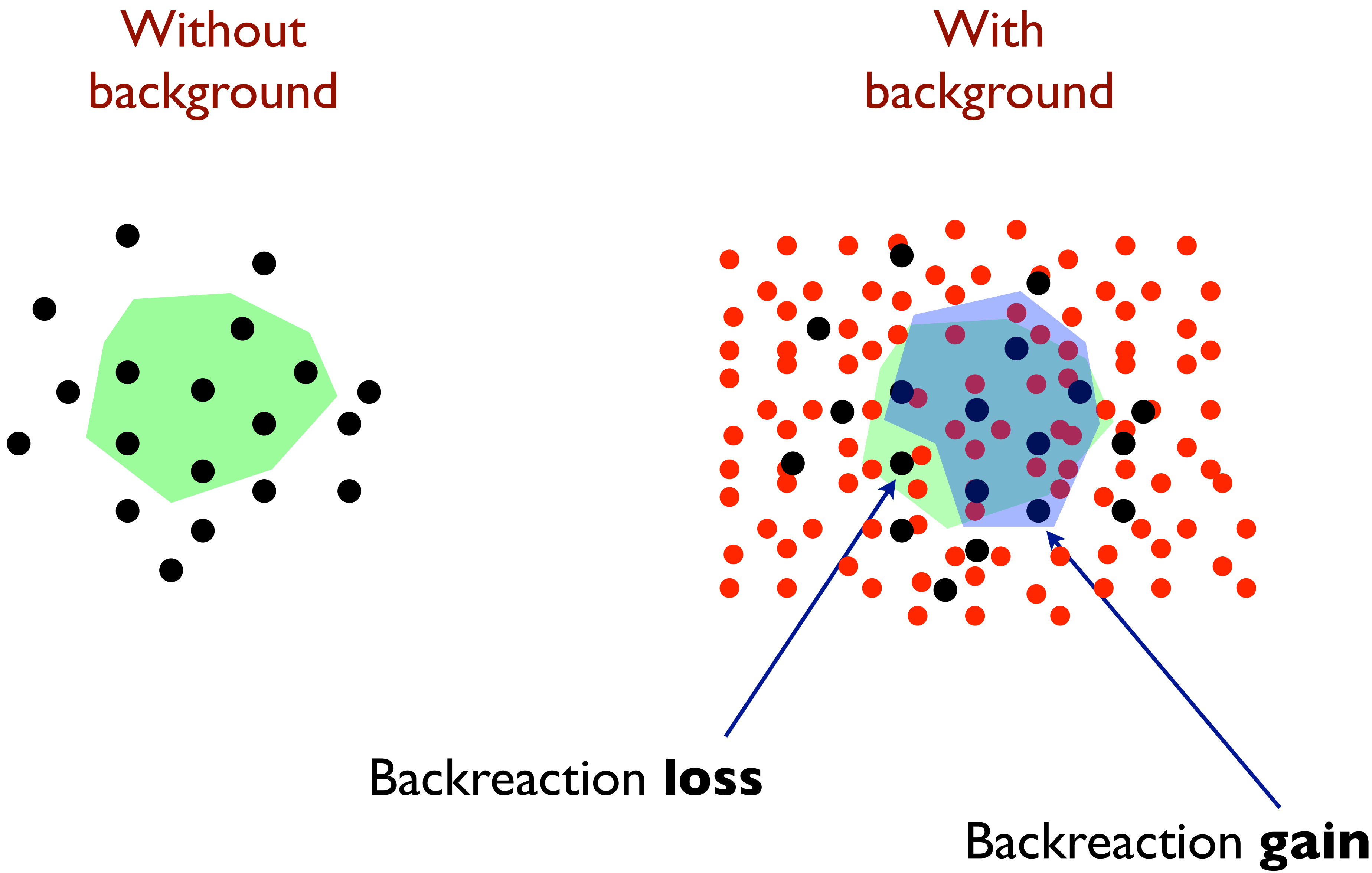


With background

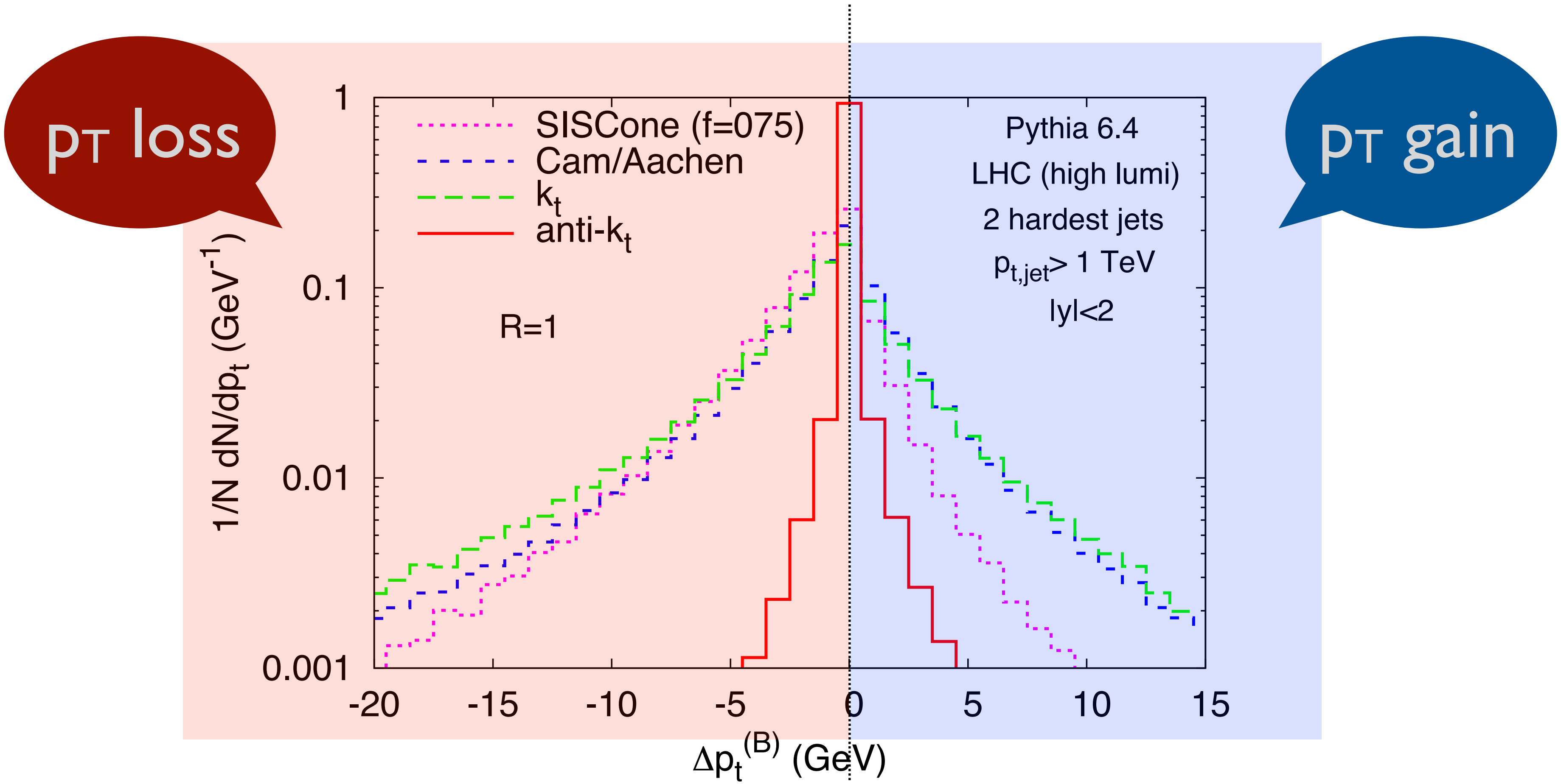


Resiliency: backreaction

"How (much) a jet changes when immersed in a background"



Resiliency: backreaction



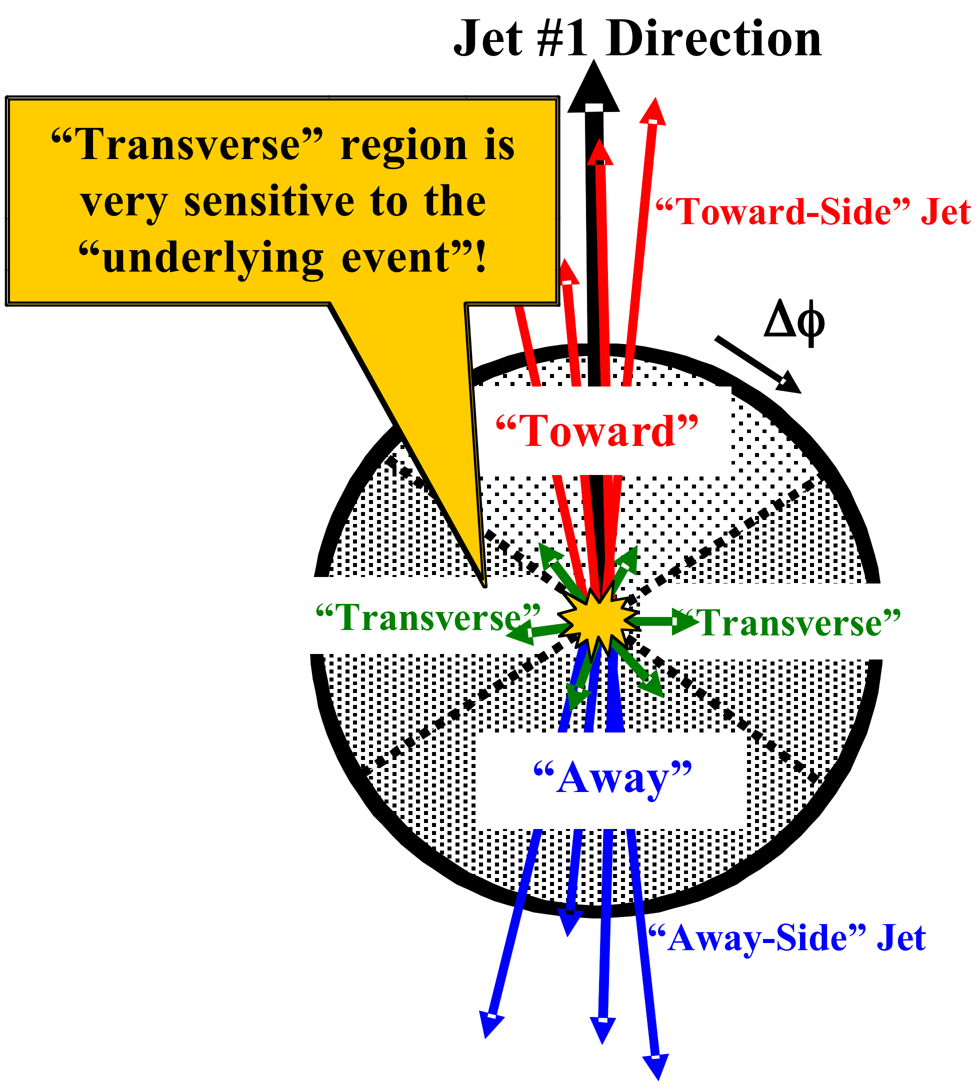
Anti-k_t jets are much more resilient to changes from background immersion

(NB. Backreaction is a minimal issue in pp background and at large pt.
Can be much more important in Heavy Ion collisions)

The IRC safe algorithms

	Speed	Regularity	UE contamination	Backreaction	Hierarchical substructure
k_t	☺ ☺ ☺	☂	☂☂	☁☁	
Cambridge /Aachen	☺ ☺ ☺	☂	☂	☁☁	
anti-k_t	☺ ☺ ☺	☺ ☺	☁ → ☺ ☺	☺ ☺	
SISCone	☺	☁	☺ ☺	☁	

Underlying event measurement



Marchesini-Webber idea:
look at transverse region to measure underlying event

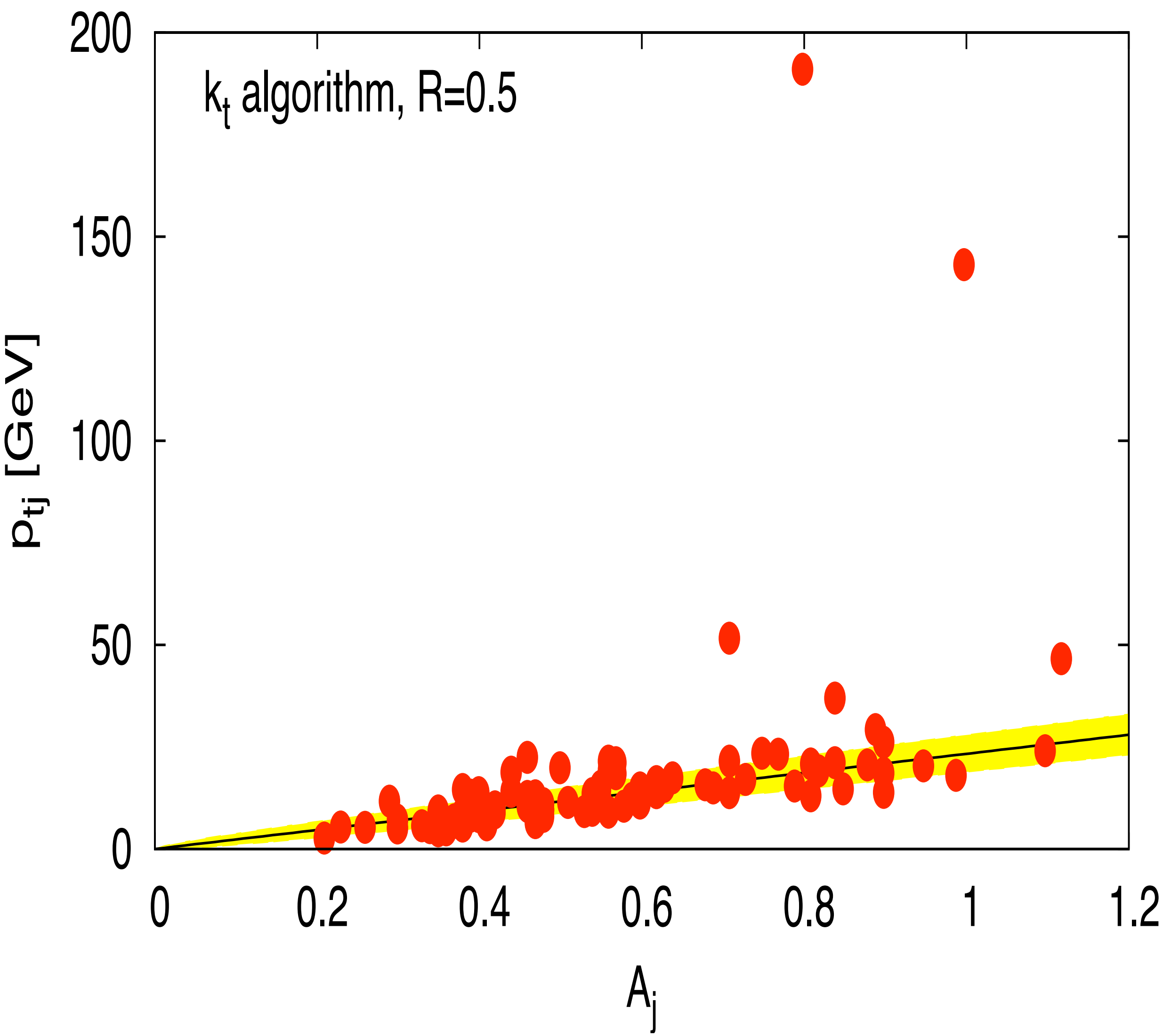
Topological selection

The jets are classified as belonging to the noise on the ground of their **position**

LHC: dijet event + high-lumi pilup

a few hard particles and many softer ones

(a similar picture applies to the Underlying Event)

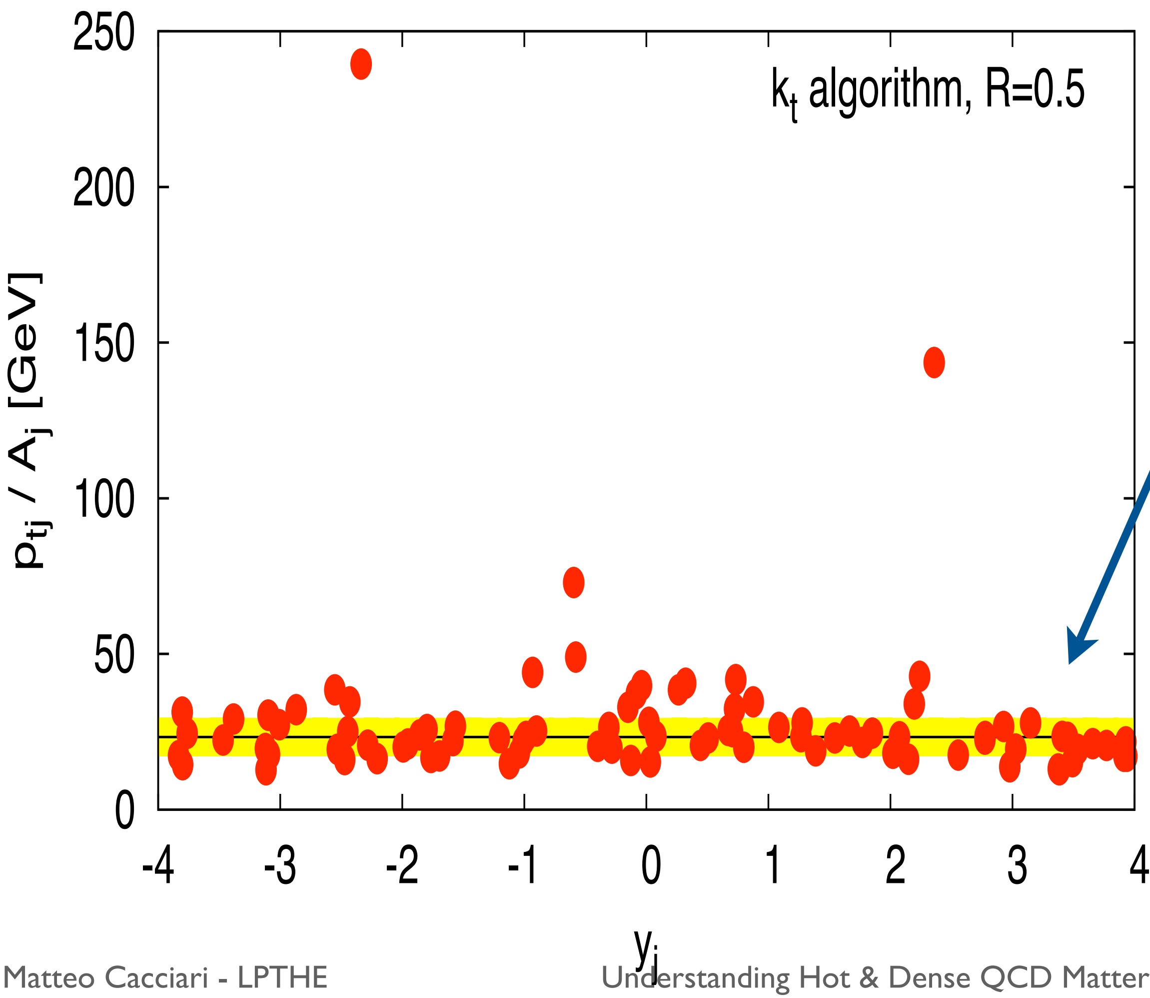


The jets adapt to the surrounding environment

They can have very different areas

The key observation

p_t/Area is fairly constant, except for the hard jets



The distribution of background jets establishes its own average momentum density ρ

(NB. this is true on an event-by-event basis)

<u>Dynamical selection</u>

The jets are classified as belonging to the noise on the ground of their **characteristics**

Background determination

Jet algorithms like k_t or Cambridge/Aachen allow one to determine
on an event-by-event basis
the **"typical" level of transverse momentum density**
of a **roughly uniform background noise**:

$$\rho \equiv \underset{\text{(over a \underline{single} event)}}{\text{median}} \left[\left\{ \frac{p_t^{jet}}{\text{Area}_{jet}} \right\} \right]$$

MC, Salam, 2007

This ρ value can, in turn, be used to characterise the UE

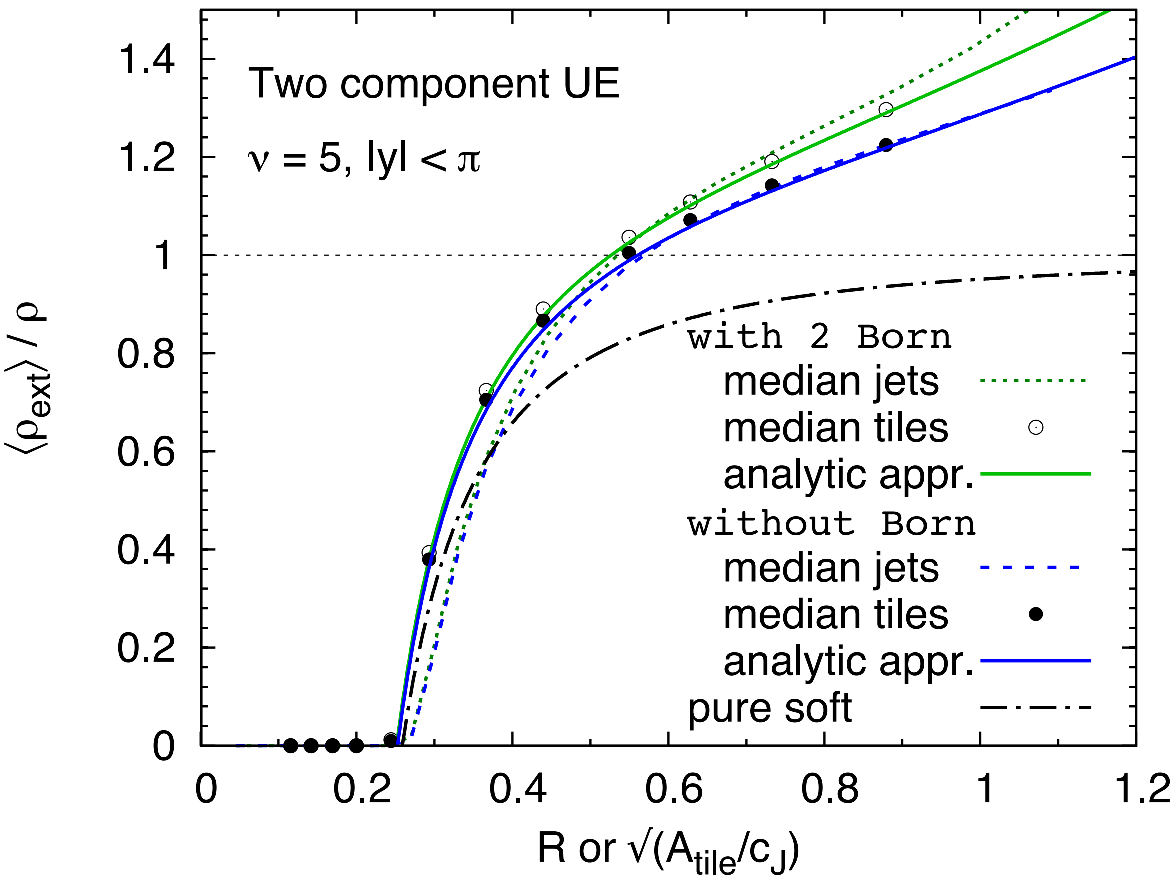
Since this measurement is done with the jets, it is alternative/complementary to the usual analyses done using charged tracks (à la R. Field)

Background determination

MC, Salam, Sapeta, 2009

How ‘right’ is ρ extracted with the area/median method?

Check with a toy model of UE + two hard particles



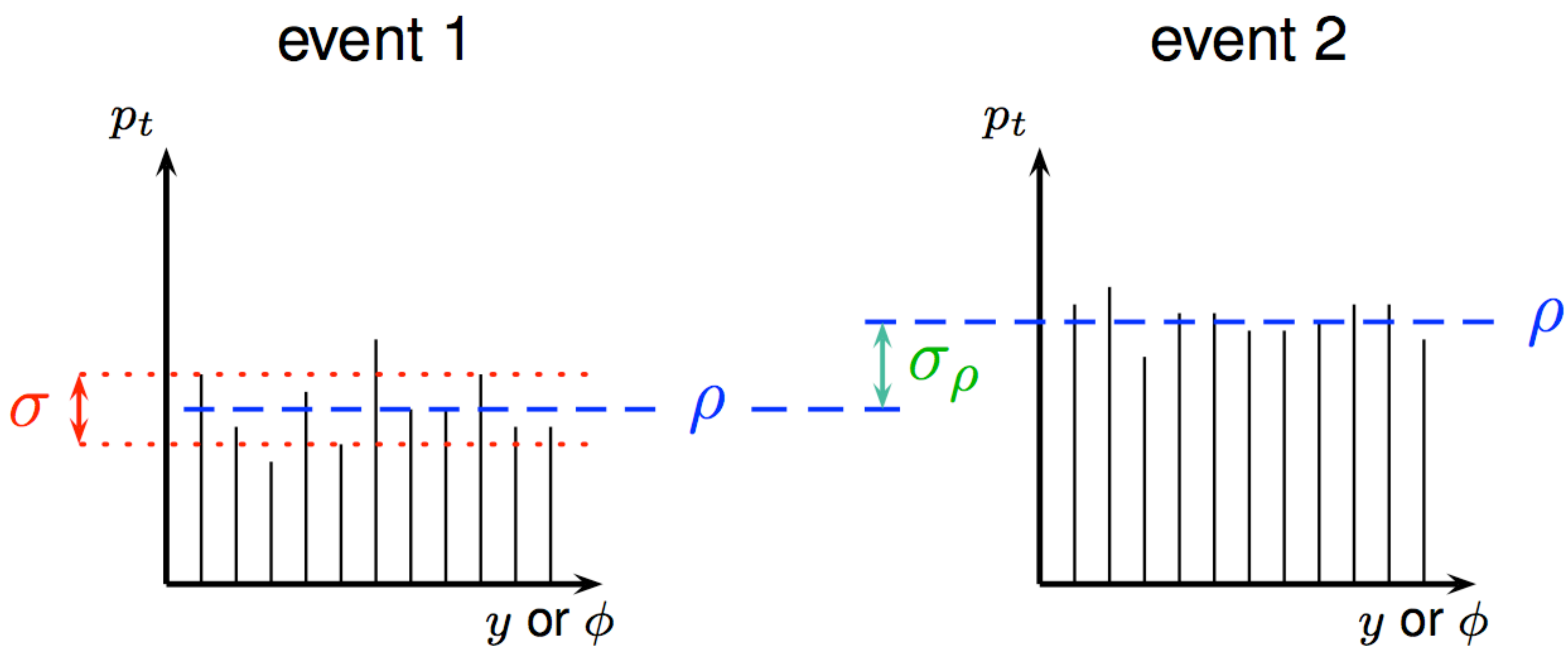
Several features to be noted

- Too small an R underestimates ρ (too many ‘empty’ jets, median biased by them)
- Too large an R overestimates ρ (too few jets, median biased by hard ones)
- A positive bias from hard jets and a negative ones from soft ones tend to compensate around R = 0.5
- A higher particle density (here 5 per unit area) would shift to the left the ‘sweet spot’ for R (and make the extraction more robust: **the more soft particles, the better it works**)

- ρ can be extracted with good precision, provided an appropriate value of R is used
- Local variations of ρ (rapidity-azimuth dependent effects due to detector and/or physics) can (and must) be accounted for by a more local determination and/or a rescaling of the extracted value of ρ

UE or pileup characterisation

- ρ : average p_t per unit area in an event
- σ : intra-event fluctuations
- σ_ρ: inter-event fluctuations



Orders of magnitude

Typical (and rough) values of ρ and σ at the LHC

	ρ	σ
pp UE	2-3 GeV	1 GeV
pileup / #	1 GeV	
PbPb (5.5 TeV, 0-10%)	200 GeV	20 GeV

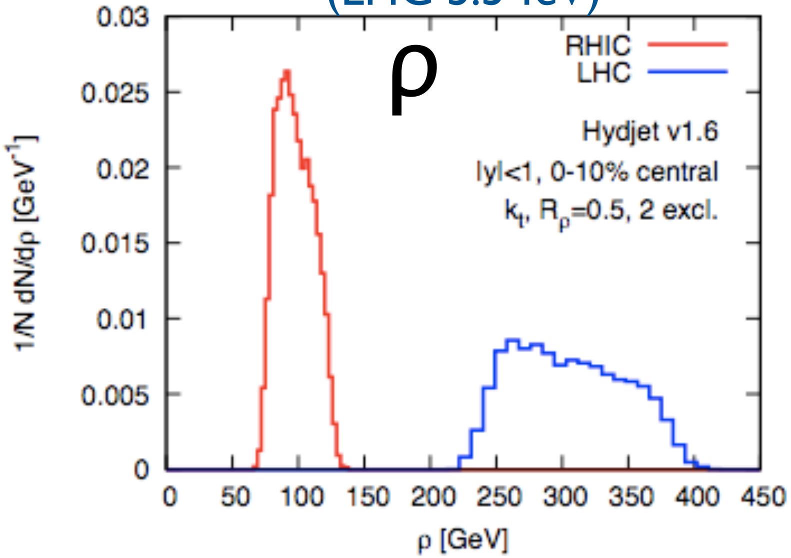
For an anti-kt jet of radius R=0.4, this translates into a background contamination of **(100 ± 14) GeV**

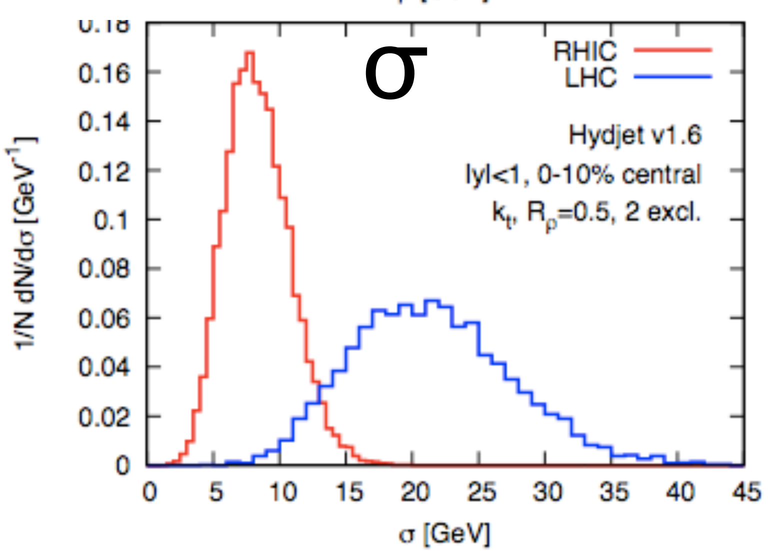
The HI background

Usefully characterized by its transverse momentum per unit area (ρ) and its fluctuations in a single event (σ)

HYDJET simulations

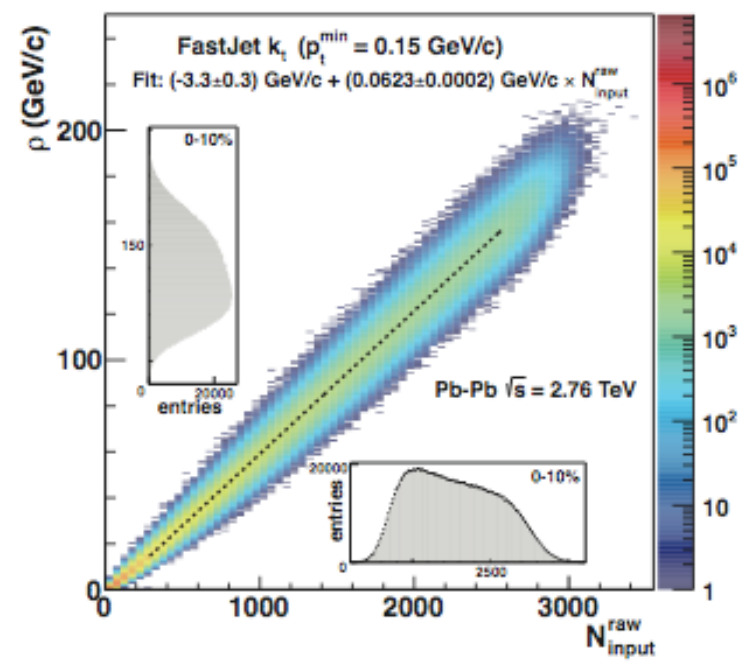
(LHC 5.5 TeV)





ρ from ALICE data

(LHC 2.76 TeV, charged only)



Hard jets and background

Modifications of the hard jet

$$\Delta p_t = \rho A \pm (\sigma\sqrt{A} + \sigma_\rho A + \rho\sqrt{\langle A^2\rangle - \langle A\rangle^2}) + \Delta p_t^{BR}$$

Background momentum density (per unit area)

background

'susceptibility'

back-reaction

'resiliency'

Background subtraction

Once the **background momentum density ρ** has been measured, it can be used to **correct** the transverse momentum of the hard jets:

$$p_T^{\text{hard jet, corrected}} = p_T^{\text{hard jet, raw}} - \rho \times \text{Area}_{\text{hard jet}}$$

MC, Salam, 0707.1378

If ρ is measured on an event-by-event basis, and each jet subtracted individually, this procedure will remove many fluctuations and generally improve the resolution of, say, a mass peak

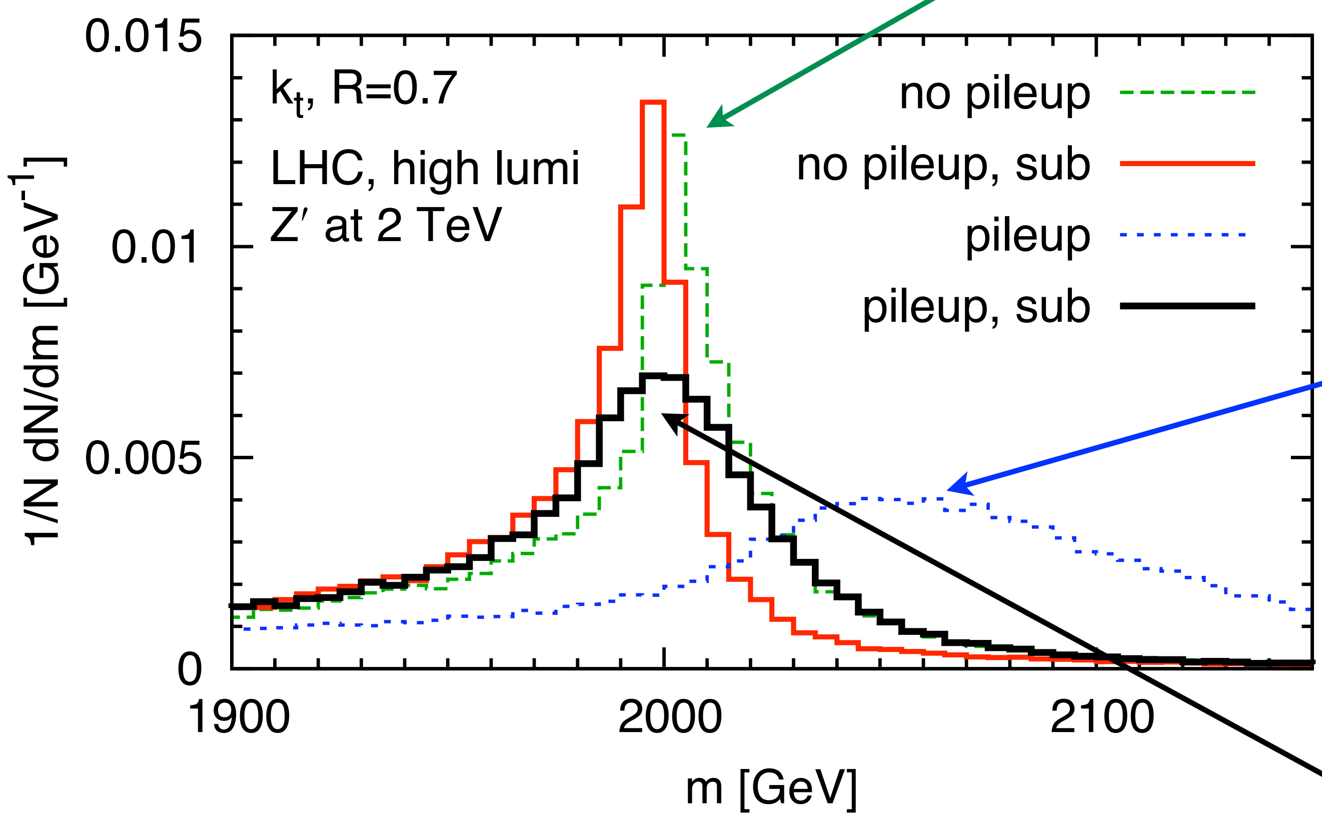
$$\Delta p_t = \rho A \pm (\sigma\sqrt{A} + \sigma_\rho A + \rho\sqrt{\langle A^2\rangle - \langle A\rangle^2}) + \Delta p_t^{BR}$$

Irreducible fluctuations:
uncertainty of the subtraction

Example of pileup subtraction

Let's discover a leptophobic Z' and measure its mass:

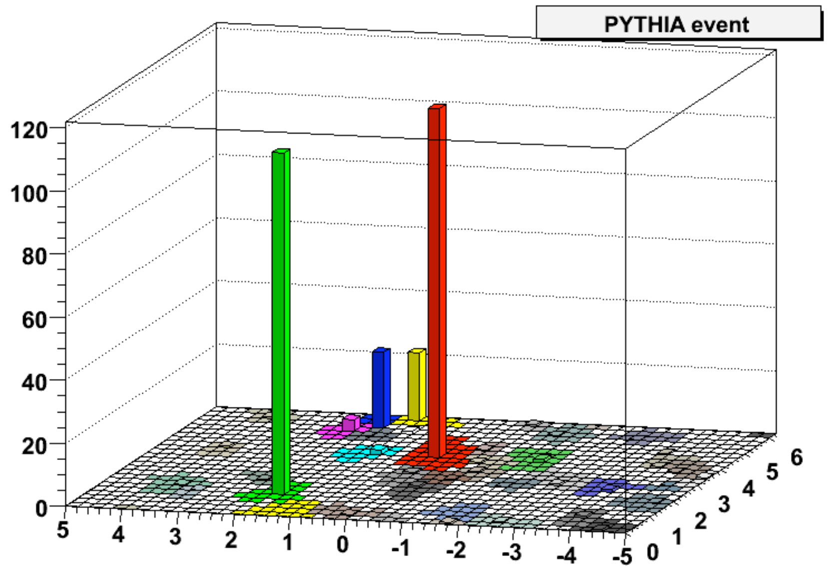
MC simulation:
m = 2000 GeV, width ~ 10 GeV

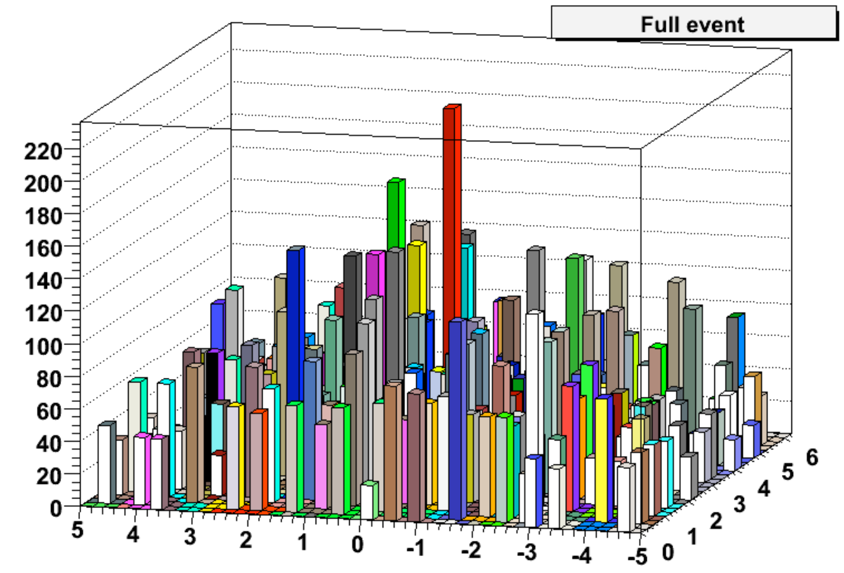


Naive measurement with PU:
m ~ 2050 GeV, width ~ 60 GeV

Measurement after subtraction:
m ~ 2000 GeV, width ~ 25 GeV

Hard jets and background





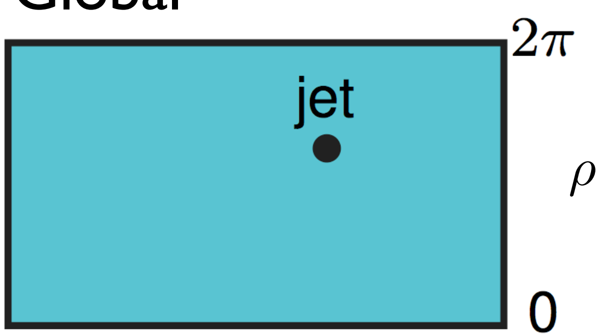
Hard jets
(pp collisions)

Hard jets + background
(AA collisions)

- In pp collisions, the background is a small correction. In heavy ions, it is overwhelming.
- It makes sense to consider background subtraction together with jet clustering: both are needed to ***reconstruct*** the jets
- As such, the same desiderata can apply: standard algorithms, well defined, with known behaviour, and well tested

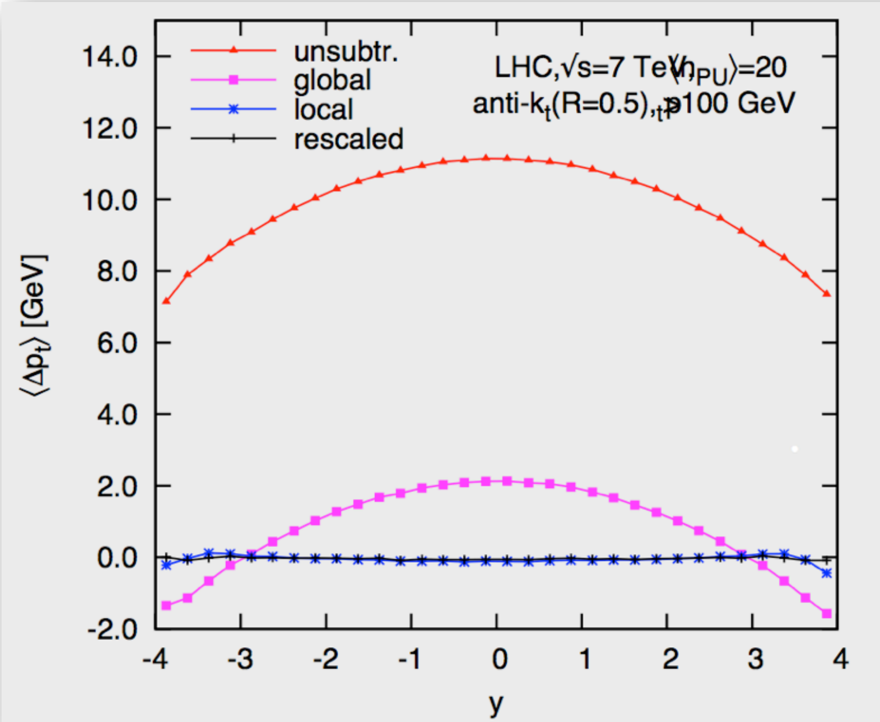
Improvements in bkgd determination

Global

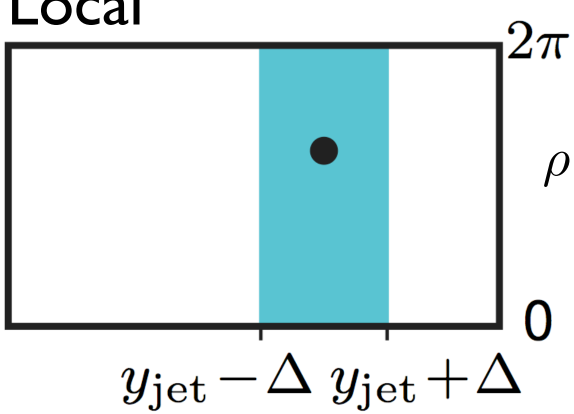


$$\rho \equiv \underset{\text{(all patches)}}{\text{median}} \left\{ \frac{p_{t,\text{patch}}}{A_{\text{patch}}} \right\}$$

$$\langle n_{PU} \rangle = 20$$

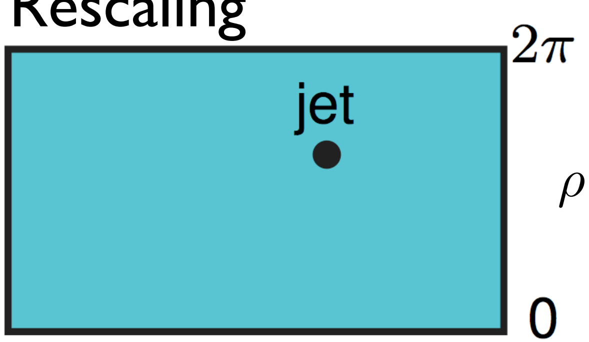


Local



$$\rho(y) \equiv \underset{\text{(local patches)}}{\text{median}} \left\{ \frac{p_{t,\text{patch}}}{A_{\text{patch}}} \right\}$$

Rescaling



$$\rho(y) \equiv f(y) \underset{\text{(all patches)}}{\text{median}} \left\{ \frac{p_{t,\text{patch}}}{f(y_{\text{patch}}) A_{\text{patch}}} \right\}$$

Jet reconstruction in HI collisions

How do the different clustering algorithms fare?

Subtract with

$$p^{sub}_{\mu,jet} \equiv p_{\mu,jet} - \rho A_{\mu,jet}$$

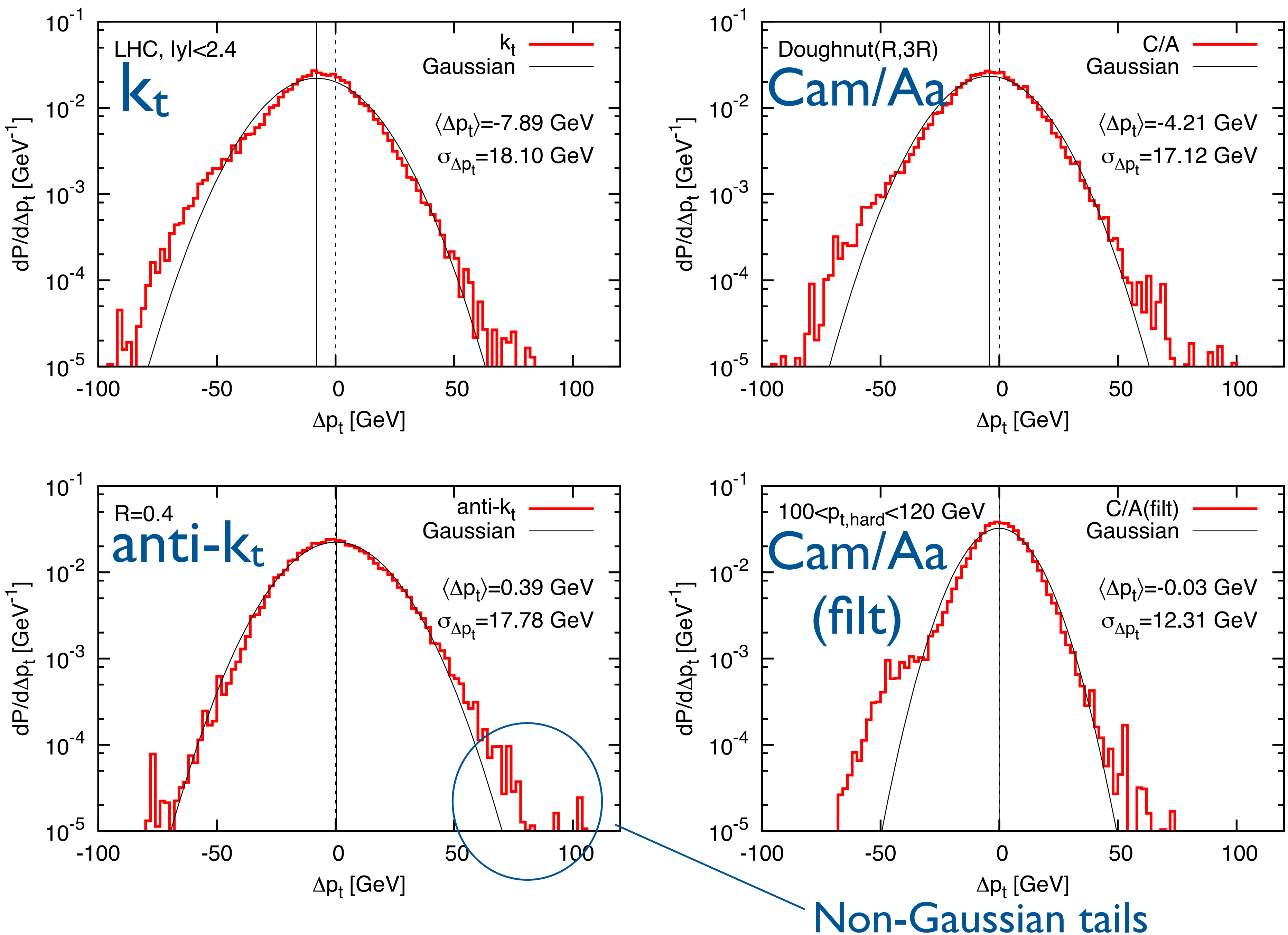
Measure quality of reconstruction looking at

Offset $$\langle \Delta p_t \rangle \equiv \langle p_t^{AA,sub} - p_t^{pp,sub} \rangle$$

Dispersion $$\sigma_{jet} \equiv \sigma_{\Delta p_t} \equiv \sqrt{\langle \Delta p_t^2 \rangle - \langle \Delta p_t \rangle^2}$$

[In the following I will use our own study (MC, Rojo, Salam, Soyez, 1010.1759)
as a source of plots, but the results should be quite generic.
NB. 'LHC' will be 5.5 TeV, but the results will be qualitatively similar at 2.76 TeV]

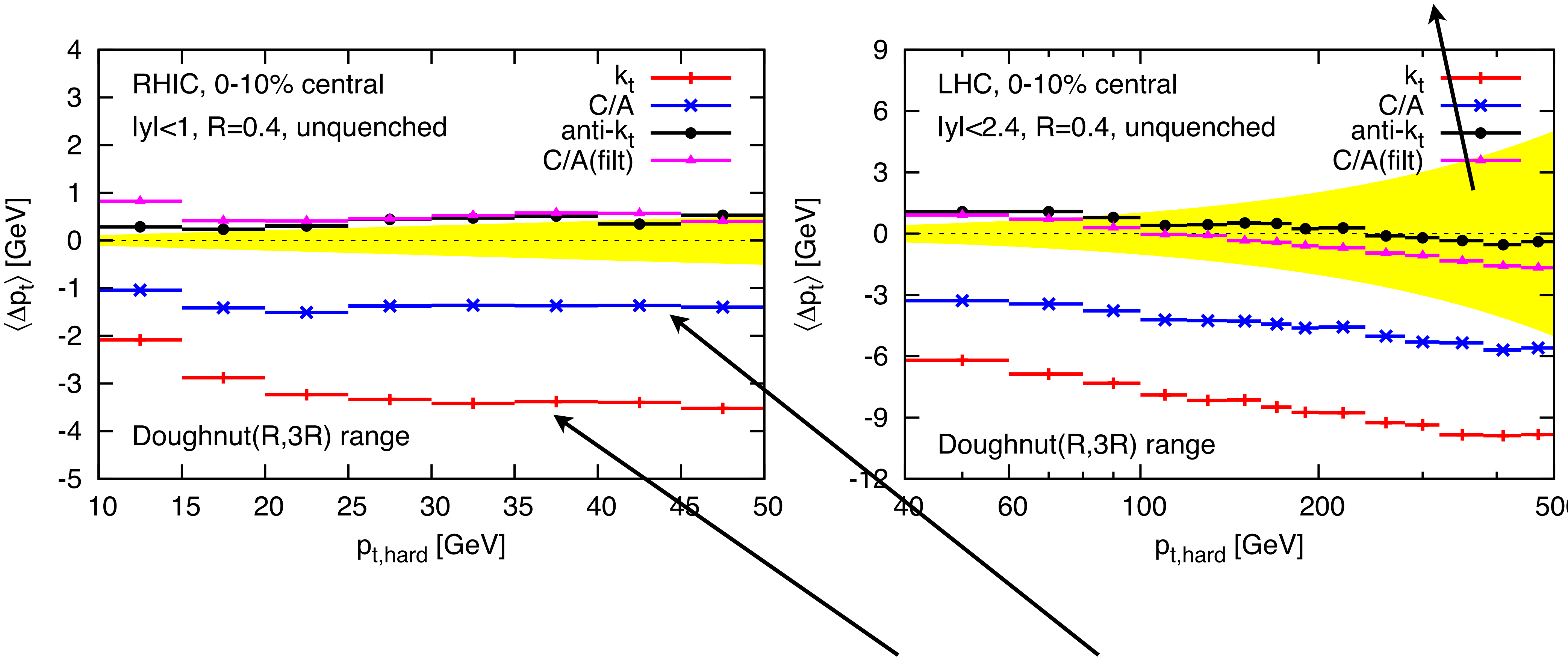
Δp_t distributions in PbPb at LHC



anti-k_t and C/A(filt) fare best

(Results are fairly independent of p_t)

Yellow band: 1% accuracy



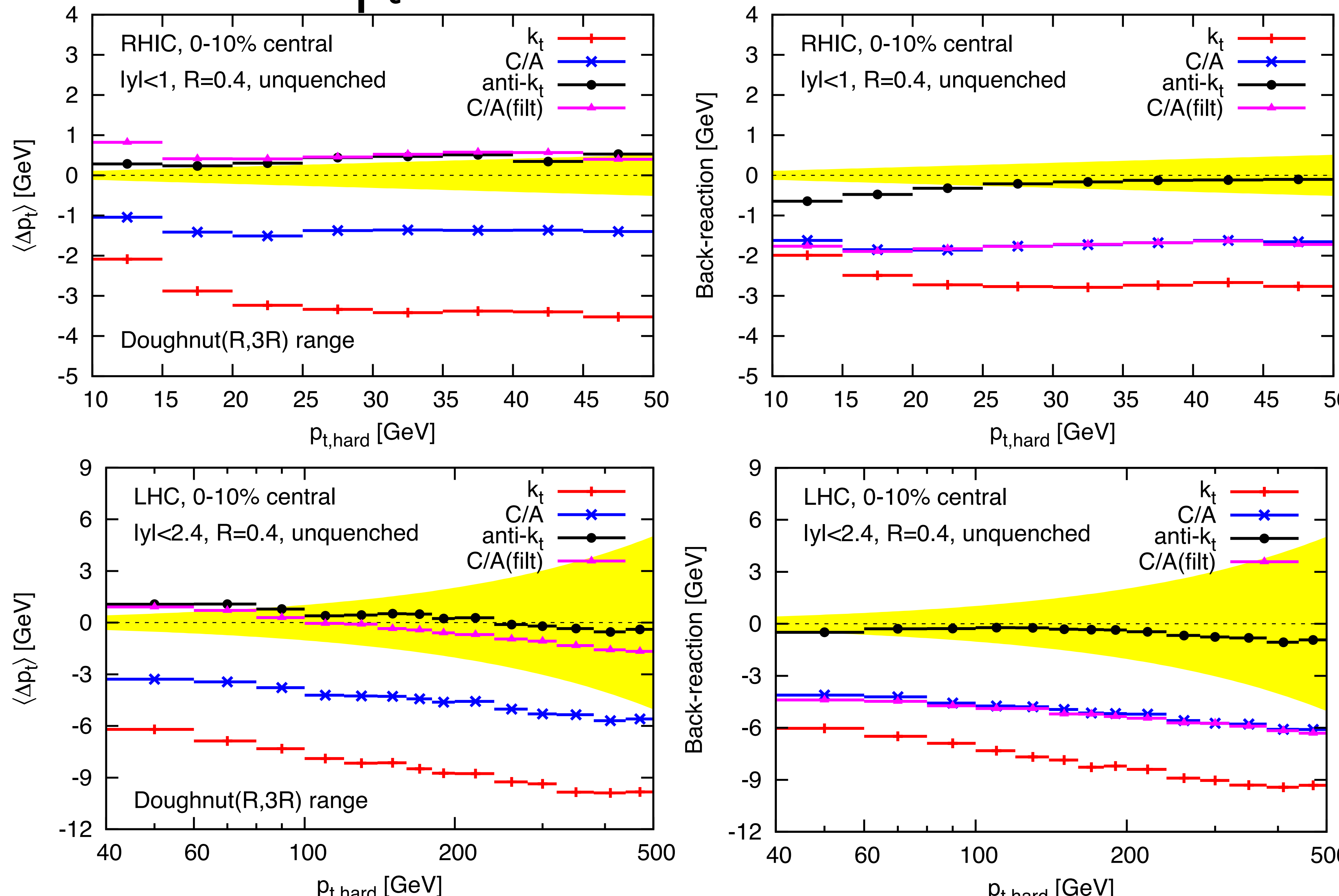
The residual offset of k_t and C/A can be interpreted as an effect of the **back-reaction**

Back-reaction contribution to <Δp_t>

<Δp_t> | Back-reaction

RHIC

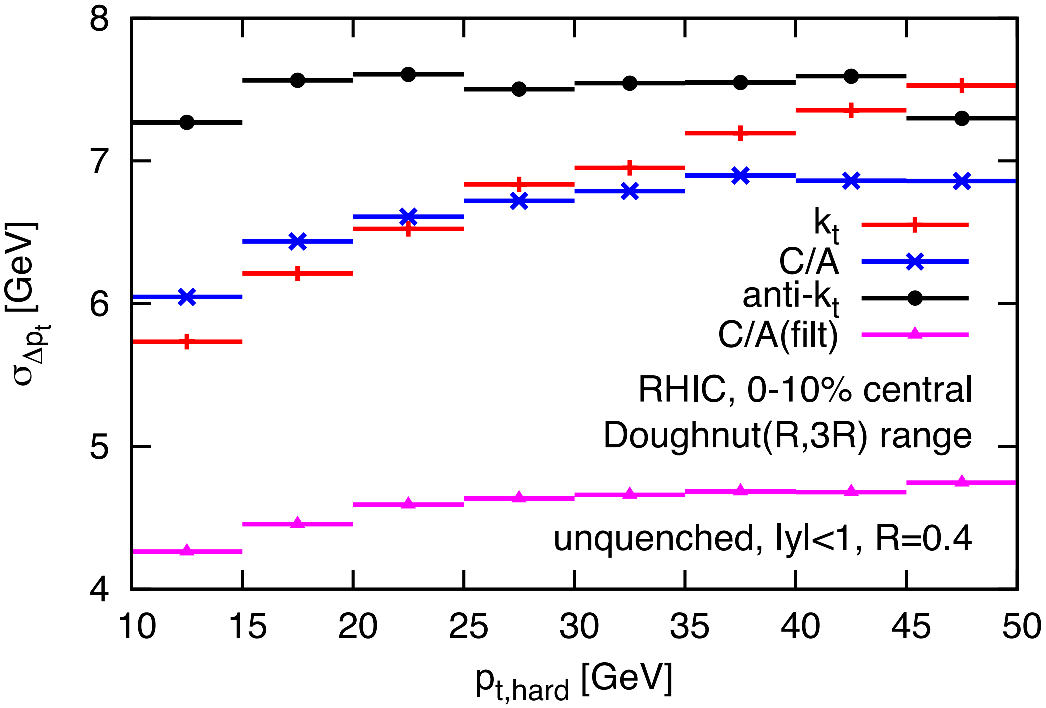
LHC

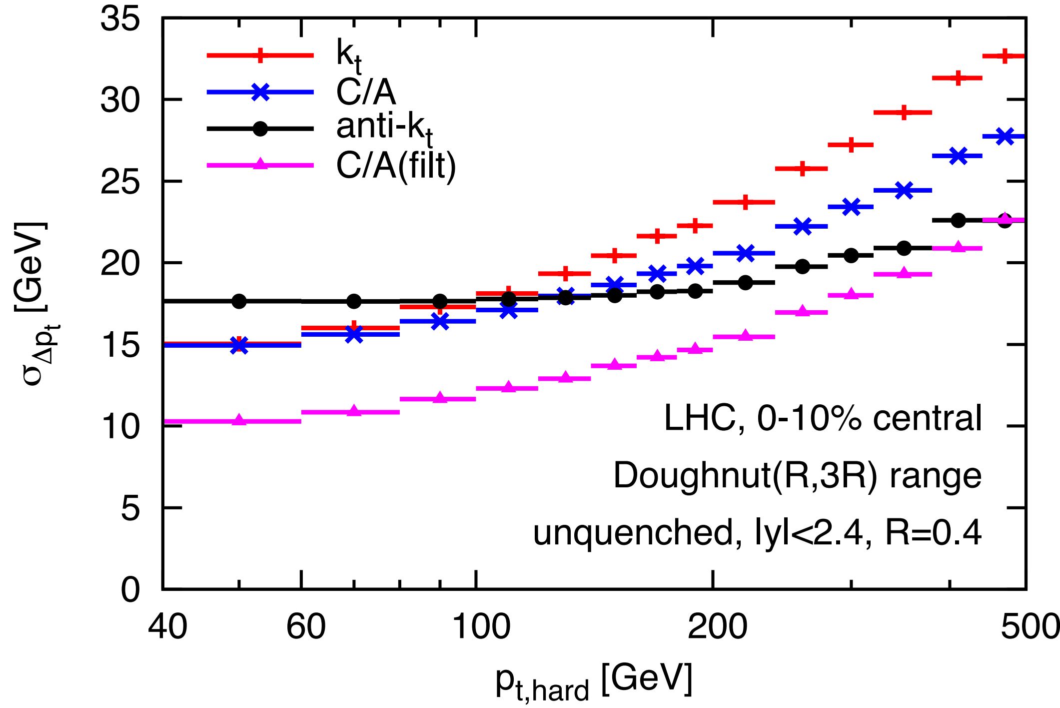


Back-reaction explains the residual offset, with the exception of C/A(filt)

(accidental compensation of back-reaction and positive offset)

Dispersion of $\Delta p_t = \sigma_{\Delta pt}$





- C/A(filt) markedly better, as a consequence of its smaller effective area
- Dispersions increase at large p_t, as a consequence of a larger dispersion of back-reaction
- anti-k_t remains fairly constant ('resiliency'), and eventually becomes better at large p_t

Jet shapes

- A jet shape is **any function of the momenta of a jet's constituents**
  - The simplest example is the **scalar transverse momentum**: sum of the p_t's of all constituents
  - Other **IRC-safe** examples: the **jet's mass**, **splitting scales**, **N-subjettiness**, **energy-energy correlation**,....
  - **Non-IRC-safe** examples are the **moments of a jet fragmentation function**
- Jet shapes can be used to characterise the jets
- Even better, if calculable they can be compared to theoretical predictions

However, they will in general be affected by background.
How to subtract its contamination?

More on pileup subtraction

The $\mathbf{p_T^{raw}-\rho A}$ technique (also called **area/median**) only corrects a jet's transverse momentum

Each jet shape has its own specific sensitivity to background contamination.

How to correct them?

- One option is to study analytically each shape [Alon et al. 1101.3002]. Can be time consuming and cumbersome
- Alternatively, determine **numerically** the *susceptibility* of any IRC-safe jet shape to contamination [Soyez et al. 1211.2811] (this generalises the jet area)

Numerical jet shape correction

Numerical derivative w.r.t. ghosts momenta

Ghosts area

Jet shape as a function of the jets's constituents momenta

$$V_{\rm jet}^{[n]} \equiv A_g^n \frac{d^n}{dr_{t,g}^n} V(\{p_i\}_{\rm jet})$$

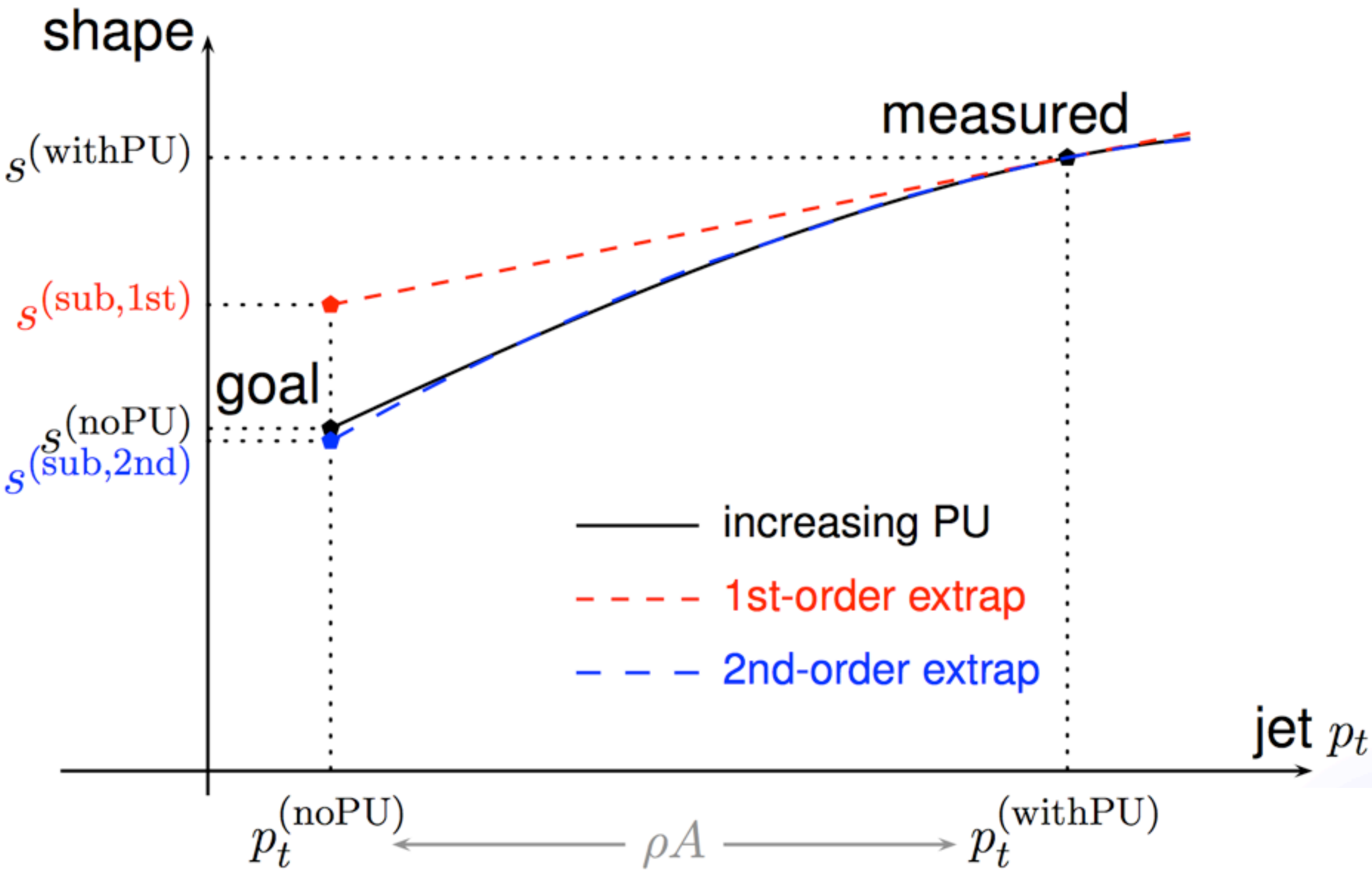
Pileup momentum density

$$V_{\rm jet,sub} = V_{\rm jet} - \rho V_{\rm jet}^{[1]} + \frac{1}{2}\rho^2 V_{\rm jet}^{[2]} + \cdots$$

Numerical derivative w.r.t. ghosts momenta

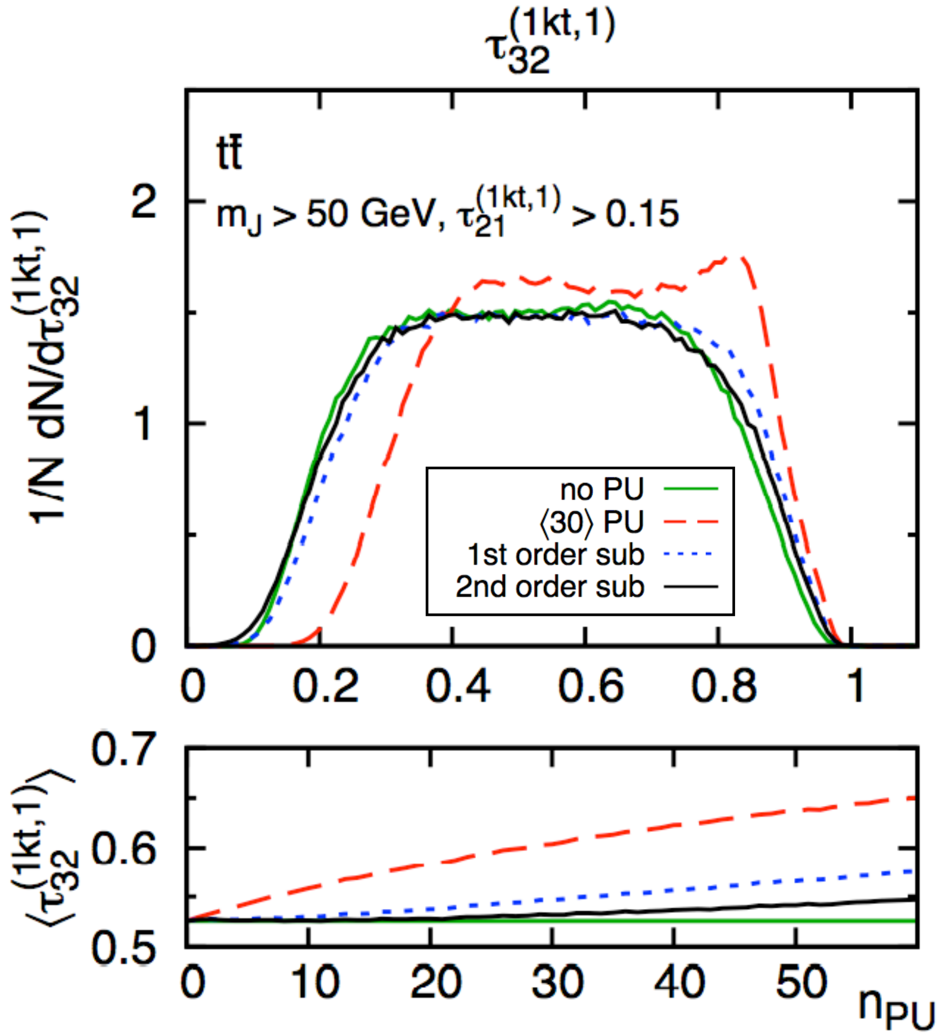
This procedure generalises the transverse momentum correction to any jet shape

Numerical jet shape correction



Numerical jet shape correction

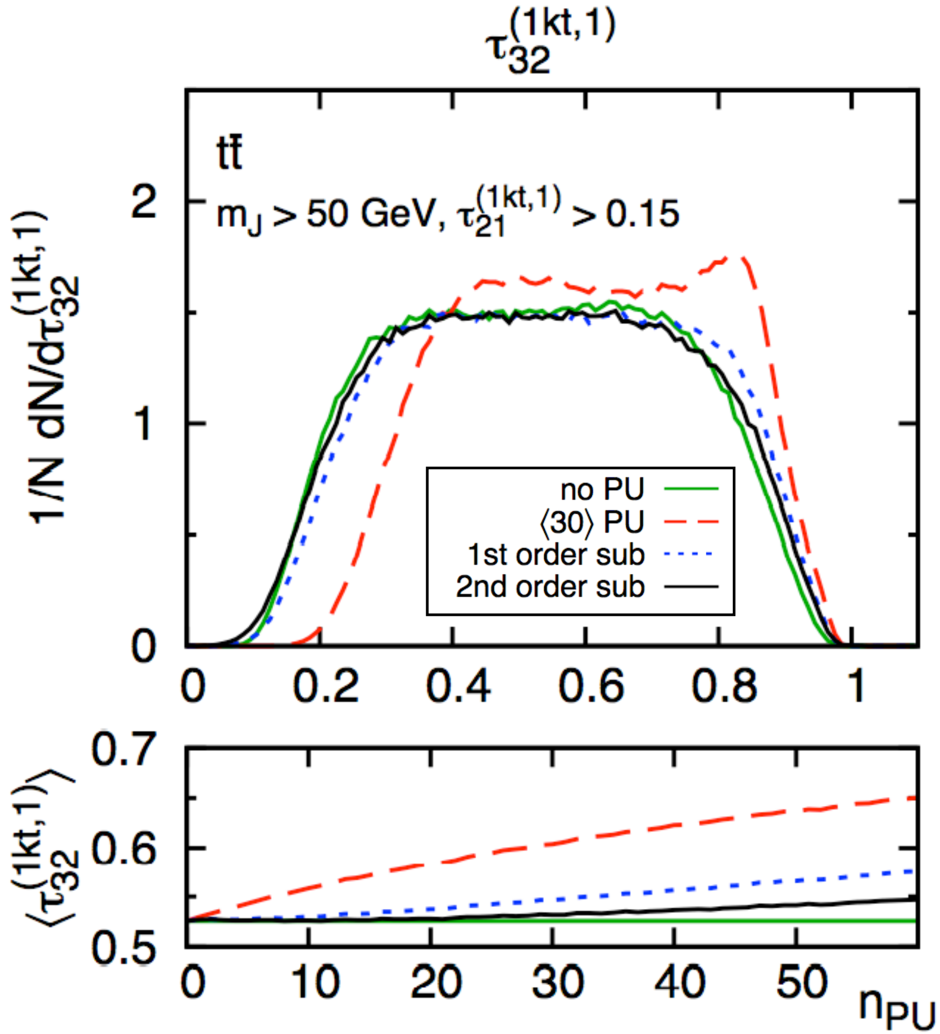
Example: τ_{32} correction and top tagging

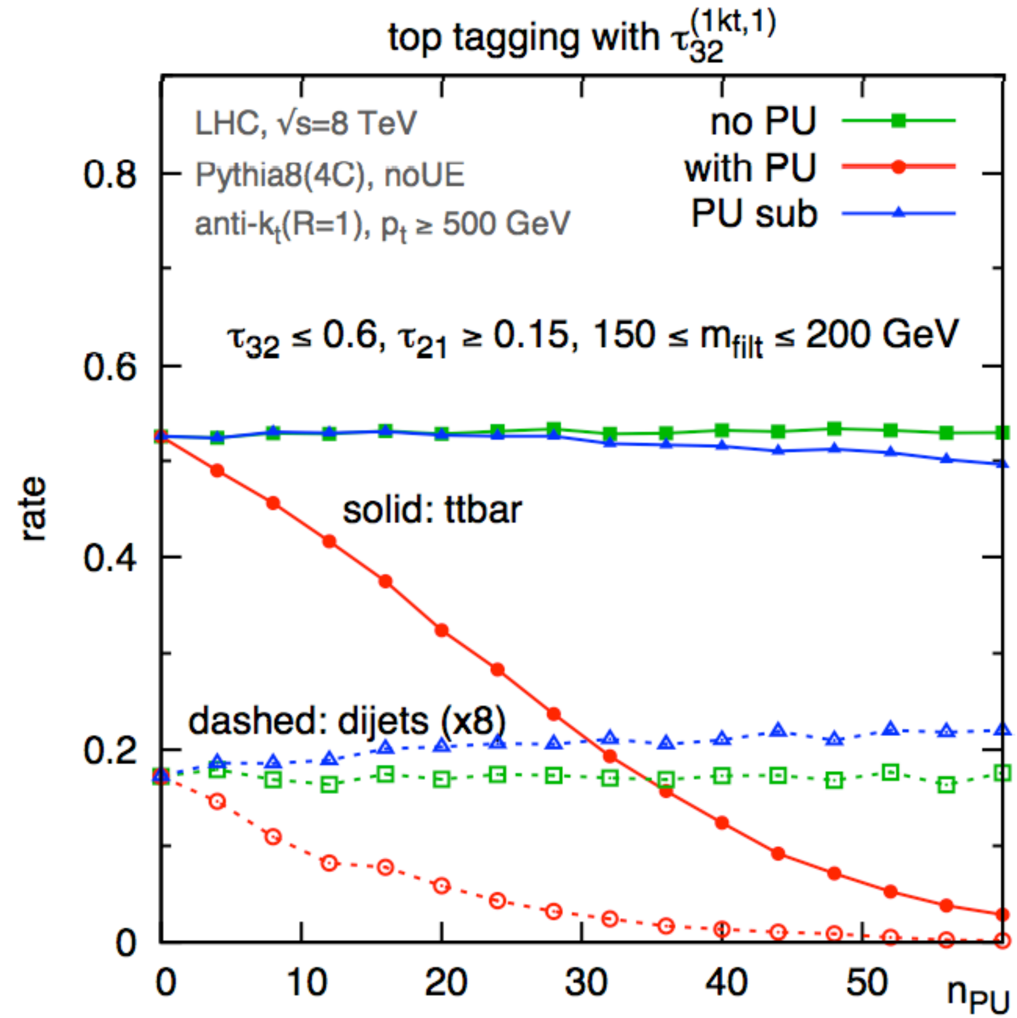


- Original distribution reproduced after pileup subtraction

Numerical jet shape correction

Example: τ_{32} correction and top tagging



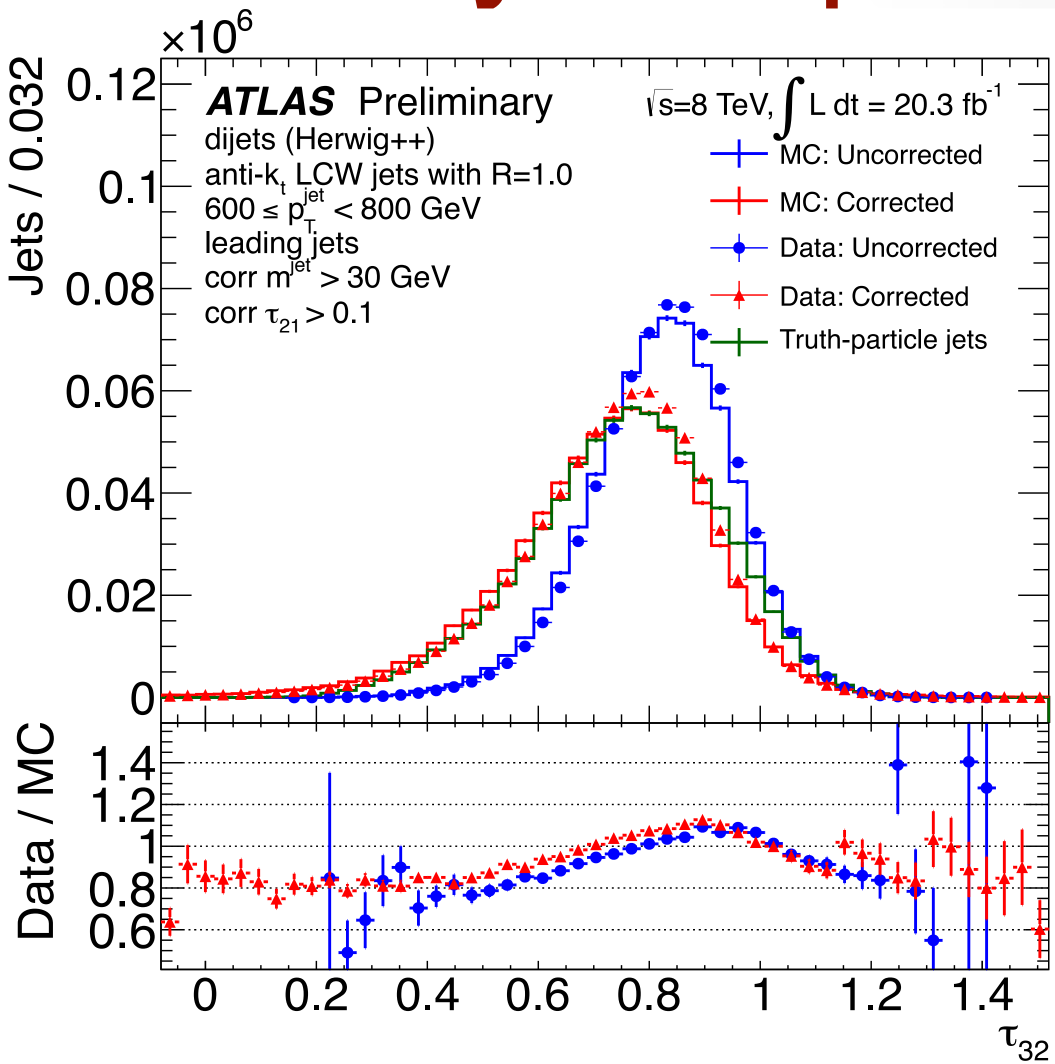


- Original distribution reproduced after pileup subtraction

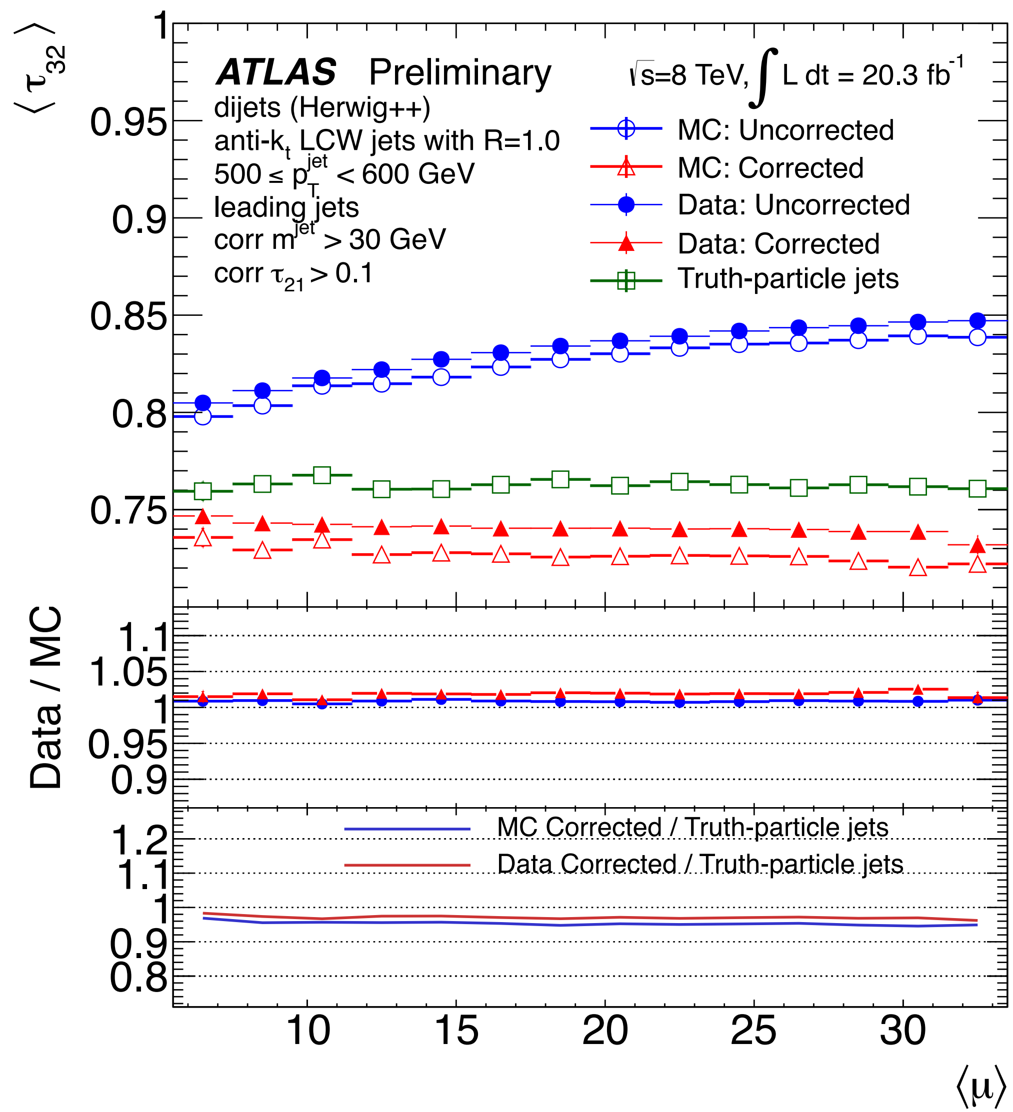
- **Tagging rates independent of amount of pileup after correction of the jet shapes involved in the tagging**

ATLAS-CONF-2013-085

Jet shape subtraction in ATLAS



τ_{32} distribution



$<\tau_{32}>$ as a function of the average number of pileup collisions

Summary

- The susceptibility of a jet's transverse momentum can be determined in terms of a jet area (or, for a generic jet shape, a numerical derivative with respect to ghosts momenta)

- Background densities can be estimated with a number of techniques, some of them jet-based. Non uniformities in the background can, up to a certain extent, be accounted for.

- The background can then be subtracted from jet momenta or from jet-shapes
  - Alternative subtraction techniques can be devised. Usually, there will be a compromise between residual bias and dispersion

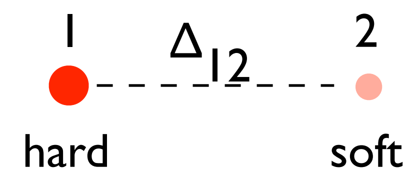
- All these tools and techniques are implemented in a standard way in Fastjet or its 3rd-party add-on fjcontrib (fastjet.hepforge.org)

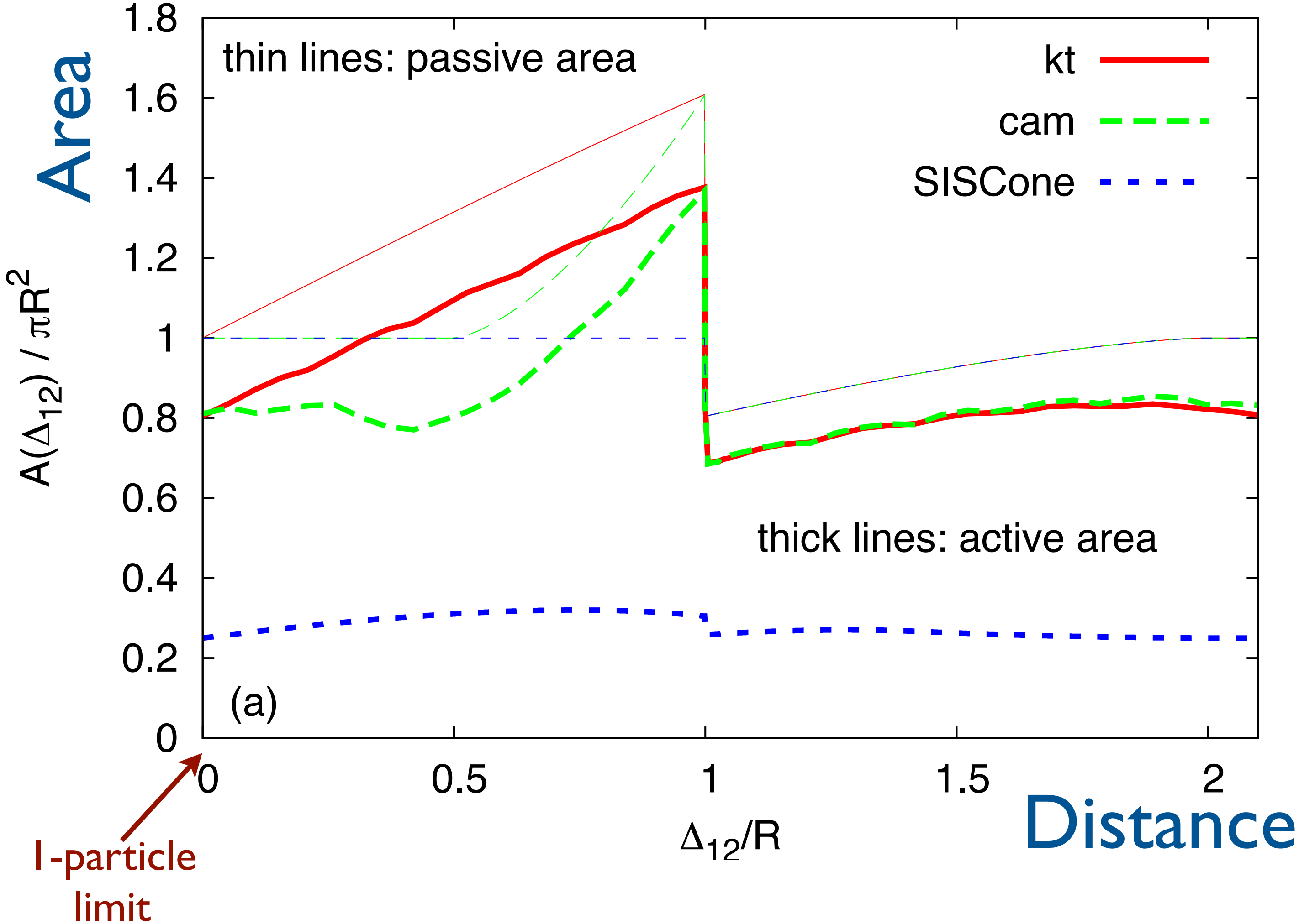
<Areas anomalous dimensions>

Jet areas: a second hard(ish) particle

Real events have more than a single hard particle.

Add a second (soft) one at a distance Δ_{12}





The jet area depends on the distance between the particles

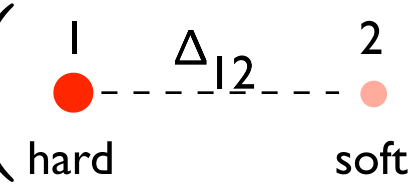
Note very small active area for SISCone!

Passive areas (and SISCone's active area) can be calculated **analytically**, while the others are obtained numerically

Jet areas: anomalous dimensions

Finally, weigh the probability of emission of the soft particle with the leading-order QCD matrix element:

$$\langle \Delta area \rangle = \int C_1 \frac{\alpha_s(p_{t2}\Delta_{12})}{\pi} \frac{dp_{t2}}{p_{t2}} \left[\frac{d\Delta_{12}}{\Delta_{12}}\right]_+ \left(\quad \right)$$



The result is an **anomalous dimension**.

Areas change with transverse momentum of the jet in a predictable way:

$$\langle \Delta area \rangle = d \; \frac{C_1}{\pi b_0} \ln \frac{\alpha_s(Q_0)}{\alpha_s(Rp_{t1})}$$

In a similar way one can also predict the evolution of the dispersion, calculating

$$\langle \Delta\, area^2 \rangle = s^2 \; \frac{C_1}{\pi b_0} \ln \frac{\alpha_s(Q_0)}{\alpha_s(Rp_{t1})}$$

Passive areas: analytical results

MC, Salam, Soyez, arXiv:0802.1188

d:

$$d_{k_t,R} = \left(\frac{\sqrt{3}}{8} + \frac{\pi}{3} + \xi\right) R^2 \simeq 0.5638\,\pi R^2\,,$$

$$d_{\mathrm{Cam},R} = \left(\frac{\sqrt{3}}{8} + \frac{\pi}{3} - 2\xi\right) R^2 \simeq 0.07918\,\pi R^2\,,$$

$$d_{\mathrm{SISCone},R} = \left(-\frac{\sqrt{3}}{8} + \frac{\pi}{6} - \xi\right) R^2 \simeq -0.06378\,\pi R^2\,,$$

Negative! SISCone jets shrink!

s^2:

$$s^2_{k_t,R} = \left(\frac{\sqrt{3}\pi}{4} - \frac{19}{64} - \frac{15\zeta(3)}{8} + 2\pi\xi\right) R^4 \simeq (0.4499\,\pi R^2)^2\,,$$

$$s^2_{\mathrm{Cam},R} = \left(\frac{\sqrt{3}\pi}{6} - \frac{3}{64} - \frac{\pi^2}{9} - \frac{13\zeta(3)}{12} + \frac{4\pi}{3}\xi\right) R^4 \simeq (0.2438\,\pi R^2)^2\,,$$

$$s^2_{\mathrm{SISCone},R} = \left(\frac{\sqrt{3}\pi}{12} - \frac{15}{64} - \frac{\pi^2}{18} - \frac{13\zeta(3)}{24} + \frac{2\pi}{3}\xi\right) R^4 \simeq (0.09142\,\pi R^2)^2\,.$$

with $\xi \equiv \dfrac{\psi'(1/6) + \psi'(1/3) - \psi'(2/3) - \psi'(5/6)}{48\sqrt{3}} \simeq 0.507471$

Jet areas: passive v. active

	area/πR^2		dispersion		d or D		s or S	
	passive	active	passive	active	passive	active	passive	active
	$a(1PJ)$	$A(1PJ)$	$\sigma(1PJ)$	$\Sigma(1PJ)$	d	D	s	S
k_t	1	0.81	0	0.28	0.56	0.52	0.45	0.41
Cam/Aachen	1	0.81	0	0.26	0.08	0.08	0.24	0.19
SISCone	1	1/4	0	0	−0.06	0.12	0.09	0.07
anti-k_t	1	1	0	0	0	0	0	0

single hard particle

emission of a second perturbative particle
(coeff. of anomalous dimension)

Jet areas: passive v. active

	area/πR^2		dispersion		d or D		s or S	
	passive	active	passive	active	passive	active	passive	active
	$a(1PJ)$	$A(1PJ)$	$\sigma(1PJ)$	$\Sigma(1PJ)$	d	D	s	S
k_t	1	0.81	0	0.28	0.56	0.52	0.45	0.41
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anti-k_t	1	1	0	0	0	0	0	0

single hard particle

emission of a second perturbative particle (coeff. of anomalous dimension)

Some remarkable features

- SISCone has very small active area
- SISCone's anomalous dimension changes from negative for passive area to positive for active area
- k_t has largest anomalous dimension
- anti-k_t has constant area (null anomalous dimension): it's a **perfect cone**

</Areas anomalous dimensions>