The impact of the slow solutions in the winds of massive stars

Michel Curé
Departamento de Física y Astronomía,
Universidad de Valparaiso, Chile

Collaborators:
Lydia Cidale, Anahi Granada **et al**. Univ. Nac. de La Plata, Argentina
Diego Rial, Univ. de Buenos Aires, Argentina
Alfredo Santíllan, UNAM, Mexico
Ignacio Araya, Universidad de Valparaíso

History

Late ‘60: first UV spectral observations

ROCKET OBSERVATIONS OF MASS LOSS FROM HOT STARS*

DONALD C. MORTON

Princeton University Observatory, Princeton, N.J., U.S.A.

Abstract. Rocket observations have shown that the far-ultraviolet resonance lines have P-Cygni profiles in the spectra of many hot stars, including of and Wolf-Rayet stars and OB supergiants. Velocity shifts as high as -3000 km sec^{-1} have been measured for the short-wavelength edges of some of the lines. Estimates of the rates of mass loss range from 10^{-8} to 10^{-6} $M_\odot$ year^{-1}.

History

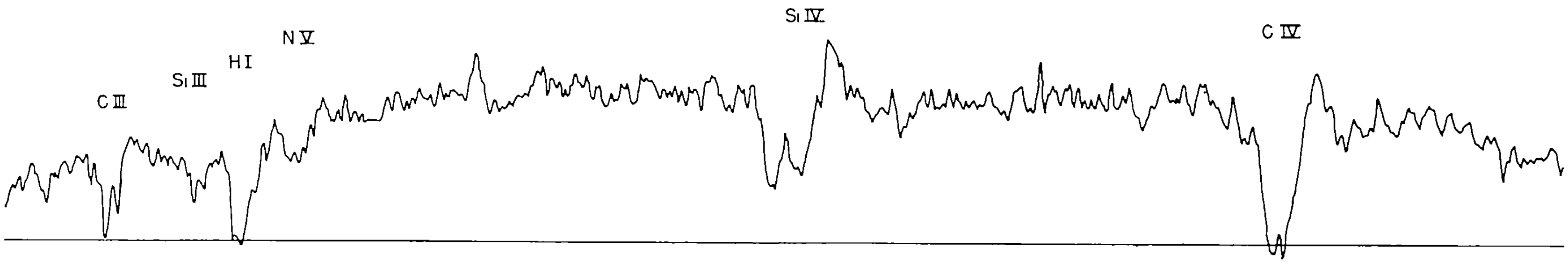


Fig. 1. Densitometer tracing, on an intensity scale, of the far-ultraviolet spectrum of ζ Orionis, photographed by Princeton on September 10, 1966. The distribution of intensity with wavelength includes the unknown response of the spectrograph. Wavelengths increase towards the right from 1140 to 1630 Å. The H I line is interstellar, but all the other identified absorption features are circumstellar with large Doppler shifts to shorter wavelengths.

History

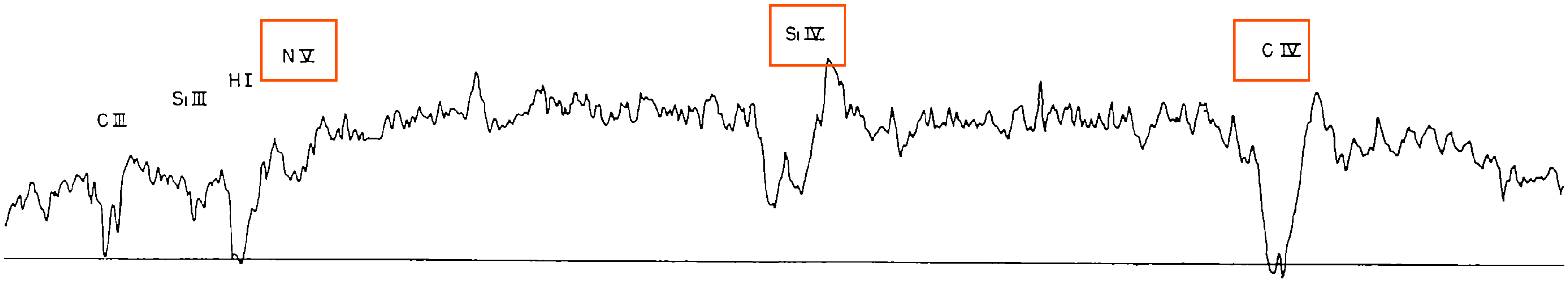


Fig. 1. Densitometer tracing, on an intensity scale, of the far-ultraviolet spectrum of ζ Orionis, photographed by Princeton on September 10, 1966. The distribution of intensity with wavelength includes the unknown response of the spectrograph. Wavelengths increase towards the right from 1140 to 1630 Å. The H I line is interstellar, but all the other identified absorption features are circumstellar with large Doppler shifts to shorter wavelengths.

Theory

Only known Theory:

Parker's Model for the Solar Wind (Parker, E.N.: 1960, ApJ 132, 821)

For O stars => Teff = 10^7 K

But at this Temperature C IV - N V - Si IV Don´t Exist

Destroyed by collisional ionization

Michel Curé
Universidad de Valparaíso

Radiation Driven Winds

Lucy & Solomon (1970, ApJ, 159, 870): Wind driven by resonance lines
Obtained only mass loss rates of about 1/100th of the observed values

Castor, Abbott & Klein (1975, ApJ, 195, 157)
Wind driven by an ensemble of lines (scattering)
They obtained a qualitative agreement with the observational values

Radiation Driven Winds

Lucy & Solomon (1970, ApJ, 159, 870): Wind driven by resonance lines
Obtained only mass loss rates of about 1/100th of the observed values

Castor, Abbott & Klein (1975, ApJ, 195, 157)
Wind driven by an ensemble of lines (scattering)
They obtained a qualitative agreement with the observational values

The Standard Model
(m-CAK)

Michel Curé
Universidad de Valparaíso

1D - Hydrodynamics

Assumptions:
Stationary - Low viscosity - Spherical symmetry - No Mag. Fields.

From Mass and Momentum Conservation laws:

1D - Hydrodynamics

Assumptions:
Stationary - Low viscosity - Spherical symmetry - No Mag. Fields.

From Mass and Momentum Conservation laws:

$$4\pi r^2 \rho v = \dot{M},$$

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM(1-\Gamma)}{r^2} + g^{\text{line}}\left(\rho, \frac{dv}{dr}, n_E\right)$$

Line Force: Very Simplified Version

Contribution by **one** line i at frequency ν_i

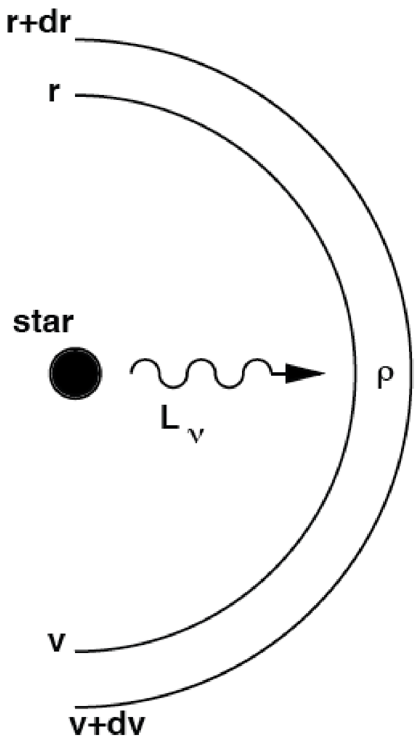
$$g^i_{rad} = \frac{photon\ \ momentum\ \ absorbed\ \ by\ \ line\ \ i}{time\ \ mass}$$

Michel Curé
Universidad de Valparaíso

Line Force: Very Simplified Version

Contribution by **one** line i at frequency ν_i

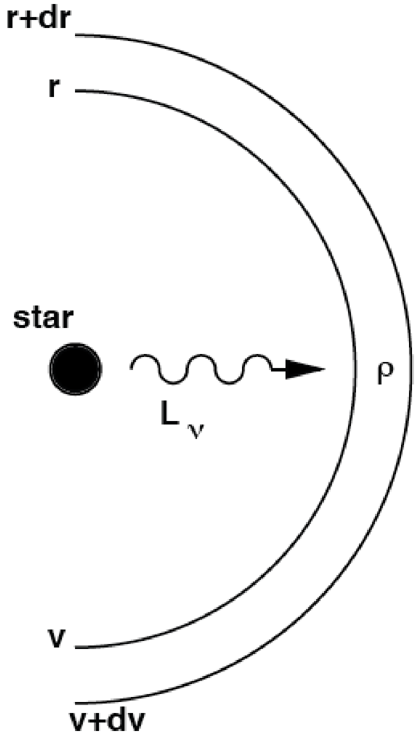
$$g^i_{rad} = \frac{photon\ momentum\ absorbed\ by\ line\ i}{time\ mass}$$



Line Force: Very Simplified Version

Contribution by **one** line i at frequency ν_i

$$g^i_{rad} = \frac{photon\ momentum\ absorbed\ by\ line\ i}{time\ mass}$$



$$dm = 4\pi r^2 \rho dr$$

Line Force: Very Simplified Version

Contribution by **one** line i at frequency ν_i

$$\frac{L}{c}\frac{L_{\nu_i}(1-e^{-\tau_i})d\nu^{\mathrm{WIDTH}}}{L} = \frac{L}{c^2}\frac{\nu_i L_{\nu_i}}{L}(1-e^{-\tau_i})dv.$$

Line Force: Very Simplified Version

Contribution by **one** line i at frequency ν_i

$$\frac{L}{c}\,\frac{L_{\nu_i}(1-e^{-\tau_i})d\nu^{\mathrm{WIDTH}}}{L} = \frac{L}{c^2}\,\frac{\nu_i L_{\nu_i}}{L}(1-e^{-\tau_i})dv.$$

total photon momentum rate provided by the star

Line Force: Very Simplified Version

Contribution by **one** line i at frequency ν_i

$$\frac{L}{c}\,\frac{L_{\nu_i}(1-e^{-\tau_i})d\nu^{\mathrm{WIDTH}}}{L} = \frac{L}{c^2}\,\frac{\nu_i L_{\nu_i}}{L}(1-e^{-\tau_i})dv.$$

$\frac{L}{c}$ ↑ total photon momentum rate provided by the star

$\frac{L_{\nu_i}(1-e^{-\tau_i})d\nu^{\mathrm{WIDTH}}}{L}$ ↑ fraction absorbed by one spectral line in an outer shell of thickness dr

Line Force: Very Simplified Version

Contribution by **one** line i at frequency ν_i

optical thickness

$$\frac{L}{c}\,\frac{L_{\nu_i}(1-e^{-\tau_i})d\nu^{\mathrm{WIDTH}}}{L} = \frac{L}{c^2}\,\frac{\nu_i L_{\nu_i}}{L}(1-e^{-\tau_i})dv.$$

total photon momentum rate provided by the star

fraction absorbed by one spectral line in an outer shell of thickness dr

Line Force: Very Simplified Version

Contribution by **one** line i at frequency ν_i

optical thickness

$$\frac{L}{c}\,\frac{L_{\nu_i}(1-e^{-\tau_i})\,d\nu^{\mathrm{WIDTH}}}{L} = \frac{L}{c^2}\,\frac{\nu_i L_{\nu_i}}{L}(1-e^{-\tau_i})\,dv.$$

total photon momentum rate provided by the star

fraction absorbed by one spectral line in an outer shell of thickness dr

$$d\nu^{\mathrm{WIDTH}} = \nu_i \frac{dv}{c}$$

Line Force: Very Simplified Version

Contribution by **one** line i at frequency ν_i

optical thickness

$$\frac{L}{c}\frac{L_{\nu_i}(1-e^{-\tau_i})d\nu^{\mathrm{WIDTH}}}{L} = \frac{L}{c^2}\frac{\nu_i L_{\nu_i}}{L}(1-e^{-\tau_i})dv.$$

total photon momentum rate provided by the star

fraction absorbed by one spectral line in an outer shell of thickness dr

$$d\nu^{\mathrm{WIDTH}} = \nu_i \frac{dv}{c}$$

$$dm = 4\pi r^2 \rho dr$$

Line Force: Very Simplified Version

Contribution by **one** line i at frequency ν_i

optical thickness

$$\frac{L}{c}\frac{L_{\nu_i}(1-e^{-\tau_i})d\nu^{\mathrm{WIDTH}}}{L} = \frac{L}{c^2}\frac{\nu_i L_{\nu_i}}{L}(1-e^{-\tau_i})dv.$$

total photon momentum rate provided by the star

fraction absorbed by one spectral line in an outer shell of thickness dr

dv/dr

$$d\nu^{\mathrm{WIDTH}} = \nu_i \frac{dv}{c}$$

$$dm = 4\pi r^2 \rho dr$$

Line Force: Super Simplified Version

Contribution by **one** line i at frequency ν_i

Dependence on the Velocity gradient

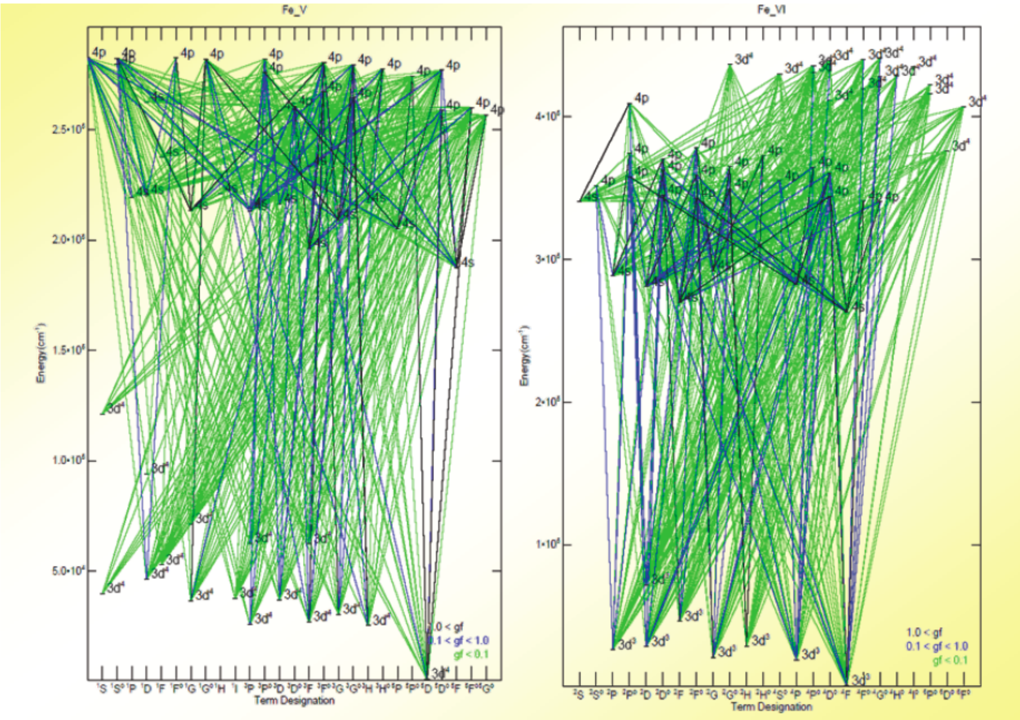
$$g_{rad}^{Th}(r) = \frac{n_e \sigma_e L}{c 4\pi r^2 \rho}$$

$$g_{rad}^{i} = g_{rad}^{Th} \frac{1}{c} \frac{\nu_i L_{\nu_i}}{L} \left(1 - e^{-\tau_i}\right) \frac{1}{n_e \sigma_e} \frac{dv}{dr}$$

FORCE MULTIPLIER

Line Force

Contribution from an ensemble of lines
Currently: 4.2 Mega lines, 150 ionization stages (H –Zn),

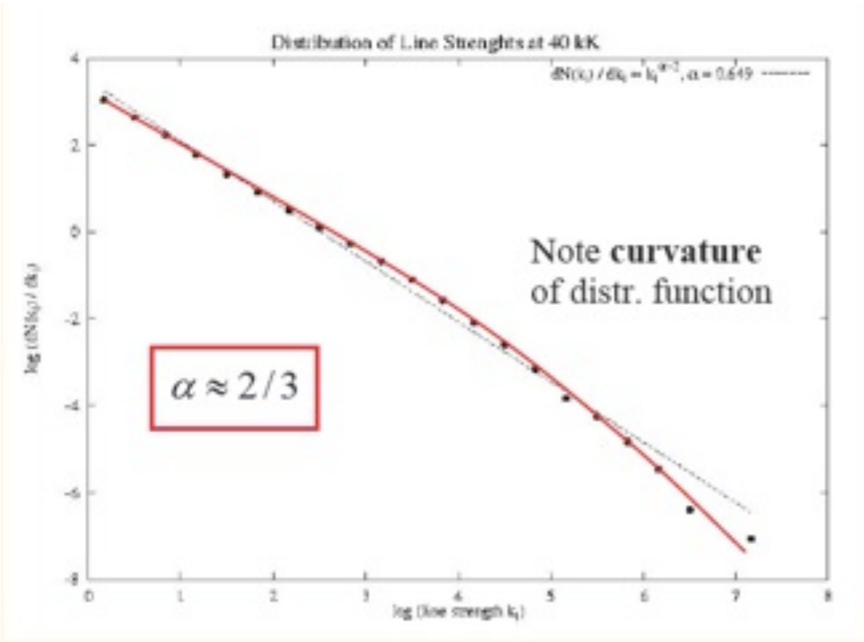


	I	II	III	IV	V	VI	VII	VIII
1	H_I							
2	He_I	He_II						
6	C_I	C_II	C_III	C_IV	C_V			
7	N_I	N_II	N_III	N_IV	N_V	N_VI		
8	O_I	O_II	O_III	O_IV	O_V	O_VI		
9	F_I	F_II	F_III	F_IV	F_V	F_VI		
10	Ne_I	Ne_II	Ne_III	Ne_IV	Ne_V	Ne_VI		
11	Na_I	Na_II	Na_III	Na_IV	Na_V	Na_VI		
12	Mg_I	Mg_II	Mg_III	Mg_IV	Mg_V	Mg_VI		
13	Al_I	Al_II	Al_III	Al_IV	Al_V	Al_VI		
14	Si_I	Si_II	Si_III	Si_IV	Si_V	Si_VI		
15	P_I	P_II	P_III	P_IV	P_V	P_VI		
16	S_I	S_II	S_III	S_IV	S_V	S_VI	S_VII	
17	Cl_I	Cl_II	Cl_III	Cl_IV	Cl_V	Cl_VI		
18	Ar_I	Ar_II	Ar_III	Ar_IV	Ar_V	Ar_VI	Ar_VII	ArVIII
19	K_I	K_II	K_III	K_IV	K_V	K_VI		
20	Ca_I	Ca_II	Ca_III	Ca_IV	Ca_V	Ca_VI		
22	Ti_I	Ti_II	Ti_III	Ti_IV	Ti_V			
23	V_I	V_II	V_III	V_IV	V_V			
24	Cr_I	Cr_II	Cr_III	Cr_IV	Cr_V	Cr_VI		
25	Mn_I	Mn_II	Mn_III	Mn_IV	Mn_V	Mn_VI		
26	Fe_I	Fe_II	Fe_III	Fe_IV	Fe_V	Fe_VI	Fe_VII	FeVIII
27	Co_I	Co_II	Co_III	Co_IV	Co_V	Co_VI	Co_VII	
28	Ni_I	Ni_II	Ni_III	Ni_IV	Ni_V	Ni_VI	Ni_VII	NiVIII
29	Cu_I	Cu_II	Cu_III	Cu_IV	Cu_V	Cu_VI		
30	Zn_I	Zn_II	Zn_III					

atomic data status:

excellent	good	poor	bad

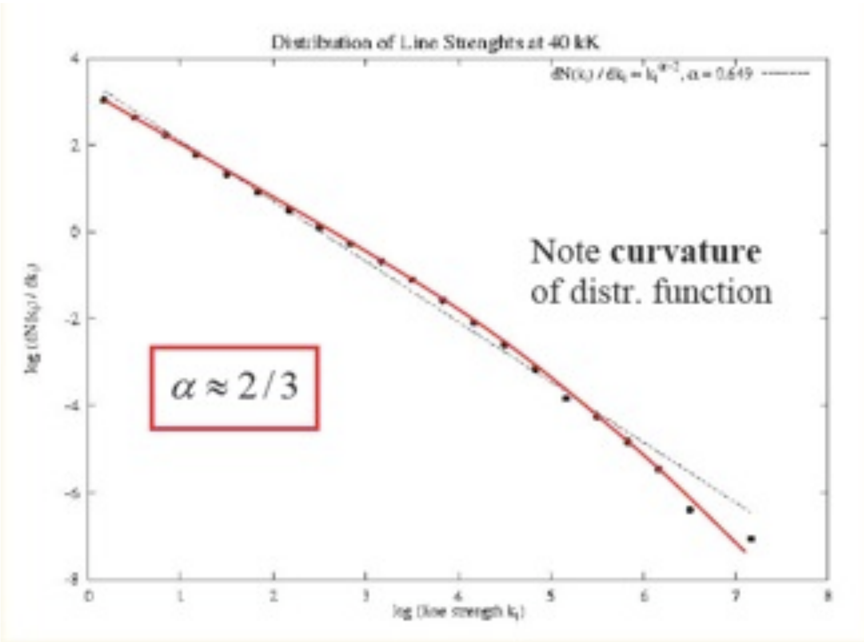
Line Force



Logarithmic plot of line-strength distribution function for an O-type wind at 40,000 K and corresponding power-law fit
(Puls et al. 2000, A&AS 141)

Line Force

$$\mathrm{n(k,\nu)}\,d\nu dk \propto k^{\alpha-2}dk$$



Logarithmic plot of line-strength distribution function for an O-type wind at 40,000 K and corresponding power-law fit
(Puls et al. 2000, A&AS 141)

Mass-loss

Ensemble of lines

$$g^{\mathrm{line}} = \frac{C}{r^2}\left(r^2 v \frac{dv}{dr}\right)^{\alpha}$$

CAK & m-CAK Models

Castor, Abbott & Klein (1975), Abbott (1982), Friend & Abbott (1986), Pauldrach et al. (1986)

CAK & m-CAK Models

Castor, Abbott & Klein (1975), Abbott (1982), Friend & Abbott (1986), Pauldrach et al. (1986)

$$g^{\text{line}} = \frac{C}{r^2} CF\left(r, v, \frac{dv}{dr}\right) \left(r^2 v \frac{dv}{dr}\right)^{\alpha} \left[\frac{n_E}{W(r)}\right]^{\delta}$$

Michel Curé
Universidad de Valparaíso

CAK & m-CAK Models

Castor, Abbott & Klein (1975), Abbott (1982), Friend & Abbott (1986), Pauldrach et al. (1986)

Eigenvalue
(Mass-loss)

$$g^{\rm line} = \frac{C}{r^2} CF\left(r, v, \frac{dv}{dr}\right)\left(r^2 v \frac{dv}{dr}\right)^{\alpha} \left[\frac{n_E}{W(r)}\right]^{\delta}$$

Michel Curé
Universidad de Valparaíso

CAK & m-CAK Models

Castor, Abbott & Klein (1975), Abbott (1982), Friend & Abbott (1986), Pauldrach et al. (1986)

Eigenvalue (Mass-loss) → C

Ensemble of lines → α

$$g^{\rm line} = \frac{C}{r^2} CF\left(r, v, \frac{dv}{dr}\right)\left(r^2 v \frac{dv}{dr}\right)^{\alpha}\left[\frac{n_E}{W(r)}\right]^{\delta}$$

Michel Curé
Universidad de Valparaíso

CAK & m-CAK Models

Castor, Abbott & Klein (1975), Abbott (1982), Friend & Abbott (1986), Pauldrach et al. (1986)

Eigenvalue (Mass-loss)

Ensemble of lines

$$g^{\rm line} = \frac{C}{r^2} CF\left(r, v, \frac{dv}{dr}\right) \left(r^2 v \frac{dv}{dr}\right)^{\alpha} \left[\frac{n_E}{W(r)}\right]^{\delta}$$

Changes in Ionization

CAK & m-CAK Models

Castor, Abbott & Klein (1975), Abbott (1982), Friend & Abbott (1986), Pauldrach et al. (1986)

Eigenvalue (Mass-loss)

Ensemble of lines

$$g^{\text{line}} = \frac{C}{r^2} CF\left(r, v, \frac{dv}{dr}\right)\left(r^2 v \frac{dv}{dr}\right)^{\alpha}\left[\frac{n_E}{W(r)}\right]^{\delta}$$

Correction Factor

Changes in Ionization

Michel Curé
Universidad de Valparaíso

CAK & m-CAK Models

Castor, Abbott & Klein (1975), Abbott (1982), Friend & Abbott (1986), Pauldrach et al. (1986)

Eigenvalue (Mass-loss)

Ensemble of lines

$$g^{\text{line}} = \frac{C}{r^2} CF\left(r, v, \frac{dv}{dr}\right)\left(r^2 v \frac{dv}{dr}\right)^{\alpha}\left[\frac{n_E}{W(r)}\right]^{\delta}$$

Correction Factor

Changes in Ionization

Radiation Driven Wind Hydrodynamics

Michel Curé
Universidad de Valparaíso

Radiation Driven Wind Hydrodynamics

Mass Conservation ⟶ $4\pi r^2 \rho v = \dot{M}$

Radiation Driven Wind Hydrodynamics

Mass Conservation ⟶ $4\pi r^2 \rho v = \dot{M}$

$$v \frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM(1-\Gamma)}{r^2} + g^{\text{line}}\left(\rho, \frac{dv}{dr}, n_E\right)$$

Michel Curé
Universidad de Valparaíso

Radiation Driven Wind Hydrodynamics

Mass Conservation $\longrightarrow$ $4\pi r^2 \rho v = \dot{M}$

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM(1-\Gamma)}{r^2} + g^{\text{line}}\left(\rho, \frac{dv}{dr}, n_E\right)$$

Inertia

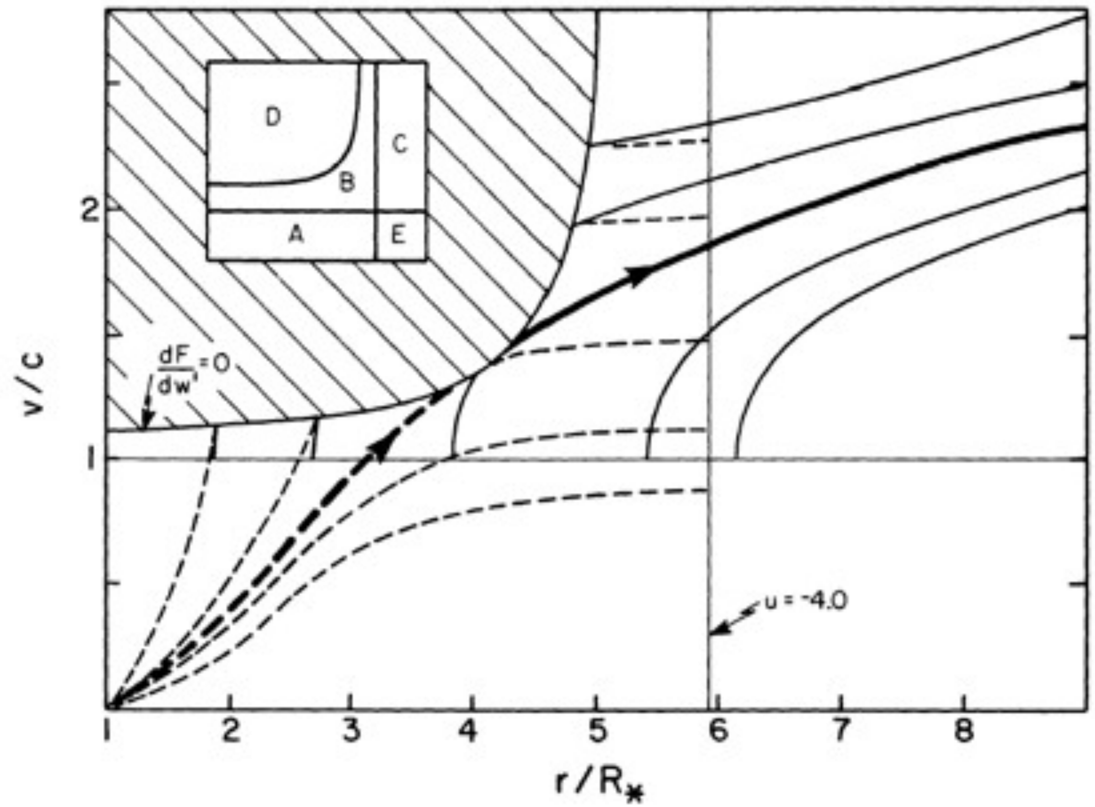
Gas Pressure

Gravity

Line Force

First Topological Analysis

Non-Rotating Solution Schema

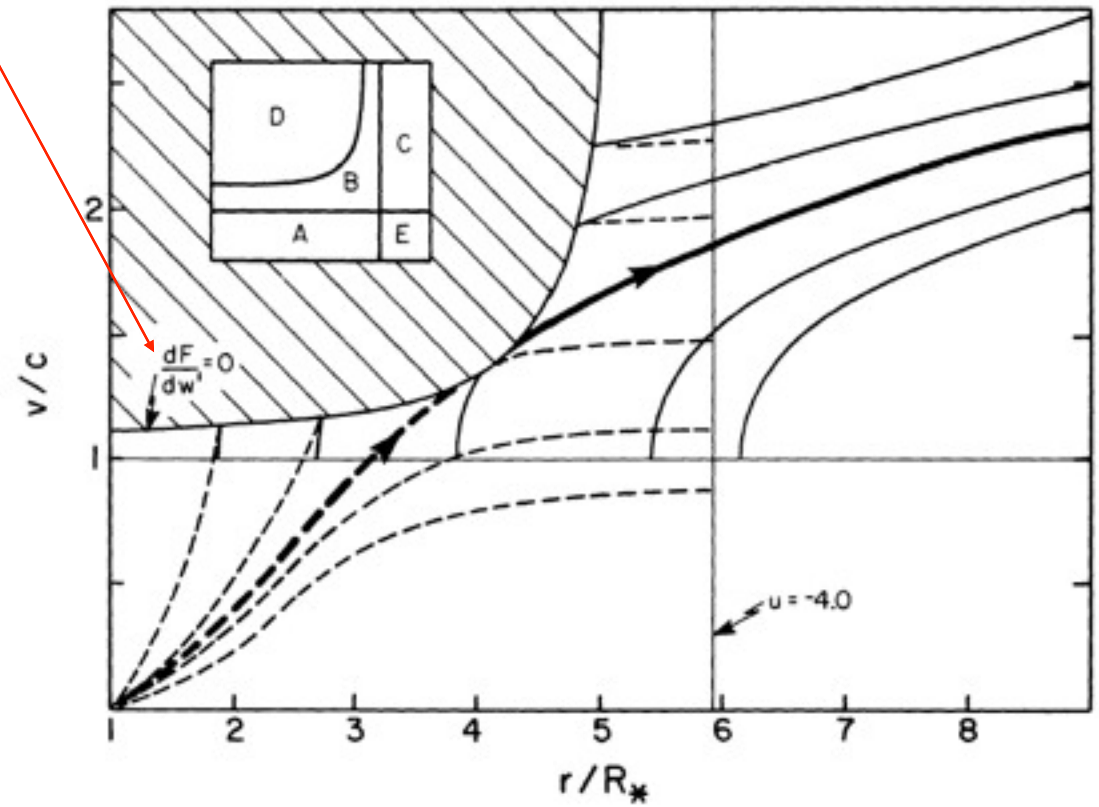


Michel Curé
Universidad de Valparaíso

Non-Rotating Solution Schema

Singularity Condition

$$\frac{\partial}{\partial w'} F(u, w, w') = 0$$



Michel Curé
Universidad de Valparaíso

First Topological Analysis

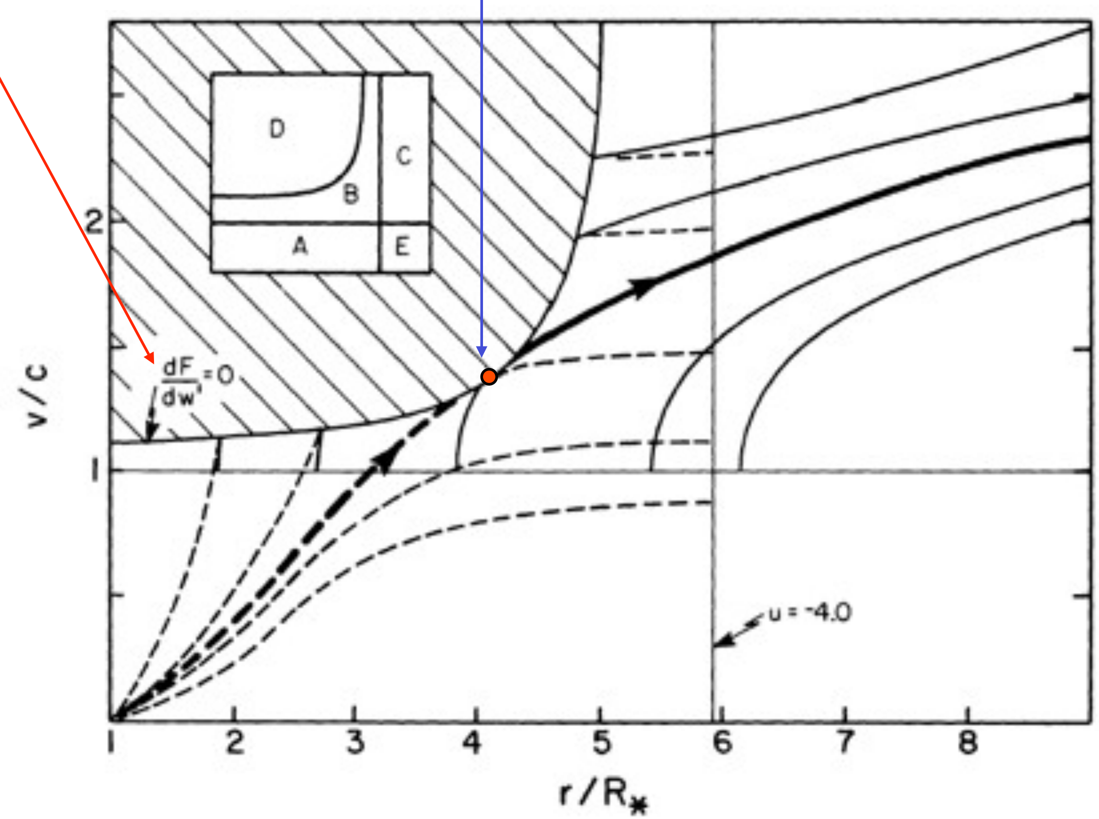
Non-Rotating Solution Schema

Singularity Condition

$$\frac{\partial}{\partial w'} F(u, w, w') = 0$$

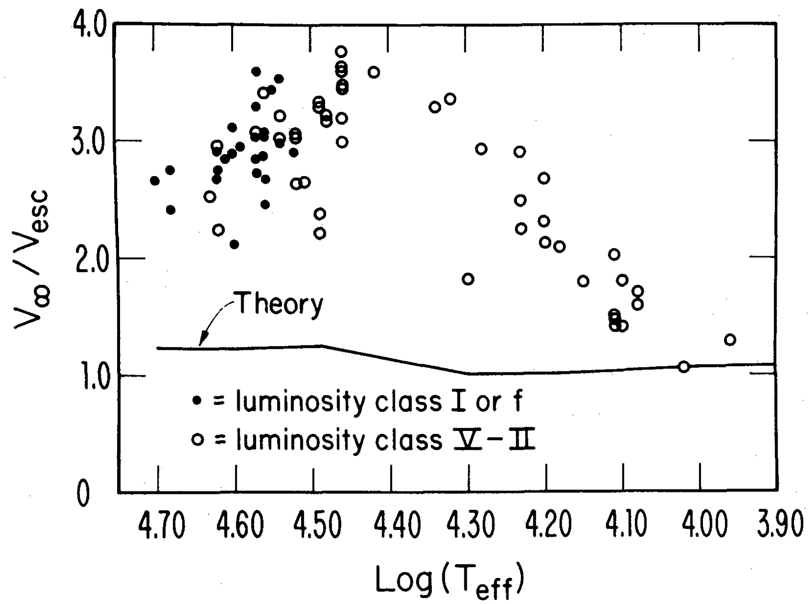
Regularity Condition

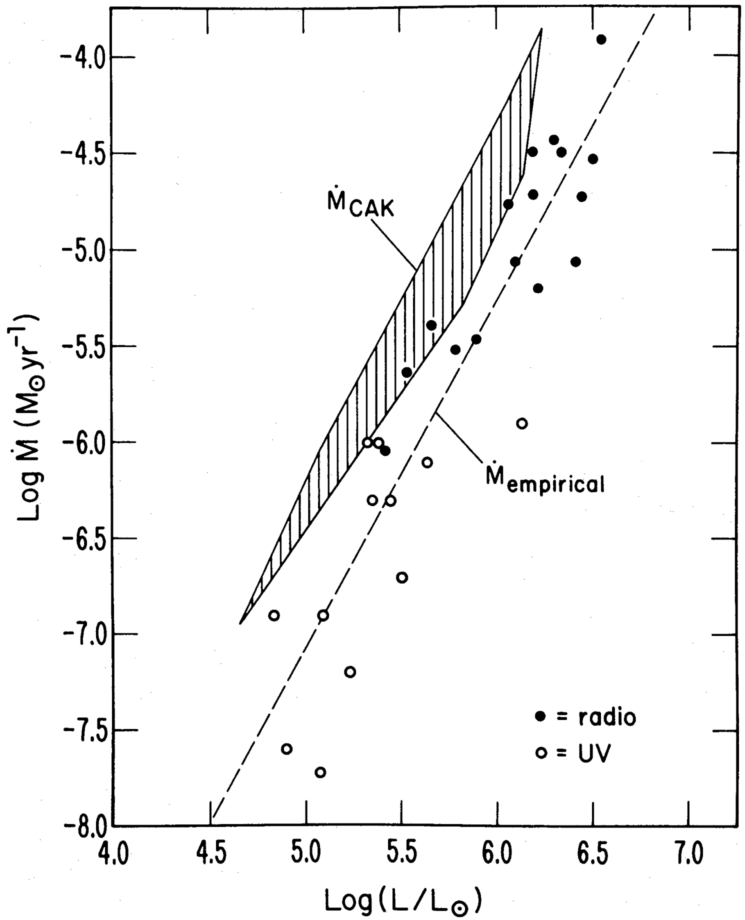
$$\frac{d}{du} F(u, w, w') = \frac{\partial F}{\partial u} + \frac{\partial F}{\partial w} w' = 0.$$



Michel Curé
Universidad de Valparaíso

CAK Model





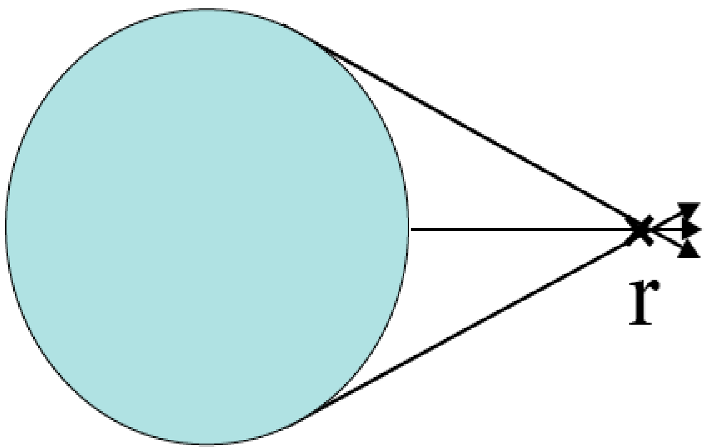
CAK Theory:

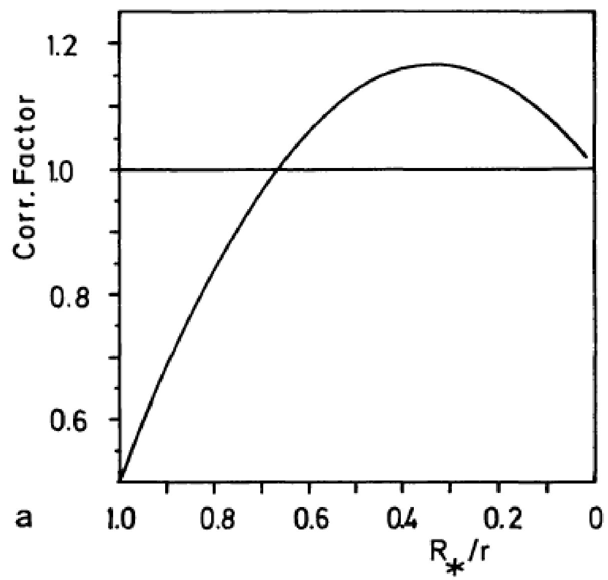
- Qualitatively acceptable agreement with the observational values
- Theory weak assumption:

Point Star approximation

m-CAK Model

modified-CAK Theory:
Finite Disk Correction Factor



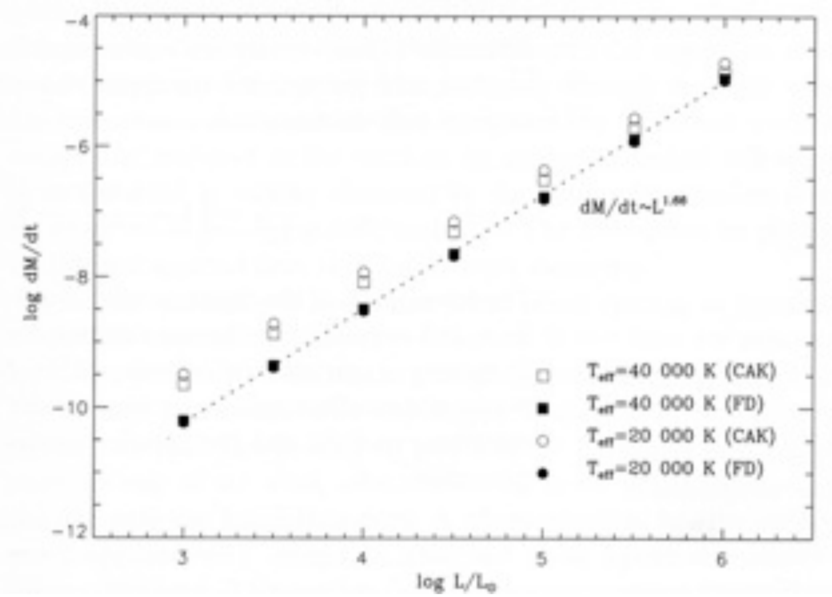


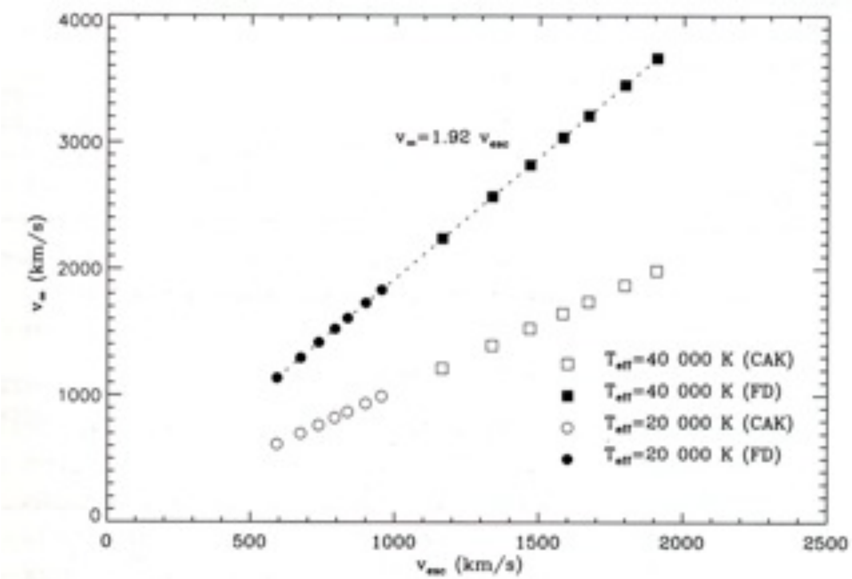
Friend & Abbott
ApJ, 311,701,1986

Pauldrach, Puls & Kudritzki
A&A, 164,86, 1986

m-CAK Model

m-CAK: better agreement with observations





Michel Curé
Universidad de Valparaíso

Thursday, March 3, 2011

The effect of Rotation in 1D models

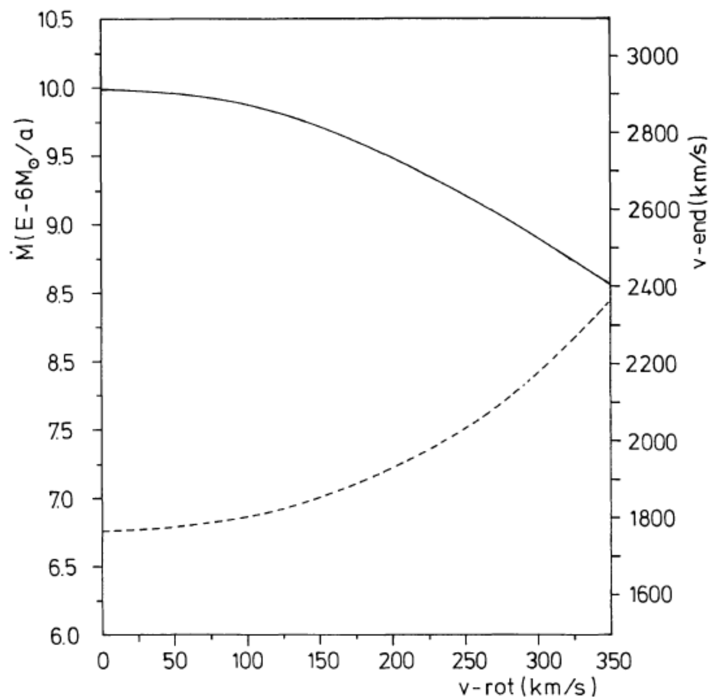
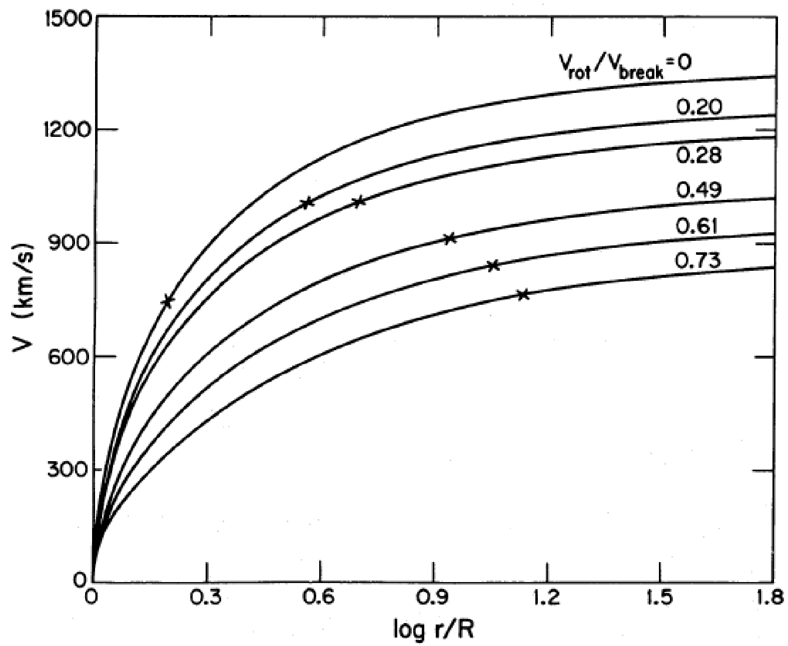


Fig. 4. The dependence of $\dot{M}$ (dashed) and v_∞ (fully drawn) on v_{rot} for the 05f-star

Pauldrach et al.
A&A, 164,86, 1986



Friend & Abbott
ApJ, 311,701,1986

The effect of Rotation in 1D models

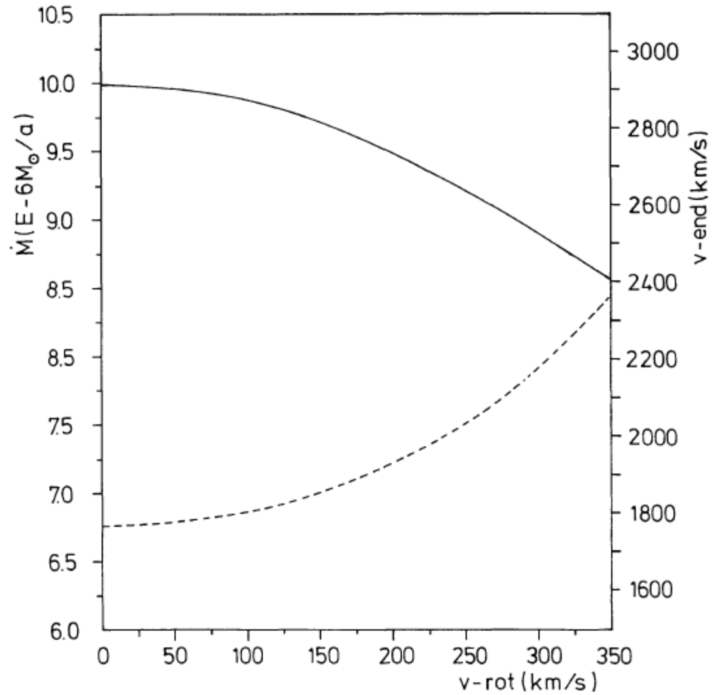
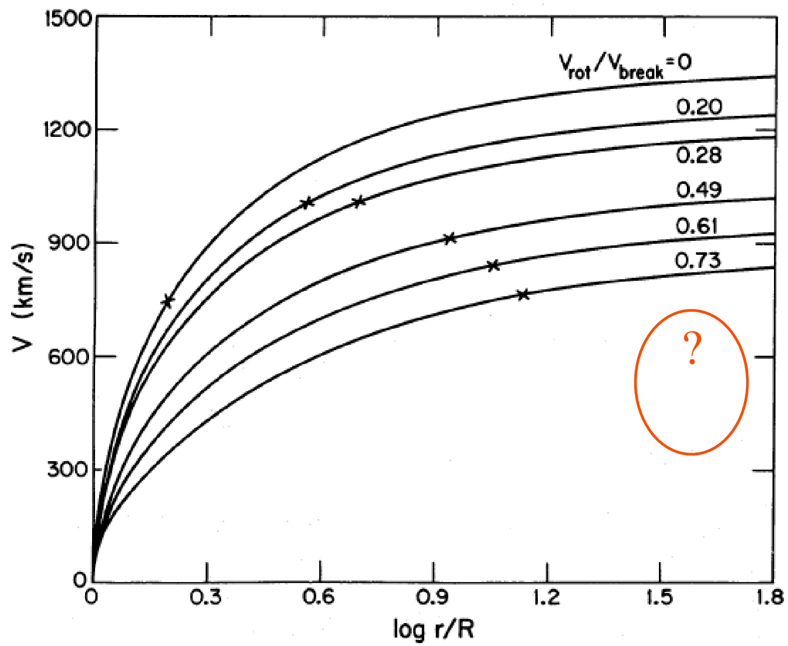


Fig. 4. The dependence of $\dot{M}$ (dashed) and v_∞ (fully drawn) on v_{rot} for the 05f-star

Pauldrach et al.
A&A, 164,86, 1986



Friend & Abbott
ApJ, 311,701,1986

Michel Curé
Universidad de Valparaíso

The effect of Rotation in 1D models

Friend & Abbott
ApJ, 311,701,1986

rotational velocities were used, the mass-loss rate might become very large. We were unable to find solutions for larger rotational velocities, mainly because of numerical difficulties involving the finite disk factor when the effective escape speed falls below some critical value. In a study of Be star winds, Poe and Friend (1986) have pushed the rotational velocity closer to the breakup value, and they find that the mass-loss rate does

506 *F. X. de Araújo, J. A. de Freitas Pacheco and D. Petrini*

$\chi = 0.7$ respectively. We have encountered severe numerical difficulties for models with $\chi \geq 0.8$. When we used $\alpha = 0.56$ we managed to obtain the solutions $v(r)$ until a certain radius $r \leq 5R$, but for the $\alpha = 0.4$ model we could not find the localization of the critical point. Somewhat analogous problems

Radiation Driven Wind Hydrodynamics with Rotation: Revisited

Mass Conservation → $4\pi r^2 \rho v = \dot{M}$

Michel Curé
Universidad de Valparaíso

Radiation Driven Wind Hydrodynamics with Rotation: Revisited

Mass Conservation ⟶ $4\pi r^2 \rho v = \dot{M}$

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM(1-\Gamma)}{r^2} + \frac{v_\phi^2(r)}{r} + g^{\text{line}}\left(\rho, \frac{dv}{dr}, n_E\right)$$

Michel Curé
Universidad de Valparaíso

Radiation Driven Wind Hydrodynamics with Rotation: Revisited

Mass Conservation → $4\pi r^2 \rho v = \dot{M}$

$$v \frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM(1-\Gamma)}{r^2} + \frac{v_\phi^2(r)}{r} + g^{\text{line}}\left(\rho, \frac{dv}{dr}, n_E\right)$$

Inertia

Gas Pressure

Gravity

Centrifugal force

Line Force

Michel Curé
Universidad de Valparaíso

Changing of variables

Equation of Motion

Changing of variables

Equation of Motion

$$u = \frac{-R_*}{r},$$
$$w = \frac{v}{a},$$
$$w' = \frac{dw}{du},$$

Michel Curé
Universidad de Valparaíso

Changing of variables

Equation of Motion

$$u = \frac{-R_*}{r},$$
$$w = \frac{v}{a},$$
$$w' = \frac{dw}{du},$$

Michel Curé
Universidad de Valparaíso

Changing of variables

$$u = \frac{-R_*}{r},$$
$$w = \frac{v}{a},$$
$$w' = \frac{dw}{du},$$

Equation of Motion

$$F(u, w, w') = 0$$

$$F(u, w, w') \equiv \left(1 - \frac{1}{w^2}\right) w \frac{dw}{du} + A + \frac{2}{u} + a_{\rm rot}^2 u - C'CF g(u)(w)^{-\delta} \left(w \frac{dw}{du}\right)^{\alpha} = 0$$

Changing of variables

$$u = \frac{-R_*}{r},$$
$$w = \frac{v}{a},$$
$$w' = \frac{dw}{du},$$

Equation of Motion

$$F(u, w, w') = 0$$

$$F(u,w,w') \equiv \left(1 - \frac{1}{w^2}\right) w \frac{dw}{du} + A + \frac{2}{u} + a_{\rm rot}^2 u - C'CFg(u)(w)^{-\delta}\left(w\frac{dw}{du}\right)^{\alpha} = 0$$

$$A = \frac{GM(1-\Gamma)}{a^2 R_*} = \frac{v_{\rm esc}^2}{2a^2},$$

$$C' = C\left(\frac{\dot{M}D}{2\pi}\frac{10^{-11}}{aR_*^2}\right)^{\delta}(a^2 R_*)^{(\alpha-1)},$$

$$g(u) = \left(\frac{u^2}{1-\sqrt{1-u^2}}\right)^{\delta},$$

$$a_{\rm rot} = \frac{v_{\rm rot}}{a},$$

Changing of variables

$$u = \frac{-R_*}{r},$$
$$w = \frac{v}{a},$$
$$w' = \frac{dw}{du},$$

Equation of Motion

$$F(u, w, w') = 0$$

$$F(u, w, w') \equiv \left(1 - \frac{1}{w^2}\right) w \frac{dw}{du} + A + \frac{2}{u} + a_{\rm rot}^2 u - C' C F g(u) (w)^{-\delta} \left(w \frac{dw}{du}\right)^{\alpha} = 0$$

$$A = \frac{GM(1-\Gamma)}{a^2 R_*} = \frac{v_{\rm esc}^2}{2a^2},$$

$$C' = C \left(\frac{\dot{M}D}{2\pi} \frac{10^{-11}}{a R_*^2}\right)^{\delta} (a^2 R_*)^{(\alpha - 1)},$$

$$g(u) = \left(\frac{u^2}{1 - \sqrt{1 - u^2}}\right)^{\delta},$$

$$a_{\rm rot} = \frac{v_{\rm rot}}{a},$$

with the influence of the ROTATION…

$$u = \frac{-R_*}{r}, \qquad w = \frac{v}{a}, \qquad w' = \frac{dw}{du},$$

Michel Curé
Universidad de Valparaíso

with the influence of the ROTATION…

$$u = \frac{-R_*}{r}, \quad w = \frac{v}{a}, \quad w' = \frac{dw}{du}, \quad \longrightarrow \quad Y = w\,w', \quad Z = w/w'$$

Michel Curé
Universidad de Valparaíso

with the influence of the ROTATION…

$$u = \frac{-R_*}{r}, \quad w = \frac{v}{a}, \quad w' = \frac{dw}{du}, \quad \longrightarrow \quad Y = w\, w', \quad Z = w/w'$$

$$\left(1 - \frac{1}{YZ}\right) Y \quad + A + 2/u + a_{rot}^2 u \quad - C' f_1(u, Z)\, g(u)\, Z^{-\delta/2}\, Y^{\alpha-\delta/2} = 0$$

$$\left(1 - \frac{1}{YZ}\right) Y \quad - C' f_2(u, Z)\, g(u)\, Z^{-\delta/2}\, Y^{\alpha-\delta/2} = 0$$

$$\left(1 + \frac{1}{YZ}\right) Y \quad -2Z/u^2 + a_{rot}^2 Z \quad - C' f_3(u, Z)\, g(u)\, Z^{-\delta/2}\, Y^{\alpha-\delta/2} = 0$$

Michel Curé
Universidad de Valparaíso

with the influence of the ROTATION…

$$u = \frac{-R_*}{r}, \quad w = \frac{v}{a}, \quad w' = \frac{dw}{du}, \quad \longrightarrow \quad Y = w\,w', \quad Z = w/w'$$

$$\left(1 - \frac{1}{YZ}\right) Y \quad + A + 2/u + a_{rot}^2 u \quad - C' f_1(u,Z)\, g(u)\, Z^{-\delta/2}\, Y^{\alpha-\delta/2} = 0$$

$$\left(1 - \frac{1}{YZ}\right) Y \quad - C' f_2(u,Z)\, g(u)\, Z^{-\delta/2}\, Y^{\alpha-\delta/2} = 0$$

$$\left(1 + \frac{1}{YZ}\right) Y \quad -2Z/u^2 + a_{rot}^2 Z \quad - C' f_3(u,Z)\, g(u)\, Z^{-\delta/2}\, Y^{\alpha-\delta/2} = 0$$

At the Singular Point: u_s, Y_s, Z_s, C'

Michel Curé
Universidad de Valparaíso

Without any Approximation!

Michel Curé
Universidad de Valparaíso

Without any Approximation!

$$Y = \frac{1}{Z} + \left(\frac{f_2}{f_1 - f_2}\right)\left(A + \frac{2}{u} + a_{rot}^2 u\right)$$

$$C'(\dot{M}) = \frac{1}{g f_2}\left(1 - \frac{1}{YZ}\right) Z^{\delta/2}\, Y^{1-\alpha+\delta/2}$$

Michel Curé
Universidad de Valparaíso

Without any Approximation!

$$Y = \frac{1}{Z} + \left(\frac{f_2}{f_1 - f_2}\right)\left(A + \frac{2}{u} + a_{rot}^2 u\right)$$

$$C'(\dot{M}) = \frac{1}{g f_2}\left(1 - \frac{1}{YZ}\right) Z^{\delta/2}\, Y^{1-\alpha+\delta/2}$$

and

$$R(u, Z) \equiv -\frac{2}{Z} + \frac{2Z}{u^2} - a_{rot}^2 Z + f_{123}(u, Z)\left(A + \frac{2}{u} + a_{rot}^2 u\right)$$

Michel Curé
Universidad de Valparaíso

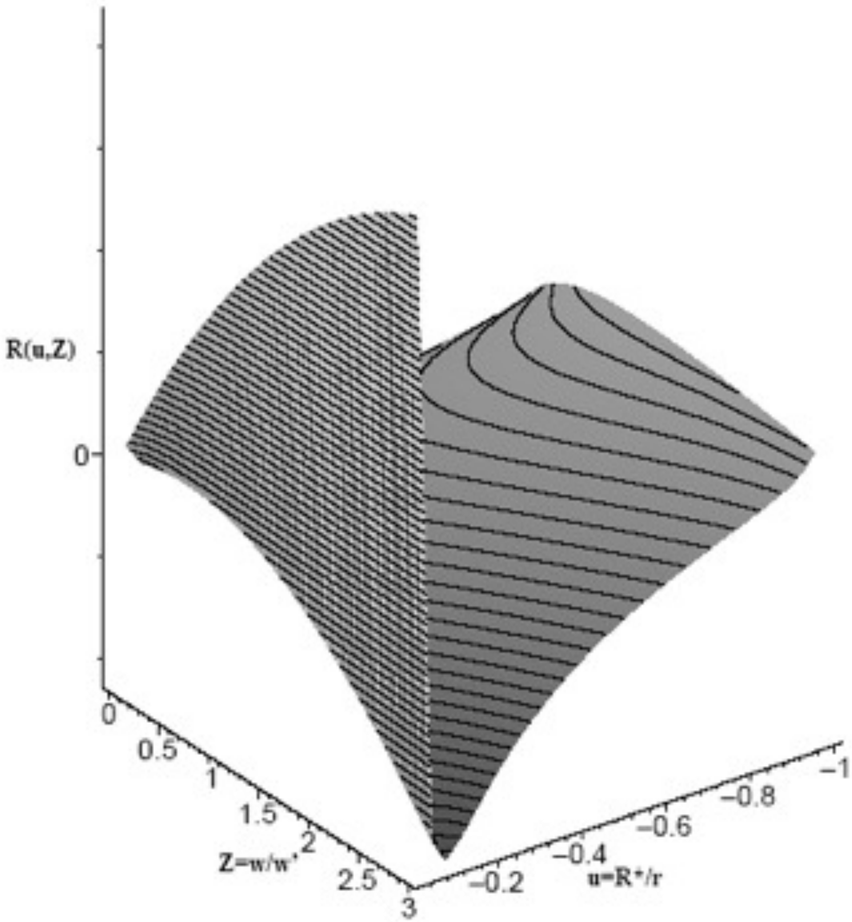
$R(u, Z) = 0$ ← singular point location

Michel Curé
Universidad de Valparaíso

$R(u\ ,Z) = 0$ ← singular point location

$$v_{\text{rot}}/v_{\text{bkup}} = 0.5$$

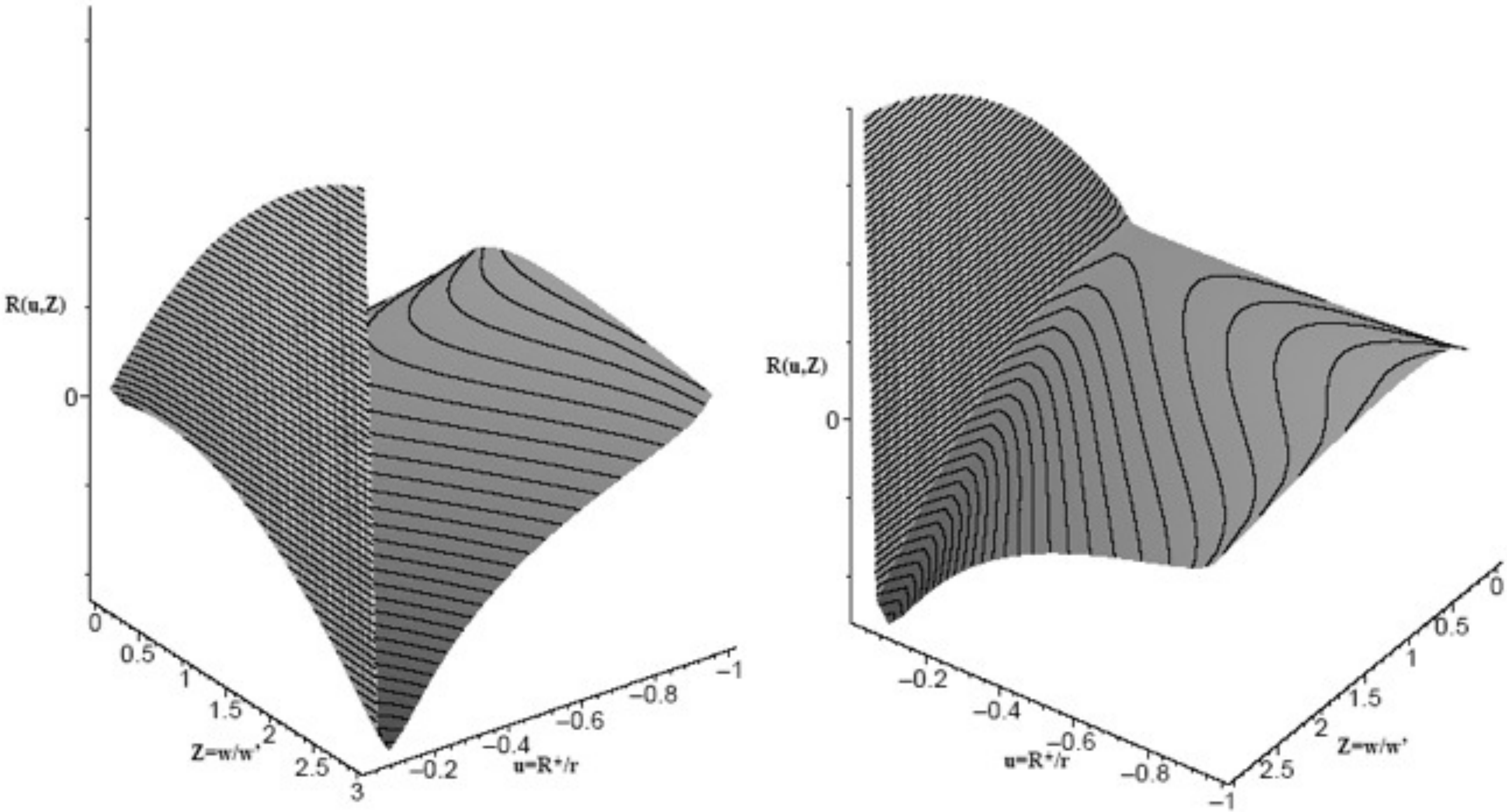
$R(u\ ,Z) = 0 \leftarrow$ singular point location



$$v_{\rm rot}/v_{\rm bkup} = 0.5$$

Michel Curé
Universidad de Valparaíso

R(u , Z) = 0 ← singular point location



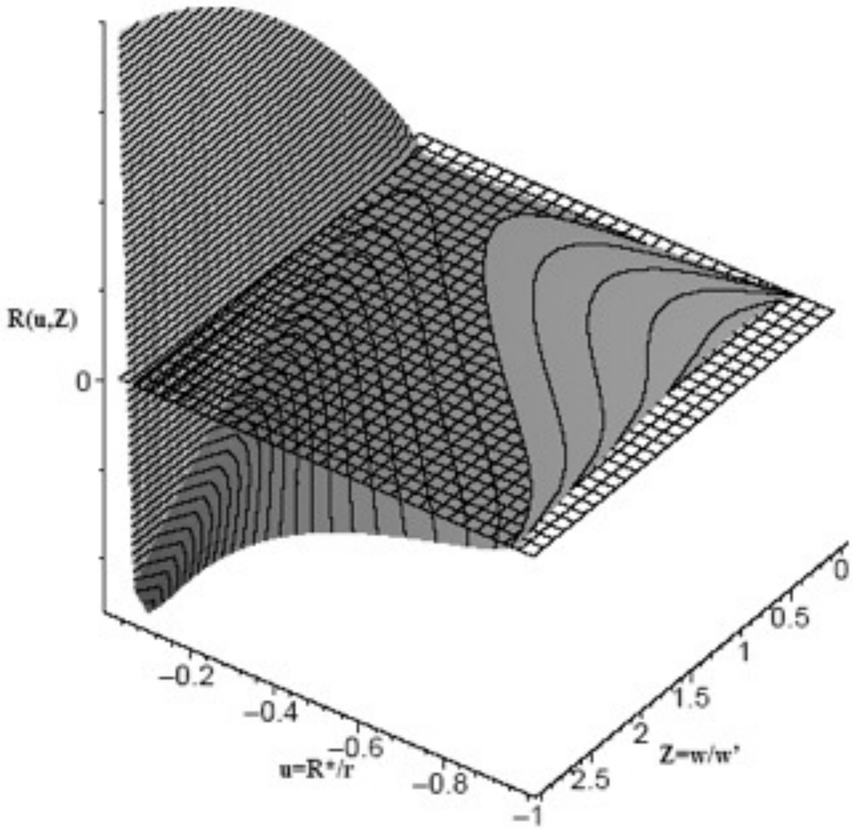
$$v_{\rm rot}/v_{\rm bkup} = 0.5$$

Michel Curé
Universidad de Valparaíso

$$v_{\rm rot}/v_{\rm bkup} = 0.5$$

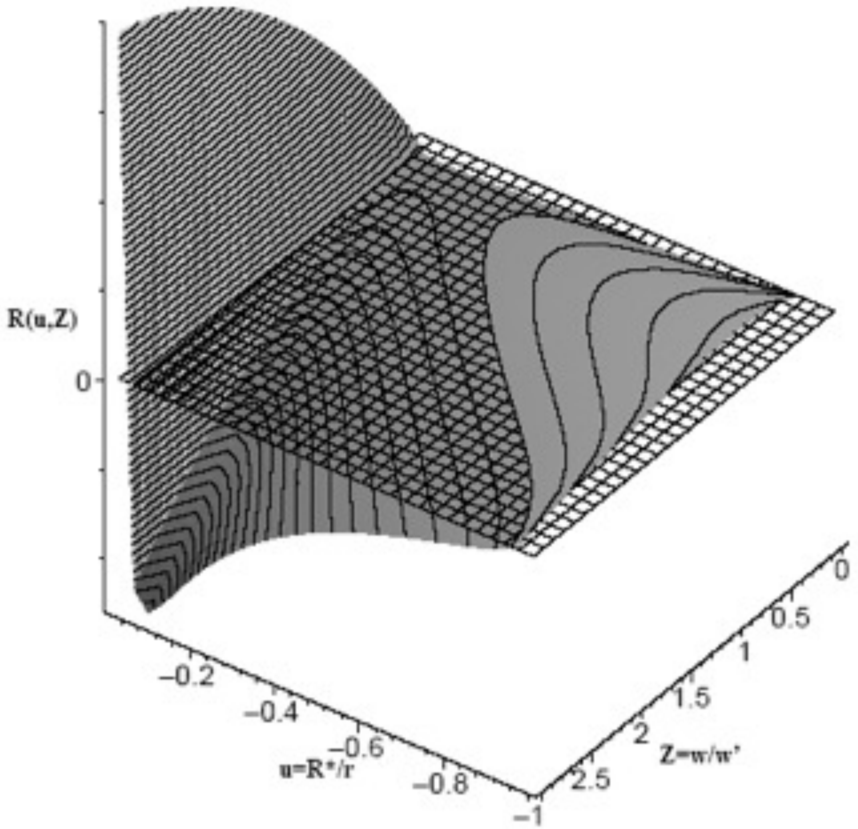
Michel Curé
Universidad de Valparaíso

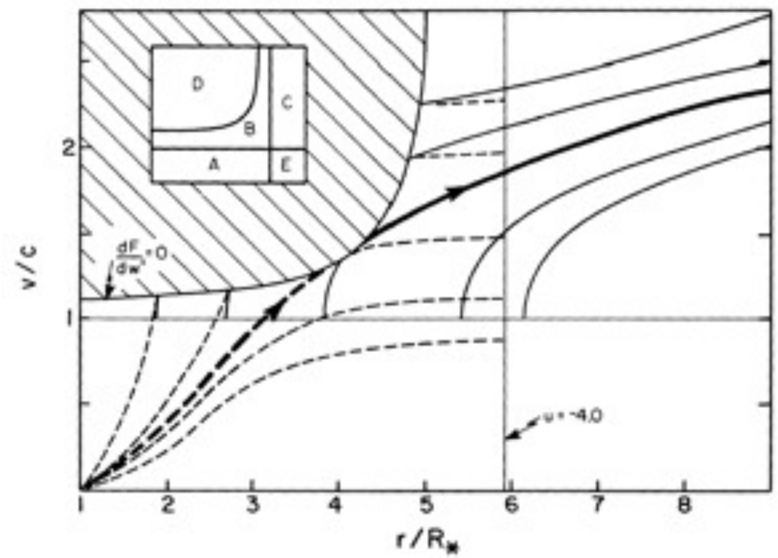
$$v_{\rm rot}/v_{\rm bkup} = 0.5$$



Michel Curé
Universidad de Valparaíso

$$v_{\mathrm{rot}}/v_{\mathrm{bkup}} = 0.5$$

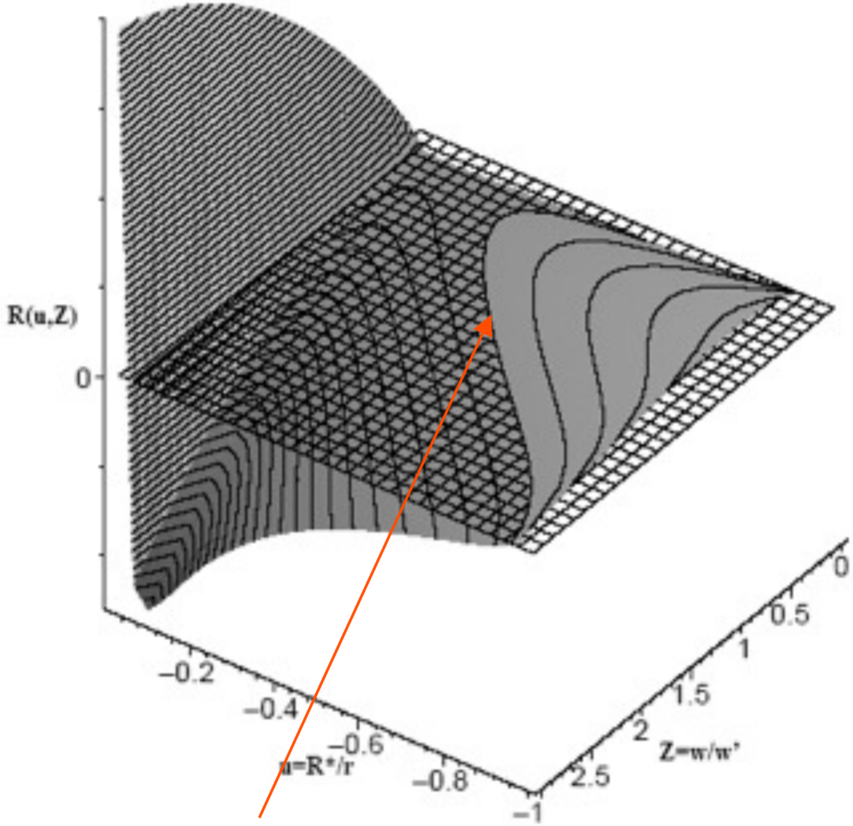


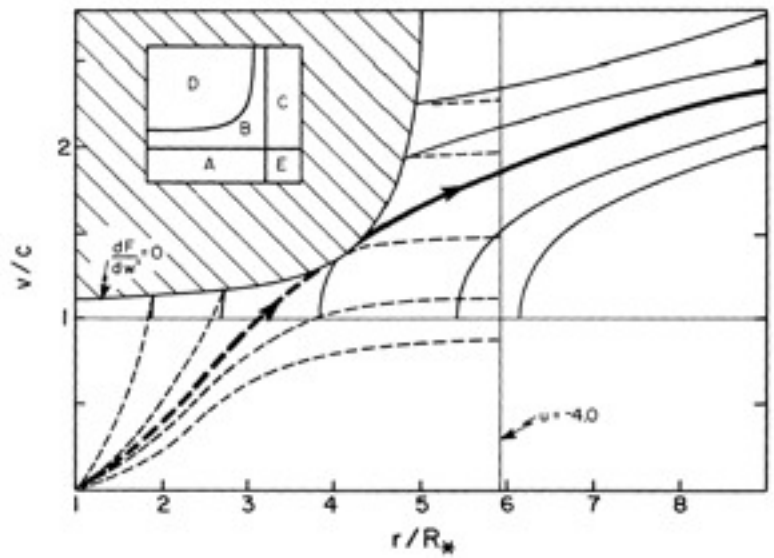


Astronomical Institute Ondrejov, 2-3-2011

Michel Curé
Universidad de Valparaíso

$v_{\rm rot}/v_{\rm bkup} = 0.5$

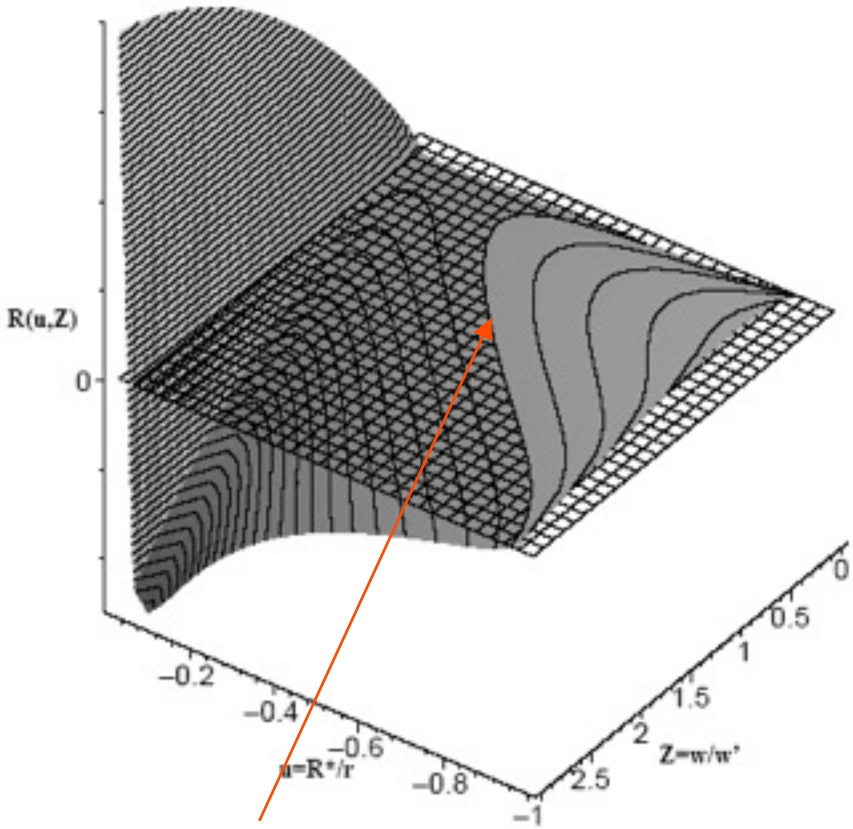




CAK family
of singular points

Michel Curé
Universidad de Valparaíso

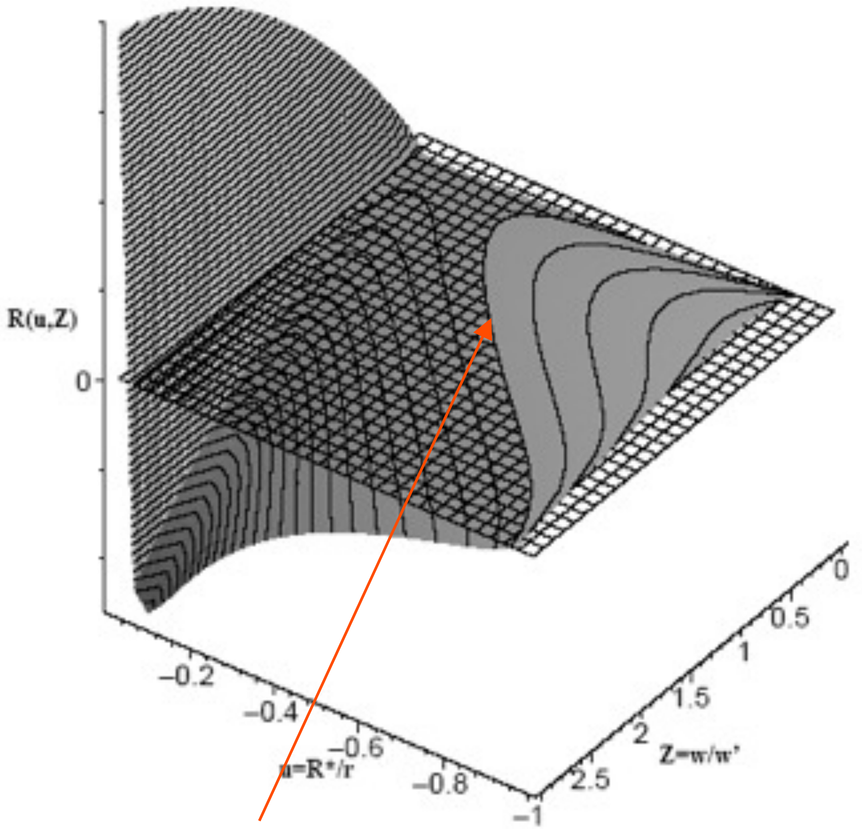
$$v_{\mathrm{rot}}/v_{\mathrm{bkup}} = 0.5$$



CAK family
of singular points

$v_{\rm rot}/v_{\rm bkup} = 0.5$

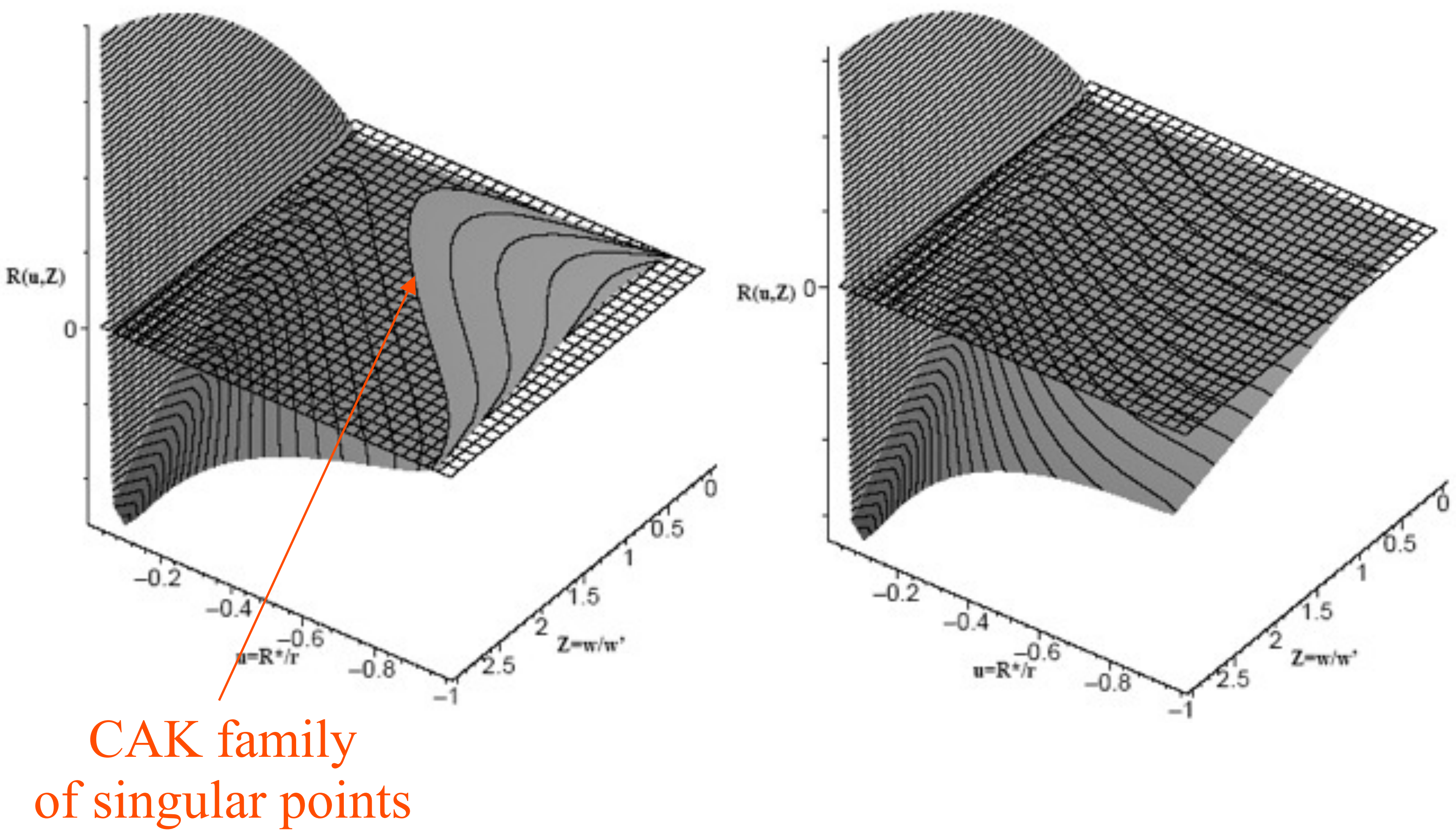
$v_{\rm rot}/v_{\rm bkup} = 0.9$



CAK family
of singular points

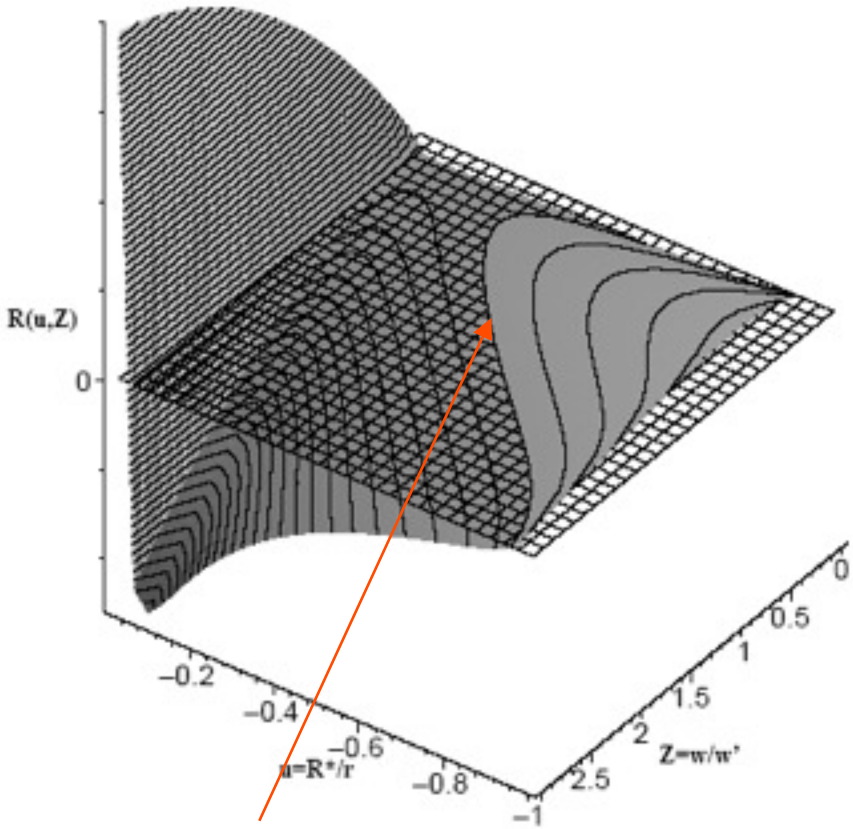
Michel Curé
Universidad de Valparaíso

$v_{\rm rot}/v_{\rm bkup} = 0.5$ $v_{\rm rot}/v_{\rm bkup} = 0.9$



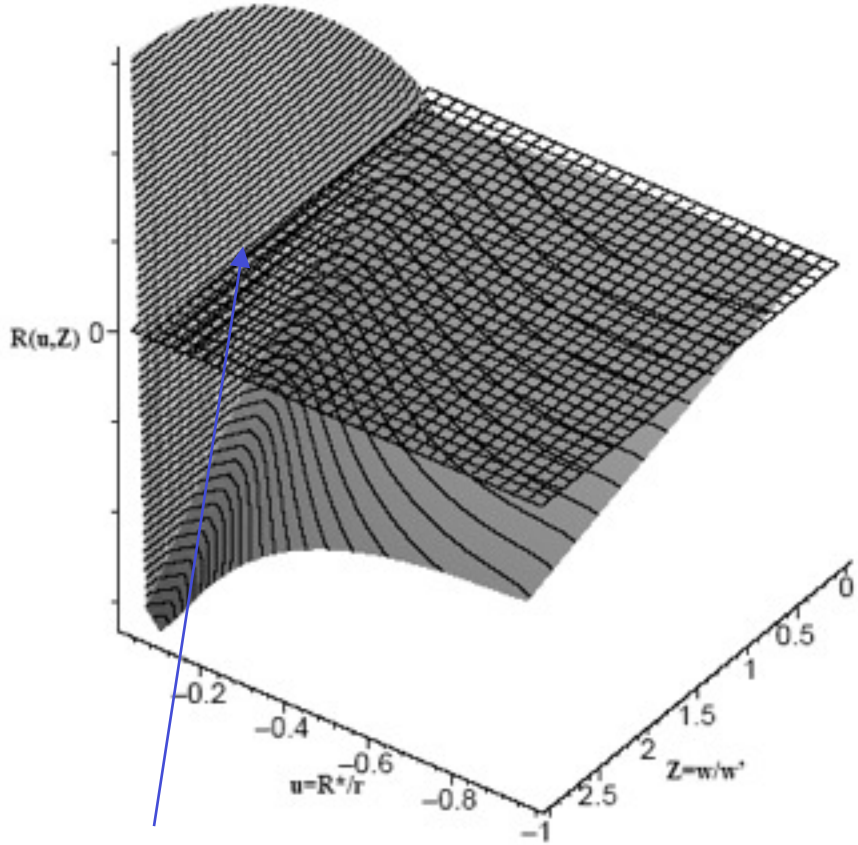
Michel Curé
Universidad de Valparaíso

$v_{\rm rot}/v_{\rm bkup} = 0.5$



CAK family
of singular points

$v_{\rm rot}/v_{\rm bkup} = 0.9$

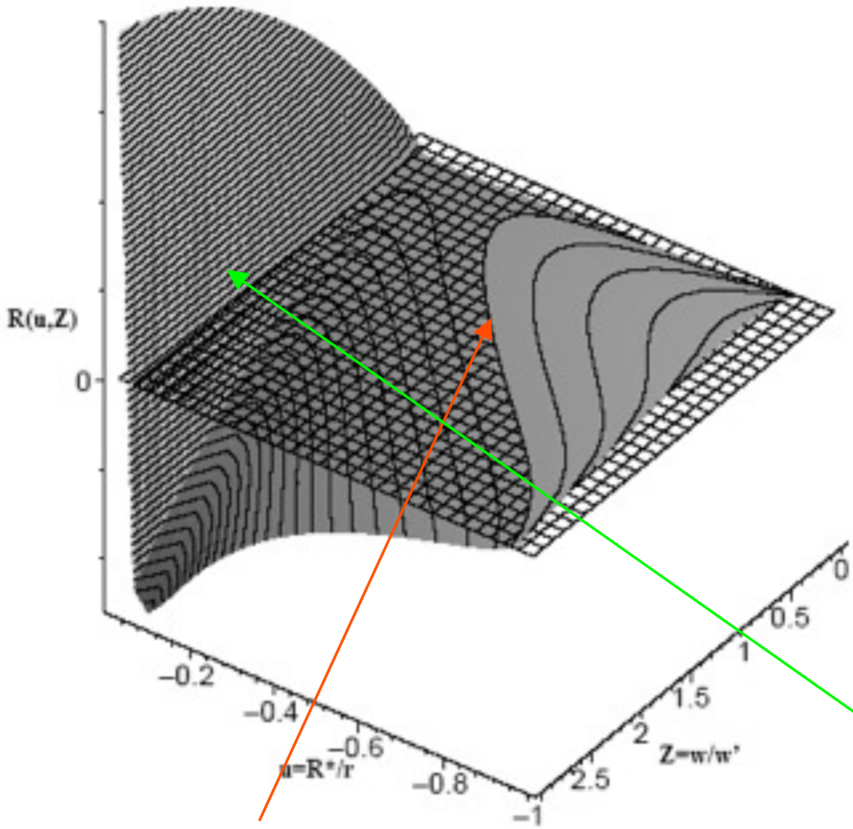


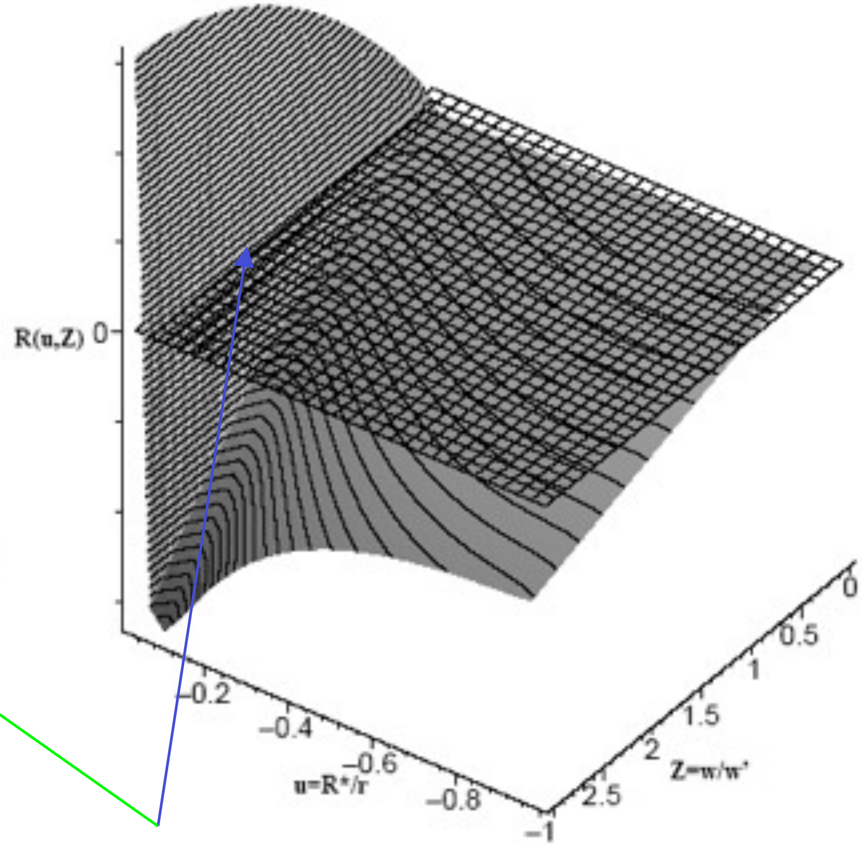
NEW family
of singular points

Michel Curé
Universidad de Valparaíso

$v_{\text{rot}}/v_{\text{bkup}} = 0.5$

$v_{\text{rot}}/v_{\text{bkup}} = 0.9$



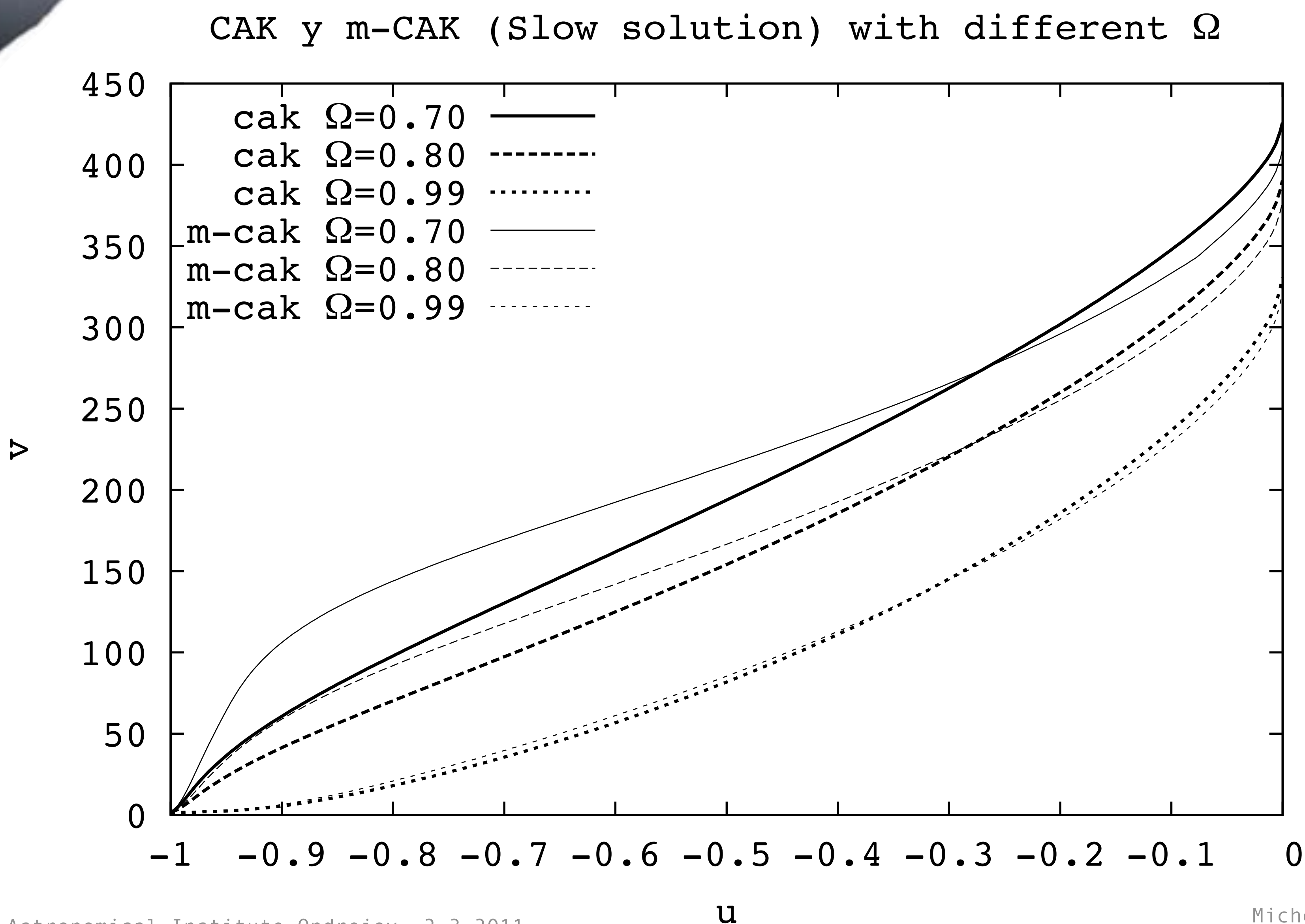


CAK family
of singular points

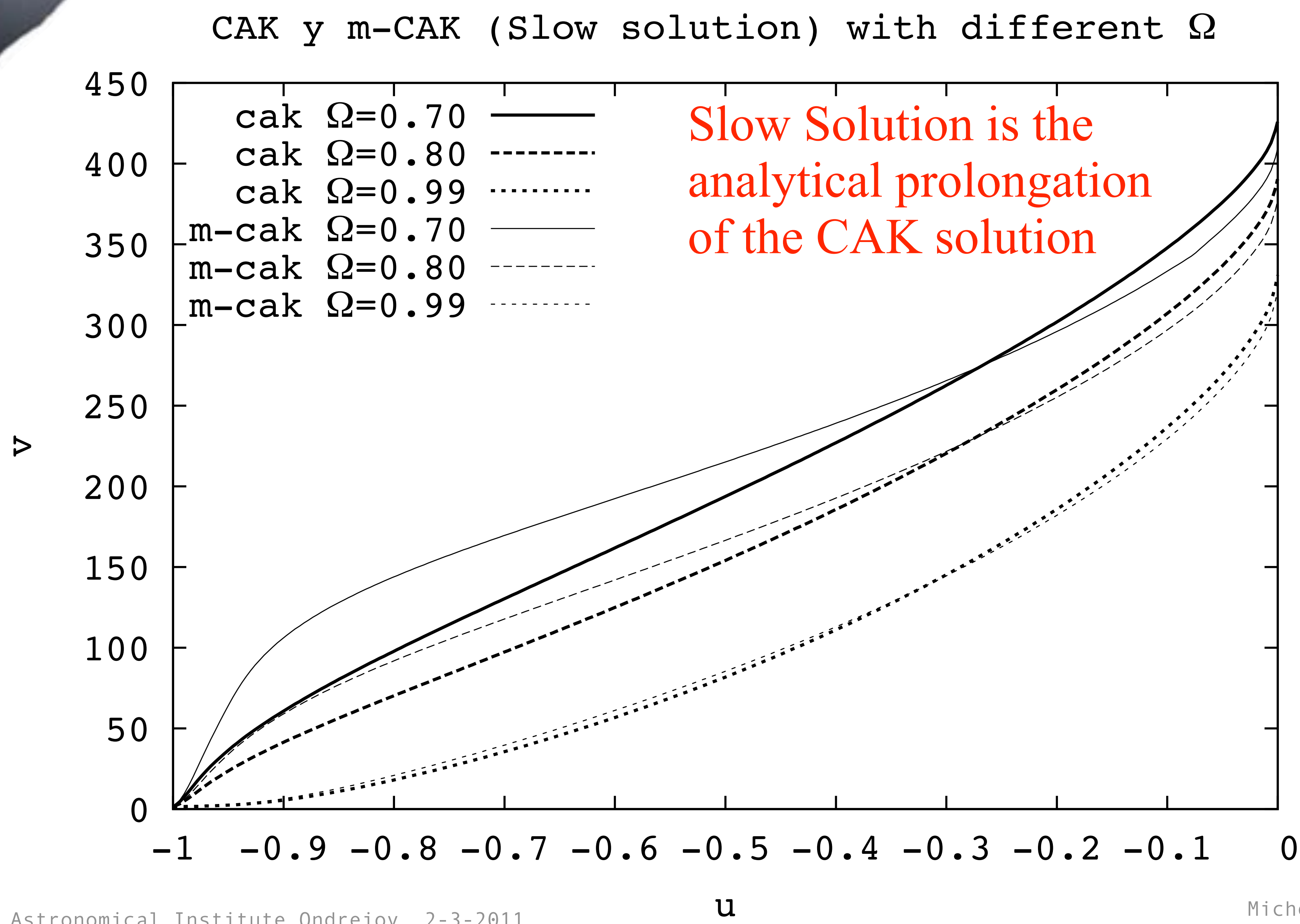
NEW family
of singular points

Michel Curé
Universidad de Valparaíso

Rotating CAK and m-CAK solutions



Rotating CAK and m-CAK solutions



Michel Curé
Universidad de Valparaíso

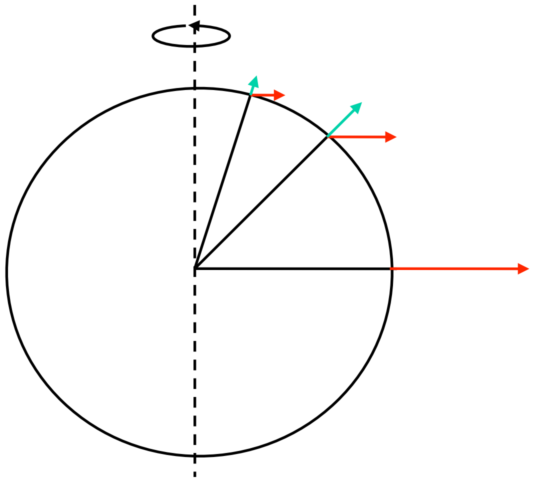
Applications of the Slow solution

Rotation:
B[e] Supergiants

Michel Curé
Universidad de Valparaíso

Applications of the Slow solution

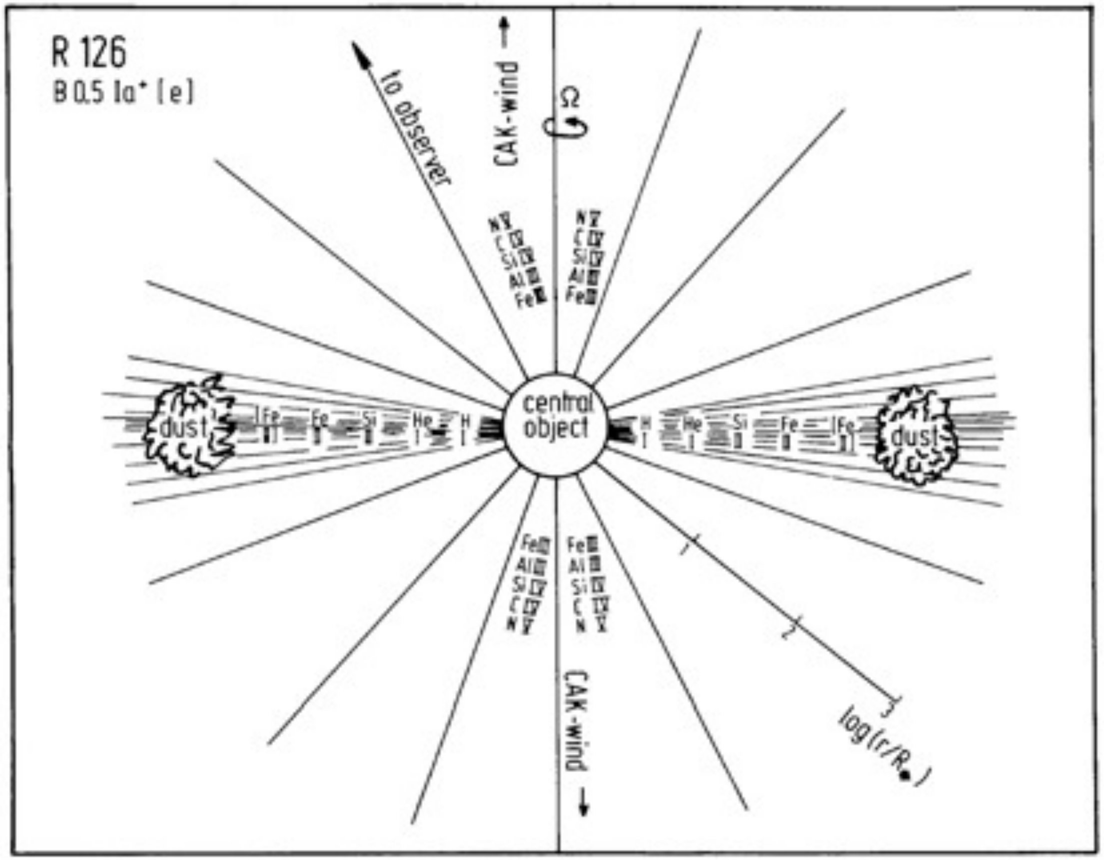
Rotation:
B[e] Supergiants



Michel Curé
Universidad de Valparaíso

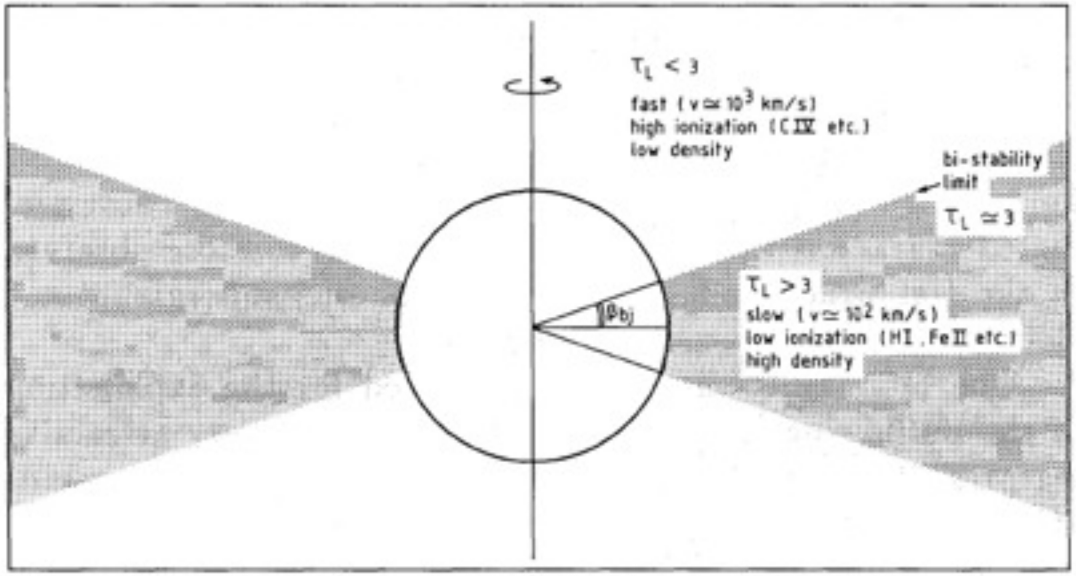
Rotating Radiation Driven Winds

B[e]-Supergiant star



from Zickgraf 1986

Bistability



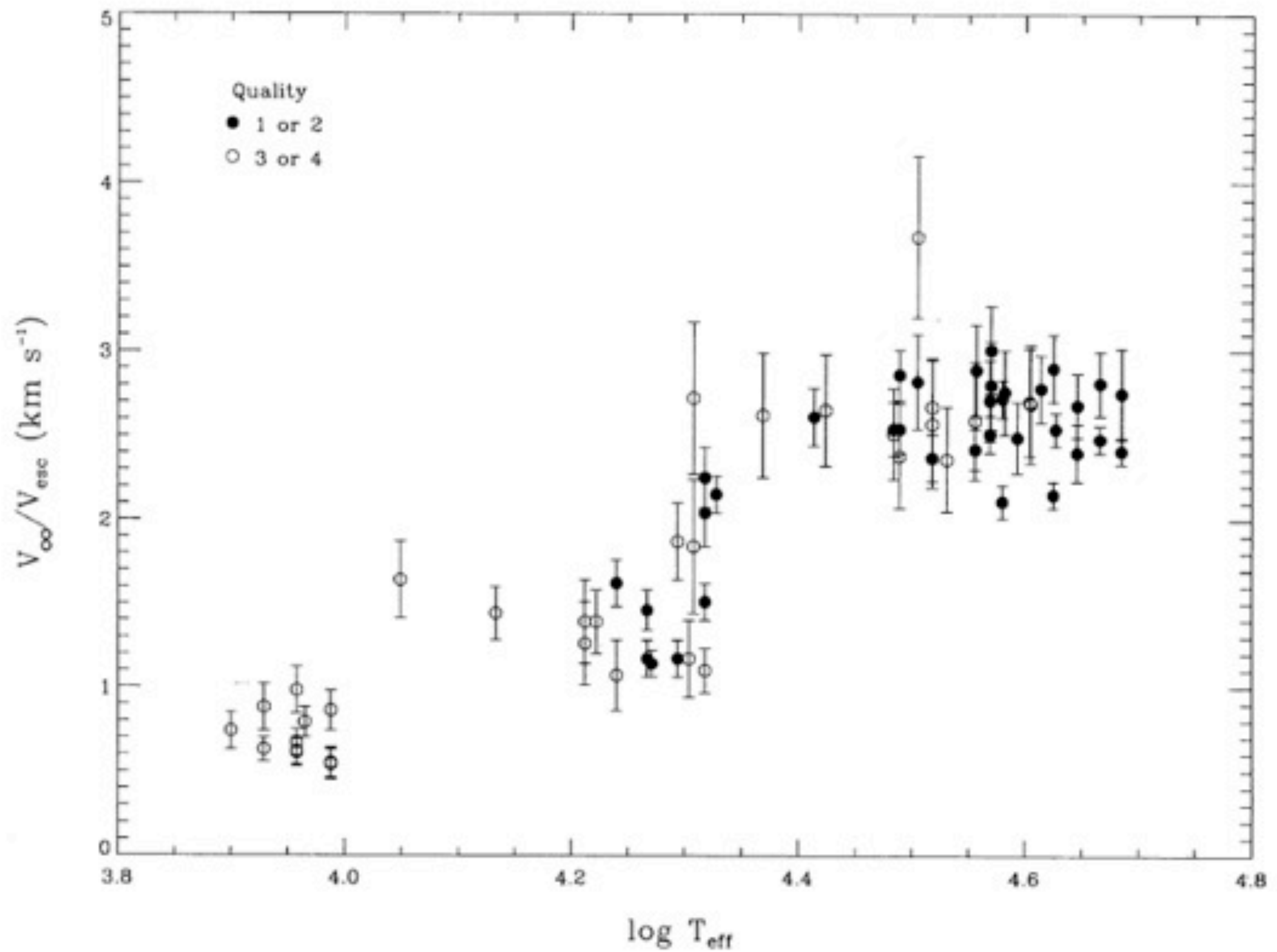
Proposed by Lamers & Pauldrach (1991)

Vink et al. (1999)
Theoreticaly showed:
Bistability Jump
T=25,000K
due to Recombination of
Fe IV to Fe III

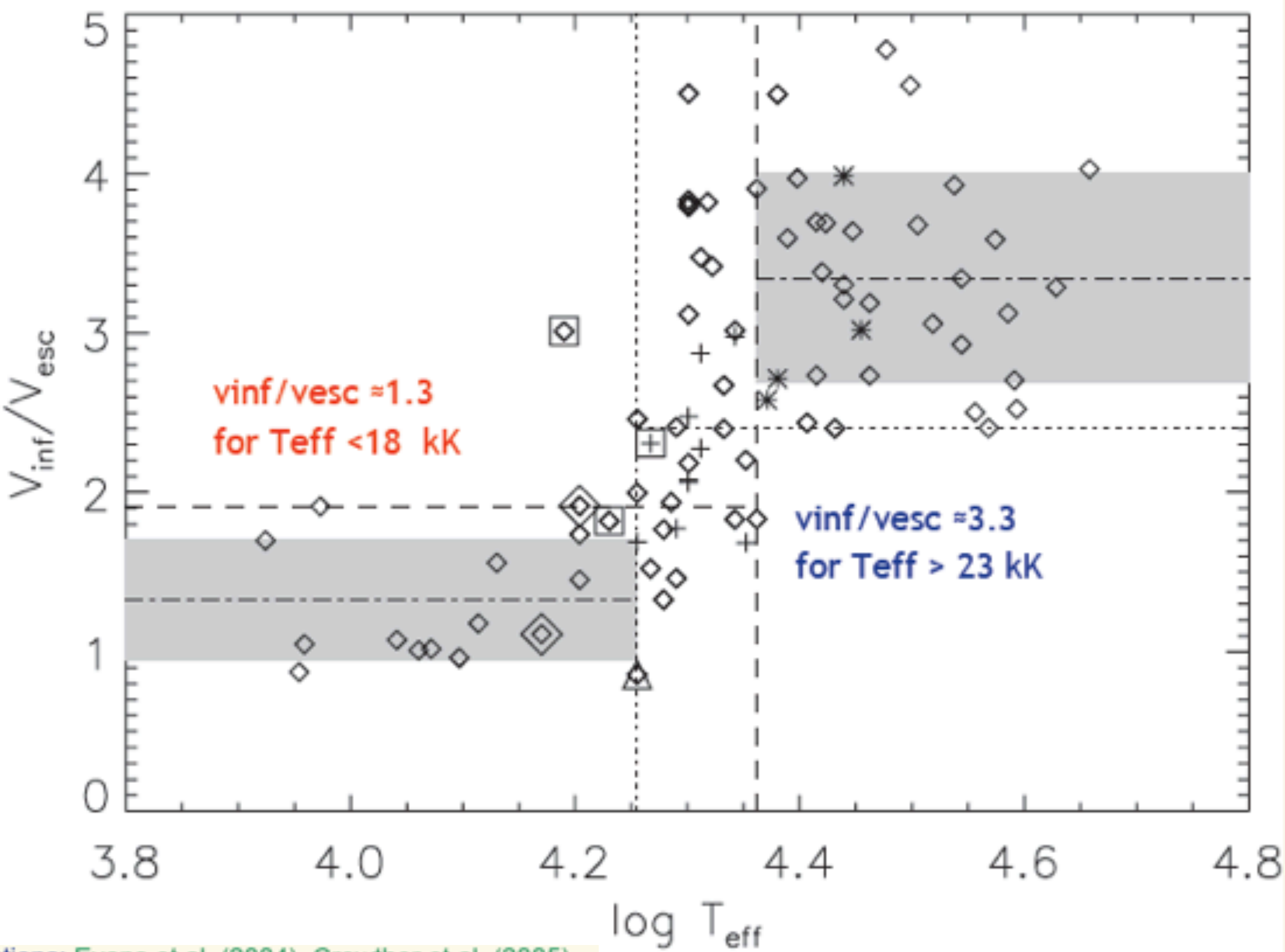
Bistability Jump

Observational determination of the Bistability Jump

Lamers, Snow & Lindholm, ApJ, 455, 269, 1995

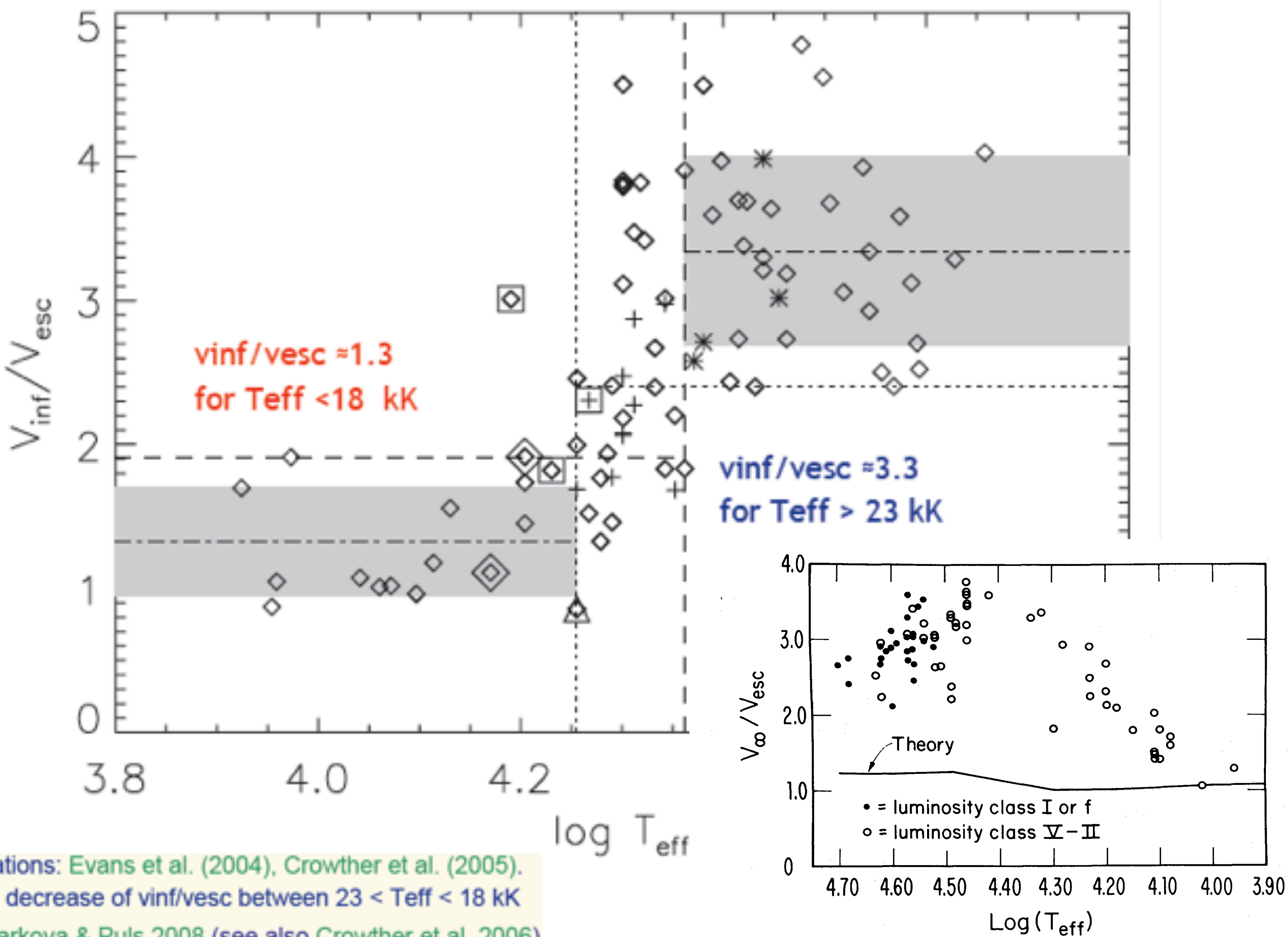


Bistability Jump



Observations: Evans et al. (2004), Crowther et al. (2005).
Gradual decrease of vinf/vesc between 23 < Teff < 18 kK
From Markova & Puls 2008 (see also Crowther et al. 2006)

Bistability Jump



Observations: Evans et al. (2004), Crowther et al. (2005).
Gradual decrease of vinf/vesc between 23 < Teff < 18 kK
From Markova & Puls 2008 (see also Crowther et al. 2006)

m-CAK

- Bistability line force parameters (T_{eff}=25,000K):
  one set for polar latitudes and
  other set for equatorial latitudes

- Fast (polar) and Slow (equatorial) solutions (m-CAK)

- Rotation parameter Ω

Michel Curé
Universidad de Valparaíso

B[e] Supergiant Wind

Stellar Parameters

$$T_{\rm eff} = 25\,000 \text{ K}$$

$$M/M_\odot = 17.5$$

$$L/L_\odot = 10^5$$

form Pelupessy et al. (2000)

Line-Force Parameters

T [K]	α	k	δ
30 000	0.65	0.06	0
17 500	0.45	0.57	0

form Pelupessy et al. (2000)

Table 17. Escape velocities, effective gravity and rotational velocities derived from $v_\infty/v_{\rm esc} = 1.3$ and stellar parameters given in Tab. 14. $M_{\rm ZAMS}$ values are from Zickgraf et al. (1986).

Star	$M_{\rm ZAMS}$	$M_{\rm B[e]}$	$v_{\rm esc}$[km s^{-1}]	$\log g_{\rm eff}$	Γ	$\Gamma_{\rm rad}$	$\Gamma_{\rm rot}$	$v_{\rm rot}$[km s^{-1}]	$v_{\rm crit}$[km s^{-1}]	Ω
Hen S22	52	35	60	0.72	0.98	0.56	0.42	240	318	0.75
R 82	30	20	55	0.65	0.98	0.32	0.66	224	283	0.79
R 50	45	30	40	0.15	0.99	0.40	0.59	204	277	0.74

form Zickgraf (1998)

B[e] Supergiant m-CAK Wind

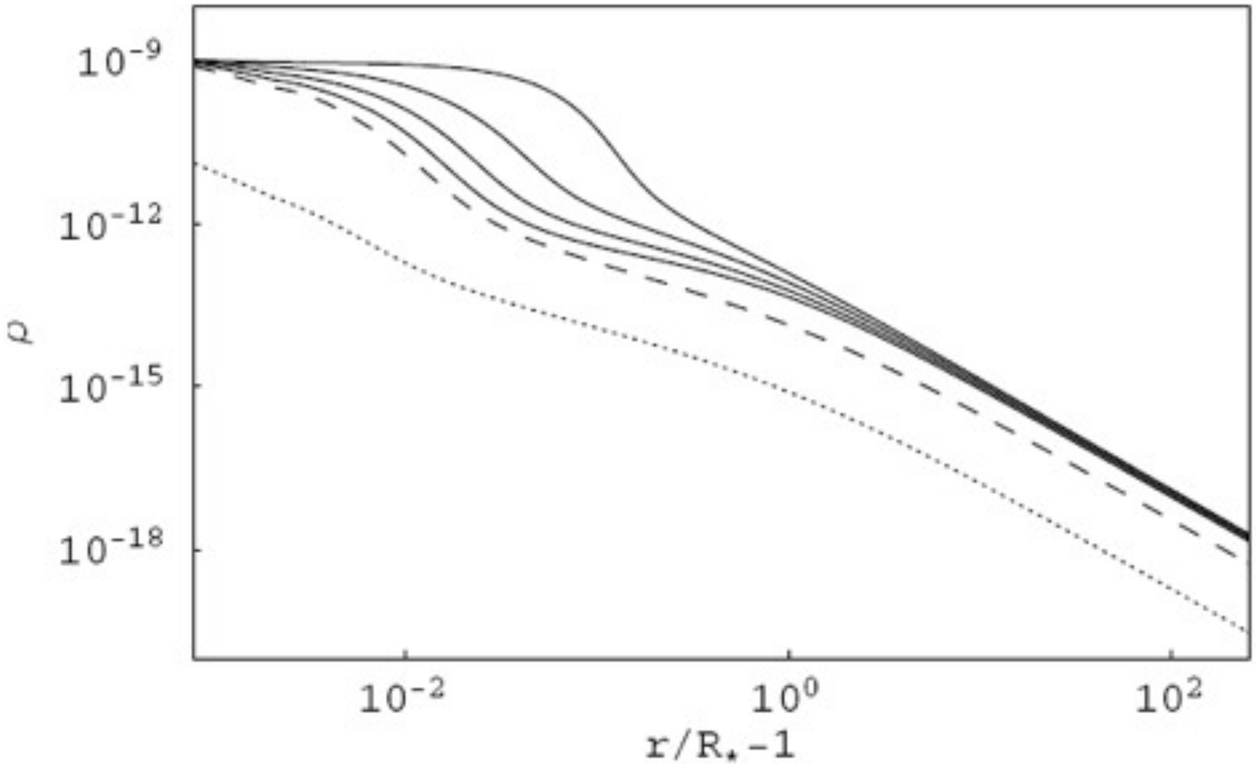


Fig. 3. m-CAK model: density (in g cm^{-3}) versus $r/R_* - 1$. Polar density is in dotted-line; equatorial density for $\Omega = 0.6$ (fast solution) is in dashed-line and equatorial densities for $\Omega = 0.7, 0.8, 0.9, 0.99$ are in continuous-line, the higher is Ω, the higher is the density.

Michel Curé
Universidad de Valparaíso

B[e] Supergiant m-CAK Wind

density contrast

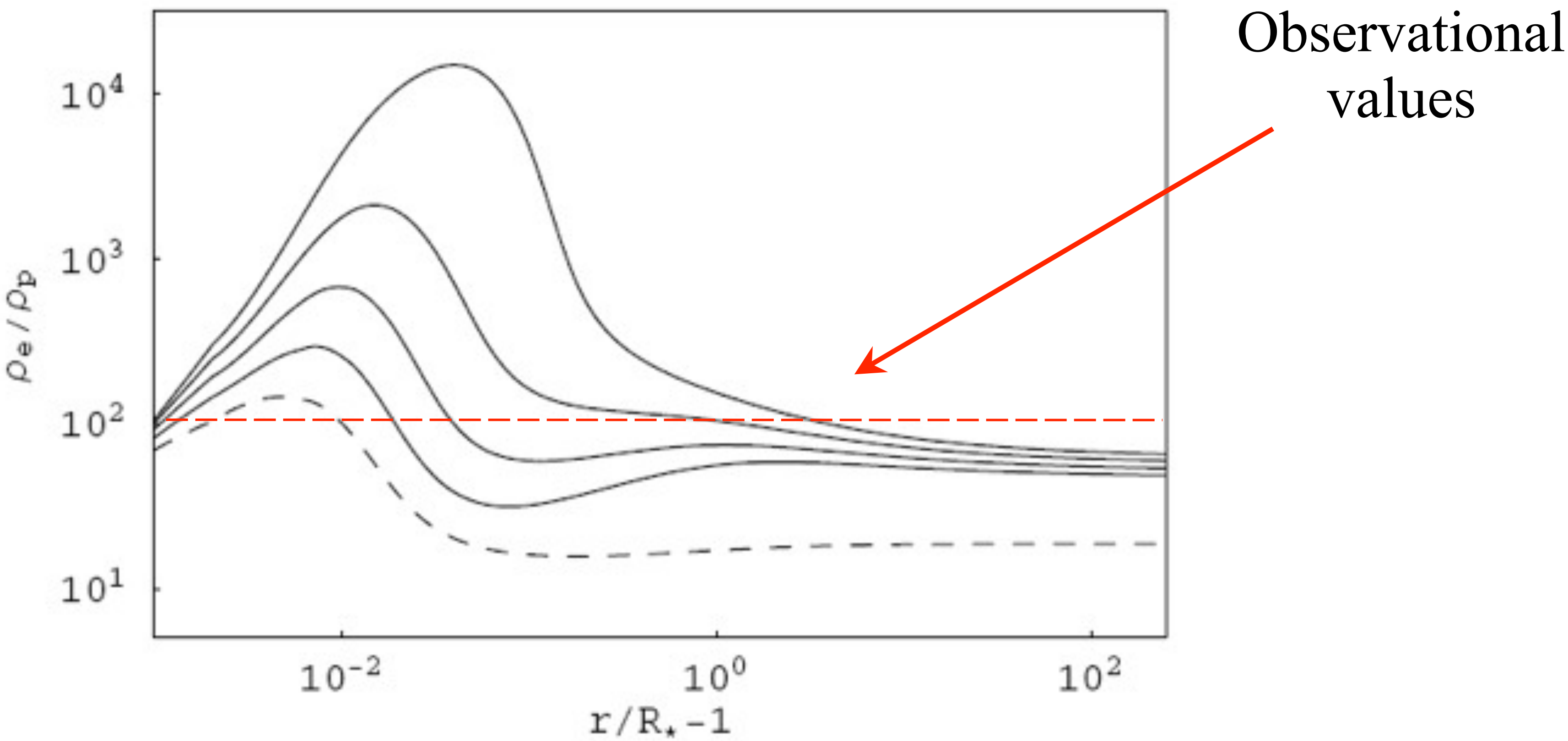
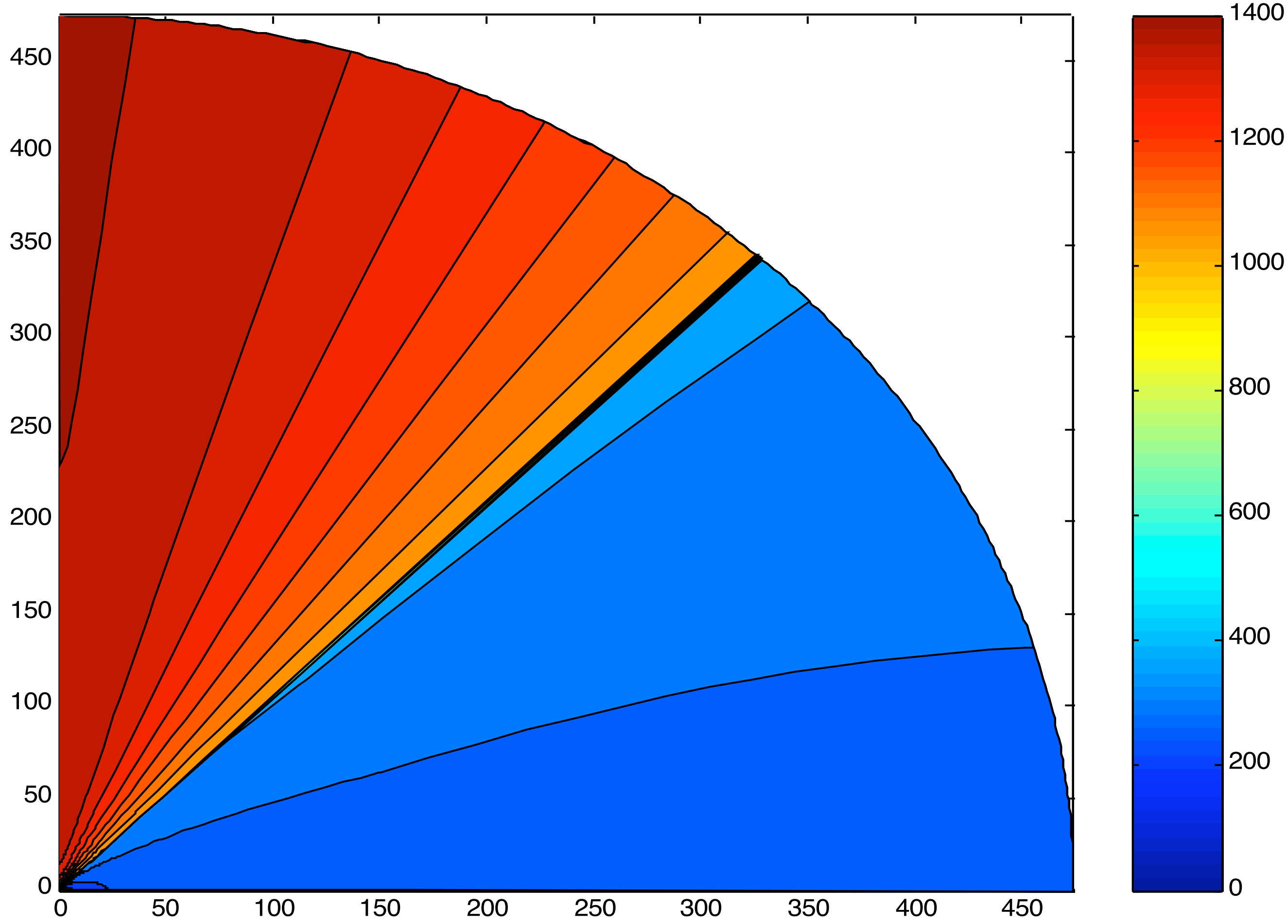


Fig. 4. m-CAK model: density contrast versus $r/R_* - 1$, dashed-line is for $\Omega = 0.6$ and continuous-line are for $\Omega = 0.7, 0.8, 0.9, 0.99$. The higher is Ω, the higher is the density contrast.

B[e] Supergiant m-CAK Wind

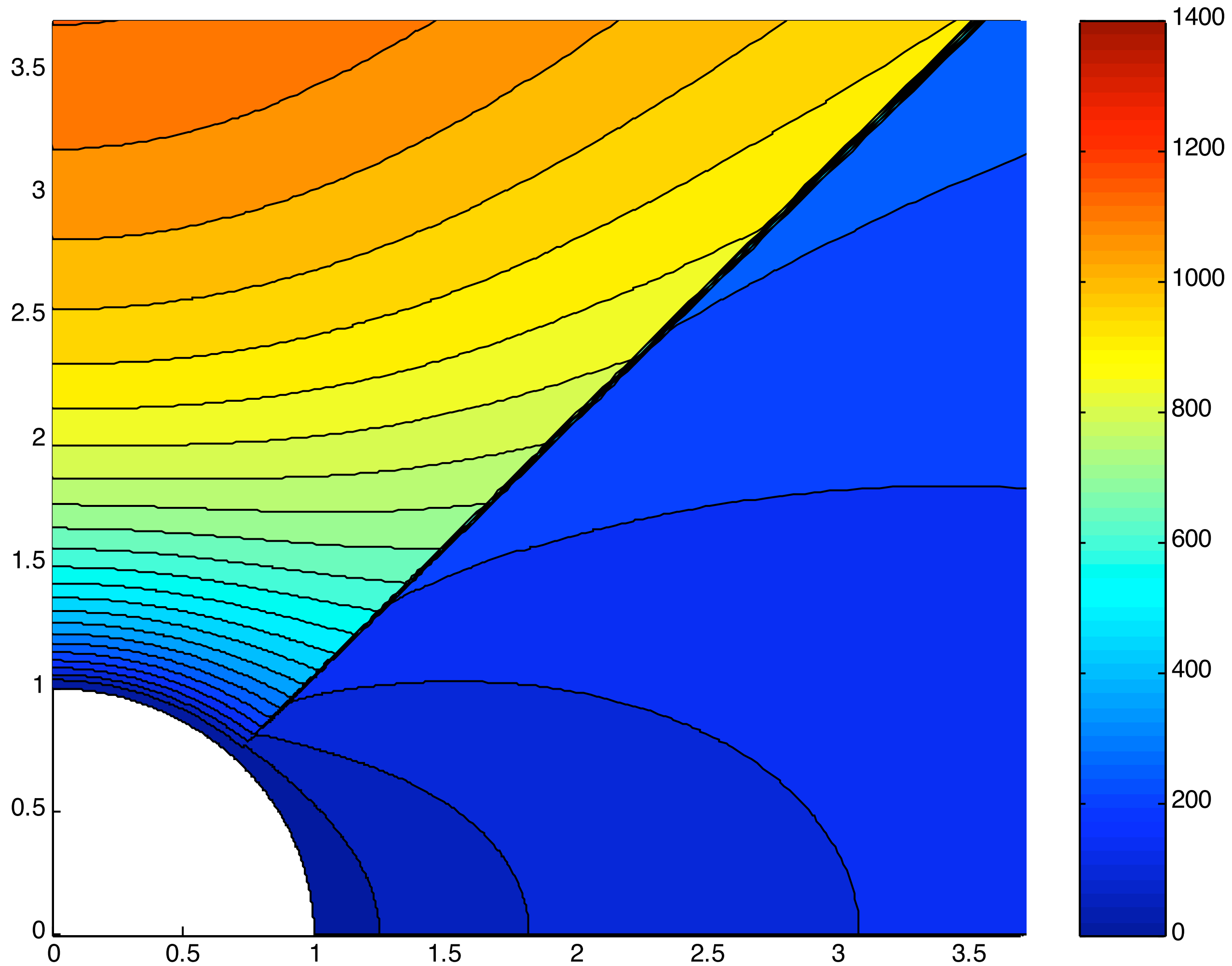
“2D” Wind



Michel Curé
Universidad de Valparaíso

B[e] Supergiant m-CAK Wind

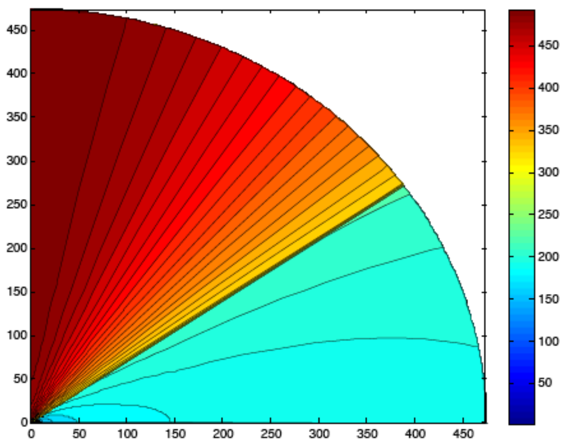
"2D" Wind



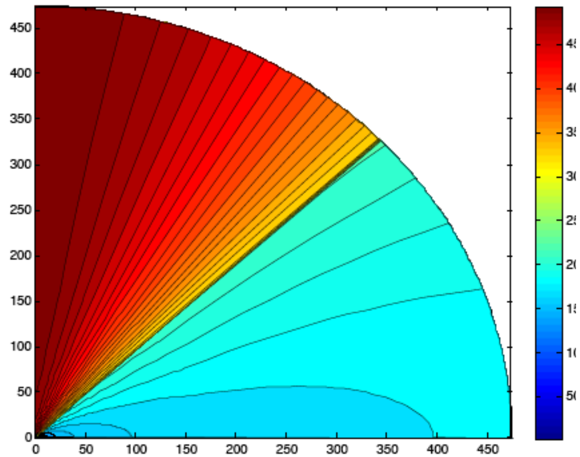
B Supergiant m-CAK Wind

Disk Aperture Angle
HD 206165

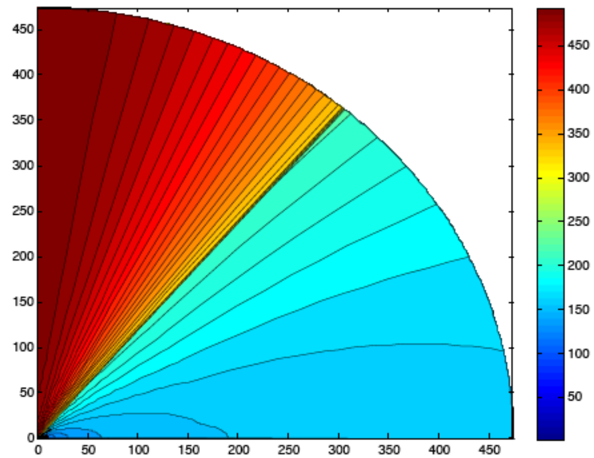
$\Omega = 0.75$



$\Omega = 0.85$



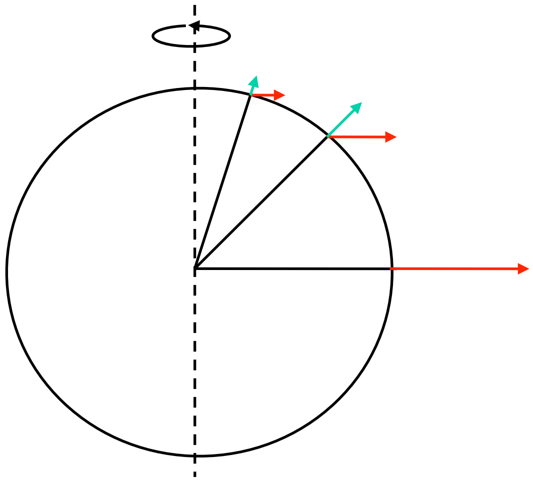
$\Omega = 0.95$



Michel Curé
Universidad de Valparaíso

Applications of the Slow solution

Rotation:
Be Stars

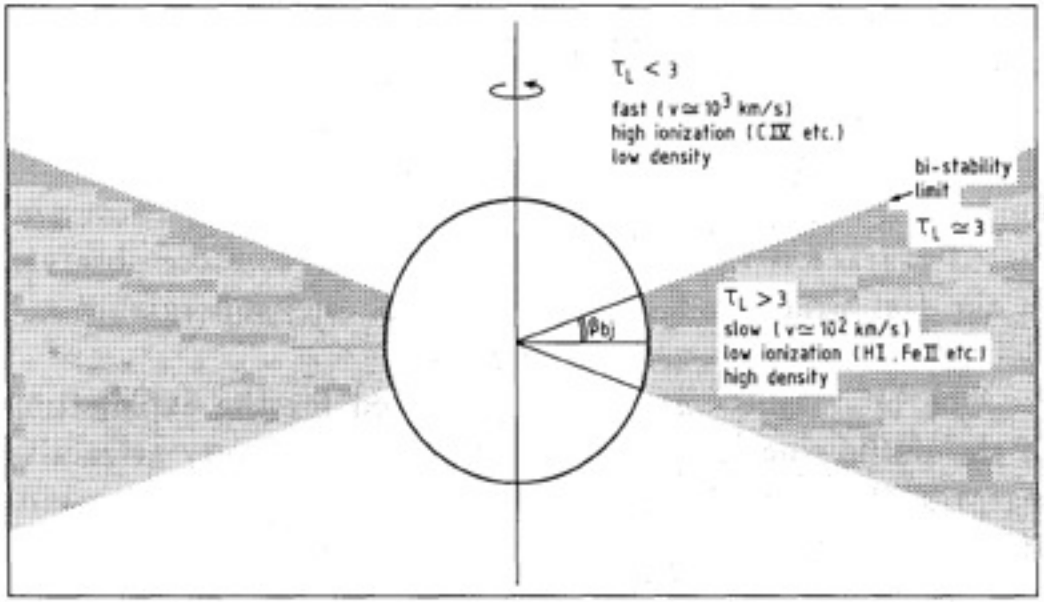


Michel Curé
Universidad de Valparaíso

Rotating Radiation Driven Winds

Rotating Stars OBJECTS

Be-Star



from Lamers & Pauldrach 1991

Be Star Wind

Stellar Parameters

B1 V star

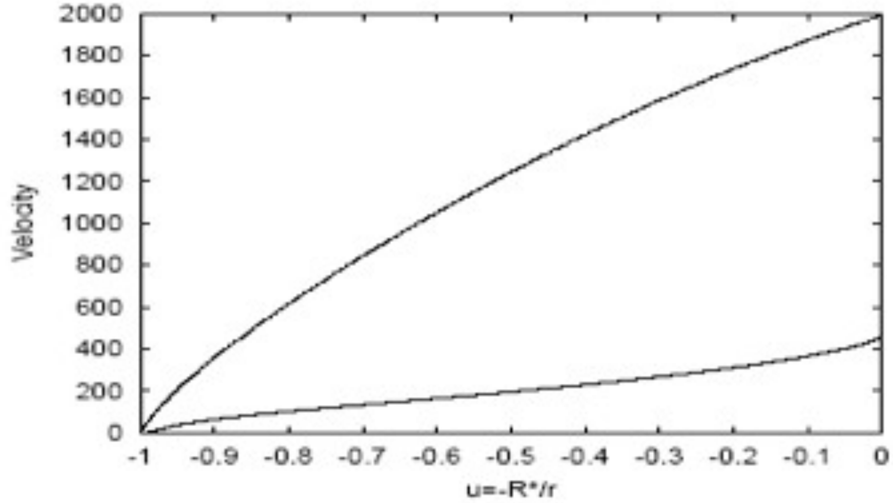
$T_{\rm eff} = 25,000$ K

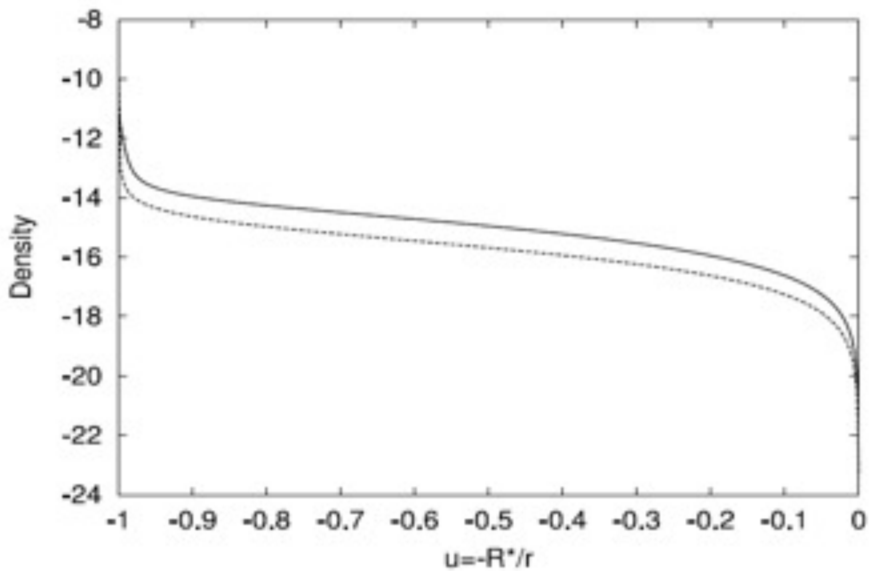
$\log g = 4.03$

$R/R_\odot = 5.3$

Line-Force Parameters

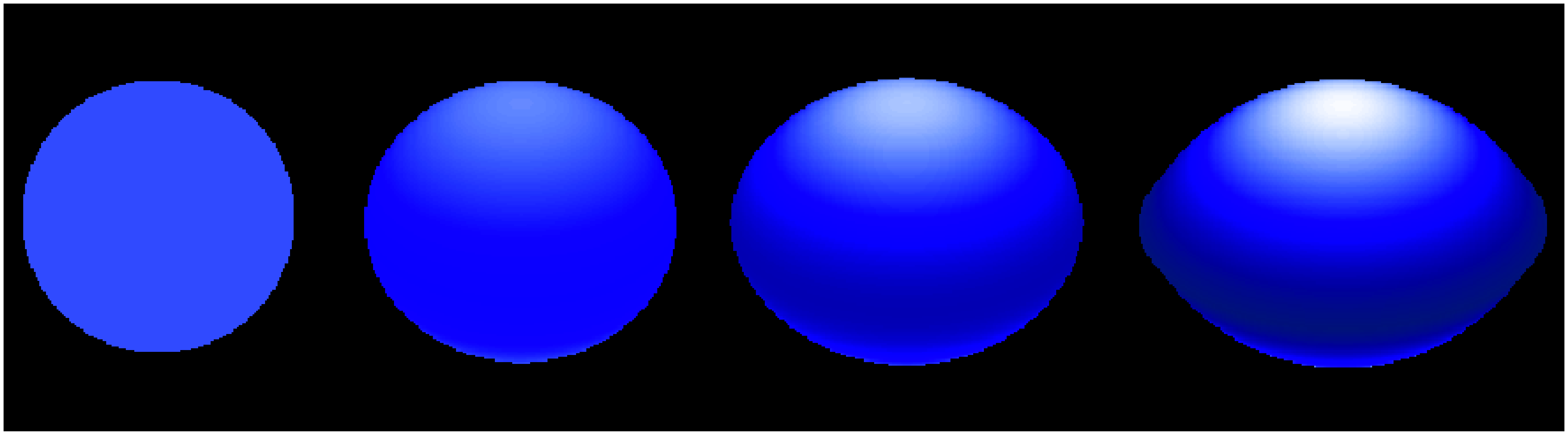
$k = 0.3$, $\alpha = 0.5$, and $\delta = 0.07$.





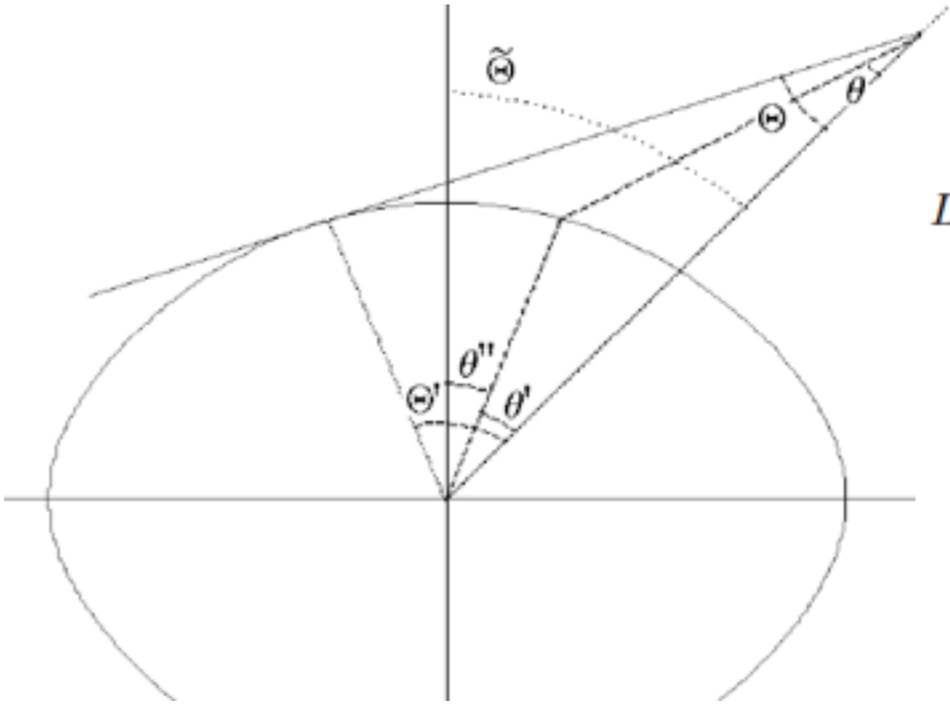
Von Zeippel Effect - Gravity Darkening

Rotational Velocity →



Michel Curé
Universidad de Valparaíso

Oblate Disk Correction Factor

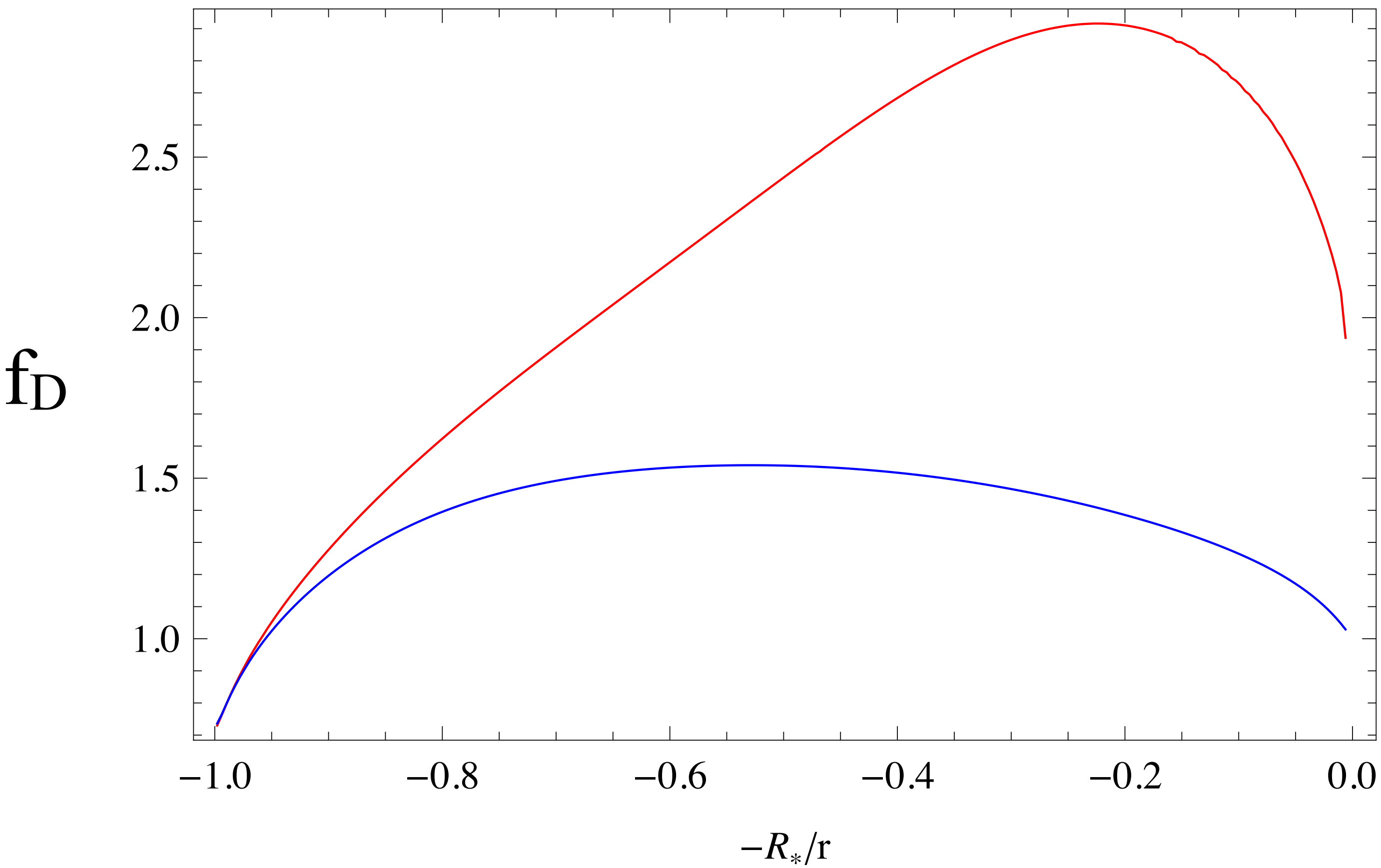


$$D_{\mathrm{fd}} = \frac{1}{N}\int_0^{2\pi}\int_0^{\Theta(\phi)} I\left(\frac{(1+\sigma)}{1+\sigma\cos^2\theta}\right)^{\alpha} \times \cos\theta\sin\theta\, d\theta d\phi$$

$$= \frac{1}{N}\int_0^{2\pi}\int_0^{\Theta'(\phi)} I\left(\frac{(1+\sigma)}{1+\sigma\cos^2\theta}\right)^{\alpha} \times \cos\theta\sin\theta\,\frac{d\theta}{d\theta'}\, d\theta' d\phi$$

Be Star Wind

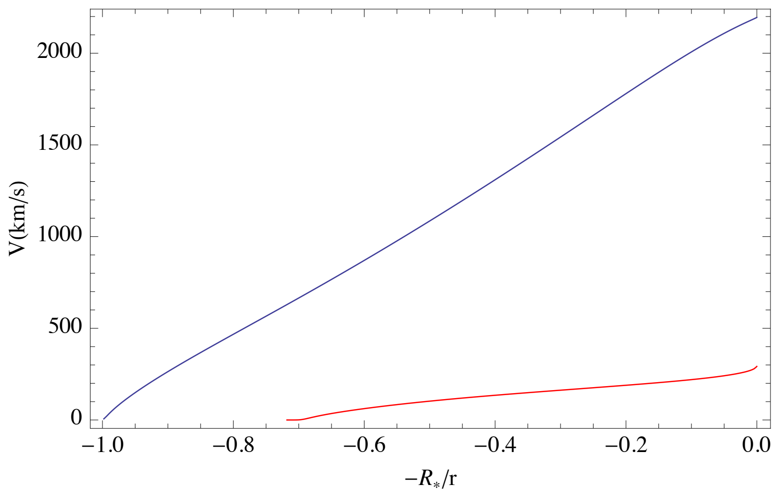
Oblate Disk Correction Factor



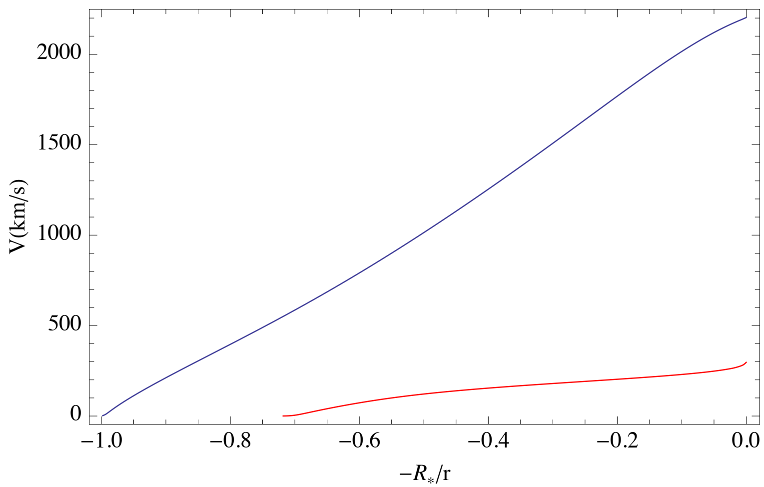
Michel Curé
Universidad de Valparaíso

Oblate Disk Correction Factor
Velocity field

$\alpha = 0.4$ and $\delta = 0.07$



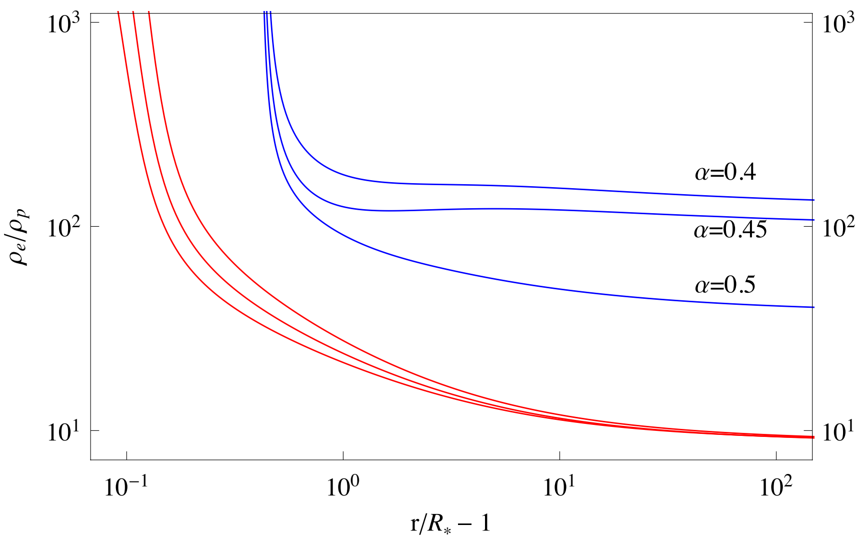
$\alpha = 0.55$ and $\delta = 0.15$



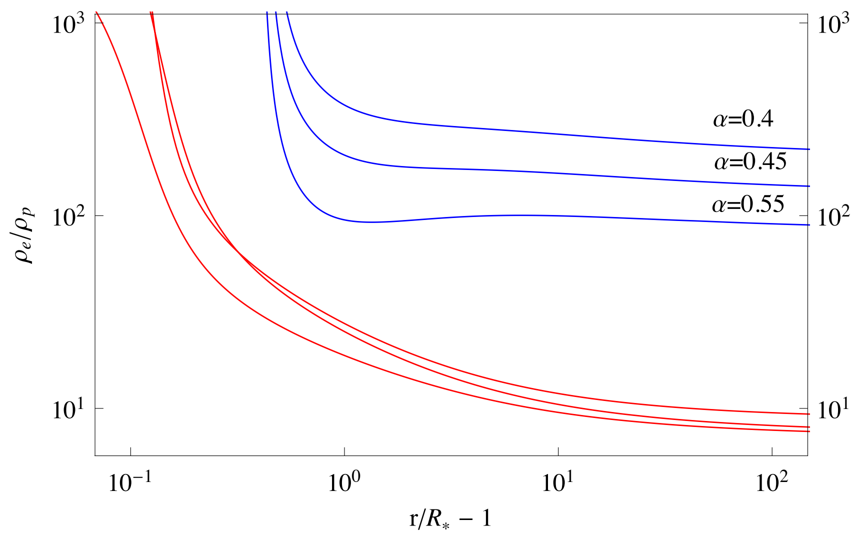
Michel Curé
Universidad de Valparaíso

Oblate Disk Correction Factor
Density contrast

$\omega = 0.99$ and $\delta = 0.07$.



$\omega = 0.99$ and $\delta = 0.15$



Michel Curé
Universidad de Valparaíso

Applications of the Slow solution

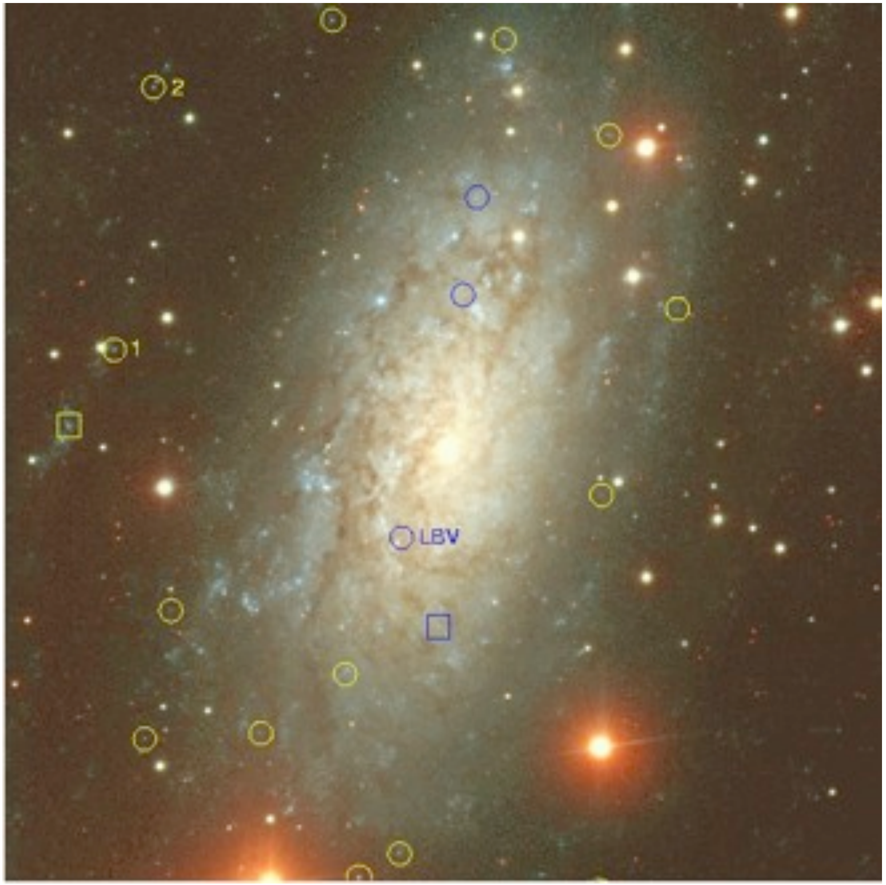
Changes in ionization: (OB)A Supergiants

Michel Curé
Universidad de Valparaíso

Wind Momentum Luminosity Relationship

A Supergiants

$$\dot{\mathbf{M}}\mathbf{v}_{\infty} \propto \frac{1}{R_*^{1/2}} \mathbf{L}^{1/\alpha}$$



NGC3621

Wind Momentum Luminosity Relationship

Hα dependence on the mass loss rate

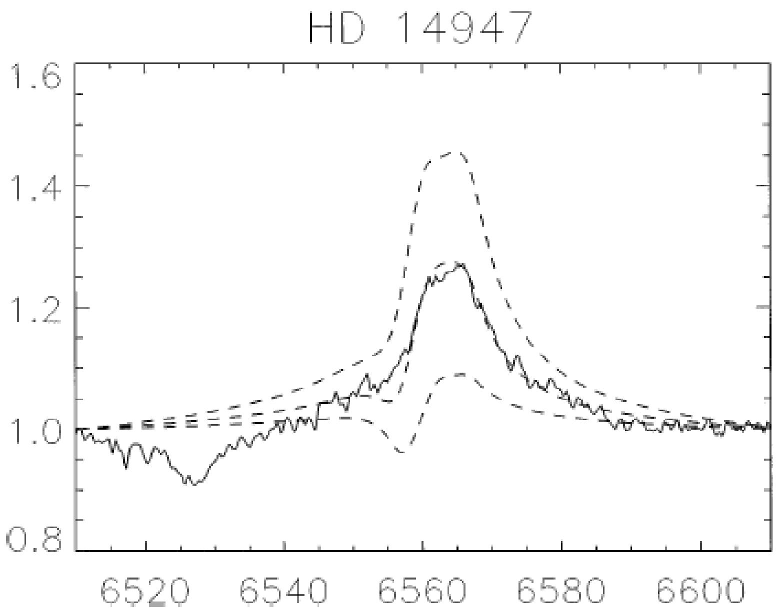


Figure 2 H_α line profile of the O 5 Ia f^+-supergiant HD 14947 compared with line fits as outlined by Puls et al (1996), adopting 10, 7.5, and 5.0 × 10^{-6} $M_\odot$/year, respectively, for the mass-loss rate. From Kudritzki (1998).

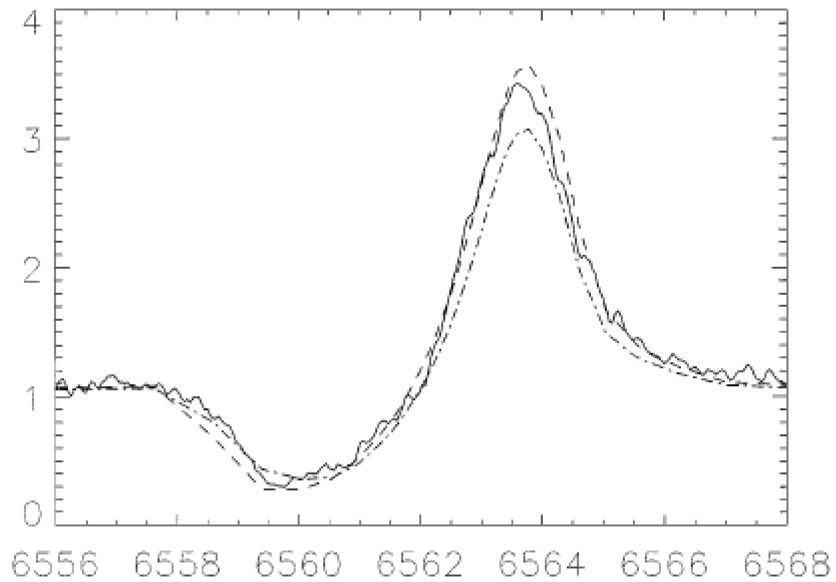


Figure 3 H_α line profile of the extreme A-supergiant 41-3654 (A3 Ia-O) in the Andromeda Galaxy M31 taken with the Keck HIRES spectrograph compared with two unified model calculations adopting $\beta = 3$, $v_\infty = 200$ km/s and $\dot{M} = 1.7$ and 2.1 × 10^{-6} $M_\odot$/year. Note the P-Cygni profile shape of H_α. From McCarthy et al (1997).

O5 Ia

A3 Ia

Wind Momentum Luminosity Relationship

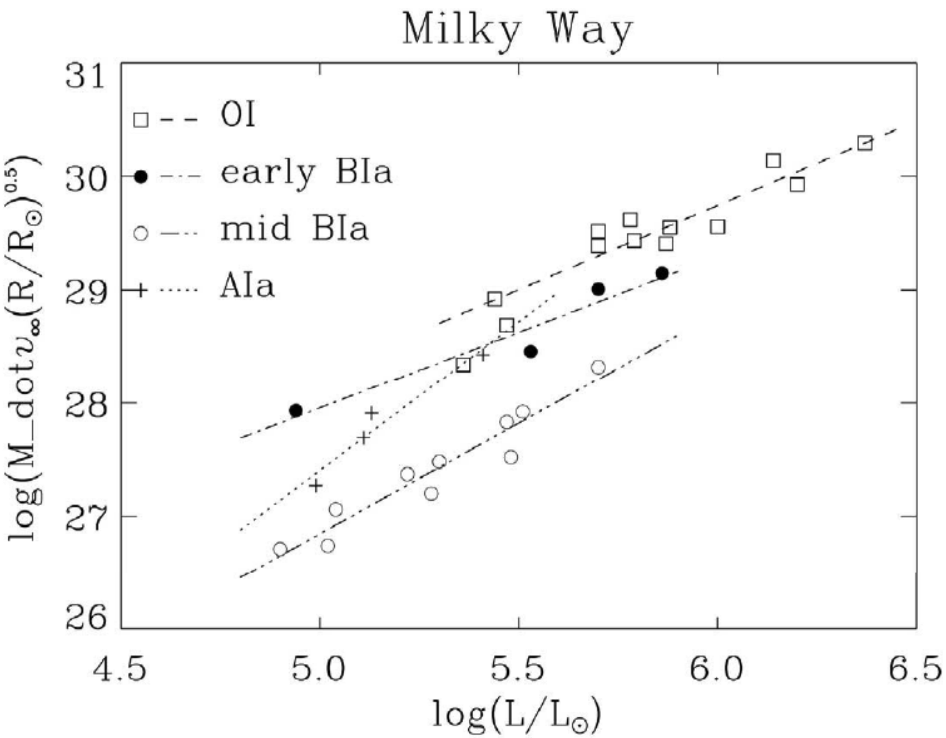


TABLE 2 Coefficients of the wind momentum–luminosity relationship for A/B-supergiants and O-stars of the solar neighborhood

Sp. type	Log D_0	x	α'
A I	14.22 ± 2.41	2.64 ± 0.47	0.38 ± 0.07
Mid B I	17.07 ± 1.05	1.95 ± 0.20	0.51 ± 0.05
Early B I	21.24 ± 1.38	1.34 ± 0.25	0.75 ± 0.15
O I	20.69 ± 1.04	1.51 ± 0.18	0.66 ± 0.06
O III, V	19.87 ± 1.21	1.57 ± 0.21	0.64 ± 0.06

Radiative Transport models do not use velocity (density) profile from CAK Hydrodynamic.

Instead: beta-profile

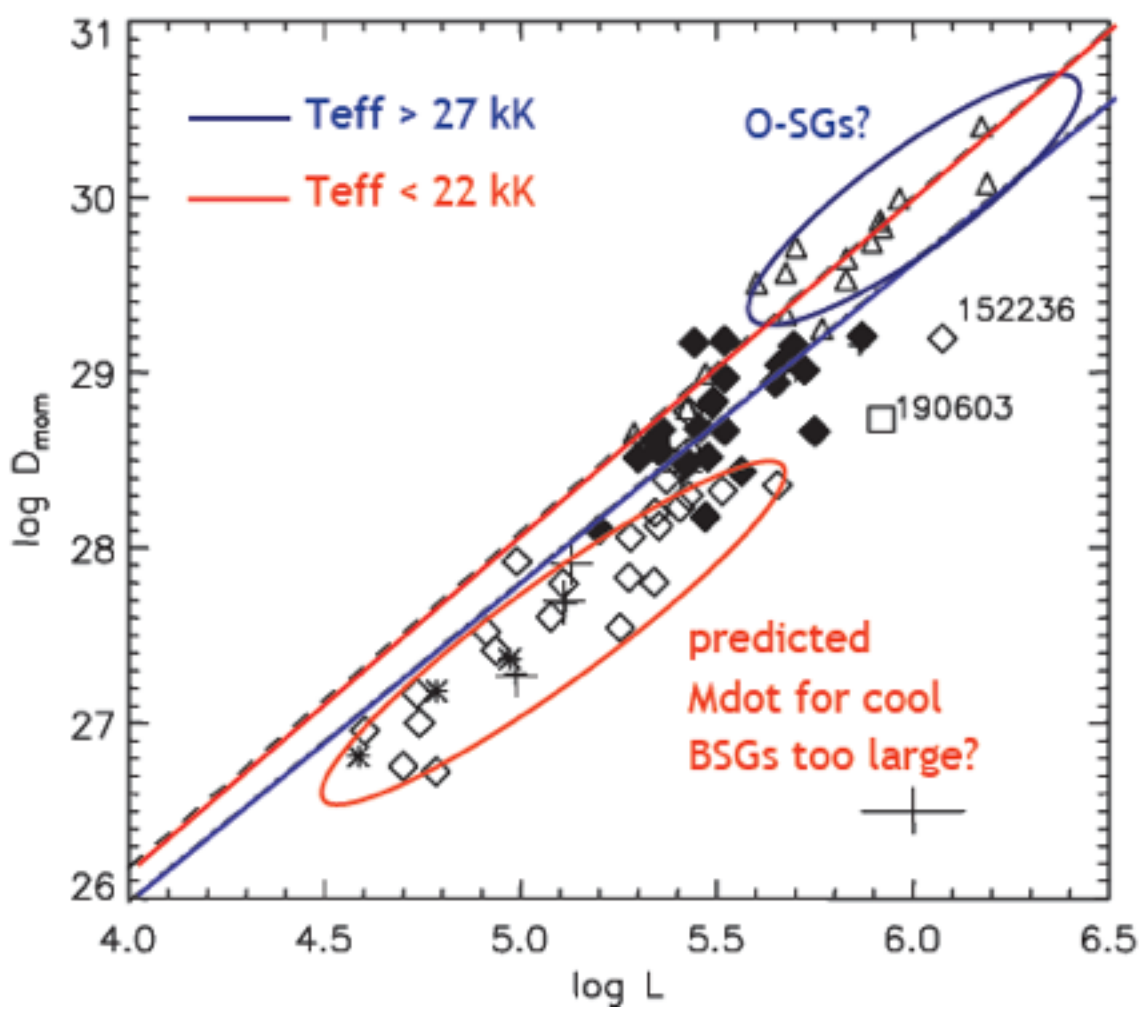
$$v(r) = v_\infty \left(1 - \frac{R_\star}{r}\right)^\beta$$

Michel Curé
Universidad de Valparaíso

Wind Momentum Luminosity Relationship

$$\dot{\mathbf{M}}\mathbf{v}_{\infty} \propto \frac{1}{R_*^{1/2}}\mathbf{L}^{1/\alpha}$$

OB Supergiants

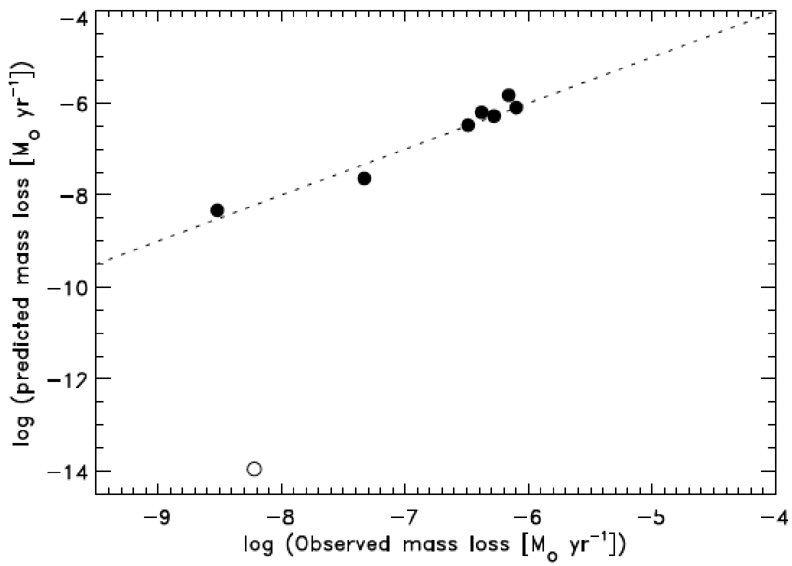


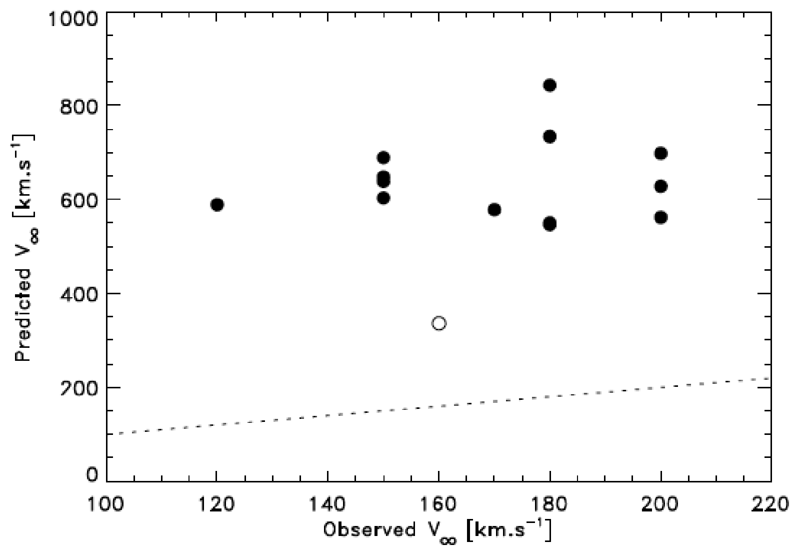
From Markova & Puls 2008

Michel Curé
Universidad de Valparaíso

A-Supergiant Models

Achmad, Lamers & Pasquini, A&A 320,196, 1997

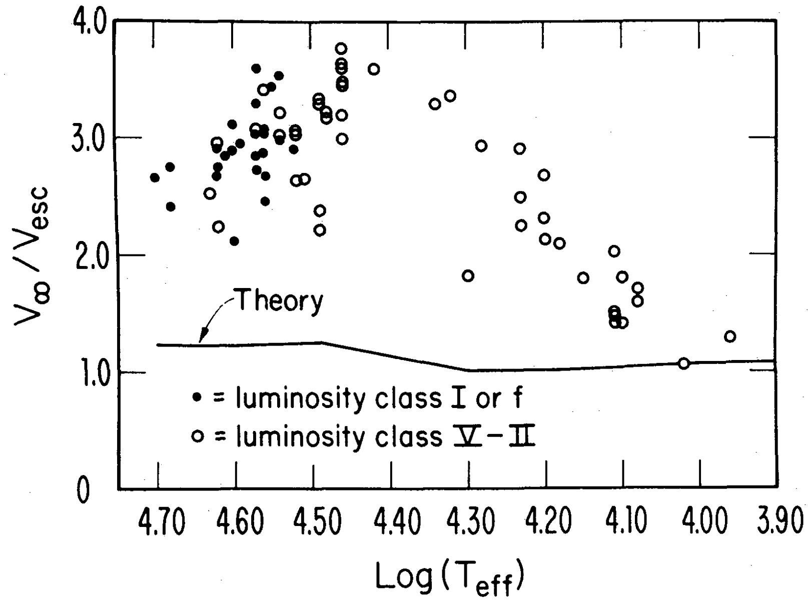


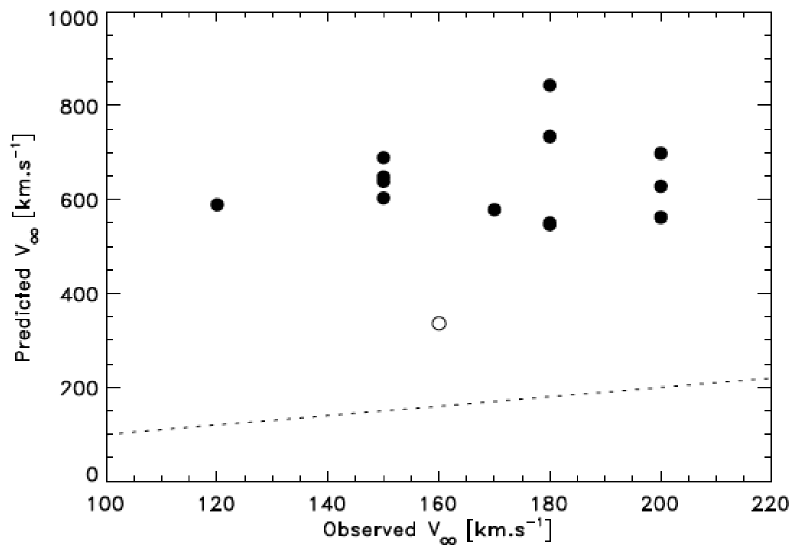


Michel Curé
Universidad de Valparaíso

A-Supergiant Models

Achmad, Lamers & Pasquini, A&A 320,196, 1997





Discrepancy

Wind Momentum Luminosity Relationship

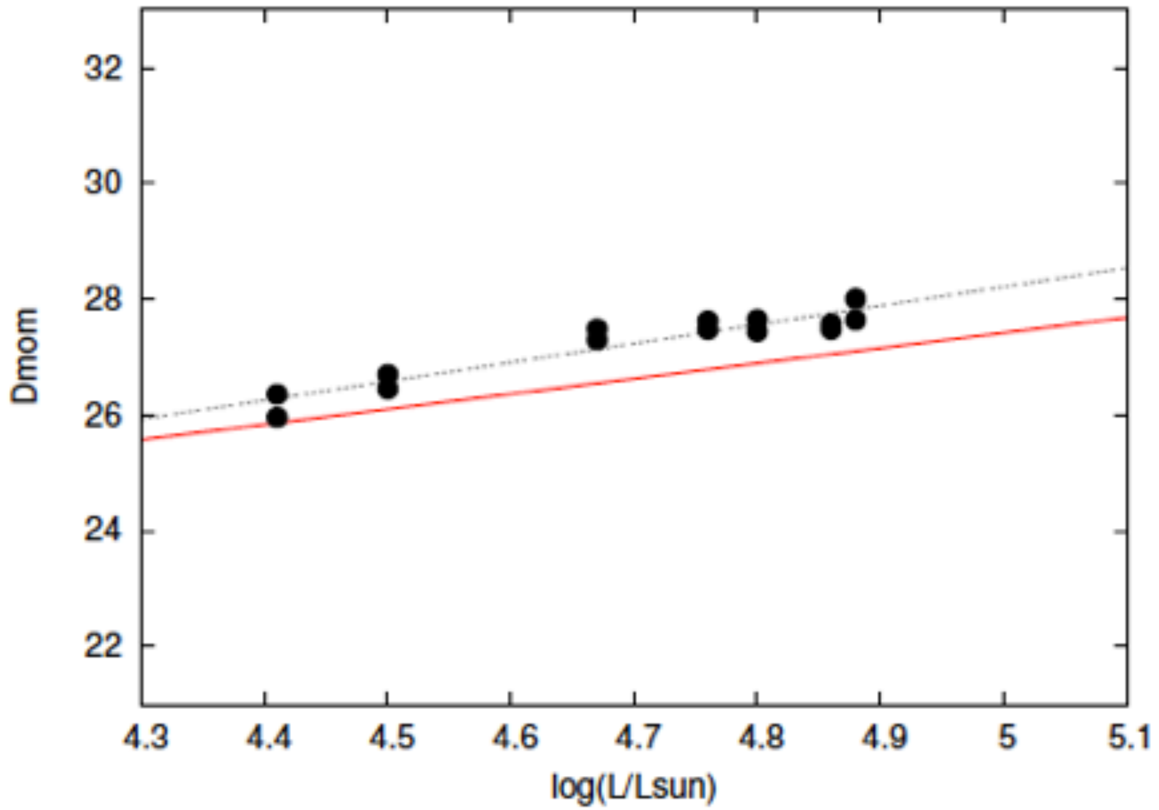
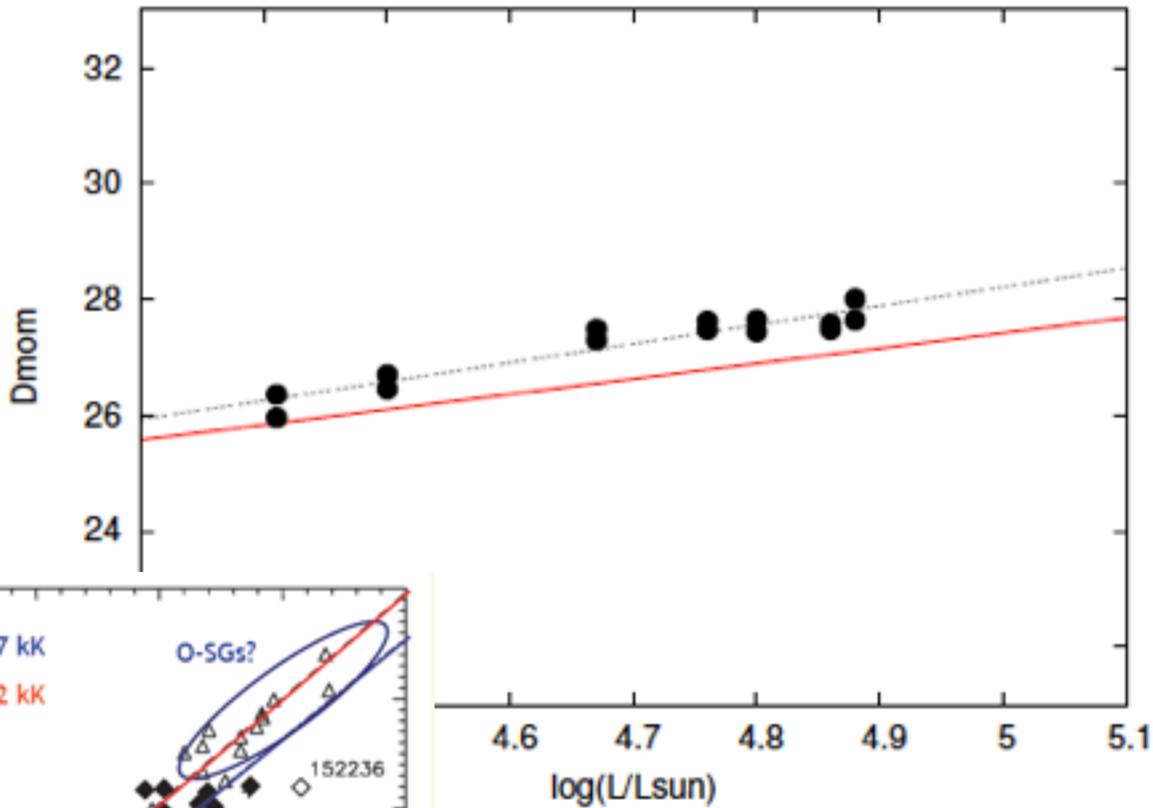
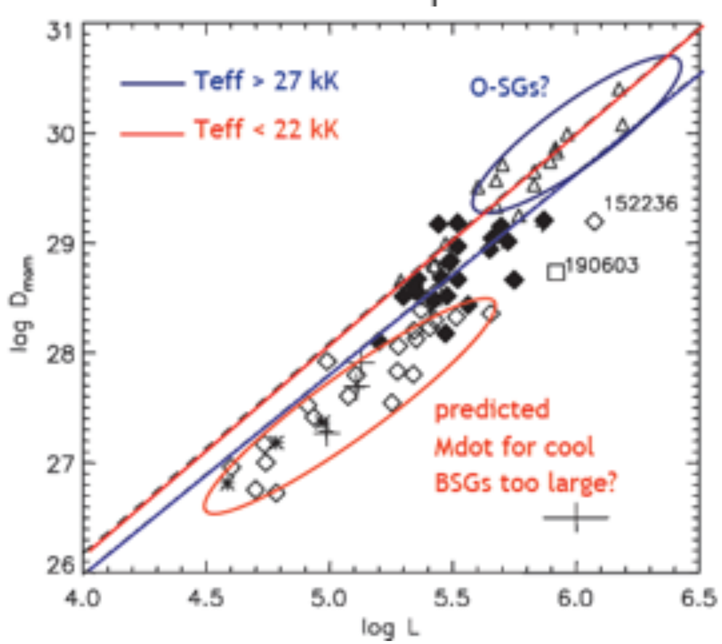


Fig. 2. Theoretical (dashed line) and observational (red solid line) WML-relationship by Kudritzki et al. (1999). Theoretical data (black circles) has been obtained from new slow wind models with $\Omega = 0.4$

Wind Momentum Luminosity Relationship





l line) and observational (red solid line) ritzki et al. (1999). Theoretical data (black from new slow wind models with $\Omega = 0.4$

Michel Curé
Universidad de Valparaíso

Wind Momentum Luminosity Relationship

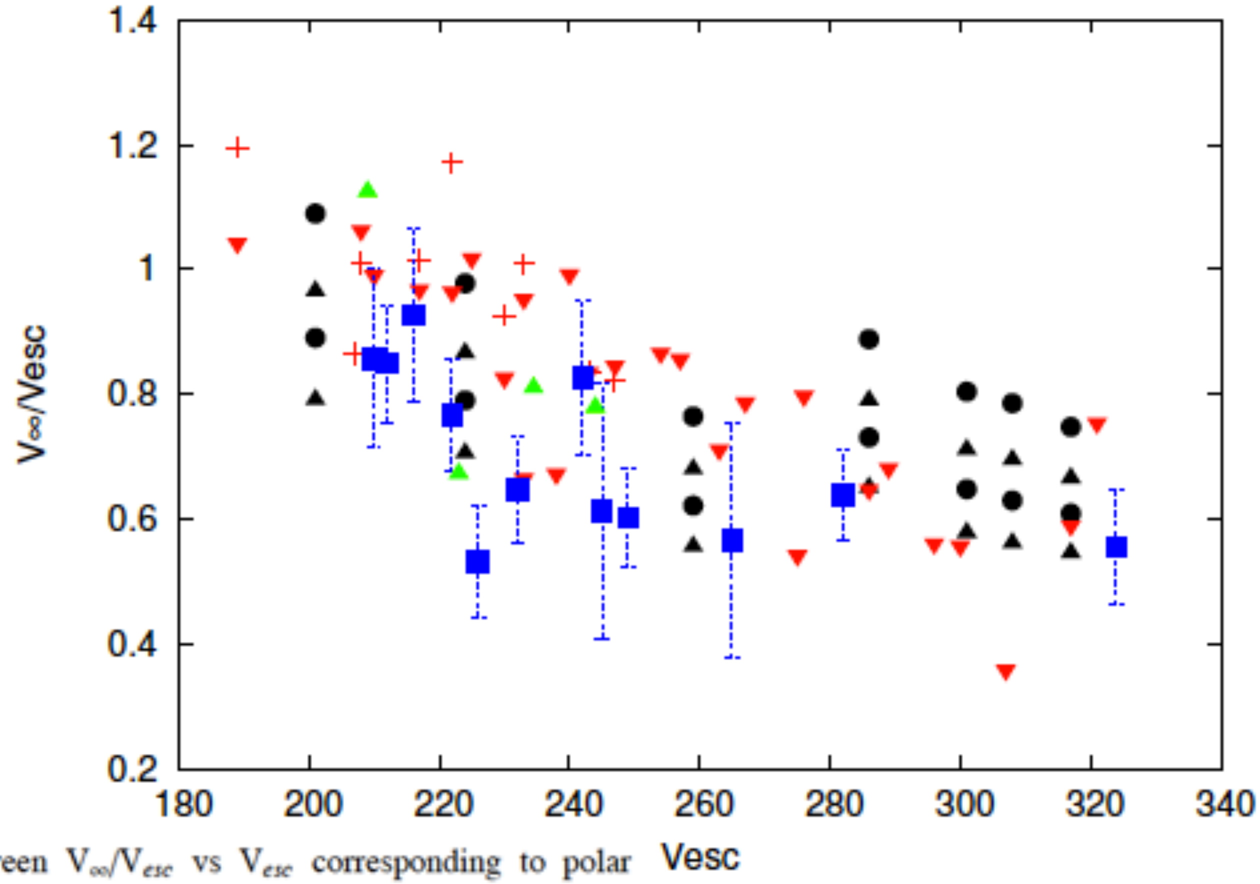


Fig. 3. Relation between V_∞/V_{esc} vs V_{esc} corresponding to polar (black circles) and equatorial (black triangles) slow solutions. Down-triangles and crosses (red symbols) represent the observational data taken from Verdugo et al. (1998b); the crosses indicate terminal velocities obtained from saturated PCygni UV lines whereas the down-triangles correspond to values determined by means of discrete absorption components; up-triangles (green) correspond to terminal velocities from Kudritzki et al. (1999); squares (blue) represent the measurements provided by Achmad et al. (1997) with their error estimates.

The slow wind solution follows the same trend of the observations

Michel Curé
Universidad de Valparaíso

Time dependent Hydrodynamic

Time dependent Hydrodynamic

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial (r^2 \rho v)}{\partial r} = 0,$$

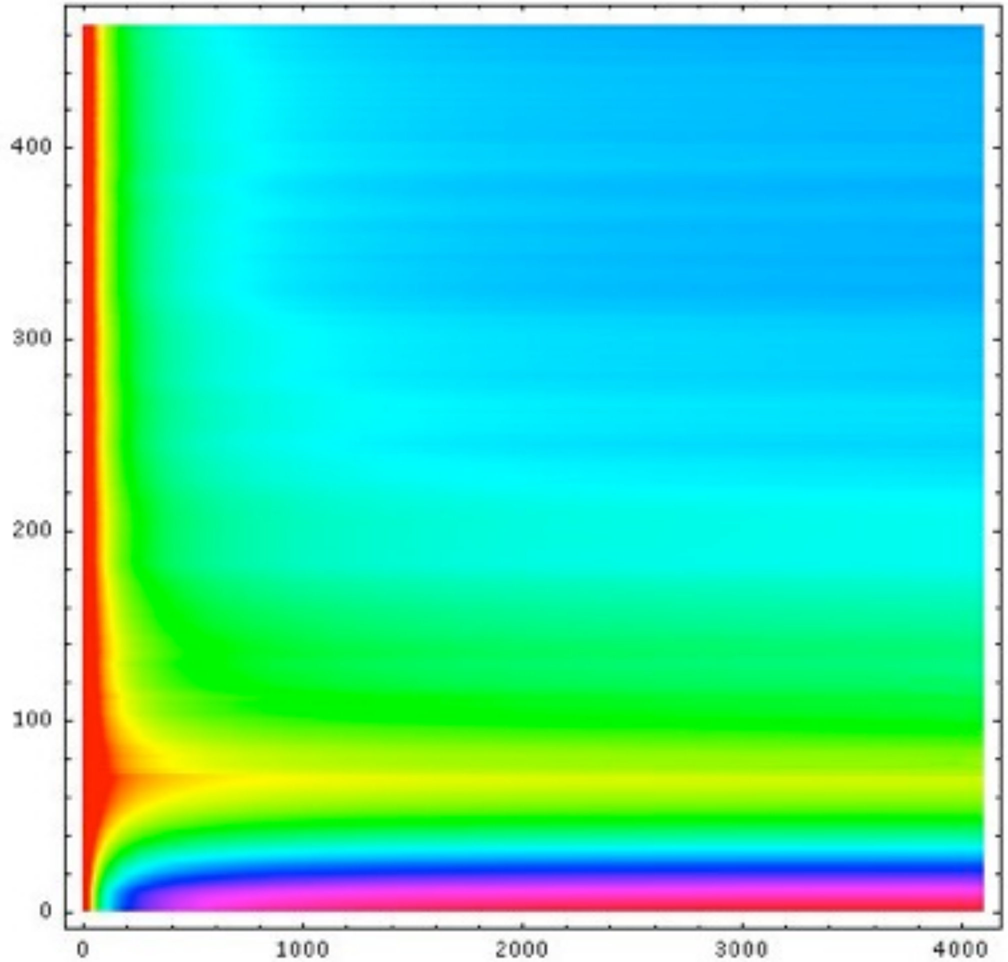
$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = \frac{v_\phi^2}{r} - \frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{GM_*(1-\Gamma_e)}{r^2} + g_{\mathrm{lines}}$$

Michel Curé
Universidad de Valparaíso

Time dependent Hydrodynamic

ZEUS-3D

CAK model

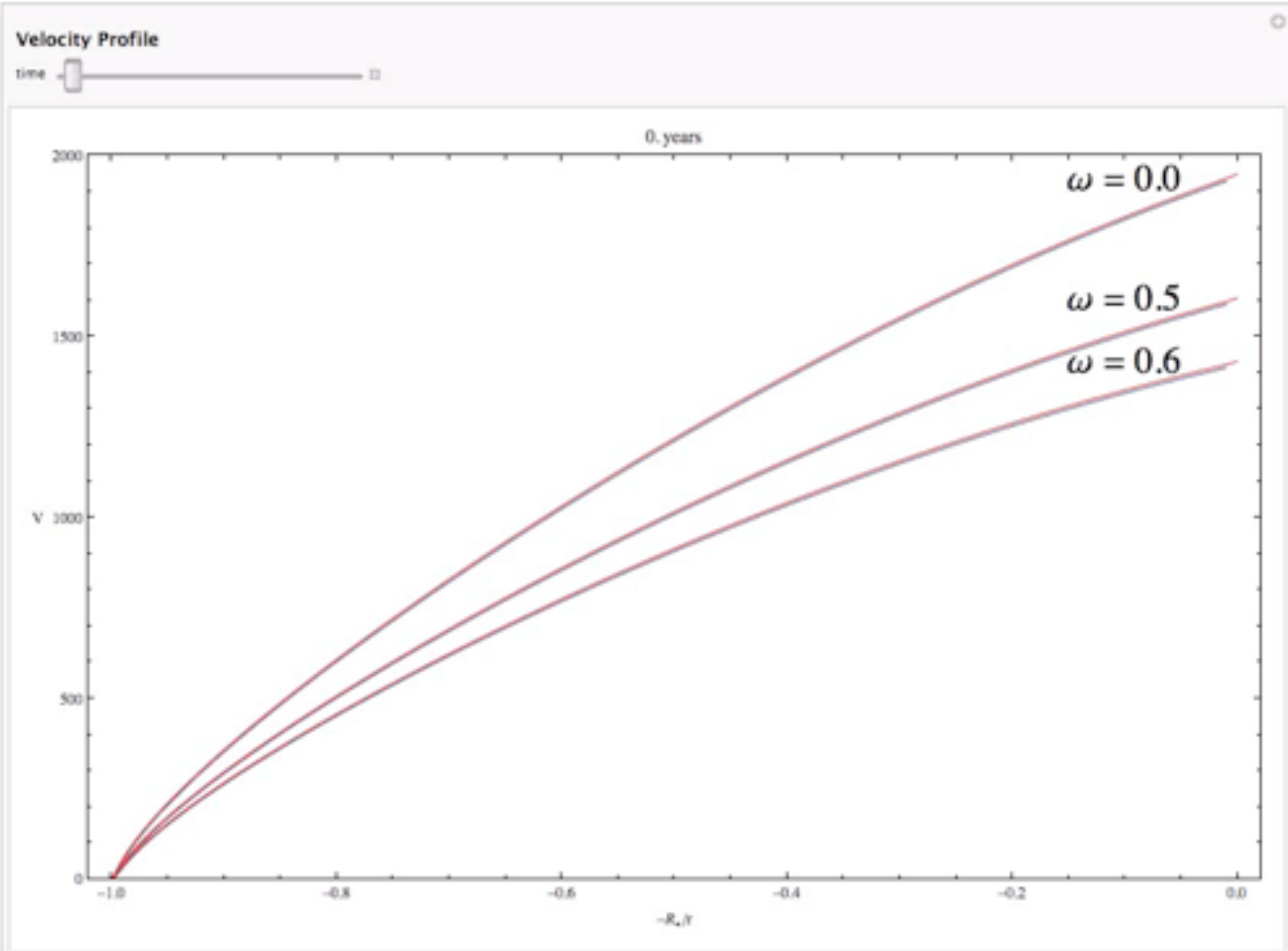


time

radius

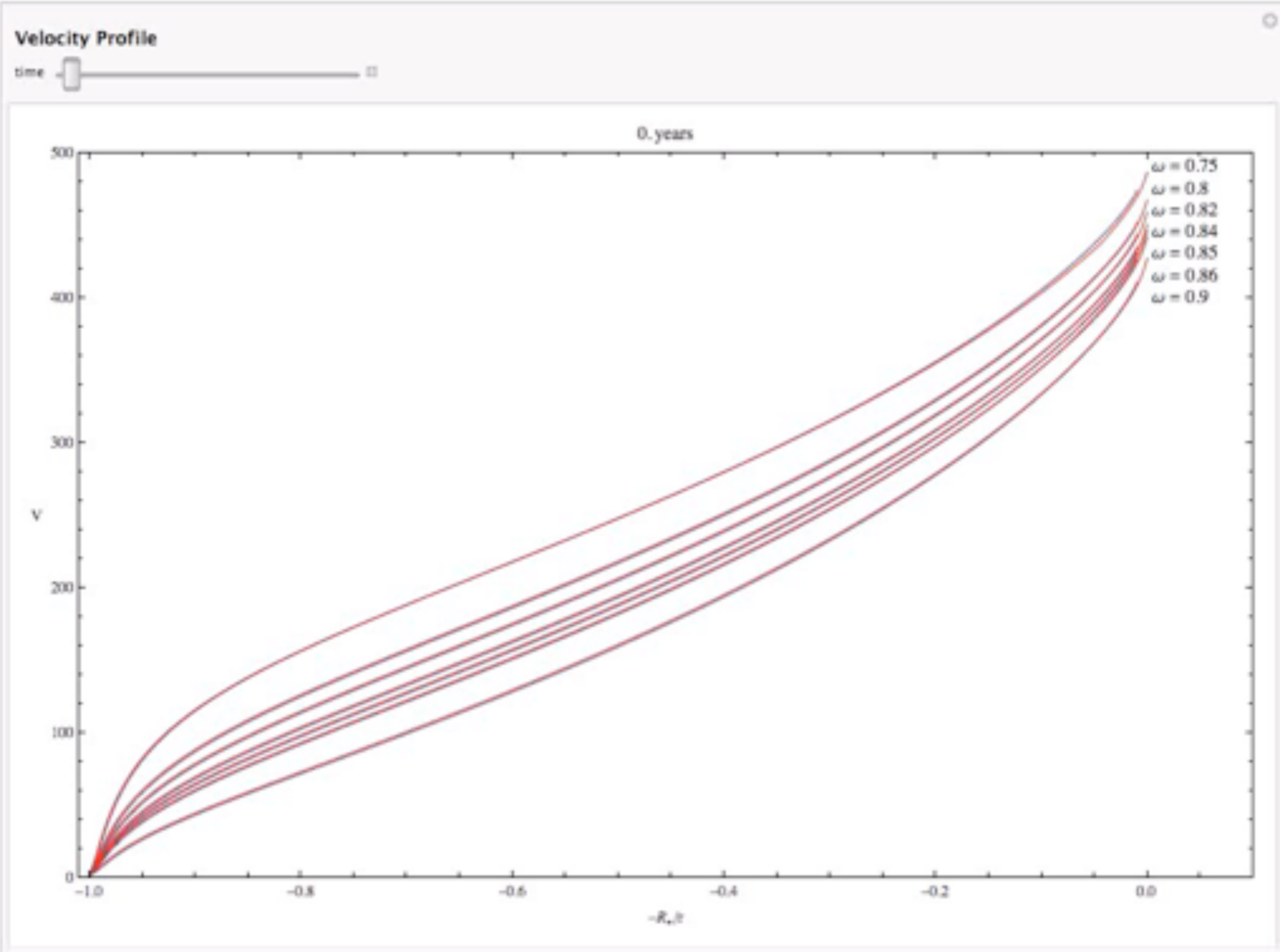
Michel Curé
Universidad de Valparaíso

ZEUS-3D m-CAK model
Fast Solutions



Time dependent Hydrodynamic

ZEUS-3D m-CAK model
Slow Solutions



Conclusions

Slow wind solutions may solve some of the problems from massive stars hydrodynamic:

- Winds from B[e] Supergiants (outflowing disk)
- Winds from BA Supergiants (WML relationship)
- Classical Be Stars (Gravity Darkening)

Future Work

- Time dependent Calculations (bifurcation, oscillation, clumping?)
- 2D-calculations
- Observations (constrains to theory)
- Magnetic Fields

Michel Curé
Universidad de Valparaíso

F I N

Michel Curé
Universidad de Valparaíso