Geometry of the Berry Phase

.... a concise μ-seminar exposition ....

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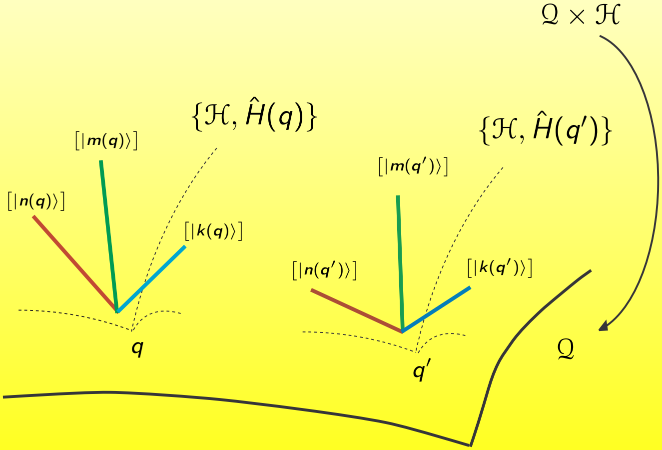
- Remark I.: (gauge transformation in given eigenvalue sector)

$$\Phi_m \mapsto \Phi'_m : q \mapsto \{|\tilde{m}(q)\rangle = e^{i\alpha_m(q)}|m(q)\rangle, E_m(q)\}$$

Geometrical overview - fiber bundle perspective



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- to find a solution $|\Psi(q_C(t = T))\rangle$ of the Schrödinger equation:

$$i\hbar\,\partial_t|\Psi(t)\rangle = \hat{H}(q_C(t))|\Psi(t)\rangle \;+\; \text{initial condition}$$

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- together with set of initial conditions: $\{B_m(t=0) = \delta_{mn}\}$

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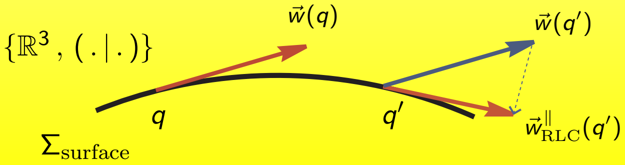

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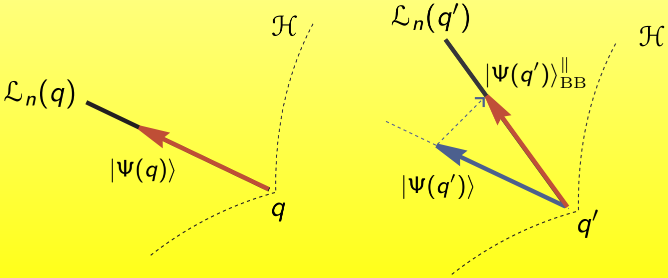

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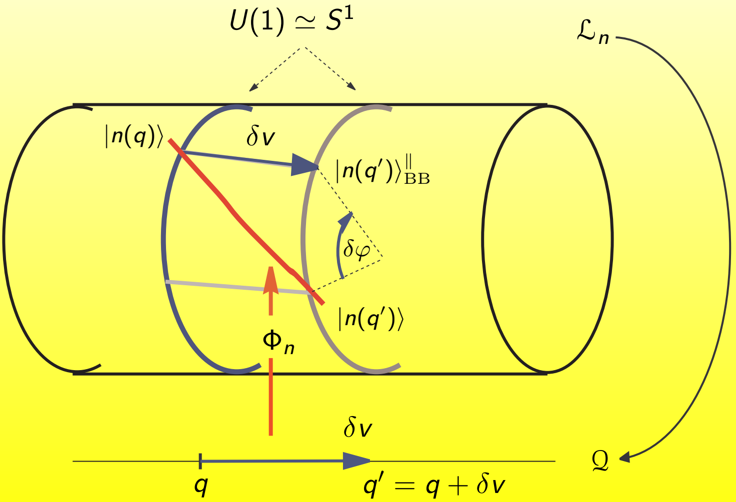
- Remark III.: $\langle n(q)|\partial_{q^\mu} n(q)\rangle$ is pure imaginary quantity, thus

$$A = A_\mu(q) dq^\mu = -\mathrm{Im}\,\langle n(q)|\partial_{q^\mu} n(q)\rangle\, dq^\mu$$

More realistic perspective



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- in classical mechanics, there is as well in adiabatic regime an analog of the Berry phase called Hannay phase (or angel)

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thanks for your attention!