Effective Field Theory for Lattice nuclei

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האוניברסיטה העברית בירושלים
The Hebrew University of Jerusalem

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The Hebrew University of Jerusalem

Trento, Italy
F. Pederiva, L. Contessi

UNIVERSITY
OF TRENTO - Italy

Orsay, France
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Les deux infinis

LQCD - The Single Baryon Case

Lattice QCD

- QCD is the fundamental theory for nuclear physics.
- It is formulated in terms of quarks and gluons.
- At low energy QCD is non-perturbative $\longrightarrow$ lattice simulations (LQCD).
- Neutron and proton masses are predictions.
- Same for pion masses.

Xui-Lei Ren *et* al., PRD **87** 074001 (2013)
L. Alvarez-Ruso *et* al., ArXiv hep-ph: 1304.0483 (2013)

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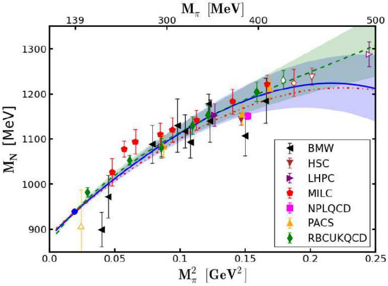

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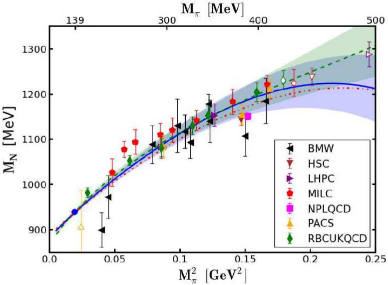
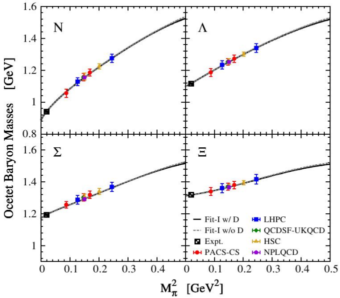
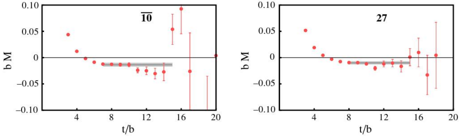


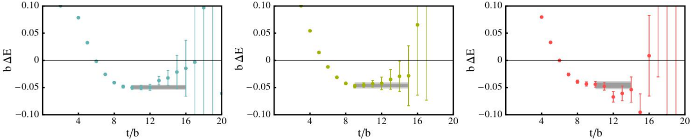


LQCD - Multi Baryon Configurations

Deutron ($\overline{10}$) and dineutron (27) simulations



Triton simulations with different lattice sizes ($24^3 \times 48, 32^3 \times 48, 48^3 \times 64$)



- LQCD simulations with $SU_f(3)$ symmetry
- Large pion mass $m_\pi = 800$ MeV
- Results with $m_\pi = 510$ MeV are already available
- Also the 2-body scattering parameters a_s, r_{eff} @800 MeV

NPLQCD Collaboration, PRD **87** 034506 (2013)

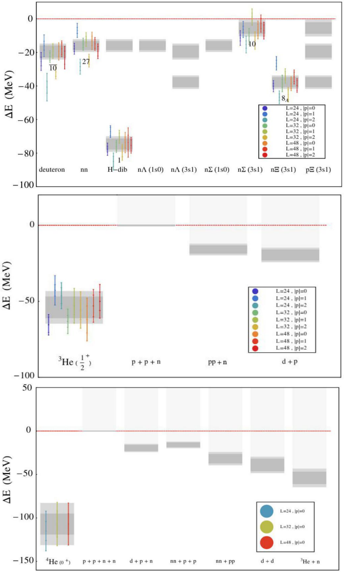
LQCD - Few-Body Baryon Spectra

2-body system - Deutron, dineutron,nΛ,...

3-body system - ^{3}He, triton,$^3_\Lambda$He,...

4-body system - ^{4}He,$^3_\Lambda$He,...

NPLQCD Collaboration, PRD **87** 034506 (2013)



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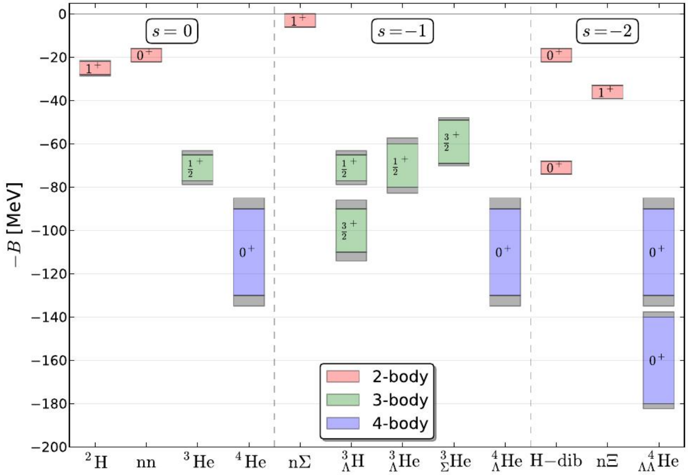

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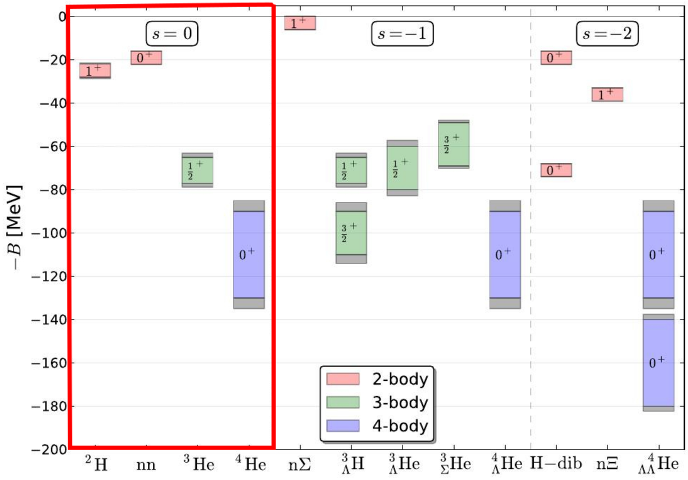
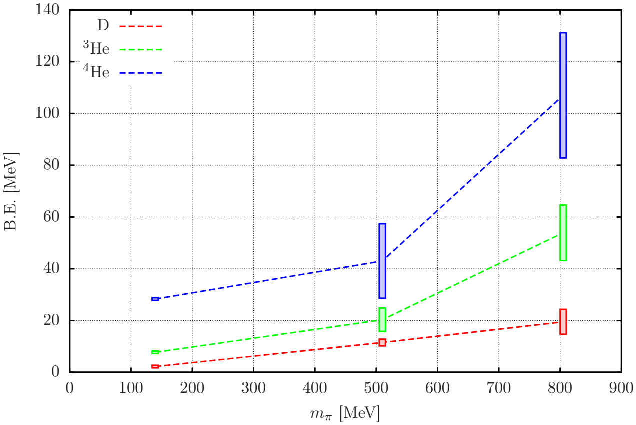


NPLQCD Collaboration, PRD **87** 034506 (2013)

The Evolution of the Nuclear Spectrum with m_π



NPLQCD Collaboration, PRD **87** 034506 (2013)
T. Yamazaki, K. Ishikawa, Y. Kuramashi, and A. Ukawa, Phys. Rev. D **86** (2012) 074514.

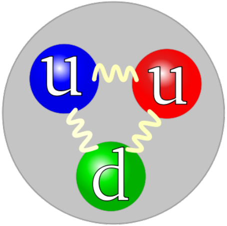
EFT in Nuclear Physics

Effective Field Theory

- At this point LQCD simulations for $A \geq 2$ nuclei are still away from the physical point.
- Currently no reliable NN interactions can be derived from lattice simulations.
- Contemporary nuclear theory is based on Effective Field Theory $\longrightarrow$ phenomenology.
- The quarks and gluons degrees of freedom are replaced by baryons and mesons.

$$\mathcal{L}_{QCD}(q,G) \longrightarrow \mathcal{L}_{Nucl}(N,\pi,\ldots)$$

- The $\mathcal{L}_{Nucl}(N,\pi,\ldots)$ is constructed to retain QCD symmetries.
- $\mathcal{L}_{Nucl}(N,\pi,\ldots)$ is an expansion in low momentum Q.
- Contains all terms compatible with QCD up to a given order.
- The low-energy coupling constants (LECs) are explicit function of the cutoff Λ.



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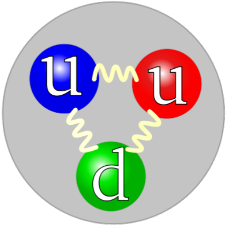

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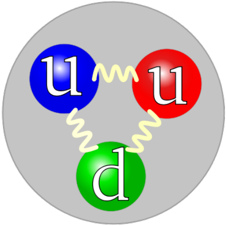

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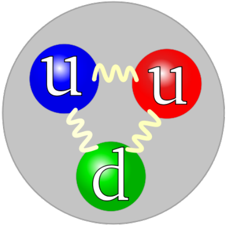

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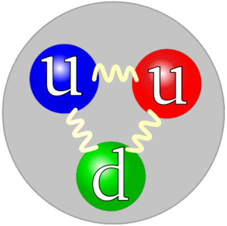

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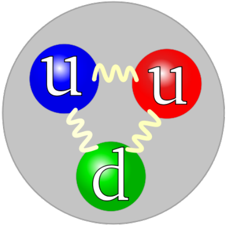

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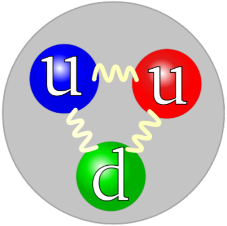

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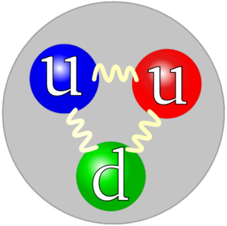
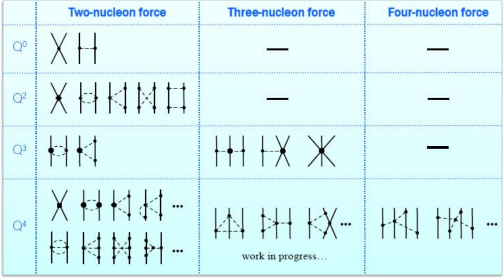


Effective Field Theory potentials

Low Eenergy Constants

- There are 2 free parameters in LO, 7 at NLO, ...
- NNN and NNNN forces come in naturally at orders N2LO and N3LO.
- The NNN force contains 2 free parameters



2 nucleon force ≫ 3 nucleon force ≫ 4 nucleon force ...

$$
\begin{aligned}
V &= -\left(\frac{g_A}{2f_\pi}\right)^2 \frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})}{q^2+m_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\
&+ C_S + C_T\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \\
&+ V_{NLO} + V_{N2LO} + \dots
\end{aligned}
$$

D. R. Entem and R. Machleidt, PRC **68**, 041001(R) (2003).
Epelbaum *et* al., EPJA **19**, 401 (2004), NPA **747**, 362 (2005).

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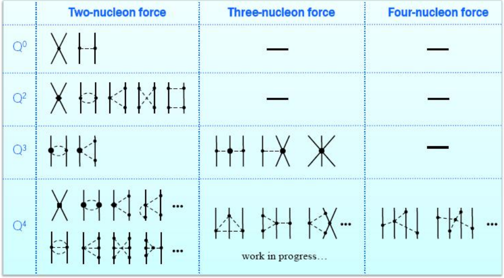

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χ^2/datum for the reproduction of the 1999 np database

Bin (MeV)	# of data	N³LO	NNLO	NLO	AV18
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04

$$V = -\left(\frac{g_A}{2f_\pi}\right)^2 \frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})}{q^2+m_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + C_S + C_T\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + V_{NLO} + V_{N2LO} + \ldots$$

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EFT for Lattice Nuclei

Energy Scales

- Nucleon mass M_n, and mass difference $\Delta = M_\Delta - M_n$
- The pion mass m_π, pion exchange momentum $q_\pi = m_\pi/\hbar c$, and energy

$$E_\pi = \frac{\hbar^2 q_\pi^2}{M_n} = \frac{m_\pi}{M_n} m_\pi$$

- Nuclear binding energy B/A

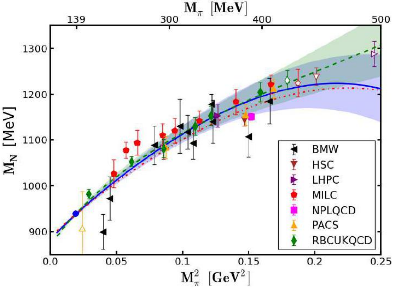
Scale	Nature	LQCD@m_π=500MeV	LQCD@m_π=800MeV
M_n	940 MeV	1300 MeV	1600 MeV
Δ	300 MeV	300 MeV	180 MeV
m_π	140 MeV	500 MeV	800 MeV
E_π	20 MeV	200 MeV	400 MeV
B/A	10 MeV	15 Mev	25 MeV

Conclusions

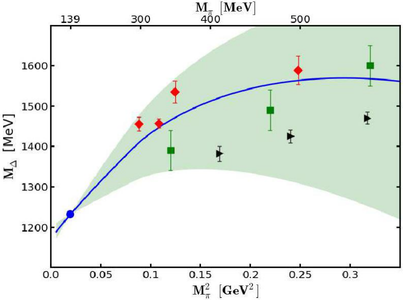
- For the Natural case $\mathcal{L} \longrightarrow \mathcal{L}_{EFT}(N, \pi)$
- For lattice nuclei at $m_\pi \geq 400$MeV $E_\pi \gg B/A$
- In this case $\not\pi$EFT is the natural theory $\mathcal{L} \longrightarrow \mathcal{L}_{EFT}(N)$

The nucleon Δ mass difference

Nucleon mass - n,p



Δ mass

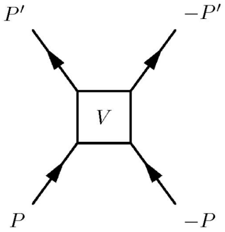


L. Alvarez-Ruso *et* al., ArXiv hep-ph: 1304.0483 (2013)

$\not{\pi}$EFT for Lattice Nuclei

- We write all possible terms in $\mathcal{L}$ ordered by the number of derivatives

$$
\begin{aligned}
\mathcal{L} \quad = \quad & N^\dagger \left(i\partial_0 + \frac{\vec{\nabla}}{2M} \right) N - a_1 N^\dagger N N^\dagger N - a_2 N^\dagger \boldsymbol{\sigma} N \cdot N^\dagger \boldsymbol{\sigma} N \\
& - a_3 N^\dagger \boldsymbol{\tau} N \cdot N^\dagger \boldsymbol{\tau} N - a_4 N^\dagger \boldsymbol{\sigma\tau} N \cdot N^\dagger \boldsymbol{\sigma\tau} N - \ldots \\
& - d_1 N^\dagger \boldsymbol{\tau} N \cdot N^\dagger \boldsymbol{\tau} N N^\dagger N
\end{aligned}
$$



- Higher order terms include more derivatives.
- Naively, the order goes as the number of derivatives.
- The 3-body term appears at LO to avoid the Thomas collapse.
- Due to Fermi symmetry the number of terms can be cut by half.
- The coefficients depend on the cutoff Λ.

Application to AFDMC

- The potential need be local.
- Avoid 3-body spin-isospin operators.

$\not\pi$EFT for Lattice Nuclei

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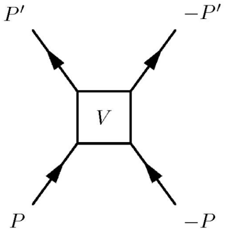
$$\begin{aligned}\mathcal{L} \;=\; & N^\dagger\left(i\partial_0 + \frac{\vec{\nabla}}{2M}\right)N - a_1 N^\dagger N N^\dagger N - a_2 N^\dagger \boldsymbol{\sigma} N \cdot N^\dagger \boldsymbol{\sigma} N \\ & - a_3 N^\dagger \boldsymbol{\tau} N \cdot N^\dagger \boldsymbol{\tau} N - a_4 N^\dagger \boldsymbol{\sigma}\boldsymbol{\tau} N \cdot N^\dagger \boldsymbol{\sigma}\boldsymbol{\tau} N - \ldots \\ & - d_1 N^\dagger \boldsymbol{\tau} N \cdot N^\dagger \boldsymbol{\tau} N N^\dagger N\end{aligned}$$



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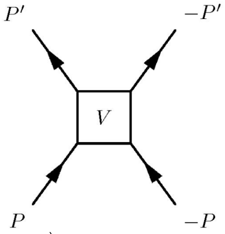
$\not\pi$EFT Potential at NLO



- At LO the $\not\pi$EFT potential takes the form

$$V_{LO}^{2b} = a_1 + a_2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + a_3\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + a_4 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$$

- The leading order also contains a 3-body term of the form

$$V_{LO}^{3b} = D_1 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \quad \text{or} \quad \mathrm{V}_{\mathrm{LO}}^{3\mathrm{b}} = \mathrm{D}_1$$

- At NLO the $\not\pi$EFT potential takes the form

$$\begin{aligned} V_{NLO}^{2b} \quad = \quad & b_1 q^2 + b_2 q^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + b_3 q^2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + b_4 q^2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\ + \quad & b_5 k^2 + b_6 k^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + b_7 k^2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + b_8 k^2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\ + \quad & b_9 i \frac{1}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\boldsymbol{k} \times \boldsymbol{q}) + b_{10}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 i \frac{1}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\boldsymbol{k} \times \boldsymbol{q}) \\ + \quad & b_{11} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) + b_{12}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) \\ + \quad & b_{13} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{k})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}) + b_{14}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{k})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}) \end{aligned}$$

- The incoming particle have relative momentum $\boldsymbol{p}$, the outgoing $\boldsymbol{p}'$.
- The momentum transfer $\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}$, and $\boldsymbol{k} = (\boldsymbol{p}' + \boldsymbol{p})$
- Nonlocalities are associated with $\boldsymbol{k}$.
- The power counting changes for a shallow s-wave dimer.

J. Kirscher, H. W. Griesshammer, D. Shukla, H. M. Hofman, arXiv: 0903.5583

A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, PRL **111**, 032501 (2013).

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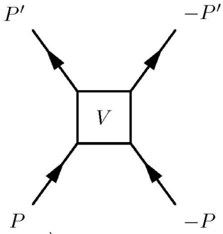

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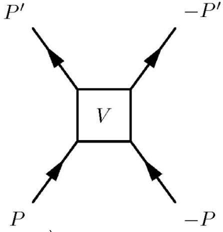
$$\begin{aligned} V_{NLO}^{2b} \;=\; & b_1 q^2 + b_2 q^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + b_3 q^2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + b_4 q^2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\ + \; & b_5 k^2 + b_6 k^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + b_7 k^2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + b_8 k^2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\ + \; & b_9 i \frac{1}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\boldsymbol{k} \times \boldsymbol{q}) + b_{10}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 i \frac{1}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\boldsymbol{k} \times \boldsymbol{q}) \\ + \; & b_{11} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) + b_{12}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) \\ + \; & b_{13} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{k})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}) + b_{14}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{k})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}) \end{aligned}$$

- The incoming particle have relative momentum $\boldsymbol{p}$, the outgoing $\boldsymbol{p}'$.
- The momentum transfer $\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}$, and $\boldsymbol{k} = (\boldsymbol{p}' + \boldsymbol{p})$
- Nonlocalities are associated with $\boldsymbol{k}$.
- The power counting changes for a shallow s-wave dimer.

J. Kirscher, H. W. Griesshammer, D. Shukla, H. M. Hofman, arXiv: 0903.5583

A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, PRL **111**, 032501 (2013).

$\pi\!\!\!/$EFT Potential at NLO



- At LO the $\pi\!\!\!/$EFT potential takes the form

$$V_{LO}^{2b} = a_1 + a_2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + a_3\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + a_4(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$$

- The leading order also contains a 3-body term of the form

$$V_{LO}^{3b} = D_1\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \quad \text{or} \quad \mathrm{V}_{\mathrm{LO}}^{3\mathrm{b}} = \mathrm{D}_1$$

- At NLO the $\pi\!\!\!/$EFT potential takes the form

$$\begin{aligned} V_{NLO}^{2b} &= b_1q^2 + b_2q^2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + b_3q^2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + b_4q^2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\ &+ b_5k^2 + b_6k^2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + b_7k^2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + b_8k^2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\ &+ b_9 i\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)(\boldsymbol{k}\times\boldsymbol{q}) + b_{10}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 i\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)(\boldsymbol{k}\times\boldsymbol{q}) \\ &+ b_{11}(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}) + b_{12}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}) \\ &+ b_{13}(\boldsymbol{\sigma}_1\cdot\boldsymbol{k})(\boldsymbol{\sigma}_2\cdot\boldsymbol{k}) + b_{14}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2(\boldsymbol{\sigma}_1\cdot\boldsymbol{k})(\boldsymbol{\sigma}_2\cdot\boldsymbol{k}) \end{aligned}$$

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- Due to antisymmetrization of the nuclear wave function

$$\boxed{V_{LO}^{2b} = C_1^{LO} + C_2^{LO}\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}$$

- The leading order also contains a 3-body term of the form

$$\boxed{V_{LO}^{3b} = D_1^{LO}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \qquad \text{or} \qquad \mathrm{V}_{\mathrm{LO}}^{\mathrm{3b}} = \mathrm{D}_1^{\mathrm{LO}}}$$

- Using the freedom to choose these parameters we set

$$\begin{aligned} V_{NLO}^{2b} = \; & C_1^{NLO}q^2 + C_2^{NLO}q^2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + C_3^{NLO}q^2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + C_4^{NLO}q^2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\ & + C_5^{NLO} i\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)(\boldsymbol{k}\times\boldsymbol{q}) + C_6^{NLO}(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}) \\ & + C_7^{NLO}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}) \end{aligned}$$

- The antisymmetric potential V_{NLO} contains
  1. LO 2-body: 2 parameters.
  2. LO 3-body: 1 parameter.
  3. NLO: 7 parameters.
- At the moment we consider only LO.

Coordinate space

- We introduce gaussian cutoff in $\boldsymbol{q}$

$$F_\Lambda(q) = e^{-q^2/\Lambda^2} \Longrightarrow F_\Lambda(r) = \left(\frac{\Lambda}{\sqrt{4\pi}}\right)^3 e^{-\Lambda^2 r^2/4}$$

- The potential matrix elements can be evaluated now

$$\begin{aligned} V(\boldsymbol{r},\boldsymbol{r}') &= N\langle\boldsymbol{r}|\int d\boldsymbol{k}d\boldsymbol{q}V(\boldsymbol{k},\boldsymbol{q})f_\Lambda(\boldsymbol{q})|\boldsymbol{r}'\rangle \\ &= N'\ V(-i\nabla_y,-i\nabla_x)\, e^{-\Lambda^2 x^2/4}\delta(\boldsymbol{y}) \end{aligned}$$

where

$$\boldsymbol{x} = \frac{1}{2}(\boldsymbol{r}+\boldsymbol{r}') \quad ; \quad \boldsymbol{y} = \frac{1}{2}(\boldsymbol{r}'-\boldsymbol{r})$$

- The LO potential contains no momentum dependence therefore

$$V^{2b}_{LO}(r) = \left(C_1^{LO} + C_2^{LO}\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right) e^{-\Lambda^2 r^2/4}$$

- With our choice of parameterization also V_{NLO} is local.
- The 3-body term takes the form

$$V^{3b}_{LO} = D_1^{LO}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 e^{-\Lambda^2(r_{13}^2+r_{23}^2)} \qquad \text{or} \qquad V^{3b}_{LO} = D_1^{LO} e^{-\Lambda^2(r_{13}^2+r_{23}^2)}$$

The Hamiltonian

At leading order the coordinate space Hamiltonian is

$$
\begin{aligned}
H &= -\sum_i \frac{\hbar^2}{2M_n}\nabla_i^2 + \sum_{i<j}\left(C_1^{LO}(\Lambda) + C_2^{LO}(\Lambda)\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right) e^{-\Lambda^2 r_{ij}^2} \\
&+ \sum_{i<j<k}\sum_{cyc} D_1^{LO}(\Lambda)\,(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\, e^{-\Lambda^2(r_{ik}^2+r_{jk}^2)}
\end{aligned}
$$

In the 3-body term the notation $\sum_{cyc}$ stands for cyclic permutation of particles (ijk).

Few-body arsenal

1. Numerov, $A = 2$
2. The Effective Interaction Hypershperical Harmonics (EIHH) method, $3 \leq A \leq 6$
3. The Resonating Group Method (RGM), $A \leq 6$
4. The Auxiliary Field Diffusion Monte-Carlo (AFDMC) method, $A \geq 2$

The "Experimental" data

There are 4 input parameters in our model:

1. The nucleon mass M_n
2. LECs: $C_1^{LO}(\Lambda, m_\pi)$, $C_2^{LO}(\Lambda, m_\pi)$, $D_1^{LO}(\Lambda, m_\pi)$

Binding energies [MeV]

	Nature	Yamazaki	NPLQCD
π	139.6	510.0	805.0
n	939.6	1320.0	1634.0
p	938.3	1320.0	1634.0
nn	-	7.4 ± 1.4	15.9 ± 3.8
D	2.224	11.5 ± 1.3	19.5 ± 4.8
^{3}H	8.482	20.3 ± 4.5	53.9 ± 10.7
^{3}He	7.718	20.3 ± 4.5	53.9 ± 10.7
^{4}He	28.30	43.0 ± 14.4	107.0 ± 24.2

Scattering data @800MeV [fm]

	Nature	NPLQCD
π	139.6	805.0
a_{31}	5.423 ± 0.005	1.82 ± 0.22
r_{31}	1.73 ± 0.02	0.91 ± 0.11
a_{13}	-23.715 ± 0.015	2.33 ± 0.33
r_{13}	2.73 ± 0.03	1.13 ± 0.10

NPLQCD Collaboration, PRD **87** 034506 (2013), hep-lat/1301.5790v1

T. Yamazaki, K. Ishikawa, Y. Kuramashi, and A. Ukawa, Phys. Rev. D **86** (2012) 074514.

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The 2-body LECs

- The 2-body potential is diagonal in the S, T basis.

$$\begin{aligned} V^{LO}_{S,T}(r;\Lambda) &= \langle S,T|V^{LO}(r;\Lambda)|S,T\rangle \\ &= \left\{C^{LO}_1(\Lambda) + [2S(S+1)-3]C^{LO}_2(\Lambda)\right\} F_\Lambda(r) \\ &\equiv C^{LO}_{ST}(\Lambda)F_\Lambda(r) \end{aligned}$$

- $C^{LO}_{ST}(\Lambda)$ are fitted to the D, nn B.E.
- Summing the bubble diagrams
- We expect to get

$$a_s \approx 1/\sqrt{m_N B}$$

- For LQCD@$m_\pi = 800MeV$

EFT	NPLQCD
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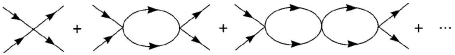

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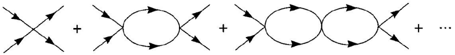

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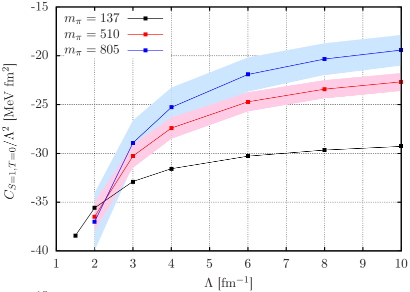
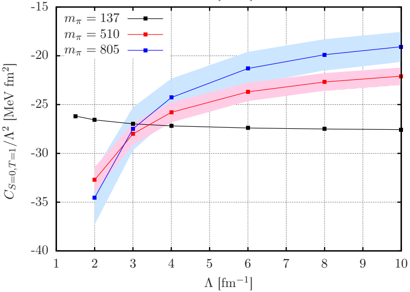
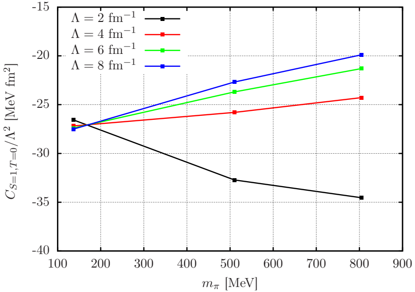
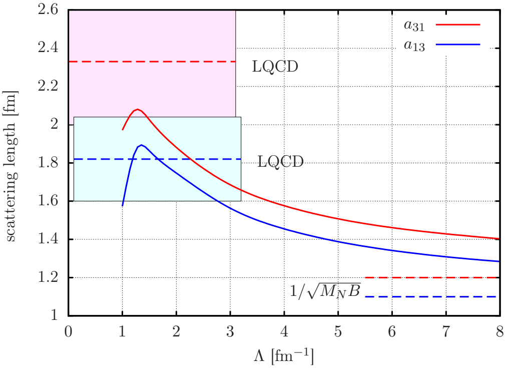




- The 2-body LECs are monotonic in Λ
- They seems to be also monotonic in m_π
- Only weak dependence on m_π
- For $\Lambda \approx 2 - 3$ fm^{-1} the potential is roughly a constant



The 2-body scattering length



Calibration of D_1

^{4}He binding energy without NNN force

Λ [fm^{-1}]	EIHH [MeV]	AFDMC [MeV]
2.0	-256.8	-256.9
4.0	-478.3	-478.2
6.0	-767.1	-766.4
8.0	-1122.9	-1120.8
LQCD	-107.0±24.2	

- The span in the binding energies is reflected in the parameters
- We shall use only the central values

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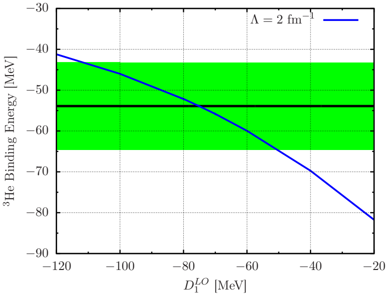

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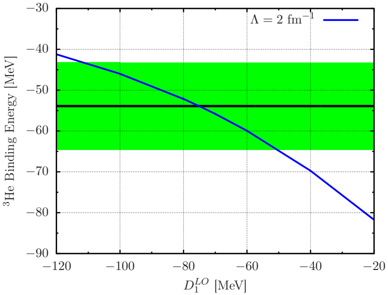

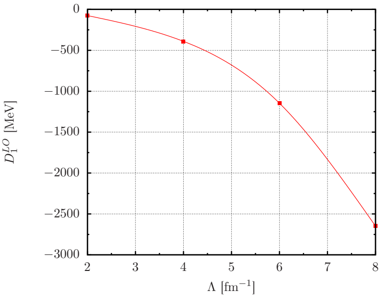

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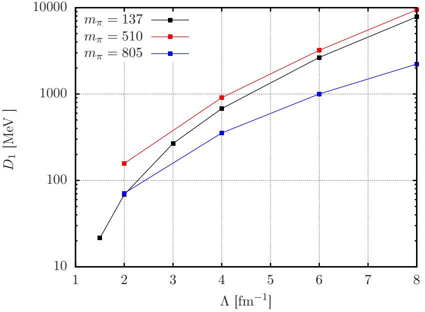
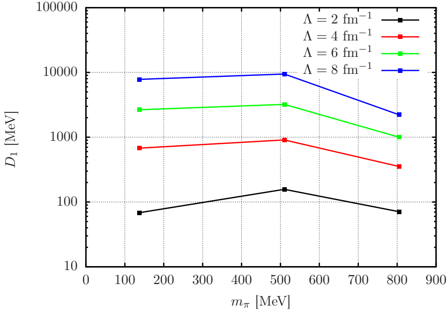
Calibration of D_1

m_π dependence





- For the various m_π's D_1 presents different Λ dependence.
- D_1 is **NOT** monotonic in m_π
- This can be an indication for a limit cycle.

Predictions/Postdictions

Possible predictions/postdictions

- The nuclear land scape **non-physical** m_π
- Form factors
- Scattering parameters
- ...

Predictions/Postdictions

The binding energy of ^{4}He

Λ [fm^{-1}]	EIHH [MeV]	AFDMC [MeV]
2.0	-89.2(1)	-87.7(1)
4.0	-93.6(1)	-93.3(2)
6.0	-99.7(3)	-99.2(2)
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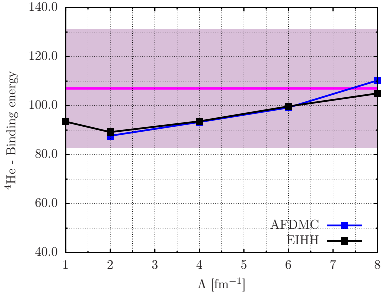

- The ^{4}He energy comes in accord with the LQCD simulations.
- It has residual cutoff dependence.
- The radii of D, ^{3}He, ^{4}He exhibits strong cutoff dependence.
- These issues might be artifacts of our "local" formalism.

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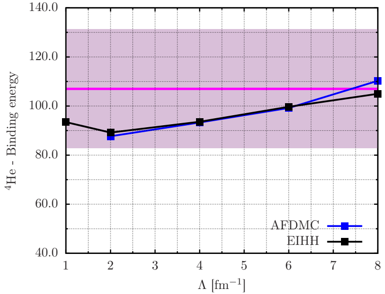
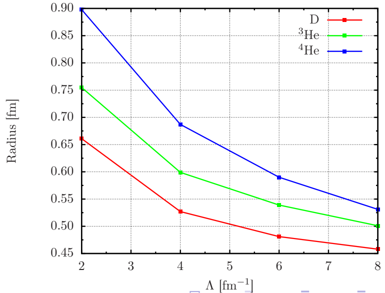
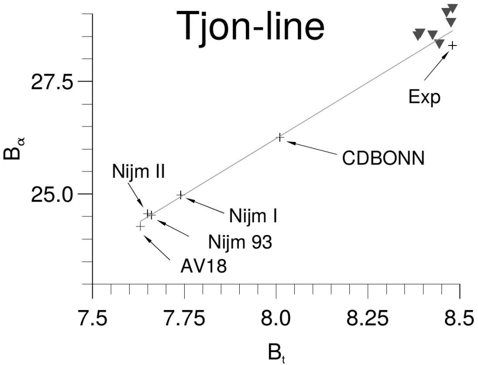




The Tjon lines



Comments

- The Tjon line is the observed correlation between the triton and ^{4}He binding energies.
- It was discovered to be a universal property of bosons with large scattering length.

A. Nogga, H. Kamada, W. Glokle, ArXiv/nucl-th/0004023v2 (2000).
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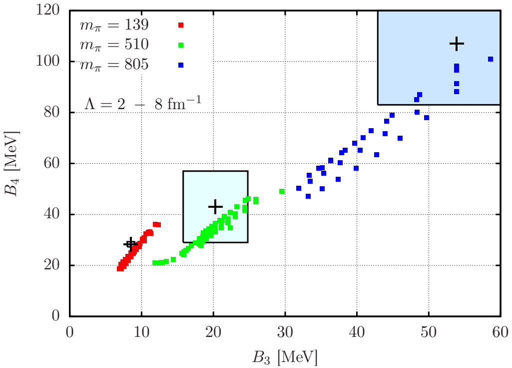

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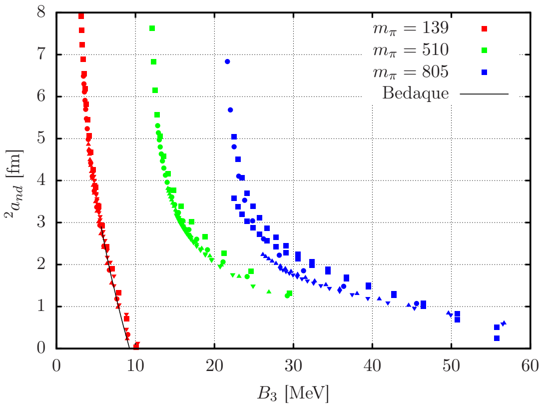
The Phillips line



Comments

- The Phillips line is the correlation between the triton binding energy and the *nd* doublet scattering length.
- Again it was proven to be a universal feature.

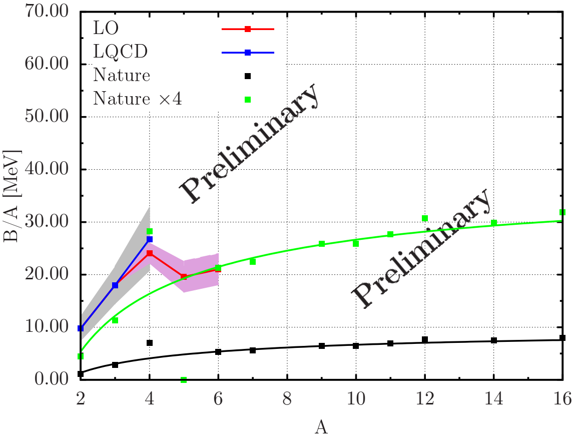
V. Efimov, Yad. Fiz. 47, 29 (1988).
P. F. Bedaque and U van Kolck, Phys. Lett. B 428, 221 (1998).

Few more predictions

Binding energies of the Light nuclei

nuclei	LQCD	EFT $\Lambda = 2\ \mathrm{fm}^{-1}$
D	-19.5 $\pm$ 4.8	-19.5
nn	-15.9 $\pm$ 3.8	-15.9
^{3}H,^{3}He	-53.9 $\pm$ 10.7	-53.9
^{3}n, ^{3}p		unbound
^{4}He	-107.0 $\pm$ 24.2	-89.2
^{4}He $J^\pi = 2^+$		-66 (?)
^{5}He		-98.2
^{6}Li		-121.(3)

Saturation Energy on the Lattice



Summary and Conclusions

- Lattice QCD simulations of few-nucleon systems open up a new front in nuclear physics.
- $\not{\pi}$EFT is the appropriate theory to study these Lattice Nuclei, down to rather small pion masses.
- Fitted to recent LQCD data we found that $\not{\pi}$EFT@LO reproduces the ^{4}He binding energy for $m_\pi = 500, 800$MeV within error bars.
- The LECs depend weakly on m_π.
- A challange for LQCD: the nd scattering length.
- At LO we see problems with the nuclear radii and 2-body scattering lengths.
- Analysis of the $s \neq 0$ sector is underway.